This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

LIBRARY OF THE
UNIVERSITY OF MICHIGAN

DEPARTMENT
OF
ENGINEERING

ENGINEERING
LIBRARY
TK
2411
.H393
1907

ENGINEERING
LIBRARY

THE DYNAMO

THE DYNAMO

ITS THEORY, DESIGN, AND MANUFACTURE

BY

C. C. HAWKINS, M.A., M.I.E.E.

AND

F. WALLIS, B A., A.M.I.E.E.

WITH FOUR HUNDRED AND THIRTEEN ILLUSTRATIONS

FOURTH EDITION

WHITTAKER & CO.
2 WHITE HART STREET, PATERNOSTER SQUARE, LONDON
AND 66 FIFTH AVENUE, NEW YORK
1907

(*The right of translation is reserved*)

PREFACE TO THIRD EDITION

SINCE the last edition of the present book great progress has been made in the science of dynamo design, and even more in the art of dynamo manufacture. In the continuous-current field the increase in the size of modern machines has necessarily brought about the adoption of multipolar forms, while the demand for power and traction generators has led to the reintroduction on a widely extended scale of the slotted armature with carbon collecting brushes. At the same time the utmost economy in the materials and labour employed in the manufacture of a machine is imperatively called for in order to meet the increasing keenness of a world-wide competition; every pound of copper and of iron must be utilised to its fullest extent, and designs involving high-priced skilled labour must give way to simple types having easily-made interchangeable parts, with the consequent result that the machines for given outputs are smaller and more standardized. While in the above changes America has led the way, the no less marked advance on the alternating-current side has been chiefly due to Continental engineers, and consists primarily in a change from single-phase to polyphase distribution. The pioneer work of the Frankfort Exhibition of 1891 reached its culminating point in the Paris Exhibition of 1900, in which the three-phase generators were in striking preponderance as compared with the single-phase alternators, only two of the latter class being exhibited.

Thus in preparing a third edition it became necessary to rewrite most of the former chapters and to add several new ones; but it remains impossible within the limits of a text-book even if intended for specialists to give more than a cursory treatment to a

Reclass 2-24-42

194963

number of questions of equal importance and interest. Inaccuracies have doubtless crept in during the process of revision and expansion, yet it is hoped that on the whole the student will find in the present book, so far as its scope reaches, a sufficiently trustworthy foundation on which to build, so that as his knowledge extends he may have little to forget or to positively unlearn.

Among the numerous sources to which the authors are especially indebted for information on many points of both theory and practice, there must now be added to the familiar names of Hopkinson, Kapp, and S. P. Thompson, those of Prof. Arnold, Fischer-Hinnen, Steinmetz, Hobart, Behrend, Rothert, Guilbert, etc. If our debt to such well-known authors should not be deemed to have been sufficiently acknowledged, our excuse is to be found in the fact that it would unduly burden the pages with foot-notes if each and every original source which has been laid under contribution were separately named.

Finally, we wish to thank the numerous Firms and Companies through whose courtesy we are enabled to give views and data of a number of typical dynamos, and among them would especially mention Messrs. W. H. Allen, Son & Co.; the Allgemeine Elektricitäts Gesellschaft; Brown, Boveri & Co.; Dick, Kerr & Co.; Ferranti, Ltd.; the International Electrical Engineering Co.; the Oerlikon Maschinenfabrik; the British Schuckert Co.; the British Thomson-Houston Co.; and the British Westinghouse Co. In the preparation of many of the diagrams we have also to acknowledge the assistance of Mr. H. H. Bentley, and of Mr. E. L. Billington.

C. C. H.

May 1903.

PREFACE TO FOURTH EDITION

THE preparation of a fourth edition has afforded an opportunity of eliminating or correcting such errors, typographical or otherwise, as have been noticed in the previous edition.

January 1907.

CONTENTS

CHAPTER I

INTRODUCTORY

Definition of the dynamo—Electrical energy—Output of dynamo—Losses in dynamo—Efficiency—Action of typical dynamo 1

CHAPTER II

THE MAGNETIC FIELD

Exploration of a magnetic field—Field of bar magnet—Lines of force—Their direction—Their sense—Their number, as representing strength of field—Field surrounding straight conductor—Field of a current-loop—Field of solenoid of many loops—Insertion of core into solenoid—Field of toroid 10

CHAPTER III

THE MAGNETIC CIRCUIT

Lines of induction—Relation to lines of force—Total flux of circuit—M.M.F.—Flux of toroid—Equation of the simple magnetic circuit—Reluctivity and permeability—Equation of an imperfect magnetic circuit—Magnetic leakage 25

CHAPTER IV

THE PRODUCTION OF AN E.M.F.

Movement of a conductor through a field—The cutting of lines—The direction, sense and value of the E.M.F.—Average and instantaneous E.M.F.'s—Equation for E.M.F.—Process for discovering whether an E.M.F. is generated in any given case—Conditions for maximum E.M.F.—Effect of width of inductor—Movement in field of varying density—E.M.F. of a loop translated in a uniform field—in a varying field—of a loop rotated in uniform field—Periodicity and phase . 46

CHAPTER V

THE MAGNETIC PULL

PAGE

Simple inductor-dynamo—Mechanical force required, when current flows—Direction, sense, and value of the magnetic pull—Lenz's law—Conversion of mechanical into electrical energy—Interaction of two magnetic fields—Magnetic pull on armature conductors—Limits of output of dynamo 66

CHAPTER VI

SELF-INDUCTION

Second consequence of flow of current—Cutting of self-induced lines, if the current varies—Law of self-induced E.M.F.—Analogy of self-induction to inertia—Inductance of loop—of solenoid—Impressed, self-induced and resultant E.M.F.'s distinguished—Storage of energy in field—Rate of change of self-induced lines—Deduction of impressed E.M.F. curve—Lag of resultant behind impressed E.M.F.—Power of alternating circuit—Measurement of alternating E.M.F.'s and currents—Output of alternator—Power-factor—Sine-law hypothesis—Reactance and Impedance 83

CHAPTER VII

CLASSIFICATION OF DYNAMOS

Bases for classification of dynamos—Heteropolar dynamos—Homopolar dynamos—(I.) Bipolar dynamos (A) with inductors parallel to axis of rotation—Functions of iron core— (1) Ring winding—(2) Drum—(B) Inductors at right angles to axis of rotation, (3) Discoidal- or Flat-ring, (4) Disc—(II.) Homopolar dynamos (*a*) with one inductor—(*b*) wound (1) with ring armature, (2) with discoidal-ring, (3) with drum, (4) with disc armature 120

CHAPTER VIII

HETEROPOLAR ALTERNATORS

Formation of a coil—Effect of width of winding—Definition of pitch—Differential action—Best ratio of widths of winding and field in ring-armatures—Connections for two ring coils in series or parallel—Best ratio in drum and disc armatures—Connections for two drum or disc coils in series or parallel—Multipolar alternators—Periodicity—Description of multipolar alternators, ring, drum, discoidal-ring and disc—E.M.F. equation of heteropolar alternator—Mechanical force in alternators 146

CHAPTER IX

HOMOPOLAR ALTERNATORS

PAGE

Difficulties in homopolar continuous-current machines—Homopolar alternators, ring, discoidal-ring, drum and disc—Useless lines entering armature—E.M.F. equation 176

CHAPTER X

OPEN-COIL ARMATURES

Two coils at right angles—Levelling of the E.M.F. curve—Combination of the two split-rings into one joint commutator—Action of the machine—Two pairs of coils at angles of 45°—Two joint commutators—Three coils 120° apart—Effect of different widths of brush 191

CHAPTER XI

CLOSED-COIL ARMATURES

Necessity for short-circuiting a coil—Two coils at right angles—Four coils at angles of 45°—Combination of four split-rings to form one commutator—Division of winding into two halves in parallel—Eight-coil bipolar armatures—Hand of armature system—E.M.F. equation—Multipolar parallel-connected ring—Rise of E.M.F. in ring armatures—Multipolar series-connected ring—Two methods of drum winding—Lap and wave winding—E.M.F.'s and resistances of parallel paths of armature must be equal—Electrical symmetry of sections—Magnetic symmetry for ring armatures—Winding formulæ—Relative advantages of series- and parallel-connected armatures . 206

CHAPTER XII

THE MAGNETIC PROPERTIES OF IRON

Permeability as depending on (1) physical and chemical state, (2) the value of the induction, (3) previous magnetic history—Ascending and descending curves of induction—Saturation—B, H curves for ingot-iron forgings, cast steel, cast iron, and stampings—Magnetic hysteresis—Dissipation of energy by hysteresis—Hysteresis loss in alternating field—Ewing's theory of magnetism—Hysteresis as affecting dynamos—Loss in a rotating field—Influence of vibration and temperature 264

CHAPTER XIII

CONTINUOUS-CURRENT ARMATURES

Eddy-currents in a solid core—Necessity for lamination—Discs for ring and drum armature—Construction of ring and drum armatures—

PAGE

Inferiority of wire armatures—Iron ribbon for discoidal-ring armatures—Strength of armature hubs—Strength of shafts—Length of journals—Friction of bearings—Pedestals and plummer-blocks—Lubrication—Insulation of armature core—Driving of the inductors—Toothed armatures—Insulation of armature wires—Eddy-currents in armature bars—Ring winding—Drum winding—Binding-wire—Commutators—Calculation of armature resistance—Peripheral speed—Ratio of length to diameter of core—Values of armature induction 299

CHAPTER XIV

FIELD-MAGNETS

Values for magnet induction—Comparison of cast iron with forged-iron and steel castings—Composite magnets—Comparison of sectional shapes—Length of magnet—Types of field-magnet—Double horseshoe fields—Multipolar forms—The design of field-magnets—Mechanical strength—Influence of joints—Proportioning of areas—Magnetic leakage—Symmetrical distribution of field—Proportioning of multipolar magnets 384

CHAPTER XV

THE AMPÈRE-TURNS OF THE FIELD

Equation of the magnetic circuit—Use of flux-density curves—Back ampère-turns of armature—Magnetic leakage—Calculation of field ampère-turns for 9″ × 12″ dynamo—Calculation of leakage permeance, P—Empirical formulæ for P—Leakage factor, ν, and its values—Flux-curves of dynamo—Air-gap permeance of toothed armatures—Reluctance of teeth—Determination of field-wire—Winding of field-magnet bobbins 423

CHAPTER XVI

SERIES, SHUNT AND COMPOUND WINDING

Methods of excitation—Separately-excited dynamo—Shunt-wound dynamo—Series-wound dynamo—Self-excitation—Stored energy of excited field—Determination of shunt and series winding—Their respective uses—External characteristics—Instability of magnetism—Compound-wound dynamo—Determination of its winding—Imperfections of compounding 478

CHAPTER XVII

ARMATURE REACTION IN CONTINUOUS-CURRENT DYNAMOS

PAGE

Diameter of commutation and brush-position—Distortion of field—Magnetic field of armature and its superposition on the main field—No diminution of flux from cross turns with iron of constant permeability—Negligible diminution in smooth armature with iron of varying permeability—Effect of forward lead of brushes—Effect of backward trail—Effect of varying permeability of teeth in slotted armature—Analysis of distribution in air-gap of toothed armature—Amount of reduction of flux—Back ampère-turns of smooth and toothed armatures—Experimental determination of them . . . 516

CHAPTER XVIII

COMMUTATION AND SPARKING AT THE BRUSHES

Nature of sparking—Adjustment of brush position—Process of short-circuiting a section—Apparent inductance of short-circuited section—The equation of the short-circuit—Effect of brush contact-resistance—The final rate of change of the current—Need of a reversing field—Angle of lead—Limitation of output by sparking—Limiting value of angle of lead—Limiting value of ampère-wires per pole in smooth armature—Advantage of multipolars—Limiting value of ampère-wires per unit of circumference—Determination of angle of lead—Comparison of copper and carbon brushes—Methods of reducing the angle of lead—Limiting value for ampère-wires in toothed armatures—Inductance of short-circuited section—Conditions favourable for sparkless commutation—Maximum value for $2LJ/T$—Number of sectors—Sparking in ring machines—Brushes and brush-holders—Flats on commutator—Brushes for open-coil machines 549

CHAPTER XIX

THE HEATING OF DYNAMOS

Rise of temperature—Increase of electrical resistance—Less accuracy of regulation—Deterioration of insulating materials—Maximum permissible temperature—Maximum permissible rise of temperature—Predetermination of rise of temperature—in field coils—Importance of eddy-current loss in armatures—Effect of peripheral speed—Loss over ohmic resistance—Eddy-current loss in armature core—in copper winding—in toothed armatures—Approximate formulæ for eddy loss—Hysteresis loss—Predetermination of rise of temperature—Calculation of cooling surface—Effect of internal ventilation—Heating of commutator—Contact-resistance of carbon brushes—Copper and carbon brushes—Calculation of commutator loss—Temperature rise of commutator—Eddy-currents in pole-pieces . . 609

CHAPTER XX

THE DESIGN OF CONTINUOUS-CURRENT DYNAMOS

PAGE

Range of speeds—Determination of length and diameter of armature—as affected by watts per rev. per min.—Design of small two-pole dynamo with smooth armature—Design of four-pole dynamo with toothed armature—Measurement of losses by (*a*) motor current—(*b*) by retardation method—Efficiency test by Hopkinson's method—Efficiency and weights of continuous-current dynamos . . . 654

CHAPTER XXI

THE WORKING AND MANAGEMENT OF CONTINUOUS-CURRENT DYNAMOS

The coupling of series-wound dynamos in series—Of shunt-wound dynamos in parallel—Of series-wound dynamos in parallel—Of compound-wound dynamos in parallel—Regulation of load between compound-wound dynamos in parallel—Foundations and erection of dynamos—Care of machine in working 709

CHAPTER XXII

ARMATURE WINDING OF ALTERNATORS

Suitability of alternators for high pressures—Single-phase drum-winding—Interlinked quarter-phase and four-phase systems—Interlinked three-phase systems—Polyphase armature windings—Connecting up of three-phase armatures—Value of K in E.M.F. equation—Form of E.M.F. curve and its delineation 721

CHAPTER XXIII

ARMATURE REACTION IN ALTERNATORS

Regulation of alternators—Values of power-factor—Fall of volts in armature—The short-circuit characteristic—Synchronous impedance and reactance—Construction of full-load characteristic—Two components of inductance of armature—Cross and back ampère-turns of armature—Calculation of back ampère-turns in single-phase alternator—in polyphase alternators—in inductor alternators—Calculation of cross ampère-turns—Armature reaction on the assumption of a fictitious armature flux—Composition of M.M.F.'s—Corrected combination of the two methods—The short-circuit diagram of ampère-turns—Wattless-current method of separating leakage reactance from back ampère-turns—Approximate estimate of leakage reactance (*a*) from experiment, (*b*) from calculation—Use of the short-circuit charac-

PAGE

teristic in designing—The synchronous reactance method in practice—Values of inherent regulation—Constant-current and constant-watt alternators 754

CHAPTER XXIV

CONSTRUCTION OF ALTERNATOR ARMATURES AND FIELD-MAGNETS

The iron core—Drum winding—Number and shape of slots—The air-gap—The external stationary armature—Discoidal ring winding—Disc armature—The magnet system of inductor generators—of heteropolar generators—The exciting coils and methods of excitation—Compounding of alternators—Choice of frequency . . . 811

CHAPTER XXV

ALTERNATOR DESIGN

Preliminary calculation of dimensions—The value of λ—The armature ampère-wires per pole—per unit length of circumference—The watts per rev. per min. as dependent upon diameter and length of core—Design of three-phase generator—Parallel working of alternators—Phase indicator—Effect of want of synchronism—of inequality of frequency—of inequality of voltage—of want of uniformity of speed—Permissible displacement of phase—Permissible variation of speed—Calculation of necessary flywheel effect—Damper winding—Testing of alternators—Efficiency 841

CHAPTER XXVI

DESCRIPTIONS OF TYPICAL DYNAMOS

Traction generator of English Electric Co.—of General Electric Co. of New York—Schuckert 900 KW. dynamo—75 KW. dynamo of International Electrical Engineering Co.—Electro-plating dynamo of W. H. Allen, Son & Co.—Three-phase generators of Brown, Boveri and Co.—of Allgemeine Elektricitäts Gesellschaft—of Westinghouse Co.—of Oerlikon Co. 885

INDEX 915

Faraday's original disc dynamo.

Frontispiece.] [*To face p.* 1.

THE DYNAMO

CHAPTER I

INTRODUCTORY

§ 1. **Definition of the dynamo.**—The theory of the dynamo is one among the many meeting-points of electricity and magnetism, and in its use the two sciences find what is perhaps their chief industrial application. From their conjunction the dynamo derives the dual nature which is characteristic of it, and which is reproduced in its structure as a machine, for it may broadly be regarded as built up of a copper or electric portion and an iron or magnetic portion. The mechanical is, however, of no less importance than the electromagnetic aspect of the dynamo. Its fundamental principles may therefore be briefly summed up as three in number—the electrical, the magnetic, and the mechanical; and its design is correspondingly based on three fundamental equations, all closely connected together. While the first deals with the electrical side, and the second with the mechanical, both also involve the facts and laws of magnetism, and need to be supplemented by a third or magnetic equation.

The dynamo may be defined as *a machine in which a conductor or system of conductors forming part of an electric circuit is given continuous motion relatively to a magnetic field or fields, and so is caused to traverse or cut the lines of the magnetic flux; an electromotive force is thereby induced in the conductors, with the result that when the circuit is closed a current flows, and mechanical energy is converted into electrical energy.*

Taking this as our text, our aim will be to establish it generally, and thence to deduce the principles which govern the design of dynamos, and determine the practical shapes which they take.

In so doing we have at the outset to explain certain technical terms which appear in the definition, and in the first place must consider what is meant by the expression '*electrical energy*.'

§ 2. **Electrical energy and its two factors, E.M.F. and current.**—For the appearance of electrical energy there are required two factors, viz., an electric pressure, or *electromotive force*, and a *current* flowing under that pressure, and until both are present there is no electrical energy. If an electrical difference of pressure has once been established between two points forming the ends of an electrical conductor or system of conductors, then when those two points are further joined by any form of electrical conductor a current flows round the completed circuit, and continues to flow if the electrical pressure be maintained.

A simple and often-quoted analogy from hydraulics will render this plainer. The electric pressure may be compared to a head of water, or a difference of level between two points, in virtue of which water will flow from the higher to the lower level, if the two are joined by a pipe. When such a channel is provided, the body of water which previously only had potential energy will do work, as in turning a water-wheel, and will realise its potential energy. If, however, this development of power is to be continuous, a pump must be provided to raise the water from the lower to the higher level—in other words, to keep up the difference of level in virtue of which the water flows, and so does work. In exactly the same way a current of electricity flowing from a point of higher electrical level or potential to a point of lower potential does work, and electrical energy is present; by its flow it tends to equalise the potential of these two points, and it will only continue to flow so long as there is some difference of potential between them. Hence, if there is to be a continuous flow of electricity we need an electric pump or machine to constantly maintain the pressure in virtue of which the current flows. Such an electrical pump is the dynamo, which is therefore primarily and essentially *a generator of electrical pressure*; its sole function is to generate an electric pressure or electromotive force, and subsequently to maintain it, when a current flows under that pressure. The fact of the current continuing to flow makes it necessary that there should be a closed circuit which throughout its entire path conducts electricity, and which contains some source of electromotive force, or E.M.F.; in the case with which we have to deal, this latter is located within the limits of the dynamo which forms part of the circuit. Hence any system containing a dynamo at work, whether lighting lamps, or charging accumulators, or driving motors, must consist of a closed circuit divisible into two portions on either side of certain points. In fig. 1 is shown diagrammatically a closed electrical circuit round which a current is flowing and part of which is a dynamo in its simplest form; the meaning

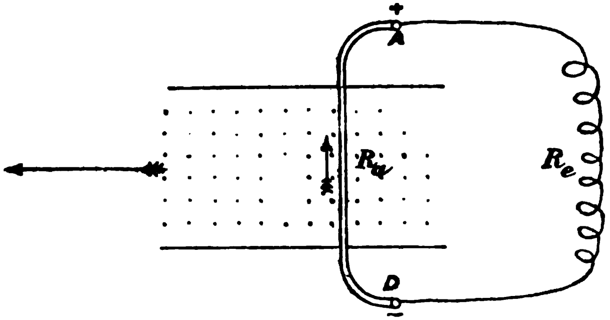


FIG. 1.

of the conventional symbols will be apparent later. The points A and D divide the entire circuit into two portions; in that to the left of our diagram, which may be called the *internal* portion of the circuit, viz. A R_a D, a certain action is set up which results in a difference of electrical pressure between A and D, A being at a higher potential than D, and therefore having a + or positive sign attached to it. When A and D are further joined by the external portion of the circuit, or, as it is called for shortness, the *external circuit*, viz. A R_e D, a current flows which would result in the equalisation of the potential throughout the whole circuit, and would then cease, were it not for the fact that R_a represents a dynamo which continuously keeps the potential of A higher than that of D. The two points A and D form at once the *terminals* of the machine and the *terminals* of the external circuit; the current flows from A to D in the external circuit, but within the dynamo itself from D to A, just as in the case of a voltaic battery, where the current flows from the positive to the negative pole outside it, but from the negative to the positive pole within the cell itself. If we replace the dynamo by a pump, and the external circuit by a pipe, through both of which water is kept continuously circulating, the hydraulic analogy will again help us; the dynamo does not, strictly speaking, generate electricity any more than the pump generates the water; it is, however, only after the production of an electrical difference of potential, and the consequent setting-up of a current, that the electricity becomes manifest. Both machines generate and maintain a certain pressure, in the one case hydraulic, in the other electrical. Neither is it the electricity that is consumed or lost in passage round the circuit, but the pressure under which it flows, just as the 'head' of the water is lost in its passage through the pipe, only to be renewed by the action of the pump. Further, the mechanical energy developed in any time in the one case is proportional to the quantity of water flowing round the system in that time and the total pressure under which it flows, *i.e.* to the product of the number of gallons passed and the pressure in pounds per square inch; the electrical energy in the other case is proportional to the quantity of electricity and the pressure, *i.e.* in the practical units of the C.G.S. system, to the product of coulombs and volts. But in both cases we are more concerned with the *rate* of development of the energy or the *power* of the machine, which is proportional to the rate of flow of the water in gallons per minute or of the electricity in coulombs per second. The rate of flow of electricity is called the current, and one coulomb passing per second is an ampère of current. Consequently the power of the dynamo is measured in volt-ampères or 'watts,' and it is in such units that its value as a machine is reckoned, just as the mechanical value of a steam or other engine is reckoned, not in foot-pounds or in horsepower-hours, but by its horsepower or rate of doing work, without reference to any particular time.

§ 3. **The output of the dynamo.**—Again referring to our system of fig. 1, it is evident that if we wish to make use of the water forced through the pipe external to the pump itself, for instance, to drive some form of water-motor, it is not the total rate of development of energy throughout the entire system that can be turned to account, but only the rate at which available energy is developed outside the pump. Of the total work done by the pump on the water some portion is spent in overcoming the friction which resists the passage of the water through the pump's own channels and valves, so that the available power is less than the total power of the pump. Similarly the *output* of the dynamo is its external power or its rate of development of *available* electric energy; that is, the product of the voltage between the points A and D, and the current flowing through the external circuit R_e; or, in symbols, if E_e = the volts at the terminals A D, and C_e = the current through R_e, the output of the dynamo is equal to E_eC_e.

Electrical energy is therefore the energy developed by a current flowing under an E.M.F., and the output of a dynamo or its rate of development of electrical energy external to itself is reckoned in units of power; the watt being but a small fraction of a horse-power, it is more convenient to express the output in *kilowatts*, or units of 1000 watts. Since the output is always proportional to the product of two factors, the volts and the ampères, the same output may be due to a great variety of different combinations; for example, 10,000 watts, or 10 kilowatts, may be the electrical power of either 10 ampères flowing under a pressure of 1000 volts, or of 100 ampères flowing under 100 volts, or again of 2000 ampères under a pressure of 5 volts. These three illustrations will serve to emphasize the important difference between machines of high voltage and small currents such as are used for arc lighting or transmission of power over considerable distances or for the distribution of electricity on the alternating-current system, and machines of moderate voltage and current, used for the direct lighting of incandescent lamps, or finally machines supplying large currents at low voltages, such as are used for electro-plating, electro-depositing, and other chemical or metallurgical processes; although their power may be the same, the construction of the machines is very widely different, owing to the different natures of the work for which they are respectively suited.

§ 4. **The conversion of mechanical into electrical energy, and the efficiency of the process.**—By the principle of conservation of energy it is impossible that any form of energy can be absolutely created; in any machine it can only be transmuted from one form to another, and in the case of the dynamo in which electrical energy is developed our definition states that the energy is supplied to it in a mechanical form. Hence while the output of the dynamo is electrical power, the *input* is mechanical power. With the latter as supplied by a steam or gas engine, turbine, waterwheel, or other prime mover, all are familiar; in any such machine motion is produced against a resistance, and our definition requires that there should be in the dynamo continuous motion of one portion relatively to another portion. The movable portion of the dynamo may be driven by means of a belt and pulley, by rope gearing, or by friction gearing; or it may be coupled directly to the main shaft of the prime mover, as on to the crank-shaft of a steam-engine or the shaft of a turbine; but in all cases when electrical power is developed, there is resistance to the motion, and mechanical power is consumed only to reappear in another form. The dynamo is thus distinguished from another electromagnetic appliance, viz., the transformer, used on the alternating-current system of electricity; in this there is no relative motion, and the input is electrical power which reappears at a different pressure but still in an electrical form. The extensive use of electrical energy for commercial purposes has in fact been rendered possible by reason of the ease and cheapness with which mechanical energy can be applied to drive the dynamo. The cost of the chemical materials which voltaic batteries require prohibits their use on a large scale, while frictional machines only yield very small currents at inconveniently high pressures, so that it was the invention of the dynamo by Faraday in 1831 which first led the way to a ready means for obtaining electrical energy economically and in a convenient form. The rate at which mechanical energy is supplied to the dynamo is usually measured in horsepowers, and the relation between the two units in which rates of development of mechanical and electrical energy are respectively measured is such that one horsepower is the mechanical equivalent of 746 watts, whence it follows that $1\frac{1}{3}$ horsepower is nearly equal to one kilowatt.

But in no machine can the conversion of energy from one form to another be carried on without some loss, by which is meant not the actual disappearance or annihilation of energy (which is an impossibility), but its appearance in a useless form or in useless places. To take the case of a centrifugal pump, the rate at which mechanical energy is supplied to it is measured by the brake horsepower spent on its shaft, but the rate at which useful energy is developed is determined by the rate of flow of water under the pressure created by the pump at its delivery outlet; some of the work done on the shaft of the pump is expended in heating its bearings by friction, and of the actual hydraulic work, as has been already pointed out, a portion is expended in driving the water through the channels of the pump against the resistance of fluid friction. Both are inevitable losses, and closely analogous to them are the losses in a dynamo; apart from the loss by friction in its bearings, it has been shown that there is a necessary loss from its total rate of development of electrical energy, due to the absorption of power in driving the current through the electrical resistance of the dynamo itself. In consequence of these mechanical and electrical losses, if the rate of supply of mechanical energy to the shaft of the dynamo be one horsepower, its output must be something less than 746 watts. Nevertheless, the *efficiency* of the dynamo or the ratio between the useful power obtained from it and the power supplied to it in another form is very high, since in all but very small machines it is practically and commercially possible to obtain as much as 90 per cent. of the mechanical input, returned at the terminals of the dynamo in its new form of electrical energy, and in large machines even higher percentages are frequently attained. Much improvement cannot therefore be expected on the score of efficiency, although the dynamos of the future may be cheaper to construct. Indeed, of all machines yet invented, the dynamo may rank as one of the most perfect converters of energy, only surpassed by the transformer, in which, however, there is no true conversion, but only a transformation of electrical energy from one pressure to another.

§ 5. **General description of action of typical dynamo.**—Let us now consider the action of the typical dynamo of fig. 2, which represents in elevation and section a two-pole belt-driven machine such as may be used for supplying a continuous current to incandescent lamps; dynamos the same or similar in design have been constructed by many makers in perhaps larger numbers than those of any other class. We have said that any dynamo consists of two structural portions, between which relative motion is produced: in this machine the stationary portion consists of an iron horseshoe *magnet*, *d d*, round the limbs of which are wound coils *e e* formed of many turns of insulated copper wire; its two pole-pieces, N and S, are bored out so as to nearly embrace the circumference of the revolving portion *a*. The latter, or, as it is called, the *armature*, consists of a cylindrical iron structure rigidly mounted on a steel shaft which is supported at either end by bearings, and at the one end carries a driving pulley. On it is wound a number of turns of insulated copper wire lying close together all round the circumference, and passing on the outside longitudinally across from one end to the other: these turns are connected at one end to another smaller cylindrical structure known as the 'commutator,' *c*, on which rest two pairs of stationary 'brushes,' *b b*, each pair being connected electrically to one of the two terminals of the machine, A D. The action of the machine is shortly as follows. By sending an electrical current round the coils *e e*, the iron horseshoe is made strongly magnetic, or, as it is termed, a 'magnetic field' is 'excited' between its poles. By means of a belt passing over the pulley, the shaft, and with it the cylinder, is caused to rotate, and so move relatively to the magnet

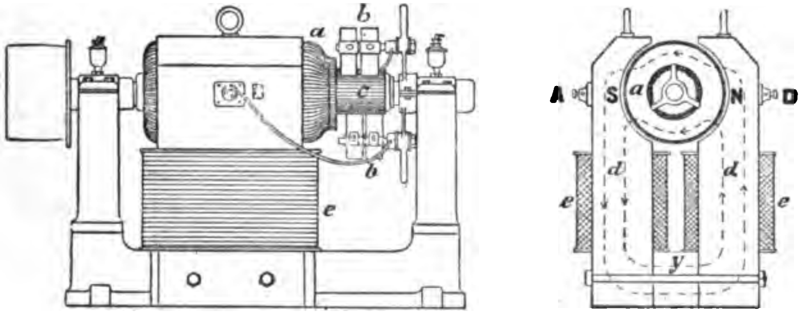


FIG. 2.—Two-pole continuous current dynamo.

at once an electromotive force is set up in each of the copper wires under the pole-pieces, the general effect of which may be broadly summed up by saying that a difference of electrical potential is produced between the points A D, just as was the case in the simple dynamo of fig. 1. If an external circuit be now closed on these terminals, this electrical pressure will cause a current to flow round the completed circuit, passing on its way through the copper wires themselves. Directly such a current flows, electrical energy is developed, and mechanical energy must be supplied to the shaft through the pulley; in other words, as soon as the circuit is closed, and a current flows, the revolving portion offers resistance to the motion, and work has to be done in overcoming this resistance. We have next to consider what is meant by the 'magnetic field' which has been alluded to as created by the passage of a current through the coils encircling the magnet limbs.

CHAPTER II

THE MAGNETIC FIELD AND ITS LINES OF FORCE.

§ 1. **The exploration of a magnetic field by a unit pole.**—Let us take an ordinary straight bar magnet and place it across an upright piece of wood, so that it is supported in the air away from any iron by means of the prop at its centre. Let us next approach it with a small 'magnetic needle,' or piece of steel magnetised along its length, so that one end is a north pole and the other a south pole; we may use for the purpose either a little compass needle delicately pivoted by a jewel or agate cap on a sharp-pointed support, so as to be free to move in a horizontal plane, or, better still, a magnetised needle suspended by its centre of gravity at the end of a thread, so as to be free to set itself in any plane, horizontal or otherwise. Now, when such a small needle is brought up near to the bar magnet, it will be found that at whatever point in the neighbourhood of the magnet it is placed it will set its length or magnetic axis in a definite direction, and if thrust aside from that position of equilibrium it will return to it and to no other. The exact direction will vary according to the place where the needle is held, but for each place there is one particular direction in which it will always set itself. And this is true whether it be held above or below, to the right or to the left of the bar magnet; it always takes up its definite position, with its north pole pointing more or less directly towards the south pole of the large magnet, and its south pole towards the magnet's north pole. We see from this that the space surrounding the bar magnet is in some way different from the same space when the magnet is removed, for then the needle will always point to the earth's north pole. Our first aim must therefore be to explore the surrounding space, and to arrive at some method by which its peculiar properties can be adequately described. To do this scientifically we require to examine its effects upon a single magnetic pole; and, consequently, the small needle which we used above, and which had two poles, one at either end, is not strictly the exploring instrument which theory demands. In nature it is absolutely impossible for the two poles to be separated so that a north pole should exist by itself and apart from a south pole, or *vice versâ*; yet by certain devices matters may be so arranged that the one pole of the exploring needle is so far from the bar magnet as to be practically uninfluenced by it, and thus a practical approximation is made to the ideal requirement of a single free magnetic pole.

Suppose that we place our bar magnet, still supported at its centre, in the middle of a large glass vessel, away from the sides, and fill up the vessel with water to the level of the magnet (fig. 3). Let us then magnetise a very thin and long piece of steel, such as a knitting-needle, and fix it vertically with one of its ends, say its north pole in a piece of cork; if we now place the cork in the water, with the needle projecting downwards like the mast of an overturned boat, the whole will float, with the top of the cork just level with the water, and the north pole of the needle projecting through it so as to be level with the bar magnet. The other, or south end of the needle, will then be so far away from the bar magnet that we may regard it as out of reach of the influence of the latter, and may therefore consider that we have a single *free north pole* floating on the water: for the cork, being non-magnetic, will in no way affect the result. Let the exploring boat be brought close up to the north pole of the magnet: as soon as we loose our hold it will start off on a voyage to the south pole, and, when it

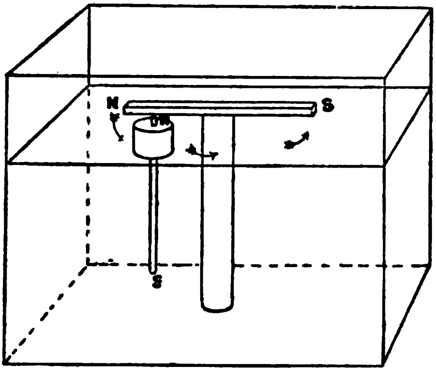


FIG. 3

has reached it, will there come to rest. In its passage from the north to the south pole it will describe a certain curved path, as shown by arrows on the top of the water in the diagram. If we take the boat back and place it at another part of the north end of the bar magnet, it will start off again, but along another path, perhaps further out from, or perhaps nearer to, the bar, according to the position of the point from which it starts. The same will be repeated also on the other side of the bar magnet; further, if between the experiments we gradually turn the bar magnet over on its side, we shall find that similar curves are invariably described by our floating north pole, until after having turned it completely over, we see that there is no difference between any of the four sides of the bar magnet as regards its action on the exploring pole.

If we place the boat far away from the magnet, near the side of the vessel, perhaps it will not move at all; yet this is only because the magnetic force which tends to drive the boat may not be strong enough to overcome the resistance of the water to the boat's motion; and if we take the needle out and can magnetise it more strongly, we shall find that the boat will start on one of its voyages to the south pole in whatever part of the water it is placed, except very close to the centre of the bar.

§ 2. **Field of a bar magnet.**—Experiment thus shows that a magnet influences a magnetic pole brought into its vicinity by acting upon it with a certain force, which at any point tends to move it in a particular direction: this force is not always in the same straight line, but gradually curves round, so that a free pole moving under it traverses a definite curved path between the north and south poles of the magnet. The space round the magnet where we can detect this effect is called the *magnetic field*, or simply the 'field' of the magnet. Further, this field has no fixed limits in space, since the distance at which the magnetic force can be detected depends upon the sensitiveness of the instruments which we employ to detect it.

§ 3. **Lines of magnetic force: (a) their direction.**—Let us next trace out the various paths traversed by the N. pole of our boat: they were all described on the horizontal plane of the surface of the water, but relatively to the magnet they were traced in every plane which contains its length, since the magnet was gradually rotated round lengthwise without in any way affecting the character of the curved paths. If, therefore, all the paths which we have seen described are combined together, they will form a complete system of lines encircling and enveloping the magnet on all sides, and all curving round from the north to the south pole. Fig. 4 gives a view from above of a bar magnet in the centre of a system of lines described in every plane containing its length, and shows how they would form a kind of spheroidal cocoon of threads stretching from one pole to the other. Now what are these paths so traced out? They are *lines of magnetic force*, being the lines or paths in which a single magnetic pole placed in the field at any point will, if left free and unconstrained, be forced to move from the one pole to the other. A tangent drawn to a line of force at any point gives the actual *direction of the magnetic force* at that point; this direction is continually changing, and the lines are curved, but the ideal pole as it follows the curves is assumed to be uninfluenced by anything of the nature of centrifugal force. Since our exploring boat would start moving from any place near the magnet (except just at its centre, where there is no magnetic force), it is evident that the number of such 'lines of force' is really infinite, yet it will be sufficient to mark down a number large enough to give us a clear idea of the various directions of the force. And this the more so because it will be found that if the lines are traced out very accurately, no two lines ever cut each other, but that they always run side by side, more or less converging or diverging, but never intersecting; in fact, it would be impossible for two or more lines to intersect, since at any point in the field a free single pole can only move in one particular direction.

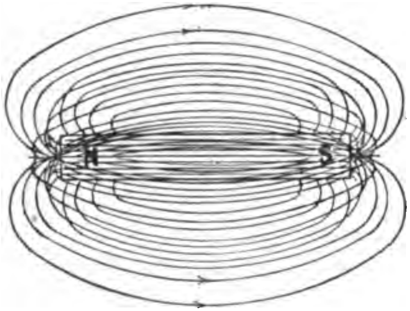


FIG. 4.

§ 4. **Lines of magnetic force: (b) their sense.**—But not only have the lines a certain position in space, they have also a certain direction in the further sense that a free north pole set down on any line will move in one direction along it, while a free south pole will move in the opposite direction. It is conventionally agreed that the *positive direction of the lines* is that in which a north pole would move, and hence in our first experiments we chose a north pole to place in our exploring boat; the direction thus given to the lines by a north pole is usually indicated on them by arrowheads. All the lines of force are thus regarded as emanating from the north pole of a magnet and passing round through the surrounding air to enter into the south pole. Further, what is their relation to the fixed directions which, as our first experiment showed, were imposed on a small compass needle when brought into a magnetic field? If we eliminate the effect upon it of the earth as a magnet, it will be found that at every point its magnetic axis is set tangentially to the direction of the curved lines of force; hence the picture given to us by our system of lines of force enables us to predict the definite position which a magnetised needle will take up at any point in the space surrounding the bar magnet. Fig. 5 shows the position and direction of a few lines separated out from the number which traverse the field of a magnet, and further shows a small compass needle setting itself tangentially to their direction. If the needle be small, and at some distance from the magnet, it does not move as a whole along the lines of force, as did our single magnetic pole, because its two poles are nearly equally affected by opposite forces, and the one tends to move in the one direction as strongly as the other in the opposite direction. Yet if placed athwart the lines the needle is acted on by a couple tending to rotate it until its axis coincides as nearly as possible with a line of force; it therefore sets itself tangentially to the direction of the lines at that

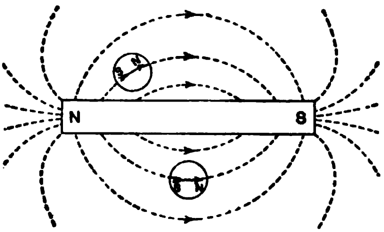


FIG. 5.—Lines of force.

part of the field where it is placed. Its north end will point along the line in the same direction as that in which a free north pole would move. By the north end or pole of a magnet we shall throughout mean that which points to the geographical north, or, as it is sometimes termed, is north-seeking; and when a compass needle sets itself along lines of force, the end which points towards the geographical north, if acted on by the earth's field alone, will indicate the positive direction of the lines of force, or point down the lines in that direction which will lead us to the south pole of the magnet.

We have now seen that one property of the space which we call a magnetic field can be usefully represented by 'lines of force'; to sum up, they themselves show the path and direction in which a free magnetic pole will move when placed in the field at any point, and a tangent to them shows the position which a small piece of magnetised material, such as a compass needle, will take up under the action of the forces on its two poles.

§ 5. **Lines of magnetic force: (c) their number as representing intensity of field.**—The reader will probably know the way in which the field of a magnet is illustrated by means of iron filings sifted on to a sheet of glass or paper held over a magnet. The little pieces of iron become magnetised along their greatest length, so that one end forms a north pole and the other a south. They then resemble our compass needle, and consequently set themselves end-on along the direction of the lines of force at any part of the field; where the plane of the paper cuts across the direction of the lines the filings stand up erect, and give, as it were, a view of the lines in section; where the directions of the lines fall in the plane of the paper they follow along them in chains, the north pole of each little magnet clinging to the south pole of its next neighbour. Thus in the case of a bar magnet laid horizontally on its side, the filings collect in crowded tufts above the poles and stand up nearly vertically; from the edges they radiate outwards, and gradually curve round, forming more or less closed chains in the horizontal plane of the paper. Meanwhile there is one point on which the picture given by the iron filings must not be allowed to mislead us. The filings arrange themselves at the sides of the magnet in curves which are to a certain extent broken and separated off by intervals in which there are no filings. But we know that an infinite number of closely contiguous lines of force could have been traced out by means of our exploring north pole, and consequently that there are in reality no gaps in the magnetic field corresponding to any gaps between the lines of the iron filings. From their mutual action on one another the magnetised filings tend to fall into lines when the paper is gently tapped, and to complete those lines, leaving interlinear bands, rather than to remain spread uniformly over the surface, even though their magnetic axis be set along some line of force.

Yet what at first sight seems a defect in the picture given by the iron filings may serve as a hint to suggest a method by which we may still futher map out a magnetic field so as to more fully exhaust its properties. At any point in the field the force acting on a pole or on a magnet as a whole, not only has a certain direction and sense, but also a certain magnitude; in other words, the magnetic field has a certain intensity or strength. This *intensity of field* must again be estimated by reference to such a simple magnetic pole as previously gave its direction, and for this purpose the pole must be of unit strength. *Unit pole* is a magnetic pole of such strength that if it is placed at unit distance from another such pole of equal strength there will be unit force acting between the two, repelling or attracting them according as they are of the same or opposite sign. Hence on the C.G.S. system a unit north pole, placed at a distance of one centimetre from a south pole of equal strength, will be attracted with a force of one dyne. The intensity of a field at any point is estimated by measuring the force with which a unit pole would be acted upon at that point, and a field has unit intensity[1] if a unit pole placed in it is acted upon by a force of one dyne. The field of a bar magnet increases in strength the nearer we approach either of its poles, as is shown by our north pole being strongly attracted or repelled when close up to the south or north pole of the magnet.

Can we, then, also represent the intensity of a field by the same geometrical picture of magnetic 'lines of force'? The experiment with the iron filings suggests a method by which this can be done.

---

[1] At the Paris Conference of 1900, it was recommended that the C.G.S. unit of intensity of field should be called a 'gauss.'

It has been stated that there are no real gaps in the magnetic field, but that the force pervades the whole of it, varying, it may be, in strength in different places, yet continuously shading off from one value to another without any sudden jumps or breaks; yet the lines of force indicated by the filings are crowded together near the poles where the force is strongest, and gradually widen out until where the force is weakest they are comparatively widely scattered. Now suppose that we cut the lines by a plane at right angles to their direction at any point; obviously we should have more lines per unit area on such a transverse plane if it is taken near to the poles than if it be placed at some distance from them; consequently the density of the lines or the number of lines traversing any unit area in a plane perpendicular to their direction might be made to serve as a measure of the relative strength of the force at different parts of the field.

If, therefore, out of the infinite number of lines of force that could be drawn at any part of the field we select a certain number of marked lines, the magnitude of the force acting on a unit pole placed in any part of the field could be represented by the closeness or sparseness of such selected lines at that part; and if each selected line represents a force of one dyne acting on a unit pole, the intensity of the field can be expressed by stating how many such lines there are per unit area of cross-section on a plane normal to their direction. *Unit field* will have one line per unit area of such a plane, because unit pole placed in it would be acted upon by unit force, or on the C.G.S. system one line per square centimetre will represent a field acting with a force of one dyne upon a unit pole.

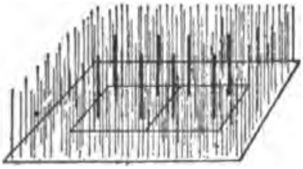


FIG. 6.

In fig. 6 let each of the small squares represent in perspective one square centimetre of area on a plane at right angles to the direction of the lines of the field: out of the infinite number of these as represented by the shading, four selected lines stand out, and if C.G.S. lines, as they are called, indicate a field of such intensity that in it a unit pole will be acted upon bv a force of four dynes. Similarly a field whose intensity or strength is 1000 in C.G.S. measures means one in which the number of C.G.S. lines per square centimetre on a plane normal to its direction is 1000, and in which, therefore, a unit pole would be acted on by a force of 1000 dynes. Thus by means of a definite system of lines the properties of a magnetic field can be accurately represented, its direction and its intensity mapped out and marked. A uniform field will be graphically represented by lines running parallel to one another and spaced at equal distances, as in figs. 14 and 20; since, then, wherever the unit area be taken on a plane normal to the direction of the field, the same number of lines traverse it. A field of varying intensity, when plotted, will appear with a varying density of lines, yet always so spaced that the number passing through unit area on the normal plane is equal to the strength of the field in dynes when acting on unit pole at that place. For example, fig. 21 shows in section and plan portions of two fields whose intensities gradually decrease as we move towards the central gap between them, the direction of one being opposite to that of the other. We may compare the method by which magnetic fields are pictorially indicated to that by which light or shadow is represented in line engraving: a strong field can be represented by a shading of densely-packed but separate lines, or by close-set dots just as is a dark shadow, although both field and shadow have no structure and are perfectly continuous, the one in nature and the other in the picture which the engraving represents in black and white. The conventionality of the method is shown by the fact that we sometimes have to deal with fractions of a line; for example, the intensity of the earth's field in England is such that it would act on a unit pole with a force of ·47 or about half a dyne; such a case would be represented by half a line per square centimetre, or, as this could not be shown pictorially, by one line passing through every two square centimetres of area on the normal plane.

§ 6. **The magnetic property of an electric current.**—It is, however, not only permanent magnets of steel which exhibit magnetic properties; a conductor carrying an electric current is also surrounded by a magnetic field. In the year 1819 a Danish physicist, Oerstedt, discovered that when a magnetic needle was brought near a wire carrying an electric current, it was deflected into a definite position relatively to the wire; in other words, between the conductor conveying an electric current and the magnet he found that there existed a certain magnetic force which, if the conducting wire were stationary, and the needle movable, would cause the latter to set itself in a definite direction. For example, if the conductor conveying the current is a long straight wire held horizontally over a compass needle, the latter tends to set itself at right angles to the length of the wire; and, further, the direction in which its north pole points when it has so set itself depends upon the direction in which the current is flowing in the wire.

In the above great fact of the *electrodynamic action of an electric current on a magnet* is involved the whole principle of dynamo-electric machinery. The simple experiment of Oerstedt was in fact the first instance that had been observed of the conversion of electrical energy into mechanical work, and as such illustrated the principle of the electric motor; for the needle as it moves could be made to do mechanical work.

§ 7. **The field surrounding a straight conductor.**—In order to map out more definitely the magnetic field due to an electric current flowing in *a straight wire*, we will again have recourse to the picture given us by iron filings. Let us pass a straight wire through a hole in a piece of cardboard or glass plate held at right angles to the vertical wire, send a current through the wire, and sprinkle iron filings on the plate; as in the similar experiment with the bar magnet, the filings will arrange themselves in certain lines, which in this case are concentric circles having the wire as their centre (fig. 7). We can slide the plate up and down, while still keeping it in a plane perpendicular to the axis of the wire, and there will be no alteration in the shape of the figures. Now the circles thus formed show us the position and distribution of the lines of force of the field, and by examination of them we find that a rectilinear portion of an electric circuit, such as a straight wire, suspended in air, and traversed by a current of which the return path is a considerable distance off, is surrounded by concentric circular lines of force, in a plane perpendicular to the axis of the wire, and having their centre in its axis.

To the right of fig. 7 the position of a few of the lines of force surrounding a straight conductor is shown more clearly, together with a small compass needle setting itself tangentially to the lines; the current is supposed to be flowing downwards through the plane of the paper, and the direction of the lines as marked by arrowheads is seen to be clockwise. By means of the compass needle, whose north pole will always point in the positive direction of the lines, it will be found that the direction of the lines is related to the direction of the current, just as the direction of rotation of an ordinary right-handed screw is associated with the direction of its forward or backward movement as viewed from its head A to which the screw-driver is applied (fig. 8).

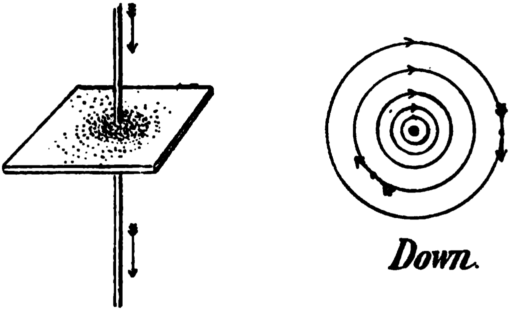


FIG. 7.—Field of straight conductor.

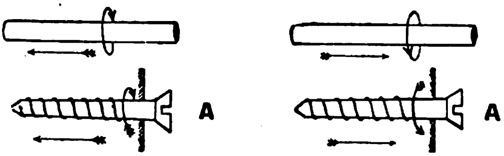


FIG. 8.

The position and direction of the lines of force surrounding a straight conductor have now been given, and their distribution is again shown by the iron filings: these are crowded together more closely near the wire, and thus show that the intensity of the field is greatest immediately round it, and gradually diminishes as the radial distance from the wire is increased. Further, if the strength of the current through the wire be increased, the rings of iron filings will crowd together still more closely in all parts of the field though still being relatively scattered at some distance away; in short, the intensity of the field varies directly as the strength of the current, and inversely as the radial distance from the wire.

One more peculiarity, which is as yet entirely new to us, remains to be noticed: it is that the lines of force are in this case *closed or endless curves*, entirely surrounding the axis of the wire. When any line is traced along its entire course we eventually arrive at the same point as that from which we started to trace it.

§ 8. **The field due to a current-loop.**—Next let us, as in fig. 9, bend the wire round so as to form a *circular loop*, the ends being twisted together and insulated from one another, so that the current passes round the entire loop; at once it will be seen that if the lines of such a current-loop retain the general character of the lines due to a straight conductor, they will, all of them, pass right through the loop, entering into one face of it and issuing outwards from the other, and such is in reality the case. The field of such a loop can be explored by means of a small compass needle, and we shall find that it not only takes up a definite direction when outside the loop, but also when placed inside it; and as so traced the lines are found to pass through the loop, and thence gradually to expand into curves which are larger and larger as we pass from the centre of the loop outwards until they become too indistinct to be traced.

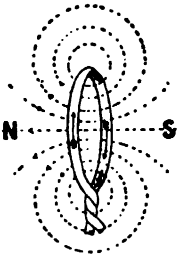


FIG. 9.—Field of loop.

Again, therefore, the lines are closed curves, interlinked with a closed electric circuit, like a number of rings threaded on to one common ring. By again applying the rule of the screw to the modified case of our circular conductor, it will be found that the direction of the lines as they pass through the loop is associated with the direction of the current round the loop, just as is the movement of a right-handed screw with the direction of its rotation, and this leads us to the following extremely simple and convenient rule: *Curve the right hand round the outside of the loop, keeping the* *palm towards its axis, so that the direction of the flow of current is from the wrist to the tips of the fingers; then the outstretched thumb will point along the positive direction of the lines within the loop, the face from which they issue being therefore on the same side of the hand as the thumb.*

Now, since all the lines enter into one face of the loop and issue from the other, the loop may be said to have a south and a north face, just as a bar magnet has a north and a south end; and the loop will, in fact, act in every respect exactly as if it were a thin circular magnet magnetised transversely along its shortest axis, or a magnetic shell, as it is termed; it will repel or attract other magnets according as the face which is presented to them is of the same or opposite polarity; and if suspended so as to be free to move, it will set itself in a plane perpendicular to the magnetic meridian, so that its north side faces the geographical north.

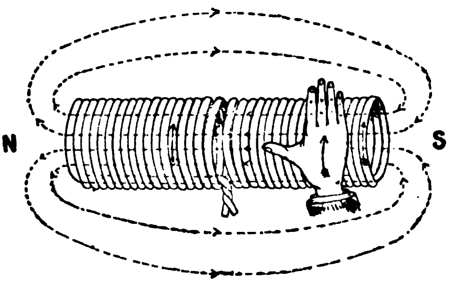


FIG. 10.—Field of solenoid.

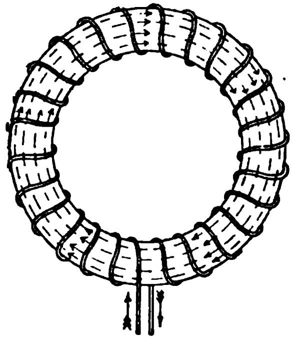


FIG. 11.—Field of toroid.

§ 9. **Field of a solenoid.**—Next let us place several such loops of current side by side in a straight line. This is best effected by coiling up one and the same length of insulated wire into a number of closely-contiguous turns, the ends being brought back through the centre of the hollow cylinder so formed (fig. 10). Such an arrangement is called a *solenoid*, and when its field is explored by a compass needle the lines (a few of which are shown in the diagram) will be found to run right through the cylinder, issuing forth from and entering into one or other end according as the current flows in one or other direction round it. Again, by the application of the *right* hand curved round the outside of the helix with the palm towards the axis, and so placed that the current flows from wrist to finger-tips, the direction of the lines through the solenoid is given by the fact that the thumb points to the north pole or end from which they issue.

§ 10. **Insertion of iron core within the solenoid.**—Let us next insert into the hollow space within the solenoid a cylindrical 'core' of unmagnetised iron or steel, extending from the one end to the other, and examine the effects thereby produced; although electrically insulated from the coil, it will be found that a north pole is developed at the one end of the iron, and a south pole at the other, so that the whole arrangement still acts externally just as did the simple solenoid (which itself was like a permanent bar magnet), only now its attractive and directive force is very much increased. Owing to the electric current circulating round the turns of the solenoid, the iron becomes magnetised, polarity being induced in it by the magnetic field in which it is placed. We have, in fact, an *electromagnet* whose magnetism depends upon the presence of the 'exciting current' circulating round it; when this current is interrupted the iron almost or entirely ceases to attract; as soon as circuit is again made, at once the iron becomes magnetised; it can therefore be made a magnet at will. The external field of the excited electromagnet is closely analogous in its distribution to that of the solenoid before the iron core was inserted, the only difference being in the greater strength of the field of the electromagnet; and, further, it is closely analogous to the field surrounding a permanent bar magnet as previously mapped out.

§ 11. **The field of the toroid.**—Finally, let the solenoid be imagined to be bent round until its ends meet; or, we may wind a number of turns of insulated wire round an endless ring-core of circular cross-section. Such an arrangement is known as a *toroid* (fig. 11), and frequent reference will be made to it in Chapter III.

If the wire is wound closely and uniformly over the whole periphery, the lines of force are closed curves, confined entirely to the inside of the turns, and there are no poles. If the core were only air, the presence of the lines of force can be made evident by the introduction of, *e.g.*, a compass needle within the current-loops, but if the turns are wrapped round an iron core, its magnetised condition is not discoverable by any action on an external magnet, although it can be recognised in virtue of another property which will be considered in the next chapter.

CHAPTER III

THE MAGNETIC CIRCUIT AND ITS LINES OF INDUCTION

§ 1. **The inductive property of the magnetic field.**—So far the condition which we call a magnetic field has been examined, and its lines of force mapped out, by means of an exploring pole. Other properties, however, belong to the magnetic field besides that of exerting force on the poles of a magnet or on electric currents brought within its influence, and of these by far the most important is its *inductive* property. In 1831 Faraday made his great discovery of the principle of *electromagnetic induction*, which may be shortly stated as follows: when a conductive circuit is placed in a magnetic field and the intensity of the field or the relative position of the circuit in it is altered, then in general an electric current is induced in the conductive circuit by, and during the continuance of, the change. This new property calls for consideration as much as the electrodynamic property which was dealt with in the preceding chapter, and its investigation will at the same time throw light on a further question. In the case of the bar magnet the lines of force were only traced as far as its poles, so that it may still be asked, What is its internal condition? Obviously an actual exploration within the mass of the iron cannot be made even with our approximation to a free unit pole, but can some similar method based on the inductive property be devised for examining its interior? A new exploring instrument is required, and such is found in an *exploring coil*; by means of this not only can we directly investigate the inductive action of the external field, but we can also answer the question as to the internal state of the bar magnet.

§ 2. **Examination of this property by an exploring coil.**—Let a conductor forming part of an electric circuit be coiled up into a plane loop (like fig. 9), and let it be placed in the magnetic field of a solenoid or electromagnet, such as has been previously described. Let the circuit of the loop be called the secondary circuit, the primary being that of the wire carrying the exciting current. Then not only can the exploring coil be moved from spot to spot in the magnetic field, but the primary circuit can be made or broken, or its current reversed in direction, and so the intensity of the field in which the coil is placed can be varied. If the coil is now set at any one spot and is turned in every direction about its centre, so that its axis or the perpendicular passing centrally through its plane occupies every possible direction in space, it will be found that when this perpendicular lies in one particular plane, no current is induced in the coil on making or breaking or reversing the primary current. For all other directions of the perpendicular, a current is induced and a certain quantity of electricity passes which can be measured by, *e.g.*, a ballistic galvanometer forming part of the secondary circuit. This quantity varies according to the direction of the perpendicular and reaches a maximum value for one particular direction, viz. when the coil is itself in the plane which has been described as containing all positions of the axis that give no inductive effect, and when in consequence the perpendicular is at right angles to this plane. As the angle which the perpendicular makes with this position of greatest inductive effect is increased, the quantity of electricity that is set in motion decreases in proportion to the cosine of the angle of inclination, so that it vanishes for all positions of the perpendicular in the plane at right angles to its position for maximum effect. The condition which we are investigating is therefore a directed quantity and must be expressed by a vector.

§ 3. **Lines of magnetic induction—their direction, sense and number.**—This quantity is called the magnetic induction, and the *direction of the magnetic induction* at any spot is defined as that of the perpendicular to the coil when in the position that gives the greatest inductive effect on making or breaking the primary current. The direction of the induction is further defined by the convention that when the primary current is broken, the *sense* of the induction is related to that of the current in the secondary in the same way as the forward or backward movement of a right-handed screw is related to its rotation (compare fig. 8). Thus on breaking the primary, if the secondary current is clockwise, the induction passes through the loop of the exploring coil in the same direction as from the face of the dial to the works of a clock. If a line be drawn so that the tangent to any point in it gives a continuous record of the direction of the induction as we pass from point to point along its length, such a line is called *a line of induction*, and if the process of drawing it be repeated for all parts of the field, an infinite number of such lines can be obtained.

It still remains to determine the *magnitude of the induction*, and for this it is necessary to make use of a unit exploring coil. Such a coil must consist of a single loop having an area of one square centimetre, and the total resistance of the entire secondary circuit of which it forms a part must be one absolute electromagnetic unit of resistance or 10^{-9} ohms on the C.G.S. system. If this be placed in any part of a field with its perpendicular in the position for greatest inductive effect, and the quantity of electricity that passes through the ballistic galvanometer when the primary circuit is broken is one absolute unit or one deca-coulomb, the induction at that spot has unit magnitude.[1] Just as in the case of magnetic force, a certain number of lines may now be marked out or labelled such that their density per unit area on a plane normal to their direction is equal to the magnitude of the quantity in question at that place, and in a field of uniform induction if any surface be taken on the normal plane the density of such lines multiplied by the area of the surface will give the total induction passing through it. Thus by means of the unit exploring coil the direction and magnitude of the induction can be surveyed, and a complete quantitative system of lines be derived by a process exactly analogous to the mapping of the lines of force by a free unit N. pole.

§ 4. **Relation of lines of induction to lines of force.**—The relation of the lines of induction to the lines of force has still to be considered. Whenever the space under investigation is a vacuum or is filled with air or other non-magnetic medium, identically the same system of lines is obtained whether the starting-point be the dynamic property or the inductive property of the magnetic field. The lines of induction are linked with the current-carrying wire, circular loop, solenoid or closed toroid when these are immersed in air, just as the lines of force which have been previously described; they follow the same paths, and are the same in number. One or other method of plotting the lines would therefore seem to be superfluous. Yet there is good reason for the introduction of both and for their distinction. The exploring coil may be wound on the outside of the iron-cored electromagnet, and still it enables us to examine the internal state of the iron; for the primary current can be made or broken as before. Or, if need be, the coil can actually be buried in the substance of the iron so as to measure the induction at any spot when the primary current is altered in value. It may even with certain modifications be applied to throw light on the internal condition of the permanent magnet. We are in fact able to trace the passage of the lines of induction through the mass of an iron magnet by direct experiment, whereas the determination of the lines of force, if such there be, *within the iron*, is only possible by theoretical and indirect methods.

---

[1] At the Paris Conference of 1900, the term 'Maxwell' was recommended for adoption to express a unit of induction or one C.G.S. line.

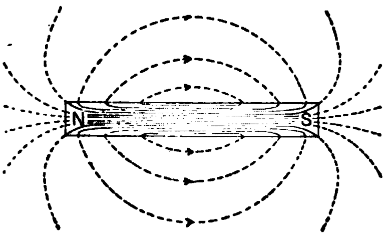


FIG. 12.—Lines of induction.

§ 5. **Lines of induction always closed curves.**—If now the exploring coil be used to map out the induction within the iron-cored solenoid or the permanent bar magnet, it will be found that as many lines of induction pass through a section of the iron taken at the centre of its length as there are in its external field, and that each line within the iron finds its appropriate continuation in one of the lines of the external field (fig. 12). Every line of induction is in fact a closed curve, which either loops round an electric current or currents (and in this case may be entirely in air or entirely in iron, or partly in one and partly in the other), or at some portion of its endless path passes through a piece of permanently magnetised steel or iron. Wherever lines of induction issue out of iron into air or other non-magnetic material, there a north pole is developed; and wherever they enter from air into iron, a south pole results; within the iron their direction is from south to north; without it, from north to south. It can next be proved that although the lines of induction and of force are identical outside the iron, they are not necessarily identical within it, save in the one case of a uniformly-wound closed toroid of which the turns encircle an endless iron core. Although this special case is one of great theoretical importance, it does not often occur in practice, and whenever we are considering the lines within any piece of iron, such as a dynamo magnet which has ends, the necessity arises for the distinction of the two systems within the iron. In dynamos the E.M.F. of the machine is, as already stated, generated by certain movements of electric conductors through a magnetic field in air; for the conductors, though wound over or laid on iron, are in effect entirely surrounded by air or other non-magnetic material. Since, therefore, in air no distinction need be made between lines of force and of induction, the E.M.F. might indifferently be said to be due to movement through either one or the other. But in every dynamo the field is produced either by one or more electromagnets or by permanent magnets of steel, and these are not in the form of closed rings; hence the lines with which we have to deal have always passed through iron at some portion of their path, and are essentially lines of induction. The latter system will therefore in future be given the first place; the curves of force as traced by the unit pole and visibly exhibited by iron filings afford the most ready means of familiarising the reader at the outset with the conception of magnetic lines and their mapping; this done, they may for our purpose be replaced with advantage by the system of lines of induction as traced by the exploring coil. But in all cases when the lines of the field are spoken of almost as if they had actual existence in nature, it must be remembered that they are only a device for expressing clearly and concisely certain physical properties of a magnet, and their meaning must always be sought for by reference back to the exploring pole or coil. The E.M.F. of dynamos will henceforth be treated as caused by movement through lines of induction, or, as our original definition stated, as due to the cutting of the magnetic flux—a synonymous expression which has now to be explained.

§ 6. **The total flux of a magnetic circuit.**—The lines of induction in the case of the uniformly-wound closed toroid such as fig. 11, are endless curves within the interior of its ring-core, whether this be made of iron or wood or simply consists of air. All the lines of the straight solenoid or electromagnet are not, however, linked with the entire number of the exciting turns; some lines escape from the surface of the bobbin without traversing its whole length. Yet every line passes through the coils for a greater or less distance in its path. And in the same way though some of the lines of the permanent bar magnet issue from or enter into the iron short of the ends, yet all pass through its cross-section at the centre (*cp.* figs. 12, 31). In every case, then, the lines of induction may be regarded as a kind of stream flowing round a closed path either in air or iron, or partly in one and partly in the other. The total number may be called the *total flux*, and the number passing through unit area on a plane normal to their direction is the *flux-density* at that spot. On the C.G.S. system the flux-density is measured per square centimetre, and is symbolised by B; it is also called 'the induction,' thereby recalling the method by which it is derived. We are thus led to the conception of a *magnetic circuit*, through which a certain flux passes under a magnetomotive force, just as an electric current flows round an electric circuit under an electromotive force. The expression 'magnetomotive force' requires to be further defined, and will be found to form a connecting link between the two systems of magnetic force and induction.

§ 7. **Magnetomotive force of a circuit.**—If any two points are taken in a magnetic field, and a free unit N. pole be moved from one point to the other along any line joining them, a certain amount of work will be done if the direction of movement of the pole has any component along the lines of force of the field. If the motion be against the sense of the lines, the work will be done on the pole, or if it be with their sense, it will be done by the pole. If the field be in vacuo or in air or other magnetically indifferent medium, this work, measured in ergs, is the *difference of magnetic potential* existing between the two ends of the length under consideration. Its value is easily calculated if the path chosen coincides throughout its whole length with the direction of the lines of force; thus if a length of one centimetre be taken anywhere along a line of force and the intensity of the field is measured by one dyne throughout the centimetre length, the work done by a unit N. pole in moving from one end to the other against the force will be one erg, and unit difference of potential exists between the two ends. Or in general, if the intensity of the field in vacuo or in air has the uniform value H, and a centimetre length be taken along its direction, the difference of magnetic potential over that length is also measured by H. If an endless line looped round an electric current be subdivided into very small elements and the successive differences of potential are calculated for each small element until the whole length of the path has been traversed, the sum of their values is the ***line-integral of the magnetic force***, or, as it is also called, the ***magnetomotive force*** along the line in question. The calculation of this again is readily effected if the line round which the differences of potential are integrated follows the direction of the field, and further, if the field has a uniform intensity throughout its length; for it is then simply the product of the intensity of the field and the length of the line, or if we use the convenient abbreviation of M.M.F. for magnetomotive force on the analogy of E.M.F. for electromotive force, we then have

$$\text{M.M.F.} = \text{H} \times l.$$

The straight wire of fig. 7, carrying an electric current of which the return is a long way off, gives us the fundamental case connecting magnetomotive force and current. If any one of the circular concentric lines be taken of which the distance from the axis of the wire is r centimetres, and the wire be carrying A ampères $\left(=\frac{\text{A}}{10}\right.$ absolute units of current on the C.G.S. electromagnetic system), the intensity of the field in vacuo or in air at radius r has the uniform value of $\frac{2\text{A}}{10r}$, and its direction is throughout that of the chosen line, of which the length is $2\pi r$; the line-integral of the force is thus simply the product of the two, or $\frac{2\text{A}}{10r} \cdot 2\pi r = \frac{4\pi\text{A}}{10}$.

In the case of the straight solenoid of fig. 10, of which the length l is great as compared with the radius of its cross-section, the intensity of the field at a point in its interior well away from its ends is $\frac{4\pi}{10} \cdot \frac{AT}{l}$, where T is the total number of turns and A is the current in ampères flowing through them. Since, however, the intensity decreases as we approach the ends of the coil or pass outside it, the magnetomotive force of the coil is not easily calculated in this particular form, and the case must be simplified by bending the solenoid round until its two ends meet and a toroid is formed. The intensity in the interior is now throughout the same as at the centre of the straight coil since it has no ends, or $\frac{4\pi}{10} \cdot \frac{AT}{2\pi R}$, where $2\pi R$ is the length of the circle corresponding to the mean radius of the ring; this length may be immediately multiplied by the intensity to obtain the magnetomotive force, or

$$\text{M.M.F} = \frac{4\pi}{10} \cdot \frac{AT}{2\pi R} \times 2\pi R = \frac{4\pi}{10} \cdot AT.$$

Thus the single current with which the line of the straight wire was looped is replaced in the coil by T turns each carrying A ampères. If now an iron core be inserted in either the toroid or solenoid, and its M.M.F. is to be determined by the imaginary operation of taking a unit pole round a circuit linked with the coil, theory shows that certain precautions are necessary in carrying out the process. If a transverse cut be supposed to be made in the iron at right angles to the direction of magnetisation at any place, and the free unit pole be brought to the place in question, within the infinitely thin crevasse formed by the cut, it will be acted upon by a certain force in dynes, while if, on the contrary, the incision into the iron takes the form of a very small hole drilled along or parallel to the direction of magnetisation, it will be acted on by a different force in dynes. It is this latter force, or the intensity of the field as measured by the unit pole within the thin tube or hole, which determines the difference of potential for each small element of length; and if the pole be moved round the circuit through the thin tube, the work done will be the line-integral of the force, or the magnetomotive force. For the exact nature and reason of the above necessary precautions in determining H in iron, the reader must be referred to larger works[1] dealing more fully with the subject; suffice it to say that when the value of H is determined in the above manner as indicated by theory, the line-integral of the differences of potential measured along any closed line passing through the iron-cored coil is $\frac{4\pi}{10} \cdot \text{AT}$, or the same as for the hollow solenoid. No difference has been introduced by the presence of the iron core, and in general the magnetomotive force of any coil, whether closed or open, and whether iron-cored or not, is equal to $\frac{4\pi}{10}$ times the ampère-turns.

The factor $\frac{4\pi}{10}$ or 1·257 results from the relation of the circumference of a circle to its radius, coupled with the fact that the unit of current on the absolute electromagnetic system is one deca-ampère, so that the number of ampères A has to be divided by 10 in order to express the current in absolute units.

§ 8. **Magnetic flux as related to the magnetomotive force of the circuit.**—The magnetic flux of induction is related to the magnetomotive force in such a definite way that the latter may be regarded as the cause of the former. In the first place, if a certain flux be produced by the *exciting power* of 100 ampères flowing through 100 turns, and these be replaced by one ampère flowing through 10,000 turns, all else remaining unaltered, the same flux will be produced, from which it is evident that the ampère-turns are the determining factor, and that the relative proportion of the ampères to the turns is immaterial. But for the same magnetomotive force the flux may be very different according to the nature of the material of which the circuit is composed. The quantitative relation between the two, or the fundamental equation of the magnetic circuit, is most easily established by again returning to the circular closed toroid uniformly wound over its whole periphery. Since the lines of induction are in this simple case entirely confined to the inside of the turns, there can be no question as to the area of cross-section of the circuit; it is $a = \pi r^2$, where r is the radius of the circular core. There is further no doubt as to the exact path of the lines; since each is circular, the mean length of the magnetic circuit is $l = 2\pi \mathrm{R}$, where R is the radius of the circular axis running through the centre of the core. Such a toroid therefore supplies us with the type of a perfect magnetic circuit. If r be small as compared with R, the flux-density may be taken as uniform over the whole area of the cross-section, and the total number of lines, Z, is then equal to the density per unit area multiplied by the area, or if a is measured in square centimetres, $\mathrm{Z} = \mathrm{B} \times a$. Now if the toroid be in vacuo, the lines of force and of induction are precisely identical in number and direction, so that $\mathrm{B} = \mathrm{H}$, and since the magnetomotive force is $\mathrm{H}l = \frac{4\pi}{10} \cdot \mathrm{AT}$,

---

[1] For a full treatment of this and other points in magnetic theory, see Prof. Ewing's *Magnetic Induction in Iron and other Metals*, and Dr. H. Du Bois, *The Magnetic Circuit*; and for a clear *résumé* of the subject within short compass, Prof. Fleming, *The Alternate Current Transformer*, vol. i. chap. ii. (2d edit.).

$$\mathrm{Z} = \frac{4\pi}{10} \cdot \mathrm{AT} \times \frac{a}{l}.$$

If the toroid be next immersed in air or be wound on a core of wood, or glass, or porcelain, or any non-metallic substance, the lines again have the same direction as in vacuo; and, further, no difference can be detected in their number save by the most refined measurements. The same, too, may be said if we try any of the metals, with the single exceptions of iron, cobalt, and nickel, and certain of their alloys and compounds. With these three exceptions, therefore, we are justified in ranking all substances as on a level, and in classing them as non-magnetic or magnetically indifferent. Even in the case of bismuth, which shows the greatest divergence as compared with a vacuum, the difference is only in the fourth or fifth place of decimals, or as 0·99982 : 1. Hence in the determination of the magnetomotive force from H the current-carrying toroid was allowed to be either in air or in vacuo.

§ 9. **Equation of the simple magnetic circuit. Reluctivity and reluctance.**—When, however, an iron ring is substituted for the air-core with the same dimensions and the same magnetomotive force, or ampère-turns, the total flux is greatly increased, and the same is true, though in a less degree, for cobalt and nickel. The new value of the flux is now $\mathrm{B} = \mu\mathrm{H}$, where μ is in each case a numerical coefficient greater than 1. The increased value of B can only be traced to some specific property of the iron as iron or of the cobalt as cobalt, and the complete equation of the magnetic circuit in its elementary form when the nature of its material is taken into account becomes

$$Z = B \times a = \frac{4\pi}{10} \,.\, AT \times \mu \,.\, \frac{a}{l}$$

or if $\xi = \frac{1}{\mu} = \frac{H}{B}$,

$$Z = \frac{1{\cdot}257 AT}{\xi \,.\, \frac{l}{a}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The denominator of the above fraction bears a close resemblance to the expression for the electrical resistance of a conductor, viz., $R = \rho \frac{l}{a}$; where ρ is the specific electrical resistivity of the material of which the conductor is composed, l and a being its length and area; and considered from this point of view, the fundamental equation is seen to be of the same form as the well-known expression for a continuous current in terms of electromotive force and resistance, viz., $C = \frac{E}{R}$. The comparison of the flow of lines of induction to the flow of a current through an electrical circuit is more therefore than a mere verbal illustration; there is a genuine analogy between the two of sufficient accuracy to guide us in the solution of many magnetic problems. The entire path of the lines forms in all cases a magnetic circuit closed upon itself, and having a certain length and area. Through this circuit under the action of a magnetomotive force there flows a stream of lines, and their total number is the quotient of the magnetomotive force divided by the magnetic resistance or (to give this property a more distinctive name), the *reluctance* of the circuit. Thus analogous to Ohm's law for continuous currents,

$$\text{electric current} = \frac{\text{E.M.F.}}{\text{resistance}},$$

we have

$$\text{magnetic flux or total number of lines} = \frac{\text{M.M.F.}}{\text{reluctance}}.$$

The magnetomotive force is proportional to the number of ampère-turns wound round the circuit, and is the equivalent of the internal electromotive force of, *e.g.*, an electric battery. Both produce a difference of potential, in the one case electric, and in the other magnetic, which is gradually expended over the resistance or reluctance of their circuits. The current or the flow of lines is in fact the rate of change of the potential over the resistance or the reluctance. The magnetic reluctance of an entire circuit or any portion of it is proportional directly to its length and inversely to its area. Further, different substances have a different specific resistivity to the passage of an electric current, and in the same way the specific reluctance per unit length and unit area of cross-section or the *reluctivity*, ξ, varies with different materials.

§ 10. **Permeability and permeance.**—The different resistivity of various substances may also be expressed in converse terms by saying that all substances do not conduct electricity equally well. In the same way we may say that all substances are not equally permeable to the passage of magnetic flux, and this specific property of *magnetic permeability* is related to reluctivity as its reciprocal, just as electric conductivity is related to resistivity. In many cases of electric circuits it is convenient to deal with conductance instead of with its reciprocal, resistance, and in magnetic circuits it is still more common to speak of the *permeance* rather than of its reciprocal, the reluctance. The fundamental equation may then also be expressed as

$$\text{magnetic flux} = \text{M.M.F.} \times \text{permeance},$$

the expression for the permeance being of the form $\mu \,.\, \frac{a}{l}$. Thus, $\mu = \frac{1}{\xi}$ represents the permeability of a substance through which magnetic flux of density B is passing, and when in the particular condition implied by the fact of such a flow of lines. On the C.G.S. system the permeability of vacuum is unity, and for all non-magnetic substances such as air, wood, copper or brass, it is also sensibly equal to unity. Thus in our first expression for the number of lines with an air-core or in vacuo, the permeability of the medium did not appear, since it was equal to 1, but the full form of the equation showed that the nature of the medium must be taken into account whenever a non-magnetic medium is under consideration, whether it forms the whole or a part of the circuit, $B = H$, and it is correct to say that the lines of induction and of force are identical in number and direction. But within iron the importance of μ may be judged from the fact that it may amount to as much as 2500, and the lines of force and of induction are not necessarily identical in either number or direction. When the magnetomotive force in an iron core is calculated in terms of H and l, H must be interpreted as the difference of potential over unit length, along a line of induction; or if a closed toroid uniformly wound is under consideration, it may be regarded as the intensity of the field, H_0, before the introduction of the iron core; it may only be taken as the intensity of the field acting on unit pole in the iron, if this is measured by the precise theoretical process described above, in which case the H obtained at any point is really the same as H_0 in the same toroid with an air-core. With this last word of caution unit pole and lines of force may be finally dismissed from consideration.

§ 11. **Comparison of various substances as to their magnetic properties.**—The difference between the high permeability of iron or steel under certain conditions and the permeability of air is of the same order as that between the conductivity of silver or copper and carbon. It is therefore evident that the great contrast between the conductivities of metals and bad conductors or insulators such as india-rubber cannot be matched by any equal difference between the permeabilities of different substances. There are in fact no magnetic insulators, bismuth itself, which is the least permeable, being only very slightly inferior to air. When air, gun-metal, brass, or wood are spoken of and often used as magnetic insulators, it must be remembered that they are so only to a comparatively small degree. At the best their permeability to magnetic flux is only some two or three thousand times less than that of iron, and at the worst this difference may sink to practically nothing. Yet it is noteworthy that in spite of this the distinction between the magnetic and non-magnetic classes is more sharply defined than between good electric conductors and insulators. Out of the three magnetic substances the magnetic quality of cobalt and nickel, though sensible, is not to be compared with that of iron or steel, so that practically the contrast lies between iron with its alloys on the one side and all remaining substances on the other side. Next it may be observed that the flux-density that can be obtained from an electromagnet with an iron core is much greater than that due to any permanent magnet, and hence the total flux from an electromagnet is much greater than can be obtained from any combination of permanent magnets, when both are of reasonable dimensions. Owing to this fact the magnetic field which must be present in any dynamo is now almost always produced by means of electromagnets, strongly excited only when the dynamo is in use; so long as the electrical engineer was dependent on permanent magnets for supplying him with a magnetic field, the powerful dynamos of the present day were impossible.

§ 12. **Analogy of magnetic reluctance and electric resistance not exact.**—On one point a warning must at once be given as to a vital distinction between the magnetic and electric circuits. The passage of an electric current through a conductor does not alter its specific resistivity; it may raise its temperature and alter its length, but when these secondary effects are eliminated or corrected for, the resistance of the conductor has been proved to be the same whether a current of a milliampère or of many thousands of ampères is passing. On the other hand, the permeability of a magnetic substance is dependent on the value of the induction, or, in other words, on the density of the flux which it is at the moment passing. It is not therefore a constant quantity, and our definition of $\mu = \frac{B}{H}$ was so worded as to imply that the value of μ could not be given unless at the same time B was specified. Further, the value of μ for the same induction depends upon how that value has been reached, and lastly, is affected by the temperature of the iron. On these and other points more will be said in Chapter XII.

§ 13. **Magnetic reluctances: (a) in series, and (b) in parallel.**—The example on which was based the fundamental equation of the magnetic circuit was a closed ring of iron, of the same nature and of the same cross-section throughout its length. But a magnetic circuit may also be made up of different materials having different permeabilities; these again may have different areas of cross-section and different lengths, while further they may be arranged either so that each line of induction in its closed path traverses them in succession (*i.e.* they are 'in series') or so that the path followed by some lines is different from that of others, either entirely or at some portion of its length (*i.e.* they are 'in parallel'). In such cases the analogy of the electric and magnetic circuits, although not perfect, yet is useful as affording a ready clue to a mode of treatment which is very closely correct in most practical instances. Just as the total resistance of a number of electric conductors in series is the sum of their separate resistances, so if a circuit or a portion of a circuit is divisible into sections having different lengths, areas, and permeabilities, and these are traversed by the same group of lines in succession, the total magnetic reluctance which must be overcome by the magnetomotive force is the sum of the separate reluctances of the different sections: *e.g.* if $l_1 a_1 \mu_1$, $l_2 a_2 \mu_2$, $l_3 a_3 \mu_3$ . . . . . . are the respective lengths, cross-sections, and permeabilities of the different portions, their total magnetic reluctance is

$$\begin{aligned} R &= R_1 + R_2 + R_3 + \dots \\ &= \frac{l_1}{a_1\mu_1} + \frac{l_2}{a_2\mu_2} + \frac{l_3}{a_3\mu_3} + \dots \end{aligned}$$

Similarly the total permeance is the reciprocal of the sum of the reluctances, or

$$P = \frac{1}{R_1 + R_2 + R_3 + \dots}$$

Next, if at any point in their path a group of lines separate and follow different parts or sections of the total area of their magnetic circuit, the law which they obey is analogous to the law of electric circuits in parallel, viz. that the number of lines flowing through any section of the magnetic circuit is the quotient of the difference of magnetic potential between opposite ends of the section, and the reluctance of the section; thence it also follows that when there are several paths for the lines between which a difference of magnetic potential exists, the total flux is divided between the several paths directly as their permeances, as determined by the respective lengths, cross-sections, and permeabilities of the different portions; and the joint permeance of the parallel paths is simply the sum of their several permeances, or

$$P = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots .$$

§ 14. **Example of a magnetic circuit with leakage.**—To illustrate these laws, let us group our ampère-turns entirely on one side of a closed iron ring composed of a curved electromagnet and an armature or keeper A (fig. 13, I); the arrangement may be likened to a battery, shown at its side, of which each cell represents a current loop, and the ends of which are joined by a thick piece of copper wire R_a: there is a rise of magnetic potential as we pass

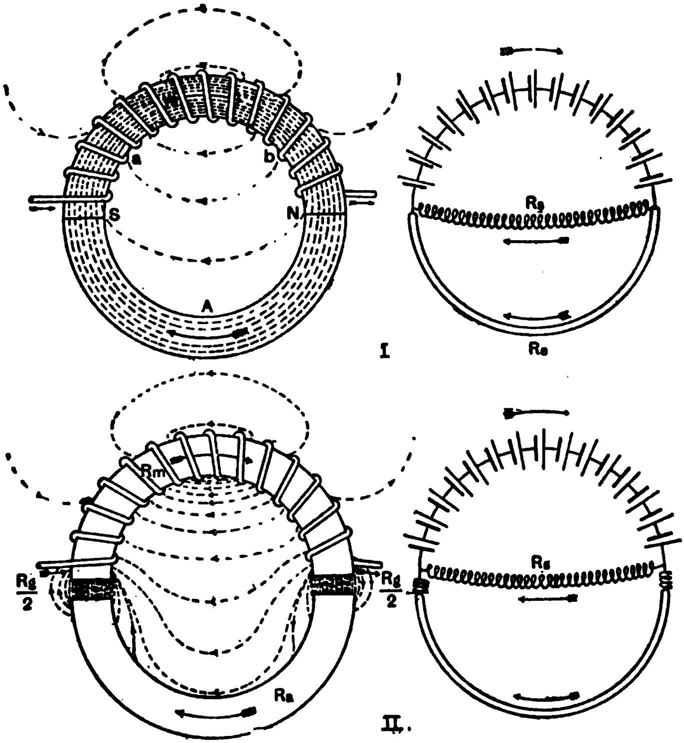


FIG. 13.—Analogy of magnetic and electric circuits.

through the loops from south to north, analogous to the rise of electric potential through the cells of the battery; and since the ends between which there is a difference of potential are joined so as to form a closed circuit, there is a flow of lines or of current round the circuit which within the magnetic or electric battery is from the – or S. end to the + or N. end, but without is from the + or N. to the – or S. end. But at this point there enters a very important difference: as there are no insulators for magnetism, it follows that, if between *any* two points there exists a difference of magnetic potential, there will be a flow of lines between those two points equal to the magnetic potential divided by the magnetic reluctance of the path, whatever be the substance of which it is composed. The lines of induction will therefore not be confined entirely to the iron, as they were in fig. 11, but will pass across from the N. end of the magnet to the S. end through the air as well as through the iron of the keeper. Thus there will be a flow from, *e.g.*, *b* to *a*, since *b* is at a higher magnetic potential than *a*, *i.e.* from one portion of the magnet to the other; such flow will increase as we pass away from the centre of the magnet towards its ends, and will again decrease as we near the centre of the keeper. If, therefore, we wish to keep up the analogy between the electromagnet and the battery, we might regard the latter as immersed in a conducting liquid or electrolyte, through which currents may flow across even from one cell of the battery to another in addition to the external current through the wire joining the terminals. Such currents would be a leakage as far as useful current in R_e is concerned, and in the same way the lines of induction in the air of fig. 13, I, and in similar cases, are frequently spoken of as *magnetic leakage*, stray lines, or waste field; but it must be remembered that such a point of view implies that only lines passing through the keeper or armature are useful and desirable. The whole illustrates the case of two or more magnetic paths joined together 'in parallel' to form a composite magnetic circuit. And this leads us to a second point which may again be emphasised: it is that, owing to the non-existence of magnetic insulators, it is impossible to define the exact limits of the magnetic circuit or the various paths followed by the lines except in a few simple cases, such as a closed toroid, and consequently the equation of the magnetic circuit, which demands a knowledge of the exact length, area, and permeability of the different portions of the magnetic circuit, only admits of an approximately accurate application. In the case of the toroid uniformly wound over its whole periphery there was no doubt about the dimensions of the magnetic circuit and its permeance, all the lines were confined entirely to the iron, and the number flowing through any section across it was identical; there was, in fact, no polarity and no 'leakage' of lines out of the loops and across the air. The reason is that all points of the ring-core are at the same magnetic potential, since the magnetomotive force supplied by the loops over any length is only just sufficient to pass the total number of lines through that length; it may be likened to a closed loop of wire in which an E.M.F. is generated, the amount generated per unit length of the wire being uniform throughout the whole circuit; a current flows, but all points of the loop are at the same potential, since the E.M.F. developed in any length is just sufficient to pass the current through the resistance of that length.

But now in our new case of an iron ring, in which the winding is grouped on one side only, the total magnetic reluctance of the circuit is a complex combination of the internal reluctance of the magnet, the reluctance of the armature, and the reluctance of the external air-paths. How, then, is the reluctance of the air to be calculated, since the paths of the lines through it are so manifold and differ so much in their length? They flow not only from the extreme ends, but across from all points of the magnet, so that the number of lines carried by the magnet, and therefore their density within the iron, is continually varying; at its centre they are a maximum, but thence they gradually leak out on all sides, especially towards the ends, so that a smaller number passes through the iron of the keeper than through the magnet. The calculation can most readily be performed by certain assumptions which are only approximately true. We must represent the whole by an electric battery between the terminals of which there are two paths, one of a low resistance and one of high resistance R_a (fig. 13, I), placed as a shunt to the other. The second represents the reluctance of the air, which is very high as compared with that of the iron.

For the sake of simplicity, therefore, the reluctance of the air is regarded as being in parallel with the reluctance of the keeper, as if all the lines passed right through the iron of the magnet from one end to the other, and then at its ends divided into two groups, some going through the keeper and some through the air, these latter being the leakage lines, which are thus supposed to issue forth from and enter into the magnet only at its ends. On this assumption, then, if R_a and r are the reluctances of the two paths, afforded respectively by the keeper and the air, the external reluctance, being their joint reluctance in parallel, is $\frac{R_a \cdot r}{R_a + r}$, and this is in series with the internal reluctance of the magnet, R_m. The total reluctance, therefore, of the entire magnetic circuit is

$$R_m + \frac{R_a r}{R_a + r}$$

and the total number of lines produced by a given number of ampère-turns encircling the circuit is

$$Z = \frac{0{\cdot}4\pi AT}{R_m + \frac{R_a r}{R_a + r}} = 0{\cdot}4\pi AT \times P$$

where P is the reciprocal of the total reluctance.

R_a is equal to $\frac{1}{\mu_a} \cdot \frac{l_a}{a_a}$, where l_a, a_a, and μ_a are the mean length, area, and permeability of the iron of the keeper, and R_m similarly $= \frac{1}{\mu_m} \cdot \frac{l_m}{a_m}$. The reluctance r of the air-paths, or their permeance p, can be calculated by assigning a certain mean length of path and a certain area of cross-section to them; or we may determine experimentally the number of lines flowing through the middle of the magnet Z_m, and the number flowing through the keeper, Z_a; the difference between the two, $Z_m - Z_a$, gives us ζ, or the number of lines which leak through the air.

On the same assumptions we can also calculate the total number of ampère-turns required in order to produce a given number of useful lines, Z_a, through the iron of the keeper; just as the product of a current and a resistance through which it flows gives us the number of volts which must be applied to the ends of the resistance in order to produce the current, so the product of a number of lines of induction and a reluctance through which they flow gives us the magnetic difference of potential which must exist between the ends of the reluctance in order that the given number of lines may flow through it, *i.e.* M.M.F. $= z\,\text{R}$. Hence the magnetic difference of potential which must exist between the ends of the armature in order that z_a lines may pass through it is $z_a \times \text{R}_a = 0{\cdot}4\pi\text{AT}_1$. But this is also the magnetic difference of potential under which the leakage lines are assumed to flow; consequently if we multiply $0{\cdot}4\pi\text{AT}_1$ by p, the permeance of the air, the product is the number of leakage lines, which we call ζ; the sum, $z_a + \zeta = z_m$, is the number of lines flowing through the magnet, and the magnetic difference of potential required to drive this larger number through the magnet is $z_m \times \text{R}_m = 0{\cdot}4\pi\text{AT}_2$. The total magnetomotive force required is therefore $0{\cdot}4\pi(\text{AT}_1 + \text{AT}_2) = 0{\cdot}4\pi\text{AT}$.

§ 15. **Example of an imperfect magnetic circuit with double air-gap.**—Next, let the keeper be held at a little distance away from the magnet, so as to interpose in the magnetic circuit two equal air gaps, one at either end of the keeper, with parallel faces and having a certain definite length (fig. 13, II).

At once the total number of lines of induction produced by a given number of ampère-turns wound on the magnet is enormously decreased, owing to the high reluctance of the two air-gaps. The effect may be repeated in the case of the battery if we add in series to each end of the external portion of the circuit a short length of wire having a very high resistivity but of the same diameter as our original external resistance: these will reduce the total current, and the current through R_s will bear a much larger proportion to the current through R_e than formerly, since the two resistances R_e and R_s have become more nearly comparable. In the same way the number of lines through the armature is greatly decreased by the interposition of the two air-gaps, but the magnetic reluctance of the rest of the surrounding air remains the same as before, and the number of lines therein will be the same if the magnetic difference of potential at the poles of the magnet remains the same; the proportion, therefore, which they bear relatively to the number in the armature or to the number in the magnet is very much increased. Before, almost all the lines passed round the magnetic circuit entirely through iron; now, a much larger proportion go partly through the surrounding air. The two air-gaps being in series with the iron of the keeper, the total reluctance of the three is the sum of their separate reluctances. Hence, if the reluctance of one air-gap be $\frac{R_g}{2}$, and they are both equal, the total reluctance, $R_g + R_a$ must be substituted for R_a in the equations for the number of ampère-turns or lines of flux. It might be thought that it would be easy to calculate R_g, since the exact length of each air-gap can be measured, as also the exact area of the parallel faces of the iron where the circuit is divided. But here again a caution is required; the lines to a great extent pass straight across from face to face of the gap in the iron, but at the edges they spread outwards, and so pass across by curved paths, which form a kind of 'fringe,' gradually shading off in density. Consequently the exact area and length of their paths is *not* known, and has to be determined by experiment, or an allowance for the fringe has to be made theoretically.

The imperfect magnetic circuit illustrated above, in which the iron is completely divided by two air-gaps, is typical of the magnetic circuit in all dynamos. For the production of the E.M.F. a magnetic field is required, and this must be in air, since a system of electrical conductors has to be moved through it; further, there should be as many lines as possible, within a small space, and produced with a reasonable expenditure of electrical energy in the coils through which the magnetising current flows. We must therefore have recourse to an electromagnet with an iron core in order to produce the field. From our equation it follows that, in order to obtain as many lines as possible round a whole magnetic circuit or through any portion of it with a given exciting power, we must make the length of the path under consideration as short as possible, its area large, and its permeability great. Iron at once presents itself as the most permeable substance, and would be used throughout were it not for the fact that we cannot have a closed magnetic circuit entirely of iron, since there must be one or more air-gaps in it. The effect of these must be minimised by making their length short and their area large; the rest of the magnetic circuit must be as short as possible, and composed of iron of high permeability. How these requirements are worked out in practice will form the subject of a later chapter.

CHAPTER IV

THE PRODUCTION OF AN E.M.F.

§ 1. **Movement of a conductor through a magnetic field, and the cutting of lines.**—It has been stated in Chapter I. that the dynamo is primarily a generator of an electric pressure or E.M.F. whence results a difference of potential at certain definite points which are the 'terminals' of the machine. The first fundamental equation upon which its theory is based must therefore deal with the origin and maintenance of its E.M.F., or, in other words, with the 'voltage' which forms one of the two elements of its output.

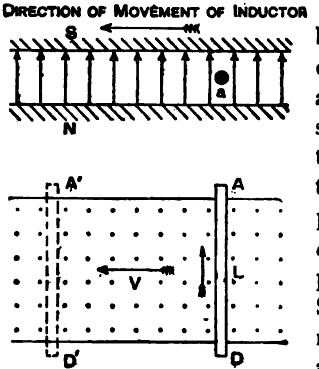


FIG. 14.

Let us suppose that we have before us a magnetic field mapped out by lines of induction, so as to at once exhibit the direction and strength of the field (fig. 14). In this field let there be placed an electrical conductor, such as a straight piece of wire, shown in section at *a*, and let relative movement take place between field and conductor. Such relative movement, it is evident, may be divided into two kinds, according as the conductor does or does not 'cut' across the lines of induction along its length. The field of lines in fig. 14 may be likened to a mass of impalpable and invisible threads passing straight across from one pole-piece to the other, somewhat like a brush thick-set with straight filaments. Now between and through these threads a needle-like conductor may be passed in any direction without in any way disturbing them so long as the direction of movement is wholly in the plane which contains at the same time the length of the conductor and the direction of the lines, *i.e.* in our diagram the vertical plane which contains the conductor. If the direction. of movement coincides with the length of the conductor, then the conductor merely passes end-on through the lines of the field; if the direction of movement coincides with the direction of the lines, the conductor merely slides up and down the lines; or while still moving in the same plane, its motion may be partly endwise through the field and partly sliding along the lines; in none of these cases is it caused to intersect the lines by passing athwart them or *cutting* them transversely with its length. On the other hand, if the direction of motion does not fall wholly in that plane which contains the length of the conductor and the direction of the field—*e.g.* in fig. 14 let it be in a horizontal plane—then it will be seen that the conductor 'cuts' the lines of induction along its length.

The above description and its pictorial representation might seem to ascribe to the lines a physical existence, each one being separate and distinct from every other, whereas we know that the magnetic field is in reality structureless. Yet, although structureless, it has direction, and by reason of this property any movement is divisible into one or other of the two kinds described above, according to the way in which its direction is related to the direction of the field and the length of the conductor. No question of the avoidance of the separate lines by the moving conductor is in reality involved, since between the pole-pieces the field is everywhere present.

§ 2. **Production of an E.M.F. due to the cutting of lines.**—Now, if the movement be of the first kind and an electrometer be applied to the ends of the moving conductor, no difference of state can be detected in it due to such motion; in fact, it is perfectly indifferent whether the magnetic field is surrounding the conductor or not. But let the movement be of the second kind; at once an entirely new phenomenon arises: the electrometer applied to its ends shows a difference of potential between them during the motion, and only so long as it lasts; in other words, an E.M.F. is set up or 'induced' in the conductor along its length in virtue of which one end of it is raised to a higher potential than the other. The conductor may now be appropriately termed an *inductor*, and there is a tendency for electricity to flow from the one end of it to the other; if these two ends are electrically joined so as to complete a closed circuit of conducting material, as in fig. 1, that tendency will be called into action, and a current will flow.

It matters not what the material may be of which the conductor is made; whether it be copper, or iron, or German silver, or a liquid electrolyte, in all alike an E.M.F. is set up when a field of magnetic induction and a conductor are so moved relatively to one another that the latter is caused to 'cut' the lines of the flux. Nor does it matter whether the conductor is moved across the field, or the lines of induction across the conductor by movement of the magnet between whose pole-pieces the field exists; or, finally, both field and conductor may be moved in opposite directions. In all cases where the conductor cuts the lines, either by its own movement or by reason of the moving magnetic field, it becomes the seat of an induced E.M.F.

§ 3. **The magnitude of the E.M.F. and its average and instantaneous values.**—Further, the magnitude of this E.M.F. along any length of the conductor is proportional to the rate at which lines are cut along that length. Just as the nature of the material of which the conductor is composed made no difference to the fact of the generation of an E.M.F., so likewise it does not in any way affect its magnitude. The larger the number of lines cut in a given time, or the shorter the time taken to cut a given number of lines, the greater is the E.M.F. If the rate of cutting is variable, then (as in the case of all phenomena which can be expressed in terms of a 'time-rate),' a distinction must be drawn between its *average and instantaneous values.*

The average rate of cutting to which the average value of the E.M.F. is proportional is the ratio of the number of lines cut in any time to the time taken to cut them; in symbols, if z is the total number of lines cut by the length of conductor under consideration in time t, the average E.M.F. induced between the ends of that length is

$$\text{E (average)} \propto \frac{z}{t}.$$

But the actual rate of cutting at any instant may vary to any degree on either side of the average value; at one moment it may be greater, at the next less, or it may vary continuously and regularly. Hence, to approximate to the actual rate of cutting when this is variable, we must take the number of lines cut in a short space of time and divide them by that small time. The smaller this period of time is, the more accurately the rate of cutting is obtained, and to be completely accurate, the actual rate of cutting at any instant is the infinitely small number of lines then being cut divided by the infinitely small time in which they are cut; or, in the notation of the differential calculus, the actual E.M.F. induced at any moment is

$$\text{E (instantaneous)} \propto \frac{dz}{dt}.$$

So far the induced E.M.F. has only been spoken of as proportional to the rate of cutting of lines, or $E = k\frac{Z}{t}$, where k is some constant whose value depends on the system of units according to which the field is mapped out in lines and time is reckoned. On the C.G.S. absolute system the magnitude of a unit of induction is so chosen that one line cut per second generates the absolute unit of electromotive force, and k is 1. As, however, unit E.M.F. on the absolute system is inconveniently small, the practical unit of the *volt* is one hundred million times greater, and consequently 100,000,000 C.G.S. lines cut per second produce one volt. If, therefore, Z be reckoned in C.G.S. lines, and t in seconds, and E is to be measured, as is usual, in volts, $k = 10^{-8}$ or

$$\text{E (average)} = \frac{Z}{t} \times 10^{-8} \text{ volts}, \quad . \quad . \quad . \quad (2)$$

The C.G.S. unit line is therefore as inconveniently small as the absolute unit of E.M.F., and a 'kiloline,' equivalent to 10^3 C.G.S. lines, has been sometimes adopted as the practical unit of induction, so that the fluxes and densities met with in dynamo practice may be expressed by more manageable quantities. The result of any such change is, of course, to alter the constant in the fundamental equation. In the following pages, however, C.G.S. lines will be adhered to, the constant 10^{-8} being sometimes omitted where such omission can easily be made good by the reader.

§ 4. **The electromotive-force equation in its simplest form.**—The elementary equation of the E.M.F. which has been given above can also be expressed in a different form. Taking the fundamental case of a straight inductor moved parallel to itself across the lines of a uniform field, first let the plane containing the length of the inductor and the direction of the movement be normal to the direction of the field, and let the direction of movement in that plane be at right angles to the length of the inductor. Such a case is that of fig. 14, where the field has a vertical direction, the inductor AD is horizontal, and is moved in the horizontal plane in a direction at right angles to itself. If L is the active length of the inductor which is within the field and is traversing the lines, and V is its velocity, the area swept through by the inductor in unit time is $LV = AA'D'D$. If B_g is the uniform density of the flux in the air-gap between the pole-pieces, the number of lines cut in unit time is B_gLV. Hence on the C.G.S. system the value of the E.M.F. induced between the points AD is

$$E = B_gLV \times 10^{-8} \text{ volts}, \quad . \quad . \quad . \quad (3)$$

when L, V and B_g are respectively in centimetres, centimetres per second, and C.G.S. lines per square centimetre.

§ 5. **The direction of the E.M.F.**—The 'sense' or direction in which the E.M.F. acts along the length AD is in fig. 14 from the observer, so that the end A is at a higher potential than the end D; and if the two ends were joined by a wire, a current would flow through the inductor in the direction shown by the arrow in the plan.

In this, the simplest case of electromagnetic induction, it will be seen that the directions of the field, of the length of inductor, and of the movement are all at right angles to each other, and form co-ordinate axes, as indicated in perspective in fig. 15; further, the direction of the induced E.M.F. is most easily remembered and discovered by means of the following rule. *Place the* **right** *hand outstretched in the line of the inductor, so that the thumb points along the direction of the field, i.e. towards the S. pole, and further so that when relative movement takes place the lines of* *induction pass across the hand from the palm to the back; then the induced E.M.F. is directed from the wrist to the tips of the fingers* (as shown in fig. 15). It will be found that, according to this rule, if it is the inductor which is moved while the field is stationary, the palm of the hand must face in the direction towards which the movement is made, as in the case illustrated; but the direction of the E.M.F. would be exactly the same if the field were moved across the inductor in the opposite direction, the inductor itself remaining stationary, in which case it is the back of the hand which must face in the direction of movement. Or, again, both field and inductor may be moved in opposite directions; but in all cases the lines of induction must by the movement traverse the right hand from the palm to the back, if we wish to discover the direction of the E.M.F. by the above rule.

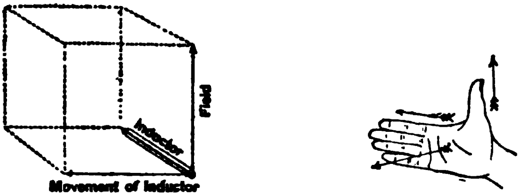
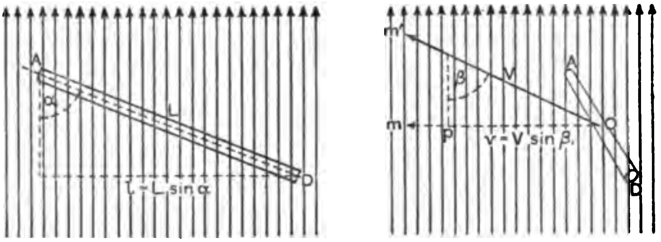


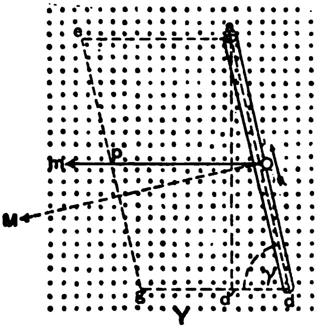
FIG. 15.

§ 6. The electromotive-force equation in its general form.—Let us next suppose that the three directions of field, inductor, and movement are not all inclined at right angles to each other. Let the length of the inductor be inclined to the direction of the field at some angle α (fig. 16, I), then the projection of the length L on the plane normal to the field is $l = \mathrm{L} \sin \alpha$, and this value must be taken instead of L as being the effective cutting length. Next let the direction of motion be inclined to the direction of the field at some angle β (fig. 16, II), and let the velocity be resolved into two components $\mathrm{V} \sin \beta$ in a plane normal to the field and $\mathrm{V} \cos \beta$ in a plane parallel to the lines of the field. In virtue of the latter component the inductor only slides along the lines, so that we are only concerned with the former component or $v = \mathrm{V} \sin \beta$, which is the projection of the actual velocity on the normal plane, and this must be substituted for V as the effective velocity. Let the projection of the direction of movement o m' on the normal plane be o m, and in this line let $op = v$. We have thus obtained on the normal plane two projections, one the effective length $l = ad$ and the other the effective velocity $v = \mathrm{v} \sin \beta = dg$ (fig. 16, III); the complete parallelogram $adge$ is then the projection on the normal plane of the actual area swept through by the inductor in unit time, or, as it may be called, the virtual area traversed. If γ be the angle at which l and v are inclined to each other, this area $= lv \sin \gamma$, and the number of lines cut per second, is the number contained therein, or $B_g l v \sin \gamma$. In this expression we may either regard $l \sin \gamma$ as a broadside projection of l on a line at right angles to the direction of movement, as in fig. 16, III, where ad' is the projection of ad on a line at right angles to om; or we may regard $v \sin \gamma$ as the component of v at right angles to l, in virtue of which it cuts the lines; while in virtue of the other component $v \cos \gamma$ the inductor simply moves endwise through the lines. Hence the number of lines cut per second is equal to the virtual area swept through by the moving inductor, when its length and its velocity are projected on to a



(I). End Elevation from M.

(II). Side Elevation from Y



(III). Plan on a plane normal to the Field.

FIG. 16.

plane perpendicular to the field, multiplied by the density of lines within that area, or $B_g l v \sin\gamma = B_g L \sin\alpha \, V \sin\beta \sin\gamma$, and the induced E.M.F. is

$$E = B_g LV \sin\alpha \sin\beta \sin\gamma \times 10^{-8} \text{ volts}, \quad . \quad . \quad (4)$$

By the rule of the right hand, when laid outstretched along the projected length *ad* of the inductor, so that the lines traverse the hand from the palm across to the back, the E.M.F. is directed from the wrist to the tips of the fingers. Such is the complete law of the E.M.F. induced in a rectilinear inductor when its movement through a magnetic field is one of simple translation.

§ 7. **The E.M.F. as due to alteration in the line-linkages.**—The above is a simple statement of one aspect of the great law of electromagnetic induction which was discovered by Faraday in 1831, and which has resulted within the short space of seventy years in the present marvellous development of electrical engineering. There are other aspects, but none so readily applied to the study of the dynamo. If the electric circuit be considered as a whole, the same law may also be expressed by saying that the induced E.M.F. is equal to *the time-rate of change of the lines of induction linked with the circuit.* If the circuit be composed of more than one loop, some or all of the lines of induction may thread through several loops (compare fig. 11 or fig. 31); under these circumstances in calculating the number of linkages of the lines with the circuit, any line which passes through n turns and so is linked n times with the circuit must be reckoned as giving n linkages. If N_0 be at any instant the number of linkages of lines of induction with the circuit reckoned as explained above, and N_1 be the altered number at the end of time t, the average time-rate of change is $\frac{N_1 - N_0}{t}$, and this is equal to the average E.M.F. induced in the circuit by this change; or if t be made small enough, the instantaneous E.M.F. is at any moment $E = -\frac{dN}{dt}$, the negative sign indicating that if the number of line-linkages is increasing, the E.M.F. is negative or in a counter-clockwise direction round the circuit as viewed from the side which the lines enter—*i.e.* when the observer is looking along the lines in their positive direction. But it is evident that $d\text{N}$ is here exactly the same as the number of lines cut by the circuit in the same small time dt, since if the lines linked with the circuit are altered in number, some portion of the circuit must have been cut by them as they passed into or out of its embrace, and this portion is our inductor or system of inductors. Thus $\frac{d\text{N}}{dt}$ is the same as the $\frac{dz}{dt}$ of our first equation when the total number of cutting inductors is taken into account. It may be argued that alteration of the number of lines linked with the circuit is still the truer view of the cause of the induced E.M.F.; and there is, in fact, no evidence that the portion of the circuit which cuts across the magnetic flux is in any way more active than the rest, for the presence of the entire closed circuit is necessary for the discovery of the existence of an E.M.F. even when an electrometer is used to indicate it. The line-linkage and line-cutting views are, however, but two ways of expressing the same phenomenon, and the one is as true as the other. While the line-linkage view is best suited to the transformer wherein there is no relative movement of the iron and copper portions and the lines simply grow and decay, the line-cutting view is at once more convenient and more lucid in the case of the dynamo. The latter is sharply distinguished from the transformer by the requirement that in it there must be actual movement in space of either the magnet or the inductor or both. Of the ultimate reason why an E.M.F. is induced by the relative movement of electric conductor and magnetic field, neither view gives any clue. The lines of the field were in the first instance mapped out by this very fact of induction; a current was induced in the exploring coil, when, as we now see, it cut or was cut by the flux, and other coils will show the same phenomenon when similarly circumstanced. The engineer in his study of the dynamo must therefore rest content with the simple fact that there is such an interaction of electricity and magnetism, even if the physicist has been able to penetrate further into the mystery of the mechanism involved.[1]

---

[1] See Prof. Fleming, *The Alternate Current Transformer*, vol. i. chap. v.

§ 8. **General process for discovering whether an E.M.F. is generated in any given case.**—The question of whether any lines of flux are or are not cut by a given length of conductive material may not always be easy to determine at first sight, but the process of projection described above affords an infallible rule by which it can be settled. Taking any two points A and D in a conductor and joining them by a line along which it is wished to discover whether there is any E.M.F. induced, let this length, which may be called the line of action, be projected on the plane normal to the direction of the lines of the field. Whatever be the nature of the movement, let its direction be also projected on the same plane. Then if these two projections are inclined to each other so as to enclose any angle other than 180°, an E.M.F. will be induced along the line of action, for a certain number of lines are cut by the given movement. If the two projections fall in the same straight line, there may be E.M.F.'s induced along other lines of action between other points, but at least there is none so far as the given line is concerned, since no lines are cut.

§ 9. **Condition for maximum E.M.F.**—When the angles α, β, and γ are all right angles, the action is evidently most direct, and reaches a maximum value for a given length of inductor, velocity, and density of lines; hence the simple case in which these directions formed co-ordinate axes, with which we started, is also the case which we should aim to reproduce in the dynamo wherein a voltage has to be set up. For example, in fig. 16 the best direction of movement would be along the line shown dotted at O M in a horizontal plane; if so moved the case becomes identical with that shown in fig. 14. It might appear that the cutting action would be greater if the direction of movement were kept unchanged along the line O *m*, and the inductor were rotated into a position at right angles to O *m*. But any such supposed advantage is purely imaginary, and suggested merely by the representation of the lines as dots on the plan. It must be remembered that the field is in reality structureless, as is shown by the fact that in the end elevation from M the lines appear at equal distances just as in the side elevation from Y, and so long as the direction of movement is at right angles to the length of the inductor, and both are in a plane normal to the direction of the field, it is a matter of in-

force equation, has now been determined, and may be conveniently brought to mind by the equation of the C.G.S. system, which gives the E.M.F. in volts, or

(I) $$E = \frac{Z}{t} \times 10^{-8}$$

with its special form

(Ia) $$E = B_g L V \times 10^{-8} \text{ volts.}$$

By means of these equations, and the methods by which they are reached, any case, however complicated, of E.M.F. set up in conductors by movement in a magnetic field may be solved; the inductors may be crooked or straight, moving with variable or uniform velocity in a field of uniform or variable strength; they may, too, be portions or not of a complete closed circuit and conveying currents or not; all that is necessary is to resolve each case into its elements to which our primary equations can be applied. For example, in the case of a complicated closed circuit it must be mentally split up into small elemental portions, and then for every small portion we have only to determine what is its rate of cutting across lines; for this is at once the E.M.F. set up in that portion due to its movement. The E.M.F. may be aiding a current or it may be opposing it; still there is that E.M.F. generated in that element of the circuit under consideration. Then by summing up all these E.M.F.'s throughout the whole of the circuit, we

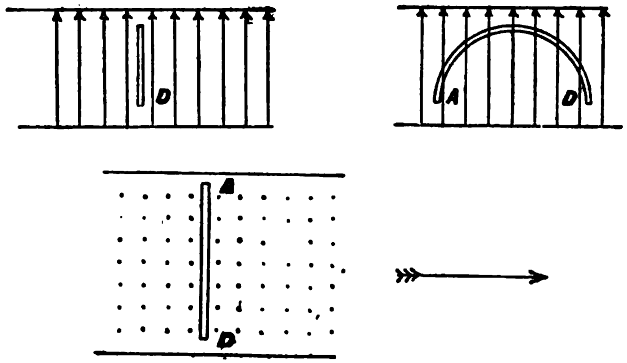


FIG. 19.

arrive at the resultant E.M.F. acting throughout the entire circuit. In so doing, however, two rules must be invariably followed: attention must be paid to the direction of these E.M.F.'s along the length of the inductor, and if we fix upon a certain direction round the circuit as positive, *i.e.* the direction in which the current round it is flowing, then all E.M.F.'s in that direction must be considered as + or positive, and all in the opposite direction as – or negative, and the E.M.F.'s must be added together algebraically, with due regard to their signs; secondly, we must consider whether any or all of the inductors into which we have divided the conducting circuit are in parallel, since then the potential difference existing at the ends of any one is also the potential difference between the ends of all alike; and although the internal E.M.F. in one or more may be higher than that induced in the others, yet so far as the supply of current to the rest of the circuit is concerned, it is only the potential difference at the ends which is effective.

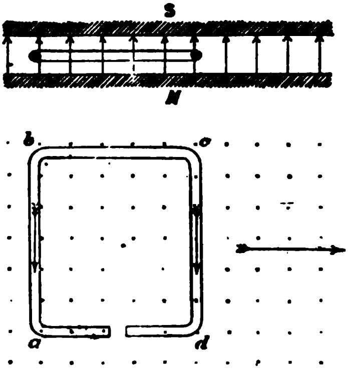


FIG. 20.

§ 13. E.M.F. of a loop translated (a) in a uniform field, and (b) in a varying field.—In order to illustrate the application of the above laws, let us consider the case of a four-sided loop moved horizontally in a direction at right angles to two of its opposite sides, across a uniform magnetic field, the direction of which is vertical (fig. 20). The two sides *a b*, *c d* will act as inductors, while in the two ends *b c*, *a d*, which are parallel to the direction of motion, no E.M.F. is generated, since they do not cut any lines. In *a b* and *c d*, the induced E.M.F.'s act in the same direction as viewed by an observer at either end, and are of the same values, since the lines are cut by each at the same rate. But the two E.M.F.'s oppose each other round the loop, and consequently if the circuit be completed from *a* to *d*, they will exactly neutralise each other, and no current will flow round the loop.

[illegible] proportional to [illegible] and [illegible] the current [illegible] in closing [illegible] we make [illegible] the direc- [illegible] the reverse [illegible] is [illegible] f [illegible] resistances, [illegible] manner at [illegible] other words, [illegible] is the sum [illegible] through the same field as in fig. 20, while we apply an external circuit to the ends *bc*, *ad* (fig. 22): the two E.M.F.'s which before balanced each other now unite to send a current through the external circuit, but being in parallel, the total E.M.F. acting is that of either of them. The arrangement, therefore, does not add to the voltage, but simply doubles the ampères that can be carried without damage resulting to the inductors owing to the heat generated by the passage of the current.

§ 14. **E.M.F. of a loop rotated in a uniform field.**—Lastly, let us rotate the loop about an axis passing horizontally through its centre (fig. 23). Now in this case the two sides, *ab* and *cd*, again act as inductors, while the ends are inactive; since the two inductors at any moment are cutting the lines of the same field in opposite directions, their E.M.F.'s assist each other round the loop, so that the total E.M.F. is the sum of the two; and if the loop is closed either immediately or by the interposition of an external circuit in *ad*, a current will flow, due to this total E.M.F.

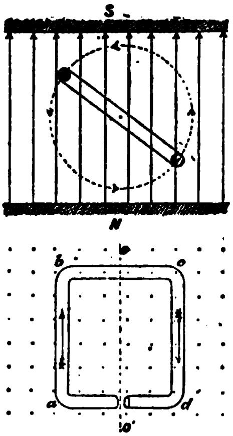


FIG. 23.

This last case leads us again to the important distinction, which has been before noted, between the average and instantaneous values of the E.M.F. If the number of lines which are being cut at any instant is the same throughout any period of time, then for so long the E.M.F. is constant, and the average and instantaneous values coincide. But if the rate of cutting varies (either from movement with varying velocity through a uniform field, or from movement through a field whose varying density is not counterbalanced by an inversely varying velocity), the whole conditions of the case can best be represented graphically by means of an E.M.F. curve in which intervals of time are represented by abscissæ or lengths measured along the horizontal axis, and corresponding values of the instantaneous E.M.F. by vertical ordinates. For example, in fig. 24 let the vertical lines represent a uniform magnetic field across which a straight inductor, shown in section at A, is moved parallel to itself with uniform velocity; then the *curve of E.M.F.* due to motion from A to B will be represented by a straight line, at some height above the horizontal axis, which is fixed by the scale on which volts of E.M.F. are plotted. The rate of cutting does not vary, and the E.M.F. is constant; while if at B the motion is instantly reversed, and the inductor travels back to A at the same speed, a constant E.M.F. of equal value is obtained, but in the opposite direction. We thus arrive at the pair of rectangles shown to the right of the figure. If, however, the inductor is rotated with uniform velocity in a circle about O, the rate of cutting will vary at every instant; as it describes the semicircle C E D, at the first instant it slides along the lines, and the E.M.F. is zero; gradually the E.M.F. rises until it reaches a maximum at E, where the motion is entirely at right angles to the lines. From this point it falls gradually to zero again at D. Now what is the actual rate of cutting at any moment? The actual velocity V at any moment is in a direc-

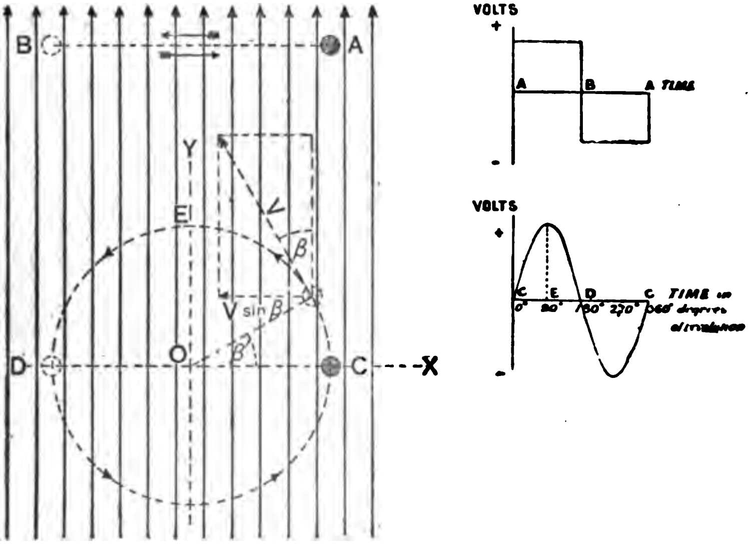


FIG. 24.

tion tangential to the circumference of the circular path moved through; in fig. 24, if V represent it in magnitude and direction, its projection on a horizontal plane is V sin β; but the angle β between the direction of the velocity and the direction of the field is equal to β' or the angle through which the inductor has been rotated, reckoned from its position when its motion was entirely parallel to the lines; hence the horizontal component of the velocity in virtue of which the inductor cuts the lines is equal to V sin β'. Viewed from a position vertically above or below, the inductor would appear to move with ever-quickening velocity from C up to the middle position at E, where it reaches a maximum, and thence with falling velocity as it passes on from E to D. If the values of this apparent velocity V sin β' are plotted, a simple sine curve is obtained whose maximum value is V. But the rate of cutting is directly proportional to this varying velocity, and therefore the curve connecting E.M.F. and time must itself be a simple sine curve like the curve connecting time and projected velocity. The exact shape of the curve is shown at the side of fig. 24; the instantaneous E.M.F. is a sine function of the angle through which the inductor has turned, reckoned from its position in a plane normal to the field; and its value is B L V sin $\beta' \times 10^{-8}$. As the rotation is continuously maintained with uniform velocity, the E.M.F. acts first in one direction, then in the opposite direction, or is alternating. When β' is 0°, 180°, or 360°, its value is zero; when β' is 90° or 270°, its value is a maximum, first in a positive and then in a negative direction. If the time taken to move from C to D be the same as that taken to move from A to B, the *average* value of the E.M.F. due to the rotation or the average height of the curve must be the same as the value due to the translational movement, or the height of the straight line above the horizontal axis.

§ 15. **Periodicity and phase of an alternating E.M.F.**— When a phenomenon which varies from instant to instant passes through a cycle of changes, such that it returns to exactly the same condition as existed at the commencement of the cycle, and then proceeds to pass through other similar cycles, it is said to vary in a periodic manner, and the time taken to pass through one complete cycle of values is termed a '*period*.' Thus the E.M.F. of the rotating inductor in fig. 24 is periodic, the time of one complete revolution being the period and giving a complete wave of E.M.F. The number of complete periods per second is called the *periodicity* or *frequency* of the alternating E.M.F. or of the inductor which yields it. In symbols, if T = the periodic time in seconds, $f = \frac{1}{T}$, and in our present case since $T = \frac{1}{n}$, where n is the number of revolutions of the inductor per second, the frequency is equal to n, or to $\frac{N}{60}$, if N be the number of revolutions per minute. Any periodic motion or E.M.F. or current which follows a sine law as above described, is known as a simple harmonic function, and may be conveniently represented by the projection on a vertical or horizontal axis of a radius revolving round a centre once in each period at a uniform speed. Thus in fig. 25 let the radius O M represent to some scale the maximum value of the function, and let it be regarded as making f revolutions per second in a counter-clockwise direction, about the point O, so that its angular velocity is $\omega = 2\pi f$ radians per second. Then the projection of O M on the vertical axis or O m will at any moment represent to the same scale the instantaneous value of the function; when the projection of the radius is above the horizontal axis, its value is to be regarded as positive, and when below, as negative. The projected length thus passes through a complete cycle of values once in each revolution corresponding to one complete period. Let time be reckoned from the instant when the radius occupies its horizontal position on O X and its projection is therefore zero; then at any moment t seconds from this starting-point O M will have turned through the angle $\beta = \omega t$, and its projection on the vertical will be $Om = OM \sin \omega t$. Thus if the radius O M represent the uniform velocity of movement of the inductor of fig. 24, the apparent velocity of the inductor as viewed along the direction of the lines of the field will be $v = V \sin \omega t$, and to this its rate of

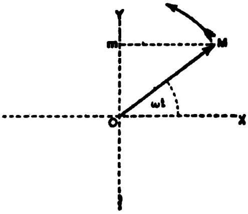


FIG. 25.—Clock diagram.

line-cutting is proportional; or if the radius represents to some other scale the maximum value E of the E.M.F., the instantaneous E.M.F. will be

$$e = E \sin \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The angular position of the revolving radius reckoned from its zero position on the horizontal axis is the *phase* of the function at that moment and is expressed in radians or degrees, 2π or 360° corresponding to one period. If two or more functions having the same frequency are combined on the same diagram, the angle between any pair of rotating radii reckoned in a counter-clockwise direction represents the *phase difference* of the corresponding functions. Such 'clock diagrams,' as they are called, are frequently used in connection with alternating-current problems.

CHAPTER V

THE MAGNETIC PULL

§ 1. **Simple inductor-dynamo.**—The principle on which the primary function of the dynamo depends has now been discussed, and the fundamental equation for its electromotive force, or the first of the two factors that go to make up its output, has been established. So far, then, the chief result contemplated in our diagrams of a simple inductor has been the production of an electric pressure at its ends or terminals; no current could flow along it so long as the ends were shown open and the circuit was incomplete, and since no electrical energy was developed, no mechanical energy was absorbed by the movement. The inductor of fig. 14 was therefore only potentially a dynamo. But now suppose that the inductor forms part of a closed conducting circuit, the remainder of which is external to the magnetic field, and is a simple electrical resistance of R_e ohms such as a piece of wire (fig. 1); then when the inductor is drawn across the field, an E.M.F. is set up between its two ends A and D; and these being points in a closed electric circuit, which contains no other source of E.M.F. besides the inductor, a current of electricity will flow through the whole circuit. The value of the current in ampères will be equal to the E.M.F. in volts divided by the resistance of the entire circuit expressed in ohms; or in symbols

$$C = \frac{E_a}{R_a + R_e},$$

where E_a and R_a are respectively the E.M.F. induced in the inductor and its resistance In short, the inductor has now become a dynamo.

But when a current is allowed to flow through the inductor, a distinction must be made between the internal E.M.F. developed in its length, or the internal portion of the circuit, and the difference of potential which exists at its terminals, A and D. The latter, which we will call E_e, is the terminal 'voltage' of the inductor-dynamo, which when multiplied by the current passing through it gives the output; it is less than the internal E.M.F. by the number of volts required to drive the current through the resistance of the inductor.

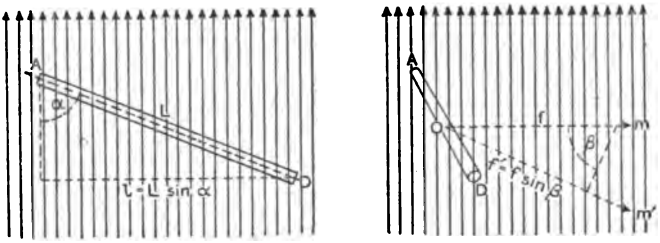
The analogy of the centrifugal pump will render this clear. We have said that the whole of the power developed by it is not available outside of it for raising water, and the reason is the same as in the case of the dynamo. The flow of water is the same inside and outside the pump, but some portion of the total pressure produced in the pump is lost in overcoming the frictional resistance of its own ports and channels to that flow, and from this loss of pressure it results that the available power for lifting water is less than the total power developed in the pump.

The *terminal voltage* of the simple inductor-dynamo is therefore

$$E_e = E_a - CR_a,$$

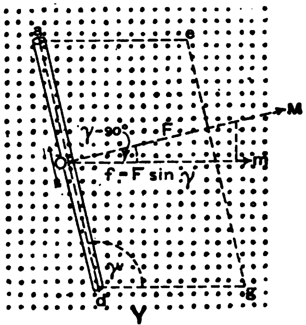
and from this it will be seen that on open circuit when no current flows $E_e = E_a$; but as the current is increased the difference between the two continually increases. The volts represented by CR_a are not available for use, being spent over the internal resistance; when multiplied by the current, their product, or C^2R_a watts, is a certain rate of development of electrical energy, but this energy simply appears as heat generated in the inductor which is not only useless but costly and inconvenient to dissipate. The smaller R_a is, the less will be this loss, and the more nearly E_e will approximate to E_a. For this reason the inductive portion of the modern dynamo is almost invariably composed of copper; since with the exception of silver (which is too expensive) copper is of all substances the best conductor of electricity, and therefore for a given length and sectional area of inductor its resistance is less than that of other metals.

§ 2. **Mechanical force required, when current flows.**—Directly, however, that we allow a current to flow in our inductor, an entirely new phenomenon comes to light. It now for the first time requires mechanical force to be applied in order to move it across the field. Magnetic field and electric current have interacted, and there is a definite resistance to the motion, which must be overcome by mechanical energy supplied from without.



(I) End Elevation from M. (II) Side Elevation from Y

The study of this second phenomenon of *electrodynamic action* will follow a course closely analogous in every particular to the previous study of the principle of electromagnetic induction. Let us place in a magnetic field a simple straight conductor A D (fig. 26) conveying a current whose direction of flow coincides with its length, and is from D to A; the rest of the circuit which must necessarily exist we will for the moment disregard. Let it occupy any position in the magnetic field, and finally let us take any line of action which we please, along which we wish to discover whether there is any mechanical force acting on the conductor and tending to move it in that direction. As, in the case of electromagnetic induction, the direction of the field and of movement were given, and any line of action forming a length within the conductor was considered with reference to the existence of an E.M.F. along it, so now with direction of field and of flow of current given, any line of action is considered with reference



(III) Plan on a plane normal to the Field.

FIG. 26.

to the question whether any force does or does not act along that line.

§ 3. **Direction and 'sense' of the magnetic pull.**—In the first place, whatever the relative positions of the lines which represent the directions of field and length of conductor, it is evident that they fall under one or other of two heads: either the length of the conductor is parallel to the direction of the field, or it is not. Now, in the first case, the conductor conveying the current is entirely unacted on by any force due to the external magnetic field in which it is placed, and it has no tendency to move in any direction. But in the second case it becomes the seat of a mechanical force which, unless resisted by an equal and opposite force, will cause it to move in a definite direction. This second case is equally well expressed by saying that, if there is to be a mechanical force acting on the conductor, the projection of its length on a plane normal to the direction of the field must be a line, and not a point. In fig. 26 let A D be a conductor whose length does not coincide with the direction of field in which it is placed, and let *a d* be the projection of its length on a plane normal to the field. At right angles to *a d* and also in the normal plane draw a line O M. Then the conductor A D is acted upon by a force equally distributed throughout its entire length (since it is entirely immersed in a uniform field), and the direction of this force at each portion of its length is in a plane normal to the field and parallel to O M; in other words, if it is free to move, the conductor will move parallel to itself through the field owing to the action of forces, which may be summed up and represented by one force at right angles to its length and in a plane normal to the field, as indicated by the line O M.

We have next to determine the 'sense' of this magnetic force or pull. In fig. 26, if the direction of the lines of the field be vertically upwards, and the current flows in the conductor from D to A, the direction of the pull on the conductor will be from left to right towards M, *i.e.* exactly opposite to the direction of movement which would produce an E.M.F. directed from D to A. Hence if, instead of the right hand, we now place the *left* hand outstretched along the length of the conductor, so that the direction of current is from the wrist to the finger-tips, while the thumb points down the direction of the lines of the field, the direction of the mechanical pull on the conductor will be across the hand from the back to the palm; in other words, the palm will face the direction towards which the conductor tends to move. The force is a mutual action between the conductor and the field, so that if the conductor is held fast, and the pole-pieces between which the field exists are free to move, the direction in which they will move is of course opposite, *i.e.* from the palm of the hand across to the back.

§ 4. **The mechanical force equation.**—Next, as regards the magnitude of the force, if l be the length of the conductor A D as projected on to the plane normal to a field of uniform density, then the force acting on A D at right angles to its length, and tending to move it parallel to itself along O M, is proportional to the product of the density of the flux, the projected length of the conductor, and the strength of the current, or

$$F \propto B_g l c.$$

But $l = L \sin \alpha$ (fig. 26, I)

$$\therefore F \propto B_g L c \,.\, \sin \alpha,$$

from which it is again evident that the conductor is only acted on by the magnetic pull when its length is inclined at some angle to the direction of the field; since if it is parallel to it, $\alpha = 0$, and F is zero.

The above forms the statement of the fact of electrodynamic action parallel to our previous statement respecting electromagnetic induction. As, in the latter case, there were aspects other than that in which it was presented, so also are there other ways of presenting the second fundamental fact, but, again, none are so suitable in connection with the theory of the dynamo. Both are laws, ultimate as regards the subject in hand, and not only verifiable by direct experiment, but also affording a sure groundwork on which to deal with any case that may arise.

The closely analogous nature of the two laws is already apparent; if the projection of the conductor or inductor on the normal plane be a point, it is entirely unacted on by any pull from the magnetic field; and further, no E.M.F. can be created in it by movement in any direction parallel to itself. When inclined to the direction of the field, the actual force acting on the conductor due to the magnetic pull a is always $F \propto B_g Lc \,.\, \sin a$ along the line O M; but the whole or part of this may be balanced by other external forces, and we have undertaken to discover the force acting on the conductor along *any* line of action which we please to consider. This we are now in a position to do, since all that is required is to resolve the force F along the given line of action, in order to find the component acting along it, which may or may not be unbalanced. In fig. 26 (II) let O m' be the line of action under consideration, its projection on the normal plane (fig. 26, III) being along O m; then the component of the force F acting along the line O m is

$$f = F \sin \gamma,$$

where γ is as before the angle which the projection of the line of action makes with the projected length of the conductor. Further, the component of f along the line O m' is

$$f' = f \sin \beta,$$

where β is the angle which the line of action makes with the direction of the field (fig. 26, II). Hence the force acting on the conductor along the given line of action O m' is

$$f' \propto B_g Lc \,.\, \sin a \,.\, \sin \beta \,.\, \sin \gamma$$

or

$$f' = kB_g Lc \,.\, \sin a \,.\, \sin \beta \,.\, \sin \gamma, \qquad . \quad . \quad . \ (6)$$

where k is some constant, depending on the system of units employed. On the C.G.S. system, the force being measured in dynes, c in absolute electromagnetic units and B_g in C.G.S. lines per square centimetre, $k = 1$, or if c be in ampères, $k = 10^{-1}$.

The expression we have now obtained is exactly analogous in form to the expression previously arrived at for the E.M.F. induced in a rectilinear conductor when moved in any direction parallel to itself, and it may be put into similar words. With any position of the conductor relatively to the line of field and the line of action, either the projections of the conductor's length and the line of action are in the same straight line, or they are inclined to each other at some angle other than 180°: in the first case there is no magnetic force acting on the conductor and tending to move it in the given line; in the second case there is a certain magnetic force or component of a force acting along that line. Whenever a conductor is subjected to a magnetic pull, the line of action along which the force is a maximum is at right angles to the projected length and in the normal plane, and hence, as we have said, it is in this direction that the conductor, if free to move, will travel parallel to itself. There is no tendency for it to rotate so as to alter the angle α. For a given length of conductor, however, this force is a maximum when the conductor is itself at right angles to the field or in the normal plane, as shown in fig. 27, and this maximum value of the force is

$$\text{(II)} \qquad F = B_g L c \times 10^{-1} \text{ dynes} \quad . \quad . \quad . \quad . \quad (7)$$

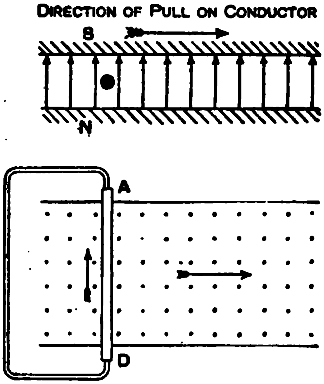


Fig. 27.

which forms the second fundamental equation of the dynamo. If F be measured in lbs. and L in inches, while B_g and c are retained in C.G.S. units and ampères respectively, the equation becomes

$$F = 57 \times 10^{-8} B_g L'' c \text{ pounds} \quad . \quad (8)$$

One feature still remains to be considered. When the conductor of fig. 26 is unconstrained and moves parallel to itself in the direction of the line O M, it passes broadside through the field, and for a given distance moved through, the number of lines cut is a maximum. Further, since the force F has no component in a vertical direction or along the length of the conductor, it is evident that there is no tendency for the conductor to move up and down the lines or end-on through them. It is essential, therefore, for the existence of force along any line of action that the conductor, by movement in that direction, should cut lines of the field along its length.

§ 5. **Lenz's law.**—Now in the simple but fundamental case of fig. 27 two corollaries attract attention. By comparing fig. 27 with fig. 14 it will be seen that the direction of the magnetic pull on the conductor, when a current flows through it from D to A, is exactly opposite to the direction of movement which would induce an E.M.F. in it from D to A. The same is true in all other cases (compare figs. 26 and 16), and is embodied in the rule that, in determining the direction of an E.M.F. along a given line of action, the *right* hand is used, while in determining the direction of magnetic pull on the conductor the *left* hand is used. Suppose that the conductor of fig. 27 conveying a current from D to A is free to move under the action of the magnetic pull; then at once it cuts the lines of the field, and the induced E.M.F., as shown by the application of the right hand, is from A to D; it is therefore a *back* E.M.F. opposing the flow of the current and reducing its strength. Whenever a conductor is left free to move under the magnetic pull on it, it always cuts lines along its length, and induces an E.M.F. so directed as to oppose the flow of current to which the movement is itself due. That it must so oppose the current is evident if we consider what would be the consequences were it to assist the flow of current; the strength of the latter would be increased, the pull would proportionately increase, and the conductor would move with ever-increasing velocity, which in turn would induce a continually increasing E.M.F. in the same direction as the current; this process would then go on for ever until both the E.M.F. and current were infinitely great. An indefinite amount of electrical energy would thus be obtained without the expenditure of any as required by the law of the conservation of energy.

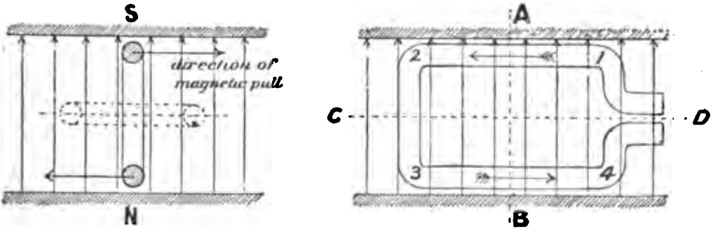
Next, let the current in A D (see fig. 27) be itself due to movement of it through the magnetic field; that is, an E.M.F. is being generated in it, acting from D to A. For this to be the case the direction of movement must be towards the left. But the direction of the force acting on it, due to its being a conductor carrying a current from D to A, is towards the right, and so is directly opposed to the movement which produces the E.M.F. Hence the movement of the conductor is resisted by a mechanical force in the opposite direction, and this fact may conversely be used to determine the direction of the E.M.F. along the length of the inductor. For, suppose a current to flow under the induced E.M.F., the direction of the two being the same; then this direction must be such that there shall be a force acting on the inductor opposing the movement which induces the E.M.F. In all cases, therefore, of electromagnetic induction the direction of the induced E.M.F. must be such that a current flowing under it tends by its electrodynamic action to stop the motion which produces the E.M.F. This general statement was formulated by Lenz in 1834, and is known as *Lenz's law*.

§ 6. **Conversion of mechanical into electrical energy.**—At once an insight is obtained into the reason why an expenditure of mechanical energy is required, if a current is allowed to flow in our inductor of fig. 1, under the influence of the E.M.F. induced in it. By the principle of the conservation of energy, no new form of energy can be made to appear, save by the transformation of some other form. In the case of the electric motor which is contained in the first of the above two corollaries, a current is passed through a system of conductors placed in a magnetic field; they are thus subjected to a mechanical pull, which will cause movement, and so give off work, electrical energy being transformed into mechanical, and the conductors becoming also inductors, or the seat of an induced back E.M.F. In the case of the dynamo which is embodied in the second of the two corollaries, a current flows in the direction of the induced E.M.F., so that the system of inductors becomes also a system of conductors; the movement which produces the E.M.F. is resisted by a mechanical force, and mechanical energy is transformed into electrical energy. The equality of the two is easily shown: *e.g.* in fig. 27 let c be the strength of current flowing through our inductor under the E.M.F. produced by movement of it towards the left with uniform velocity V, all units being on the C.G.S. absolute system. The rate of development of electrical energy throughout the circuit is equal to the product of the E.M.F. and the current flowing round it, *i.e.* $= Ec$. But $E = B_g LV$; therefore electrical energy is developed at the rate of $B_g LVc$ ergs per second. Now with the current flowing in the direction shown in the inductor, it is acted on by a force at right angles to its length, and tending to draw it towards the right. The magnitude of this force is $F = B_g Lc$. But this force is overcome through a length V in unit time, so that mechanical energy has to be expended at the rate of FV ergs per second. This is again equal to B_gLVc, and is therefore identical with the expression for the rate of development of electrical energy.

Or, again, take the case shown in fig. 15, and compare it with fig. 26; the E.M.F. produced by movement of the inductor in the direction Om' is equal to $B_gLV . \sin \alpha . \sin \beta . \sin \gamma$ in the direction from D to A. Let this produce a current of strength c round the circuit when closed; then the inductor is acted on by a magnetic pull, $F = B_gLc \sin \alpha$, in the direction OM (fig. 26). But the whole of this is not overcome by the force which produces the movement; it is only the component of F along the line Om' which acts as a resistance to the movement, and this is equal to $F \sin \beta . \sin \gamma = B_gLc . \sin \alpha . \sin \beta . \sin \gamma$. The product of the resistance overcome, and the velocity V, is therefore again equal to the rate of development of electrical energy, or Ec.

§ 7. **The interaction of two magnetic fields.**—But by what mechanism, it is naturally asked, is this resistance to a movement inducing an E.M.F., and which only arises when a current flows, brought about? It is by the interaction of the magnetic field due to the current, with the inducing magnetic field. It has been shown that a conductor conveying an electric current is itself the centre of a magnetic field, and it is through its magnetic properties that the current can react on the original field. This interaction can be treated in several different ways: these consist in attributing either to the magnetic lines themselves or to the closed conducting circuit certain properties, by means of which all the phenomena attendant upon the superposition of two or more magnetic fields can be correlated and brought into harmony. Of such methods two principal ones may be mentioned; but it must be remembered that the ultimate nature of the action is not more explained by them than by the primary law under which the subject was first presented.

Take the case of a rectangular loop of wire round which a current flows, and placed so that its plane is parallel to the direction of the field between two external pole-pieces (fig. 28). Then it follows from the electrodynamic law that if the direction of the current round the loop be as shown by the arrow, the side 1–2 will be subjected to a force tending to move it towards the right, as seen from D, while the side 3–4 will be subjected to an equal force tending to move it towards the left; the ends 2–3 and 1–4 will while parallel to the field have no force exerted on them, and when in a plane normal to the field, or in any intermediate position, the force tending to move 2–3 in one direction will be counterbalanced by the force tending to move 1–4 in the opposite direction. The whole loop will therefore be acted upon by a couple tending to rotate it in a clockwise direction, and the position which it will take up if free to move is one in which its plane is normal to the direction of the field, as shown by dotted lines. When the loop is vertical and is subjected to a magnetic pull, the lines that would be due to the current alone would pass through it from right to left, as seen from D. Such a supposed field must, however, be compounded with the original vertical field, and the result of this composition is that the lines of the actual field in nature are twisted out of their direct path. When the loop has rotated into its position of rest, the lines that would be due to the current alone coincide in direction with the lines of the original field, *i.e.* pass vertically upwards through the loop. Hence, as the loop rotates, the lines of the resultant field shorten their path, and further become a maximum. Thus not only may the lines of a compound field be said to tend to shorten themselves, but every closed circuit conveying a current tends to set itself so that the lines embraced by it are a maximum.



Section at A.B. seen from D.

FIG. 28.

Again, it has been shown (Chap. II. § 8) that a loop of wire conveying a current may be likened to a magnetic shell, the two faces of which are respectively N. and S. Consequently, in fig.

28 the loop tends so to set itself that it presents its N. face to the S. pole of the external magnet, and its S. face to the N. pole-piece, since unlike poles attract, like poles repel. If the loop be forcibly rotated in an anti-clockwise direction, an E.M.F. will be set up which would be the cause of an alternating current; but with such a direction of rotation the N. face of the loop would be continually presented to the N. pole of the magnet, and therefore there will be repulsion resisting the motion.

§ 8. **Magnetic pull on armature conductors.**—Let us now consider the case of an iron cylinder along the outside of which and parallel to its length are arranged a number of conductors conveying currents, the whole being placed in a magnetic field between two pole-pieces (fig. 29). The direction of the lines in the air-space between the iron poles and the iron cylinder will be nearly radial, except for a small fringe of lines near the interpolar gaps. Let the direction of the currents in the one half of the conductors (marked with a dot) under one pole-piece be towards an observer, looking at the cylinder from its end, and in the other half (crossed) away from the observer. Such is the case of the armature of the machine in fig. 2. By application of the left hand it will be found that each conductor is subjected to a pull exactly tangential to a circle passing through the centre of the conductors, save in those which are nearly or quite vertical. The directions of a few of

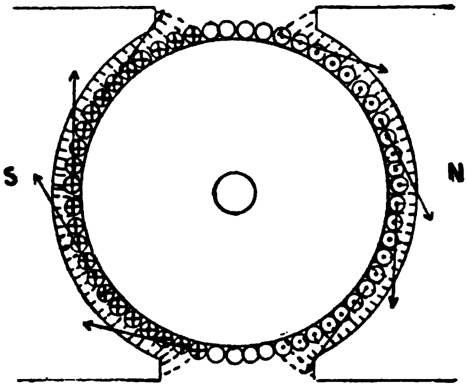


FIG. 29.—Magnetic pull on inductors of dynamo armature.

these forces are shown on the diagram. We may consider the whole as a number of loops all tending to set themselves vertical, so as to embrace all the lines of the field, while the ends of the loops will exert neither an upward nor a downward pull. The total result of the action will be that, unless otherwise constrained, the whole system of conductors will rotate in a clockwise direction. If the above represents an electric motor, the conductors will so rotate, and therefore induce in themselves an E.M.F. opposed to the flow of current.

But if the conductors are rotated in a counter-clockwise direction at a uniform speed, they will induce E.M.F.'s in the directions which the currents are supposed to have, and the whole becomes a dynamo; hence the mechanical resistance to rotation which arises when a continuous current flows under those E.M.F.'s, takes the form of a *tangential drag*, acting on each of the conductors round the circumference; and this must be overcome by the mechanical force applied to the shaft of the armature. The tangential drag is removed and put on again twice in each revolution as the conductor passes over from under one pole to the other; it is, however, always in the same direction relatively to the rotation.

Assuming the induction B_g in the air gaps to be practically uniform under the pole-faces, if L = length in cm. of each conductor within the influence of the field or the length of the iron cylinder, the force on each conductor within the polar field is $B_g Lc \times 10^{-1}$ dynes, where c = the steady current in ampères carried by the conductor. If τ = the total number of conductors each carrying the same current c, and ϕ = the angular width of the polar field in radians, the number of conductors within one polar field at any moment is $\tau \times \frac{\phi}{2\pi}$; the combined tangential pull due to the conductors on the one half of the armature under one pole-face is thus

$$F = \tau \frac{\phi}{2\pi} \cdot B_g Lc \times 10^{-1} \text{ dynes} \quad . \quad . \quad . \quad (9)$$

and this acts at radius r in cm. drawn to the centre of the conductors. The total torque from the two halves of the armature is therefore

$$T = \tau \cdot \frac{\phi}{\pi} \cdot B_g Lcr \times 10^{-1} \text{ dyne centimetres.}$$

or more generally, for p pairs of poles

$$\mathrm{T} = p \cdot \tau \cdot \frac{\phi}{\pi} \cdot \mathrm{B}_g \cdot \mathrm{L}cr \times 10^{-1} \text{ dyne-centimetres.}$$

Now the area of the field under one pole-face is $2\pi r\mathrm{L} \times \frac{\phi}{2\pi} = r\mathrm{L}\phi$, whence if $z_a =$ the total number of lines that pass into the armature from one pole or pass out of it into the other pole, $\mathrm{B}_g = \frac{z_a}{r\mathrm{L}\phi}$. Substituting this value, the torque is

$$\mathrm{T} = \frac{p \cdot \tau \cdot c \cdot z_a}{\pi} \times 10^{-1} \text{ dyne-centimetres,}$$

or

$$= \frac{p \cdot \tau \cdot c \cdot z_a}{4 \cdot 26 \times 10^8} \text{ pound-feet} \quad . \quad . \quad . \quad (10)$$

Multiplying by ω, the angular velocity in radians per sec., $\left(= \frac{2\pi\mathrm{N}}{60}\right)$ we obtain the rate of absorption of mechanical energy in the machine either in ergs per second in the first case, or in foot-pounds per second in the second case.[1]

In dynamos such as that shown in fig. 29 the density of field in

---

[1] The equation $\mathrm{B}_g = \frac{z_a}{r\mathrm{L}\phi}$ assumes that the lines are confined entirely within the area of the pole-face, and is not therefore strictly true, since the fringe of flux surrounding the pole-edges is neglected. The correction for this error slightly decreases the value of B_g for a given value of z_a, but correspondingly increases the number of inductors which are subjected to the drag; hence the above final equation for T is strictly true, as may be proved by equating the mechanical horsepower that is absorbed to the rate of development of electrical energy. The latter is $\mathrm{E}_a \mathrm{C}_a$ watts; $\mathrm{C}_a = qc$, and $\mathrm{E}_a = \frac{2}{q} \cdot z_a \cdot \frac{p\mathrm{N}}{60} \cdot \tau \times 10^{-8}$ volts (eq. 14); whence

$$\mathrm{E}_a \cdot \mathrm{C}_a = 2z_a \cdot \frac{p\mathrm{N}}{60} \cdot c\tau \times 10^{-8} \text{ watts,}$$

Converted into horsepower, this is

$$\frac{2p\mathrm{N}}{60} \cdot \frac{c \cdot \tau \cdot z_a}{746} \times 10^{-8},$$

which is equal to the mechanical horsepower absorbed, or

$$\frac{p \cdot c \cdot \tau \cdot z_a}{4 \cdot 26 \times 10^{-8}} \times \frac{2\pi\mathrm{N}}{60} \times \frac{1}{550}.$$

the air-space through which the inducing system of conductors is moved is usually at least 5000 C.G.S. lines per square centimetre. The force acting on each conductor to resist its motion per foot of active length is then ·0342 lb. for every ampère of current flowing through it, or nearly 3½ lbs. for 100 ampères. The shaft and framework of the dynamo must be strong enough mechanically to withstand the drag on the inductors, and they must be prevented from slipping on the armature. The second fundamental equation has now, therefore, introduced the mechanical element which is required in order to complete our first view of the dynamo as a machine, and it can be readily seen why the knowledge both of the electrician and of the mechanical engineer is required for the design of the dynamo; neither can be neglected, as indeed the very word 'dynamo-electric' foreshadows.

§ 9. **The limits of the output of a dynamo.**—This leads us to the next question, viz.: What determines the amount of current which may be taken out of any dynamo? We have seen that when once an electric pressure or electromotive force has been generated by the simple inductor (fig. 1) the current which will flow through it is determined by the resistance of the inductor itself (R_e) and that of the external circuit (R_a), or

$$C = \frac{E_a}{R_a + R_e},$$

and consequently for given values of E_a and R_a it is a maximum when the external resistance is negligibly small, as will be the case if the terminals of the inductor are joined together by a short piece of stout copper, or, as it is termed, if the dynamo be short circuited.

But for every ampère of current flowing through the inductor our second equation shows that a definite force is brought into play which resists the motion. Hence, theoretically, the maximum rate at which electrical energy can be developed by a dynamo is limited by the horsepower of the steam-engine, or other prime mover employed to overcome the resistance to the motion, when taxed to its utmost. But for practical purposes it is not simply a maximum rate of development of electrical energy which is required, but a maximum rate of development *in the external circuit*, since here alone it is useful; in other words, it is the output or the product $E_e C_e$ which has to be a maximum. When the machine is short-circuited, although the rate of internal development of energy may be a maximum, there is *no* output, for the number of volts used in driving the current through the internal resistance of the inductor, or CR_a, is equal to E_a, its internal E.M.F., and therefore E_e is zero. If the horsepower at our disposal suffices to maintain the speed and E_a constant, then the output is a maximum when the loss of volts over the internal resistance is equal to the loss of volts over the external resistance, and $E_e = \frac{1}{2} E_a$. This implies, however, a progressive lowering of the terminal voltage as the current is increased, but in practice it is necessary to maintain at the terminals of the machine a certain voltage suited to the purpose in view. If by increasing the field or the speed, E_a can be raised so as to keep E_e constant at the required value, then with sufficient mechanical power obtainable from the prime mover, the output of the dynamo may be indefinitely increased by lessening the external resistance, and so increasing the current. In either case, however, the output that can be obtained is determined by the amount of current that may be permitted to pass through the machine, and this is in practice limited by certain considerations, of which the most important is the heating of the inductor by the passage of the current. Unless the heat which is continuously generated at the rate of $C^2 R_a$ watts can be dissipated quickly enough, the temperature of the inductor will rise so high that we run the danger of fusing it or at least of burning its insulation. Given, therefore, sufficient horsepower applied to a dynamo constructed to give a certain terminal voltage, the greatest current which it will be able to *continuously* maintain is in general determined by the rate at which the heat produced in it can be dissipated, and so indirectly by the relation existing between the electric resistance of its armature and the area and effectiveness of the surface exposed to the cooling of the air.

The two equations of the dynamo have now been discussed, and expressed symbolically, *i.e.* on the C.G.S. practical system

(I) $E = B_g L \times 10^{-8}$

(II) $F = B_g L c \times 10^{-1}.$

As the law of electromagnetic induction yields the first fundamental equation of the *electromotive force*, so the law of electrodynamic action yields the second fundamental equation of the *mechanical force*. With these may now be placed the third or *magnetomotive-force* equation in its simplest form, as given in Chap. III. § 9, viz.:

$$\text{(III)} \qquad \frac{4\pi}{10} \cdot \mathrm{AT} = \mathrm{Z} \cdot \frac{l}{\mu a} \cdot$$

CHAPTER VI

SELF-INDUCTION

§ 1. **Second consequence of flow of current.**—One of the consequences which ensue when a current is permitted to flow through our inductor wire, viz., the magnetic pull on it, has now been considered, and it has been shown that it is connected with the interaction of the magnetic field which surrounds the current-carrying wire with the original inducing field. Another consequence of great importance still remains to be traced. When the inductor is carrying a current, we have spoken of its own magnetic field as being superposed upon that already existing, so that we now have to ask, What is the effect of the co-presence of these two fields as regards the E.M.F. induced in the inductor? In actual nature at any point in space there can never be more than one magnetic field as mapped out by lines of induction; but this magnetic field may for purposes of investigation be regarded as the resultant of two or more fields; and hence it is legitimate to consider these as acting separately on the inductor, and so arrive at the resultant E.M.F. set up in it. In the case here to be considered the actual field in which the inductor is immersed may be regarded as compounded of two sets of lines—one due to an external magnetomotive force, and one due to the current in the inductor itself; each produces its own proper effect, which can be considered entirely apart from the effect of the other.

§ 2. **The cutting of self-induced lines if current varies.**—When the current flowing through a conductor is a steady current in one direction and of constant value, the field due to it remains unvarying in intensity and position relatively to the conductor. Hence, if it be a steady current which we have supposed to be flowing in our conductor, then, even though it be moved in space, it carries its own field along with it, and therefore can never by its own movement cut *its own lines.* At any point lying in its path, and through which it either will pass or has already passed, the number of lines transversal to its length may be more than the number due to the external field within any given area, but it cannot either cut or have cut this increased number, since it carries along with it its own lines, which have so increased the original field. But a further possibility still remains, and this is that the lines due to the current may by their own movement cut the conductor; and this does, in fact, take place whenever the current begins, or ends, or varies in strength. When a current starts to flow in a conductor the magnetic field surrounding it has to be called into existence; and we may picture to ourselves this process by means of the conception of the circular lines, shown in fig. 7, being thrown off from the central axis of the conductor and gradually expanding outwards like the circular ripples caused by the dropping of a stone into still water. As the ring-lines first formed expand outwards and pass laterally across the substance of the conductor out into the medium surrounding it, new ones are continually being formed at the centre, themselves in turn to expand outwards. Whenever the current increases in strength, more rings are thrown off from the centre of the conductor, and the field external to it becomes more crowded; while if the current decreases in strength all the rings contract inwards and some pass into the conductor and disappear by absorption at the centre. The magnetic circles behave like elastic rings stretched outwards by some internal pressure acting all round them. Finally, when the current ceases to flow, the rings, being in a state of strain, and now being left unsupported, as it were, by the falling current, collapse inwards, cut the conductor transversely and disappear entirely. All such changes are not instantaneous, but take a certain time—short it may be, but still a definite period of time.

§ 3. **Law of self-induced E.M.F.**—Here, then, we have a case of relative motion of field and conductor by reason of which the lines cut the conductor at a certain rate; consequently the conductor conveying a current of varying strength must also act as an inductor, and an E.M.F. is set up in it proportional to the time-rate at which the lines, expanding or contracting, cut the conductor. Such cutting is not due to any movement of the conductor itself, but to the variation in the strength of the current flowing in it, which causes its own magnetic field to vary, and the E.M.F. so produced is called the *E.M.F. of self-induction*, as being induced by the conductor upon itself. Since the circular lines of force surrounding a solitary rectilinear conductor are in a plane perpendicular to the direction of the flow of the current at any point, the line of action of the E.M.F. which they produce must necessarily be the axis of the flow of current in the conductor. Its direction along this line of action has next to be considered. In fig. 7 let the current passing downwards through the plane of the paper be increasing in value, then circular lines having the direction shown are expanding outwards. Now, if we place the outstretched right hand along the conductor, so that the thumb points down the lines of induction, and so that their radial direction of expansion passes outwards across or through the hand from the palm to the back, *i.e.* with the palm facing inwards towards the centre, it will be found that the direction of the self-induced E.M.F., being by the rule of § 5, Chap. IV., from the wrist to the finger-tips, is opposed to the direction of the current, and resists the increase of its strength. If, however, the strength of the current is decreasing, and the circular lines are contracting inwards, the palm of the hand must face outwards, and the self-induced E.M.F. being in the direction of the current, tends to keep up its strength. Again, therefore, the self-induced E.M.F. resists the change in the current by which it is itself produced, and this is in fact the universal law which governs its nature.

Consider the case of a loop of wire (fig. 30) which is rapidly pushed up to the pole of a magnet from some position outside its field; by the cutting of the lines of the external field an E.M.F. is set up in the direction of the arrow on the loop: this increases in strength, since more and more lines are cut as it moves through the denser field near to the pole. Under the action of the E.M.F. in the closed loop a current begins to flow; but as soon as this current begins, lines of induction due to it spread outwards from the conductor, and form loops linked with the electrical circuit. A second field is thus superposed on the first, and the direction of the lines of this second field as they pass through the loop is exactly opposite to that of the first: this is roughly indicated in the diagram by the dotted lines lying counter to the full lines of the original field. It follows that the rising current tends to crowd the loop full of lines opposite in direction to those of the external field; or, in other words, it tends to reduce the resultant flux-density through the loop, just at the time when it is being increased by the motion. Consequently the rise of current in the loop is not so rapid as it would be if the current had itself no magnetic effect, or we may say that the current by reason of the magnetic qualities of its circuit reacts on the original field. At any moment the resultant field in which the loop is moving has a certain value and distribution, and the rate at which the lines of this field are cutting the conductor loop gives the E.M.F. to which the current strength at that moment is proportional; but this actual field may also and more conveniently be resolved into two—the original field, and the current's own field, the effect of the two being considered separately. At any instant the lines induced by the current on itself are increasing at a certain rate or being thrown off round the loop and cutting it, but the effect of this opposes the effect due to the cutting of the external field. If we place the right hand along the conductor on the inside of the loop, it is the direction of the two sets of lines which is different; if we place it on the outside, it is the direction of movement of the lines which is different. Hence the movement of the second field, due to the fact that the current is rising, induces a counter or back E.M.F. opposed to the rising current, just as was previously the case with the simple straight conductor; but now further in this case, where the change in the current strength is due to change in the E.M.F. induced by

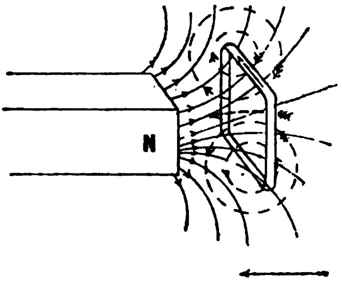


FIG. 30.

movement through the external field, this latter E.M.F. as it rises in value is continually opposed by a counter E.M.F. of its own creating, and the current cannot rise in strength as rapidly as the E.M.F. imposed by the external field rises.

If the movement of the loop is suddenly reversed, and it is made to recede from the magnet, the direction of the E.M.F. will be reversed, and so also eventually will the current. But not at once; the lines of its own field have to collapse upon the wire, to be succeeded by fresh lines passing in the reverse direction through the loop. The collapse of the former lines inwards and the expansion outwards of the new lines both cause a self-induced E.M.F. in the same direction as that in which the current was flowing; this for a time tends to counterbalance the new E.M.F. impressed on the loop by the external field, and hence the current only gradually sinks to zero and finally becomes reversed in direction.

§ 4. **Analogy of self-induction and inertia.**—From the universal law that the self-induced E.M.F. always opposes the change in the current strength of which it is itself the effect, the real nature of self-induction becomes more apparent. It is a certain property attaching to a circuit or to a conductor regarded as forming a possible part of a circuit in virtue of which no change in the current flowing through it can be instantaneously produced. If a steady electromotive force of E volts be applied to the ends of a conductor of resistance R, it takes time for the current to rise to its full value, as given by Ohm's law, viz. $C=\frac{E}{R}$. Even if the resistance R be negligibly small, a finite current cannot be instantaneously produced, since at the very moment when it starts to flow, it calls into existence a back or opposing E.M.F. dependent on its own rate of increase; its rise from zero can therefore only take place gradually, and before it reaches C, it passes through all intermediate values successively in point of time. Similarly if the E.M.F. be instantaneously removed, the current shrinks to zero, not instantaneously, but by a gradual decrease of strength. The reason must be sought in the fact that the current is indissolubly connected with the presence of a magnetic field due to it, and when the current is started, or stopped, or altered in value, this field has also to be created, destroyed, or altered. The analogy of the electrical property of 'self-induction' to the mechanical property of 'inertia' at once suggests itself; in virtue of the inertia which attaches to any mass of matter, a finite velocity cannot be given to it instantaneously, or destroyed instantaneously, nor can it be instantaneously altered to another value; a fact which is most familiar to us in the case of the starting from rest, or stopping, or altering the speed of, a heavy body such as a flywheel. When, however, the term electromagnetic inertia is applied to the property of self-induction, it must be clearly understood that the current itself has no quality analogous to inertia; it is only to the current as producing a magnetic field that the property attaches. For with the same current flowing round a circuit, the self-induction can be altered very greatly by any change which affects the magnetic field surrounding the circuit; and it is therefore a truer view to regard every circuit, or portion of a circuit, as possessing a definite property other than its electrical resistance, but which in conjunction with its electrical resistance determines the strength of current flowing through it at any instant under any E.M.F.; and this quality of the circuit is briefly called its *inductance*, since upon it depends the E.M.F. of self-induction. Obviously the inductance of a circuit, or portion of a circuit, will be dependent on the magnetic conditions which determine the number of lines of induction connected with it, when a given current is flowing, and hence is governed, not only by the geometrical form or shape of the conducting path, but more especially by the presence of iron within or near it.

§ 5. **Inductance.**—Two simple cases will, however, serve to indicate the nature of the property known as 'inductance,' and for this purpose let us first consider a single loop of wire surrounded by air, such as fig. 9, in which case the lines of induction due to a current round it traverse a medium of constant permeability. They all pass through the loop, and are linked with the electrical circuit once, so that when the E.M.F. causing the current is removed from the loop, and the current is diminished to zero, each line vanishes into the wire, cutting it once; and this process takes place with a certain time-rate of disappearance, depending on the rapidity of the change in the current. The inductance of such a loop is equal to the number of linkages of lines with the wire when unit current is flowing round the loop, so that if Z = the total flux through the loop when i = the strength of current in it, the 'inductance' of the loop is $L = \frac{Z}{i}$; and this quantity is so related to any change of current strength that if the latter be varied, and the rate of change be $\frac{di}{dt}$, the E.M.F. of self-induction is $e_s = -L\frac{di}{dt}$, the negative sign signifying that with an increasing positive current the direction of the self-induced E.M.F. is negative or opposed to its increase.

In the case of a helix or solenoid, such as fig. 31, where most of the lines are linked with or thread through a number of loops, each line that passes through n loops must be reckoned as giving n linkages (Chap. IV. § 7); so that the E.M.F. due to an alteration in the current strength will depend on the number of times that the lines are linked with the circuit, and not merely upon the total flux. Thus in the diagram, where batches of lines are indicated by single dotted curves, 100 lines are supposed to be linked with all four loops, and 50 with two loops; if the current ceases to flow, each of the hundred lines vanishing by contraction cuts through the electrical circuit four times, and each of the fifty lines cuts through two loops, so that the number of linkages is $(4 \times 100) + (50 \times 2) = 500$. The general formula, therefore, for the inductance of such a circuit is in absolute units $L = \frac{N_s}{i}$, where N_s is the total number of linkages and i is the current flowing in C.G.S. electromagnetic units; or it may be defined as the number of linkages of self-induced lines with the circuit when unit current is flowing round it. The self-induced E.M.F. $e_s = -\frac{d(Li)}{dt}$, or if L be constant, $= -L\frac{di}{dt}$ is given in absolute units, when L is in C.G.S. measure; but if L is in practical units of

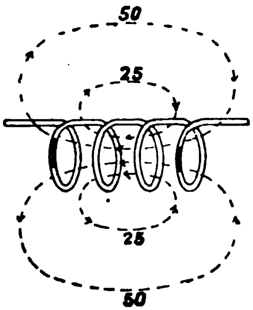


FIG. 31.

inductance (to which the name *henry* has been assigned, in honour of the American scientist Prof. Joseph Henry), e_s is given in volts when i is reckoned in ampères, the practical unit of the henry being 10^9 times the absolute unit. Thus a circuit has an inductance of one henry if one ampère flowing round it gives rise to 10^8 linkages of C.G.S. lines with the circuit; or if Z lines are all linked with t turns, carrying A ampères, L (henrys) $= \frac{tZ}{A} \times 10^{-8}$.

If a coil be composed of a number of turns closely wound together, so that by far the larger number of the lines pass through all the loops, and it be immersed in air or other medium of constant permeability, the inductance of the coil is approximately proportional to the square of the number of turns, since for the same current both the lines and the number of loops with which they are linked are proportional to the number of turns.

Next if we insert an iron core into the solenoid, or in any way modify the case by the presence of iron near the solenoid, the inductance is no longer a constant quantity, but varies with the strength of current under consideration, and the variations in its values can only be adequately represented by some graphical method. More than this, the term 'inductance,' when applied to circuits embracing iron or other magnetic media, admits of more than one definition, and these result in different values under the same circumstances. In dynamo-electric machines iron is almost invariably present, and in consequence the inductance of circuits forming part of them is often a subject of no little intricacy. Yet even in such cases it usually suffices as a first approximation to consider L as the ratio between the E.M.F. of self-induction and the time-rate of change in the current strength which produces it, and further as being a constant quantity.

The product Li or N_s may be called the *electromagnetic momentum* of the circuit, and the time-rate of change of this or $L\frac{di}{dt}$ is the electromotive force of self-induction; just as the time-rate of change of the momentum of a body is the measure of the mechanical force acting on it.[1]

---

[1] For fuller treatment of the subject the reader may be referred to *The Alternate Current Transformer* (Fleming), vol. i. chap. iii.; also to a paper,

§ 6. **Impressed, self-induced, and resultant E.M.F.'s distinguished.**—It is evident from a consideration of the case of an increasing current which is directly opposed by the self-induced E.M.F. that the latter cannot be the cause of the flow; there must be another E.M.F. in the same direction as the current, which is greater than the back E.M.F.; further, that the actual E.M.F. causing the flowof current is the difference between the two. We must therefore, in cases where the current is altering in value, distinguish between (1) the *impressed* E.M.F.; (2) the counter E.M.F. of *self-induction*; and (3) the *resultant* or *active* E.M.F. which immediately causes the flow of current, and the value of which at any moment is equal to the algebraic sum of the values of the other two. In any conductor forming part of a closed circuit and conveying a current there must be a certain E.M.F. 'impressed' upon it, and to which the current through it is ultimately due; this impressed E.M.F. may be a difference of potential applied to it from without, or existing within its own limits, as in a battery; or, again, it may be an E.M.F. generated in it by movement relatively to a magnetic field, as in the loop of fig. 30, in discussing which we have already distinguished between the E.M.F. impressed on it by its movement through the external field and the E.M.F. of self-induction. The general law by which the three E.M.F.'s are related is that if e_i = the impressed E.M.F. at any moment, and e_s = the self-induced E.M.F. at the same moment, the resultant E.M.F. to which the current is then proportional is

$$e = e_i + e_s,$$

the actual algebraic sign of e_s depending on the question whether the current is increasing or decreasing in strength. Or if e_2 is the E.M.F. consumed by the self-induction and so is the exact opposite of e_s, *i.e.* $= -e_s$,

$$e = e_i - e_2 \text{ and } e_i = e + e_2.$$

At any instant, and under all circumstances, the current flowing in any conductor is $c = \frac{e}{R}$, as given by Ohm's law[1]; but it is only

---

'Inductance and its proposed Unit, the Henry' (Kenelly), reprinted in the *Electrical Engineer*, January 2 and 9, 1891.

[1] R being interpreted, as will be explained later, as an *effective resistance*

when the current is constant in direction and value, and therefore $e_s = 0$, that $e = e_i$, and therefore $c = \frac{e_i}{R}$.

Take the case of a steady E.M.F., such as that of a battery, impressed on a simple electrical circuit of resistance R which contains no other source of impressed E.M.F. As soon as the circuit is made, a current begins to grow, evoking a back E.M.F. opposed to its growth; hence e_s is negative and e_i is the sum of two positive quantities e and e_2.

§ 7. **Storage of energy in the field.**—Now to what does this division of the impressed E.M.F. into two portions correspond? The answer is that two different kinds of work are being done. In the case of a motor to the armature of which electrical energy is being supplied at the rate of EC watts, the impressed E.M.F. is divisible into two portions, the one equal to the E.M.F. which is required to drive the current through the resistance R_a, *i.e.* equal to CR_a, and the other equal and opposite to the back E.M.F. developed by the motor, or $E = CR_a + e_2$: the work done is similarly divisible into two portions, one corresponding to C^2R_a, which appears as heat, and the other corresponding to e_2C, which appears as mechanical work done in turning the motor against the resistance of its load. In a similar way, when an impressed E.M.F. causes a current to grow in a circuit of simple ohmic resistance R, the rate of expenditure of energy may be divided into two portions; the one corresponding to the rate at which energy is being dissipated in the circuit in the form of heat, and the other to the rate at which energy is being stored up in the magnetic field. If e_i, e_s, and c be the values at any one and the same moment of the impressed E.M.F., self-induced E.M.F., and the actual current then flowing, the one portion is equal to c^2R, the other to $-ce_s = ce_2$, but the total rate of expenditure of energy is ce_i; consequently when the field is being created

$$ce_i = c^2R + ce_2;$$

or

$$e_i = e + e_2.$$

which takes into account any back E.M.F. by reason of which chemical or mechanical or electrical work is usefully done in addition to the mere dissipation of heat over the ohmic resistance of the circuit. See Steinmetz, *Alternating-Current Phenomena*, chaps. i. and x.

The creation of a magnetic field, therefore, demands energy, and the total amount absorbed in the process of bringing the current up from zero to its steady value C can be shown to be $\frac{1}{2}\text{LC}^2$. This amount represents the electromagnetic energy of the field, corresponding to the kinetic energy or $\frac{1}{2}mv^2$ of a moving body; it is measurable in ergs or joules according as the absolute or practical system of C.G.S. units is employed. But when once the field is established, *i.e.* when a steady current is set up, no further expenditure of energy is required to maintain it. That a certain amount of energy is in reality absorbed in the creation of the magnetic field round the conductor is evident from the fact that, although so absorbed, it is not irrecoverable, but is as it were stored up and can be liberated. For suppose e_t to be instantaneously withdrawn; then the self-induced E.M.F. tends to keep the current flowing, and does actually do so, since the current only falls to zero after a certain period of time. During this time work is being done in the circuit, the energy stored up in the magnetic field reappearing as heat, *e.g.* in the spark which occurs when the circuit is opened. The same is true if the impressed E.M.F. is gradually withdrawn, for the current falls less rapidly than the impressed E.M.F.; and in general, when the magnetic field is being destroyed, $e_t = e - e_s$ where e_s is positive. The analogy of a flywheel revolving between bearings is exact; when being started work has to be done upon it, which is greater than the work absorbed by friction by the amount which is required to impart a certain angular velocity to it. When once its speed has been got up, and is kept constant, all the work done is spent in overcoming its frictional resistance; the frictional resistance of the bearings corresponds to the electrical resistance of the circuit, the work done against either the one or the other being dissipated as heat in an irrecoverable form. But when the turning force is withdrawn, the kinetic energy possessed by the wheel keeps it rotating until it is itself absorbed by the friction, and during this period work is done by the wheel against its frictional resistance.

In the above case of a simple circuit to which an E.M.F. is applied, it is easy to see that energy is stored up when a magnetic field is created, and, since the amount of the flux depends on the magnetic qualities of the circuit, that the inductance is a property belonging to the circuit or to the conductor to the ends of which a difference of potential is applied. But in the case of a conductor in which an E.M.F. is generated by movement through a magnetic field such as the loop of fig. 30, the magnetic effect of the current shows itself by modifying or *reacting* on the external field, and such expressions as the inductance of the loop and the energy stored up in its field are only legitimate owing to a mental separation of the actually existing field into two component parts. It is not that the loop first reacts on the field, and then cuts the resultant field by its own movement; the resultant field moves relatively to the conductor, partly by reason of the latter's own movement, and partly by reason of change in its own current, which affects the strength and distribution of the actual field. To this movement of the resultant field relatively to the conductor the resultant E.M.F. is due; and if we were to consider this resultant field, we must not also credit the loop with self-induction or inductance:. it must then be considered as a circuit possessing only resistance. It is, however, simpler at the outset to treat the conductor as possessing inductance, and so to deal with the external field unmodified by reaction of the current on it.

§ 8. **Self-induced E.M.F. as due to the variation in the impressed E.M.F.**—Two different causes have now been considered, either of which may cause a variation in the strength of the resultant E.M.F. acting on a conductor; if e_i is steady and constant, the circuit may be made, altered in resistance, or broken; that is, the resistance of the circuit may be varied within the limiting values R and ∞; or while R remains constant, e_i may itself be varied in direction or value, or both. The first case deals with the growth and decline of current in a circuit to which a steady E.M.F. is applied. The second case is the one with which we are chiefly concerned in the study of dynamos, for upon it depends the theory of alternators. As before, e_s always opposes a rising current and assists a falling current; but now, as distinct from the former case, we have also to consider its relation to the varying e_i. Reverting to the rotating inductor of fig. 24, it was shown that the impressed E.M.F. varied not only in direction but also in value according to a certain function of the time, and these variations were graphically represented by a single-valued curve which during one complete period gradually rose from zero to a positive maximum, then as gradually fell to zero, and lastly repeated both rise and fall on the negative side of the zero line. Now if the circuit of an alternator formed out of one or more such rotating inductors is closed, as in fig. 45, and a current is allowed to flow, how will the current vary in strength? If the circuit have no inductance, but be composed entirely of a certain ohmic resistance, there will be no self-induced E.M.F., and consequently the current will alternate not only after the same law as the impressed E.M.F., but exactly simultaneously with it. In other words, there will be no difference of phase between the impressed E.M.F. and the current. The value of the latter at any moment will be given by dividing the impressed E.M.F. at that moment by the resistance of the circuit, so that the curve which represents to one scale the varying impressed E.M.F. will also represent to some scale the variations of the current. But such a case of a circuit with no inductance is only possible theoretically; physically it will always have some inductance, although it may be so small as to be negligible. We have therefore to consider the effect of self-induction on the current curve; and since this latter will always be identical in shape and law with the resultant E.M.F. curve, we require to compare together the curves of impressed E.M.F. and of resultant E.M.F., and until we have done so we cannot assume that they will be identical; they may not be coincident in phase, their maximum values or heights may not be alike, or they may even be dissimilar in shape. The necessity of this caution will be apparent from the following considerations.

§ 9. **Deduction of impressed E.M.F. curve.**—In fig. 32 let O P M O′ be any single-valued curve; in this curve take any point, P, and through P draw a line P T tangential to the curve and making an angle a with the horizontal axis O O′. Then it can be shown mathematically that the rate at which the ordinate Q P is increasing or decreasing with respect to O Q is represented by the trigonometrical tangent of the angle of inclination a; or, as it is termed, the 'slope' of the tangent at the point P represents the rate of change of Q P with respect to O Q. Let a second curve, $opmo'$, be plotted whose ordinates at any point represent the slope of the tangent at the corresponding points on the first curve, the horizontal axis being retained unchanged; the ordinates of the second or derived curve so obtained will represent the rate of change of the ordinates of the first curve at the same points along the horizontal axis.

Now we may reasonably assume that the current curve of an alternator will alternate, and rise and fall somewhat after the same fashion as the impressed E.M.F. curve. Let us suppose, therefore, that in fig. 33 the thick line is the curve representing graphically the varying current of an alternator for a complete period, time being represented by abscissæ measured from the origin O on the left-hand side of the diagram. This curve need not be symmetrical or expressible by a simple mathematical function, the only assumption being that it is single-valued and divisible into two half-periods, during which it is in opposite directions; and, further, that the areas of the two half-waves when plotted as in fig. 33 are equal. From this curve can be obtained a second derived curve (shown by a dotted line below), the values of which represent to some scale the slope of the tangents to the first curve; the two curves of fig. 33 will therefore represent respectively the strength of the current at any instant and the rate of variation of its strength with respect to time or $\frac{di}{dt}$. It should be noticed how greatly the two derived

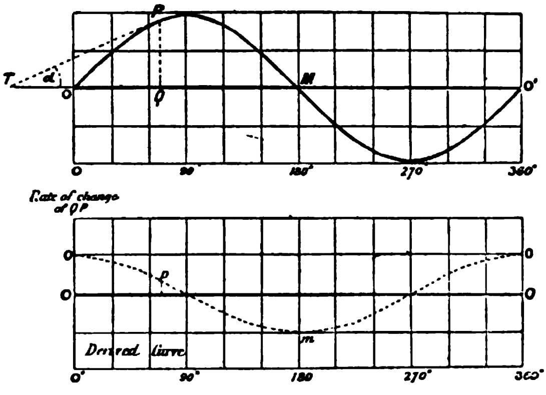


FIG 32.

curves of figs. 32 and 33 differ from each other in shape and character, although the two original curves both show a wave-like form; this difference is due to the fact that their rates of change are different at different points of time, an almost imperceptible difference in the shape of the two originals completely altering the shape of the derived curves. Further, in fig. 32 the original curve is a sine curve, and therefore symmetrical, whereas in fig. 33 the two sides of each half-wave are not identical, the fall being shown more gradual than the rise. Since the original curve of fig. 32 is a sine curve, it follows from its mathematical nature that its derived curve is also a simple periodic curve exactly analogous to its original, and, further, a sine curve whose phase precedes the phase of the original curve by 90°. As the point P in fig. 32 is moved away from the origin up the curve, the angle a gradually diminishes in value; at the top of the curve, $a = 0$ and $\tan a = 0$; therefore the

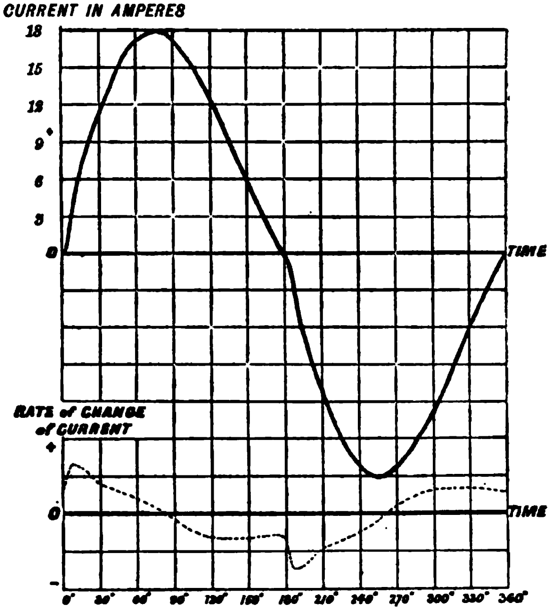


FIG. 33.

rate of change of the ordinate is zero; but as the point P descends to the zero line, tan α increases until the rate of change reaches a maximum at O or M. The crests and hollows therefore of the derived dotted curve do not occur at the same moment as those of the original curve, since the higher the ordinate Q P, the less is its rate of variation, but are shifted forwards (towards the left) by one quarter of a complete period, or 90°. The zero ordinate of one curve corresponds to the maximum ordinate of the other, and *vice versâ*. Something of the same nature is seen in the derived curve of fig. 33, but a comparison of the two curves will show that the maximum ordinate of the dotted or derived curve does not exactly correspond in point of time to the zero of the original, and that there may be several crests and hollows in the half-period of the derived curve, even though the original curve only shows one, since the rate of rise or fall of the latter need not vary after any simple law.

Now the varying current produces a magnetic flux looped or interlinked with its conductors, and this flux or its number of linkages also varies; if the magnetic circuit remains unaltered during a period, the number of self-induced lines depends upon the strength of the current, and also the magnetic circuit presented to the loops of current may itself be varying at any instant in length or nature. Consequently, analogous to the two curves of current and rate of change of current there must be two other curves; one representing for a complete period the number of linkages due to the lines induced by the current itself through the magnetic circuit, as it is at any instant presented to the current loops; and the other representing the rate of change of these self-induced linkages with respect to time, and derived from the former. In fig. 34 let the full line be the linkage curve of the self-induced lines plotted on the same horizontal axis of time as the current curve of fig. 33; these two are not by any means necessarily of the same shape, since the number of self-induced lines may not vary directly as the current under all circumstances, nor need the phase of the current curve and induction curve be the same; there may be a lag of induction behind the magnetising current in respect of time, and, as will be shown hereafter, where a portion or the whole of the magnetic circuit through which the self-induced lines pass is composed of iron, there will be such a lag, although it may be very slight. Hence in fig. 34 the full line curve is shown passing through zero at an instant slightly later than that at which the current curve passes through zero in fig. 33. From the curve of self-induced linkages the second dotted curve of fig. 34 is derived, representing to some scale the time-rate of change in the number of linkages or $\frac{dN_s}{dt}$, where N_s is the total number of self-induced linkages; and this, as

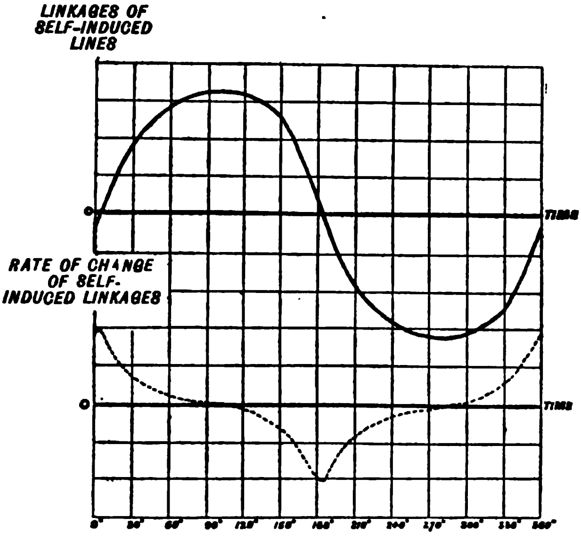


FIG. 34.

we know, may be unsymmetrical if the original curve is unsymmetrical. But the E.M.F. of self-induction is proportional to $-\frac{dN_s}{dt}$, so that, if we invert the derived curve of fig. 34, it will also represent to some scale the E.M.F. of self-induction at any instant, and at the same time indicate the direction of this E.M.F. When the number of self-induced lines is increasing in the positive direction, or decreasing in the negative direction, the E.M.F. is negative or in a direction opposing a positive current; and when they are decreasing in the positive direction, or increasing in the negative direction, the self-induced E.M.F. will be in the positive direction. Thus if the curve of linkages shows, as in fig. 34, a continuous increase of lines, and then a continuous decrease of lines, although the rate of this increase or decrease may be very different at different points, the curve of self-induced E.M.F. will pass through zero when the total number of self-induced linkages

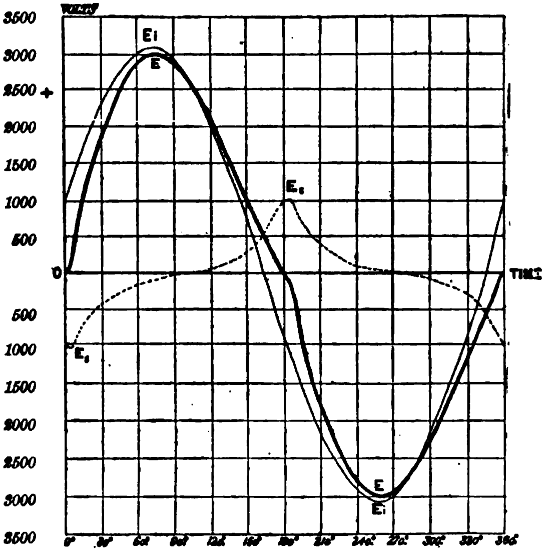


FIG. 35.

is a maximum. and for a whole period will be divisible into two portions, below and above the horizontal axis.

If the magnetic circuit has a constant inductance L, the self-induced E.M.F. can be derived immediately from the dotted curve of fig. 33, since $e_s = -L\frac{di}{dt}$; but in the general case where the inductance may be variable, e_s must be taken as $= -\frac{dN_s}{dt}$, and the intermediate curve of N_s is required.

In fig. 35 let the dotted curve E_s be the dotted curve of fig. 34

when inverted, being, therefore, to some scale the curve of the self-induced E.M.F. in volts, and upon the same horizontal axis of time let us reproduce by the thick line (E) the current curve of fig. 33. It will be seen that, roughly speaking, the phase of the self-induced E.M.F. lags 90° behind the phase of the current, but owing to the slight lag of the induction behind the magnetising current, the curve of self-induced E.M.F. does not reach its maximum until after the current curve has passed through its zero. But the thick-line current curve will also represent the curve of resultant E.M.F. (in the circuit as a whole), provided that the scales be so chosen that the same height represents indifferently either one ampère or one ampère multiplied by the resistance of the circuit. In fig. 35 this is supposed to be the case, so that the volts of resultant E.M.F. can be read off the curve E. We are now, therefore, in a position to deduce the curve of the impressed E.M.F., which must have acted on the circuit in order that with the assumed conditions the curve of resultant E.M.F. should have the shape shown in the diagram. Since we have obtained curves which determine for us the signs at any moment of e and e_s, the universal equation of § 6, $e_i = e - e_s$ shows us that we have only to subtract the ordinates of the self-induced E.M.F. curve from the ordinates at the same points of the resultant E.M.F. curve and plot their difference as a third curve, due regard being paid to the algebraic signs of the ordinates; the third-line curve so obtained (E_i) will be the required curve of impressed E.M.F. in volts. Or if we had been given the curves of E_i and E_s we could have deduced the curve of E.

§ 10. **Lag of resultant behind impressed E.M.F.**—Two striking facts now come to light: the first is that the result of combining the curve of self-induced E.M.F. with the impressed E.M.F. curve may make the shape of the resultant E.M.F. curve differ very materially from that of the impressed E.M.F.; for example, the curve of impressed E.M.F. shown in fig. 35, and deduced logically from the previous curves, is strictly a sine curve, yet the curve of resultant E.M.F. is not a sine curve, and is not even symmetrical on its ascending and descending sides. Secondly, while the self-induced E.M.F. always opposes a rising e_i, it does not always assist a falling e_i. From this, two important consequences follow: (1) Since the self-induced E.M.F. does not always assist a falling e_i, but does always assist a falling current or falling resultant E.M.F., the curve of resultant E.M.F. cannot coincide in phase with the impressed E.M.F. curve, but, on the contrary, the current curve lags behind the curve of impressed E.M.F. by a certain time or angle depending on the inductance of the circuit. (2) So long as the self-induced lines continue to rise, and their number of linkages to increase, however slowly, there is some self-induced E.M.F. opposing the impressed E.M.F.; but at the instant when they are at their maximum, and the self-induced E.M.F. is zero, the curve of resultant E.M.F. cuts the curve of impressed E.M.F. Where this point of intersection is, entirely depends upon the curve of self-induced linkages; and this depends, not only on the curve of magnetising current, but also on the magnetic circuit through which at any instant it is inducing lines. Hence, if the curve of self-induced linkages continues to rise after the current curve has begun to fall, owing to a more than proportionate decrease in the reluctance of the magnetic circuit, the curve of resultant E.M.F. will cut the impressed E.M.F. at a point after it has reached its own maximum, as in fact is shown in fig. 35. If, however, the maximum number of self-induced lines coincides in time with the maximum strength of the current, the highest value of the resultant E.M.F. will be its point of intersection with the curve of impressed E.M.F. Further, a fixed magnetic circuit always permits of an increase of the number of lines of induction through it, when the magnetising current is increased. If, therefore, the impressed E.M.F. is never constant, but *always* altering in value, the curve of self-induced lines never becomes a straight line (unless the magnetic circuit be altered so as to exactly counterbalance the changing current—a rare possibility); hence there is a definite self-induced E.M.F. at the moment when the impressed E.M.F. reaches its maximum, and the point of intersection must be subsequent to the point of highest impressed E.M.F.; in other words, E can never attain as high a value as E_i.

The whole may easily be illustrated by the case of a flywheel to which is applied a turning force, which not only alternates in the direction in which it tends to turn the wheel, but also varies in value from zero to a maximum strength, and thence, passing through zero, to a maximum in the opposite direction. Suppose that the wheel is rotating in the positive direction, and the turning force changes to the negative direction; the wheel cannot instantly change its direction, and it continues to rotate for some little time in its old direction; consequently its speed does not become zero until the turning force has acted for a certain time in the negative direction, and so for each change. The phase, therefore, of the wheel's velocity would lag behind the phase of the turning force. Further, since the turning force is never steady, but varies continuously, the flywheel would never reach the maximum velocity corresponding to a steady turning force equal in strength to the maximum value of the alternating force, since before reaching such a velocity the value of the turning force has already begun to decrease, and in just the same way the resultant E.M.F. lags behind the impressed E.M.F., and its maximum value is less than the maximum value of the impressed E.M.F.

§ 11. **Power in an alternating circuit as determined graphically.**—It has been stated previously in general terms that the rate of development of electrical energy throughout an entire circuit or in any portion of it is equal to the product of the volts of E.M.F. acting in it, and the ampères of current which flow through it under that pressure. But since in the case of an alternator or an alternating current, the voltage and strength of current are continuously varying after some periodic law, the question arises, What number of volts and what strength of current are we to multiply together in order to arrive at the power which is being developed? Obviously not the maximum values of E.M.F. and current, since at another portion of the period they both fall to zero: it must be some mean value of the two which will accurately represent their effective magnitudes throughout an entire period. In any portion of a circuit the rate of development of electrical energy at any instant is equal to the product of the current which is then flowing in that portion of the circuit and the E.M.F. which is impressed upon it at that instant. Thus, in fig. 36, if the thick line represents the curve of current in ampères flowing through any portion of a circuit, and the thin line represents the corresponding curve of E.M.F. in volts which is impressed upon that portion, the rate of development of energy in watts at any instant *a* is equal to the product of the ordinates *a b*, *a c*, which represent the ampères flowing at that instant and the impressed volts. By thus multiplying together a number of simultaneous values of the impressed E.M.F. and current, and plotting their products along the same horizontal axis, a third curve (shown with a shaded fringe in fig. 36) is obtained, representing the instantaneous rate of development of energy in watts throughout an entire period, and the area which it encloses, being the product of power and time, represents work done. In so doing we must pay attention to the algebraic signs of the E.M.F. and current, all ordinates above the horizontal line being reckoned as + and all below as − ; and, further, if their product be positive, it must be plotted above the horizontal line as positive work ; if negative, below it, as negative work. Since the product of two quantities, one + and the other −, is negative, the product of two ordinates is negative unless both are either above or below the horizontal line, *i.e.* positive work is done only when current and impressed E.M.F are in the same direction. Now to what does this distinction

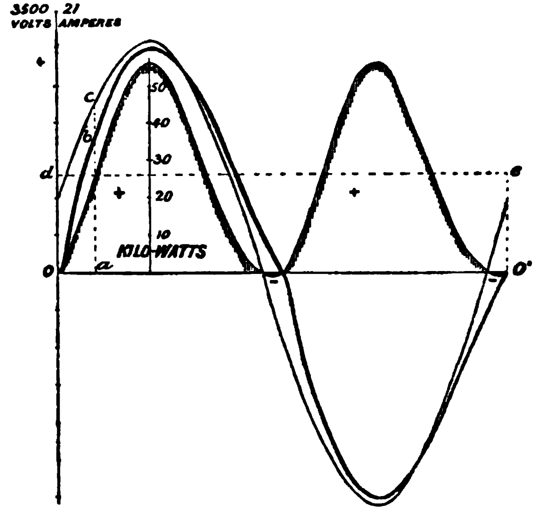


FIG. 36.

between positive and negative work correspond? Again the flywheel analogy will help us: whenever the rotation of the flywheel is in the same direction as that of the turning force, the turning agent is doing positive work on the wheel; but whenever the direction of rotation of the wheel is opposite to that in which the turning force is endeavouring to rotate it, the flywheel is doing work on the turning agent, or negative work is done. In the same way, whenever the current is in the same direction as the impressed E.M.F., positive work is done by the impressed E.M.F.; but whenever the current is flowing against the impressed E.M.F., negative work is done, the current being forced against the impressed E.M.F. The possibility of this is due to the fact that the positive work of an alternator is divisible into two separate portions; one part is expended in heating resistances or in forcing a current against a back E.M.F. due to causes external to itself, as in transformers, both being useful work so far as the alternator is concerned; the other part is work done on the magnetic field of the circuit. This division of the work corresponds to the division of the impressed E.M.F. as the sum of $\mathrm{R}i$ and $\mathrm{L}\frac{di}{dt}$ or $\frac{d\mathrm{N}_s}{dt}$, but here, as hinted in note to § 6, R is to be interpreted not merely as the ohmic resistance of the circuit, but as the *effective resistance*. In the case of the establishment of a continuous current, some part of the energy may be expended in chemical or mechanical work, and all such expenditure must be grouped with the energy dissipated in heat, when the distinction is drawn between the two classes of work, the one dissipated in heat or transmuted into a different form, and the other stored in the magnetic field, and immediately recoverable therefrom. So in an alternating circuit, some part of the energy may be expended not directly as heat, but in external mechanical work, or in work against the back E.M.F. due to the mutual inductance of transformers; all these may be expressed in terms of an effective resistance R, so that at any instant $i=\frac{e}{\mathrm{R}}$, where e = the active or resultant E.M.F. as in Ohm's law for an ohmic circuit. Thus $\mathrm{R}i$ represents the energy component of the E.M.F. and corresponds to the actual expenditure of energy, while e_s refers to the energy which surges to and fro between the conductive circuit and the medium which surrounds it. The work done on the magnetic field is stored up, and, at a later portion of the periodic time, is again given out when the magnetic field itself does work on the circuit, this being negative work so far as the impressed E.M.F. is concerned. Consequently the total useful work done by the alternating current and E.M.F. in one complete period, or the total energy transformed from electrical energy into heat or other useful work, is measured by the net area enclosed by the fringed curve in fig. 36, when the two negative areas (shown black and marked –) are added together, and their sum subtracted from the two positive areas. It will be seen that the wavy character of the power curve is due to the fact that current and E.M.F. are continually varying, but the appearance of the negative work (shown black) is solely due to the 'lag' of the current curve behind the impressed E.M.F. curve, by reason of which the impressed E.M.F. and current can be in opposite directions; and this lag is solely caused by the inductance of the portion of the circuit considered, which therefore indirectly determines the respective amounts of the positive and negative areas. The greater the lag, the smaller the net amount of work done. If the lag of the current curve were to amount to as much as a quarter of a period, the entire current curve being retarded in phase by an angle of 90° as compared with the E.M.F. curve, so that the current value is zero when the impressed E.M.F. is a maximum, and *vice versâ*, then the two negative areas would be exactly equal to the two positive areas, and the net work done in a period would be nil, the explanation being that the magnetic field gives back as much energy in the second and fourth quarters of a period as was previously developed in the first and third quarters by the source of the impressed E.M.F. On the other hand, if there is no inductance and no lag, there is no negative work done, since the phases of E.M.F. and current coincide. Both cases are ideal, but serve to indicate the theoretical limits to which practical cases approximate, and as a circuit realises one or other ideals more or less closely, it is classed as either an *inductive* or a *non-inductive* circuit.

Given, therefore, the two curves of impressed E.M.F. and current during one complete period, we can deduce a curve whose area represents the work done, and whose ordinates represent the power developed at any instant. The mean power or mean rate of development of energy will be the mean of all the values of the product of current and impressed E.M.F. taken over a sufficiently long time; since we have supposed that the E.M.F. and current vary periodically, so that the curves continually repeat themselves after each period, it will suffice to consider the mean of all the values during one period. The mean value of the power during one period will evidently be represented by the mean ordinate to the curve of power, *i.e.* an ordinate o *d* (fig. 36) of such a height that when multiplied by the length o o′ the area of the rectangle so formed, o *d e* o′, is equal to the net work done, or the difference between the areas of positive and negative work.

The whole of the above is applicable not only to any portion of a circuit, but also to the circuit as a whole; in the case of an alternator supplying energy to the external circuit, its output or rate of development of energy in the external circuit is equal to the mean ordinate of a curve formed by multiplying together simultaneous values of the E.M.F. impressed on the external circuit from its terminals and of the current flowing in that circuit. Thus in fig. 36 the output of the alternator, if the curves give simultaneous readings of the volts and ampères in the external circuit, as shown by the mean ordinate to the power curve, is 26 kilowatts. It must be remembered that the E.M.F. impressed on the external circuit need not be identical with the curve of resultant E.M.F. for the external circuit, and in general will only be so if the external circuit be a simple ohmic resistance without inductance.

§ 12. **Measurement of the power in an alternating circuit by wattmeter.**—The determination of either the output of an alternating dynamo, or the power in an alternating circuit by the graphic means described above, involves a knowledge of the exact curves of E.M.F. and current—neither of which are usually known—the multiplication of their simultaneous values, and the discovery of the mean ordinate to the curve so formed. The same process is, however, automatically carried out for us in a *wattmeter*. This instrument is a 'split' dynamometer, of which one coil, made of thick wire, is placed in series with the main circuit wherein the power is expended, while the other coil of fine wire is connected as a shunt across the ends of the circuit between which the power is to be measured. The thin-wire coil is arranged to have a negligibly small inductance so that the instantaneous value of the current which it carries is proportional to the instantaneous value e of the E.M.F. impressed on the circuit. The thick-wire coil carries the full current of the circuit, of which the instantaneous value is i. The force acting between the two coils, of which the thick-wire one is stationary, and the thin wire movable, is proportional to the product of the simultaneous values of e and i at any moment, and the movable coil is brought back to its zero position by torsion applied to a spring. During a complete period the movable coil is thus subjected to a continually varying force, but if the period of vibration of the movable coil is much in excess of the period of the alternating current, the force will have a steady mean value proportional to the mean ordinate of the power curve.[1] The true power in watts which is given to the circuit or developed by the alternator can thus be determined by comparison with the watts in direct-current form which gave the same reading.

§ 13. **Measurement of the effective value of an alternating E.M.F. or current.**—It is still, however, desirable to separate the two components of the power, viz., the volts and the ampères, and the question therefore arises: How are we to measure any alternating E.M.F. and current the values of which are continually varying? Given a current fluctuating in value and flowing through a fixed resistance of R ohms, the rate at which electrical energy is transformed into heat over the resistance is at any instant equal to the product of the ohmic resistance and the square of the current strength at that instant. If, therefore, being given the curve (I) of an alternating current (fig. 37) during one complete period, we square the ordinates and plot these squares as a second curve on the same horizontal axis, the total area enclosed by this second thick-line curve $O\,I^2\,O'$ is proportional to the total heat developed in the resistance in one period. But there is some one ordinate to this second curve whose height multiplied by the horizontal line o o′ gives a rectangle equal in area to the total area of the curve o I^2 o′. This mean ordinate or mean square is equal to the square of a certain current which would produce the same heating effect if it flowed for the same time through the resistance with unchanging strength. Hence the *square root of the mean square* is the value in ampères of that steady current whose rate of production of heat in a given resistance is equal to the mean rate at which heat is actually developed by the alternating current in passing through the same resistance; and

---

[1] For the theory of the wattmeter and the corrections necessary for its accurate use, see Fleming, *The Alternate Current Transformer*, vol. i. chap. iii. §§ 24-28.

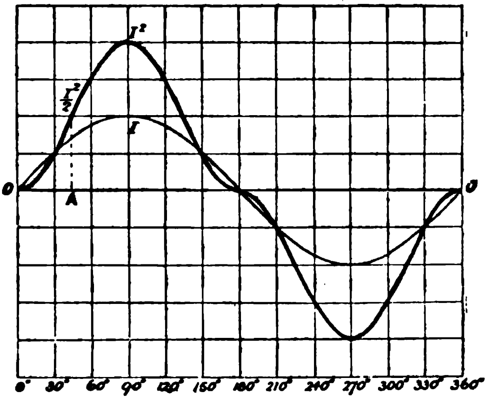


FIG. 37.

this square root of the mean square is called the *effective current*, since, as regards heating, the alternating current is equal in effect to a steady current of that number of ampères.

In a hot-wire voltmeter the heating effect of a current passed through it causes the needle to take up a definite position, which indirectly measures the rate of development of heat in the instrument. When, therefore, an alternating current is passed through the instrument, the position taken up by the needle measures the square root of the mean square of the current strengths, or the effective current. But if the wire through which the current flows has inductance so small as to be negligible—and this is arranged to be the case—the current strength is at any instant proportional to the E.M.F. impressed on the wire from the terminals of the instrument, so that the curve of impressed E.M.F. is identical in its law and phase with the current curve. Hence the position taken up by the needle of the voltmeter not only determines the square root of the mean square of the current, but also the square root of the mean square of the E.M.F. impressed upon the instrument. This square root of the mean square of the E.M.F. is called the *effective E.M.F.*, since it is that value of a steady, unidirected E.M.F. which would when applied to a given resistance produce the same amount of heating as the alternating E.M.F. produces.

The effective E.M.F. can also be measured by an electrostatic voltmeter, or by an electrodynamometer used as a voltmeter; in each case the deflection can be calibrated against that due to a continuous steady E.M.F., and the reading of the instrument furnishes us directly with the square root of the mean square of the alternating E.M.F. For the measurement of the effective ampères an electrodynamometer can be employed in series with the circuit. Finally, for the measurement of either the volts or the ampères an electromagnetic voltmeter or ammeter can be used, provided the instrument is suitable to the frequency of the alternating current.

§ 14. **Determination of effective values from a curve by means of a polar diagram.**—If the curve of E.M.F. or current is of irregular shape, the process of finding the effective value by squaring the ordinates and integrating the area in order to find the mean square is tedious, and a simpler method has been devised by Prof. Fleming.[1] It consists first in plotting the instantaneous values as a polar diagram instead of in wave-form by rectangular co-ordinates. In a clock diagram, instead of rotating a radius of fixed length equal to the maximum value of the function, let the length of the rotating radius increase or diminish as the instantaneous value of the function. It then becomes a radius vector, and the curve traced by the extremity of the radius will be a polar diagram of the alternating function, one revolution corresponding to one period, and half a revolution, or the passage through 180°, corresponding to a half-wave when plotted by rectangular co-ordinates. Thus from a centre O draw a number of radii preferably spaced at equal distances apart, say, every 10°, and upon these radii mark off lengths representing to some scale the instantaneous value of the function taken at the same angular intervals of 10°, reckoning time as an angle say in a clockwise direction. The half-wave is now represented by a closed curve, since at 0° and 180° the radii will coincide, and in fig. 38 the polar curve has been drawn for the half-wave of the current in fig. 33. Let the area of this closed curve be measured by a planimeter, and from a table of areas of circles find the circle whose area is double that of the polar curve. Then to the same scale as the radius vector measures

---

[1] Fleming, *Alternate Current Transformer*, vol. i. chap. iii. § 32.

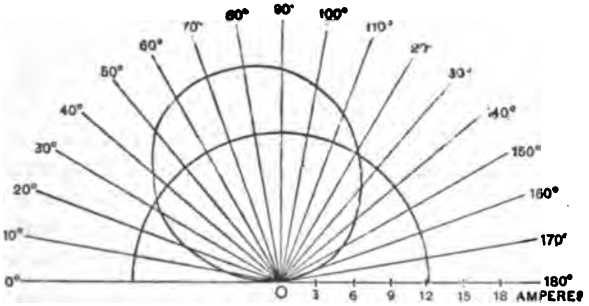


FIG. 38.—Polar diagram and equivalent semi-circle of alternating function.

the instantaneous value, the radius of the equivalent circle measures the square root of the mean square of the periodic function. In fig. 38 is shown the semicircle enclosing with the base line an area equal to the area of the polar curve of the instantaneous current. The radius of the semicircle is seen to represent an effective current of 12·1 ampères, and in a similar way the effective volts of fig. 35 or 36 are found to be 2255.

§ 15. **The power factor of an alternating circuit.**—We are still, however, not in a position to determine the mean value of the power. In the theoretical case of an alternating circuit having no inductance or capacity but simply ohmic resistance, the product of the effective volts and ampères will measure directly the mean power therein developed. But in the more general case of a circuit having inductance and capacity, the phases of the impressed E.M.F. and the current may be different. Hence just as it is not admissible to multiply together the maxima values of the E.M.F. and current to discover the maximum power, since these may not coincide in point of time, so also it is not admissible to multiply together the effective volts and ampères to discover the mean power developed. The product of the $\sqrt{\text{mean square}}$ or R.M.S. values of the E.M.F. and current or EI is known as the *apparent power* of the circuit, and will differ from the true power if there be any phase difference of the impressed E.M.F. and current. The ratio of the true power or watts to the apparent power or watts is called the *power factor* of the circuit. In the case of a circuit which is either entirely non-inductive or in which the E.M.F. and the current are in phase by reason of the capacity effect exactly balancing the inductance effect, the power factor is unity, but in all other cases it is some fraction less than 1. Its value may be obtained from the ratio of the wattmeter reading described in § 12 to the product of the ammeter and voltmeter readings for the same circuit. Thus the apparent power of the alternator of figs. 36 and 38 is 2255 volts × 12·1 ampères = 27·3 kilovolt-ampères, while the true power is 26 kilowatts, whence the power factor is $\frac{26}{27 \cdot 3} = 0 \cdot 95$. A readier means for calculating the power of an inductive circuit traversed by an alternating current is still desirable, and for this purpose it is necessary to assume some simple law to govern the periodic variations of the E.M.F. and current.

§ 16. **Simplification by assumption of a sine-law hypothesis.**—In the previous explanation of the behaviour of alternating currents, the effect of self-induction has been expressed in more or less general terms, independent of the exact type of the curves drawn and assumed, but for the purpose of the simpler process now required, the curves are assumed to obey a *sine law*. It has been said that if the full-line curve of fig. 32 is sinusoidal, it follows, from the mathematical nature of a sine function, that the curve derived from it and expressing its rate of change will also be a sine curve of the same periodic time, but differing by 90° in its phase. If, therefore, the current curve in any piece of alternating machinery follows a sine law, and if its inductance be strictly constant throughout the whole periodic time, and the lines of induction due to the current are strictly proportional to the current, and follow simultaneously upon its variations, the curve of self-induced linkages (fig. 34) will be a sine curve; consequently the dotted curve derived from it, which, when inverted, represents the self-induced E.M.F., will also be a sine curve of the same periodic time, but differing by 90° in its phase. Further, if two sine curves be compounded together, as was done in § 9, the third curve so deduced is a sine curve of the same periodic time, but differing in phase from either of the two component curves, and therefore the curve of impressed E.M.F. (in fig. 35) will be such a curve. Fig. 39 shows the combination of two sine curves to form a third, and it is, therefore, the counterpart of fig. 35 on the supposition that the curves of current and E.M.F. are sine curves. From a comparison of the two figures it will be seen that

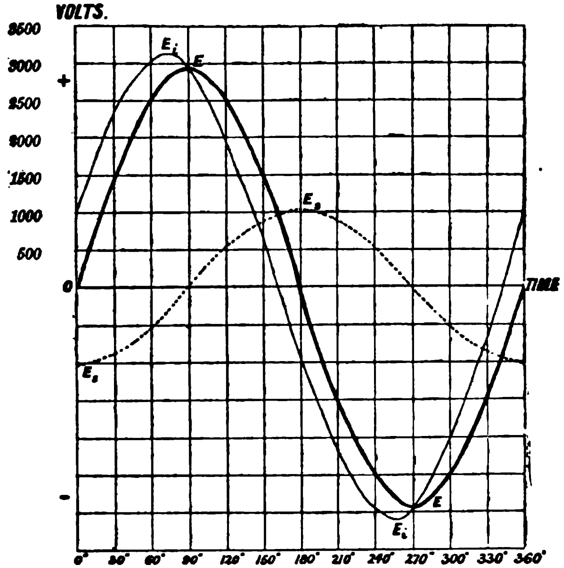


FIG. 39.

the chief difference is that in fig. 39 there is a definite angle of lag (18° in the diagram) between the current and impressed E.M.F. curves. Not only does the current pass through its zero at a certain angle behind the zero of the impressed E.M.F., but it passes through its maximum with the same angle of lag. Further, the maximum ordinate of the current curve coincides with its point of intersection with the impressed E.M.F., and lastly the maximum value of the resultant E.M.F. is necessarily less than the maximum ordinate of the impressed E.M.F. The effect, therefore, of constant inductance in any portion of a circuit traversed by an alternating current is to shift the entire current curve backwards by a constant angle of lag, and to reduce the height of all the ordinates of the resultant E.M.F. curve as compared with those of the impressed E.M.F. curve. We thus obtain three sine curves, representing respectively impressed E.M.F., self-induced E.M.F., and resultant E.M.F. or current, mutually related to one another by simple mathematical laws, so that from any two the third necessarily follows, and from the mathematical nature of sine curves we can go on to deduce other facts respecting the three related factors. Now the assumption that in any given alternator the three curves are sine curves is not by any means necessarily true, but experience has shown that in modern machines no great error arises in theory or practice from assuming that they *are* sine curves—at least for a first approximation to enable practical problems to be readily treated mathematically and graphically. The curves of impressed E.M.F. of modern alternators do closely resemble sine curves in their general nature, and so also but to a less degree do the curves of current under normal conditions of working.

§ 17. **Effective value of E.M.F. and current on sine-law hypothesis.**—If the first curve o 1 o′ of fig. 37 be a sine curve, as in fact is shown in the diagram, it can be proved mathematically that the mean of the values of all its ordinates squared, *i.e.* the mean ordinate to the second curve $o\,1^2\,o'$ is half the square of the maximum ordinate of the first curve. Hence if I be the maximum value of the current, the mean square is $\frac{I^2}{2}$, and the square root of this mean square, which has been before defined as the effective value, is $I=\frac{I}{\sqrt{2}}\cdot$ Similarly if E be the maximum value of the E.M.F., the effective value is $E=\frac{E}{\sqrt{2}}$, and in general for any function that obeys a sine law, the effective stands to the maximum value as $\frac{1}{\sqrt{2}}$ to 1, or is 0·707 of the maximum value. Thus in the case of the rotating inductor of fig. 24, which gave a sine curve of impressed E.M.F., its instantaneous value is $B_gLV \sin \beta \times 10^{-8}$ volts, of which the maximum value is B_gLV; but $V=2\pi rn$, where r is the radius of rotation, and if Z be the number of lines cut as the inductor passes once from C to D, $B_gL2r=Z$; hence the maximum value of the E.M.F. is πZn, and its effective E.M.F. is $\frac{\pi Zn}{\sqrt{2}}\cdot$ The arithmetical mean value or the average of all the instantaneous values of an alternating E.M.F. or current during one complete period is, strictly speaking, zero, but the term is usually extended to mean the average value of one half-wave only, or of all the values without regard to their sign. The average value as thus defined possesses but little interest in alternating problems, and may here be dismissed with the incidental remark that for a sine function it stands to the maximum value as $\frac{2}{\pi}$ or as 0·637 to 1.[1] Thus the inductor of fig. 24 in one revolution cuts $2Z$ lines, and the time taken to cut them is $\frac{1}{n}$ seconds, so that its average E.M.F. is $2Zn \times 10^{-8}$ volts, or equal to $\frac{2}{\pi}\times$ maximum value.

§ 18. **Power factor given by the cosine of angle of lag on sine hypothesis.**—Next, if we have two simple sine curves of equal period, but of different amplitude and phase, and simultaneous values of the ordinates of the two curves be multiplied together, it can be shown mathematically[2] that the mean of all the products so obtained is equal to half the product of the maximum values of the two sets of ordinates multiplied by the cosine of the angle which expresses their difference of phase. Hence if the two curves of E.M.F. and current be sine curves as in fig. 39, the mean ordinate to the power curve can be thus directly obtained; if e and i be the impressed E.M.F. and alternating current at any instant, and E and I be their maximum values, the mean value of ei or the mean rate of expenditure of energy on the circuit during a complete period or any number of complete periods is $\frac{EI}{2}\cos\phi$, where ϕ is the angle of lag or difference of phase between the impressed E.M.F. and current. If E_e and I_e be the effective values of the terminal E.M.F. and external current of the alternator, their product is equal to half the product of the maximum E.M.F. and current in the external circuit; for

---

[1] See Fleming, *The Alternate Current Transformer*, vol. i. chap. iii. §§ 8 and 9.

[2] *Ibid.*, § 22.

$$E_e\,I_e = \frac{\sqrt{2}\,.\,E_e\,.\,\sqrt{2}\,.\,I_e}{2} = \frac{EI}{2}.$$

Hence the output of the alternator is equal to *the product of the effective E.M.F. and effective ampères multiplied by the cosine of the angle of lag*; or $E_e\,I_e\cos\phi$, and in this form it is usually expressed, $\cos\phi$ being the power-factor or the ratio of the true to the apparent watts. It will be seen later that the value of the power-factor varies with the nature of the load, *e.g.* whether the alternator is lighting lamps directly or through transformers, or is driving motors. So small an angle of lag as that which has been illustrated above, or so high a power-factor as 0·95, would in actual working correspond to the case of a machine lighting lamps directly or connected to fully-loaded transformers.

§ 19. **Reactance and impedance.**—When the three E.M.F.'s are simple harmonic functions having the same period, they may be graphically represented by rotating radii and then combined in a single clock diagram as mentioned in Chap. IV., § 15. Thus in fig. 40 let O E, O R, and O S represent to some scale the respective maxima of the impressed, active or resultant, and self-induced E.M.F.'s. The relation of the rotating radii to the curves when plotted by the method of rectangular co-ordinates is easily followed from the diagram which repeats the waves of fig. 39. Their relative phase difference may also be directly determined by

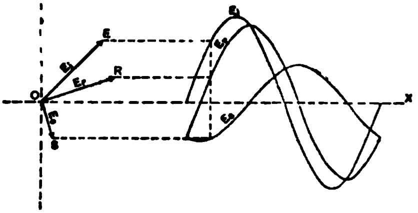


FIG. 40.

the following considerations. Let time be reckoned from the instant when the current, or the active E.M.F. which is in phase with it, is zero, *i.e.* from the horizontal axis O X. Then at time t the instantaneous value of the current will be $i = I \sin \omega t$, and of the self-induced E.M.F. will be $e_s = -\text{L}\dfrac{di}{dt} = -\text{L}I\dfrac{d\,(\sin \omega t)}{dt}$. We know that the rate of change of a sine curve is another sine curve preceding the original by 90°, or $\dfrac{d\,(\sin \omega t)}{dt} = \omega \cos \omega t = \omega \sin (90° - \omega t)$. Therefore $e_s = -\text{L}\,I\,\omega \sin (90° - \omega t)$ is represented by the projection on the vertical axis of another rotating radius of length $\omega \text{L} I$, which lags 90° behind O R, or $90° - \omega t$ behind the horizontal O X at the instant t. That this position fulfils all the requirements is evident from the consideration that when the radius O R is in the first quadrant, *i.e.* so long as the current is increasing in a positive direction, the projection of O S is negative and decreasing, *i.e.* the E.M.F. of self-induction is opposing the increase of the current; but when the current, although still positive, is decreasing, the projection of O S is itself in the first quadrant and so is positive or in the same direction as the current and increasing. Since O R is the resultant of O E and O S, we have only to complete the parallelogram having for its side O S and for its diagonal O R to find the direction and length of the radius O E. Or produce O S to give the E.M.F. consumed by self-induction O S′, when the impressed E.M.F. becomes the resultant of O R and O S′. The radii O R and O S being at right angles,

$$\text{OE} = \sqrt{\text{OR}^2 + \text{OS}^2},\ E_t = \sqrt{E_r^2 + E_s^2};\ \text{since } E_r = I\text{R},$$
$$\text{and } E_s = \omega \text{L} I = 2\pi f \text{L} I,\ E_t = I\sqrt{\text{R}^2 + 4\pi^2 f^2 \text{L}^2}.$$

Here R is the effective resistance, and $2\pi f\text{L}$ is a quantity known as the *reactance* of the circuit (also measured in ohms), the square root of the sum of the squares of the resistance and reactance, or $\sqrt{R^2 + 4\pi^2 f^2 L^2}$, being known as the *impedance* of the circuit. The relative position of O E is given either by the relation $\tan\phi = \frac{\omega L}{R}$, *i.e.* the tangent of the angle of lag of the current behind the impressed E.M.F. is equal to the ratio of the reactance to the resistance; or by the relation $\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$, *i.e.* the power factor is equal to the ratio of the resistance of the circuit to its impedance. On the sine-curve hypothesis, therefore, with a constant inductance and a constant angle of lag, a right-angled triangle (fig. 41) with the impedance as hypothenuse gives the necessary relations between resistance and reactance, or between the maxima of the three E.M.F.'s. Since the effective values are in each case equal to the maxima values divided by $\sqrt{2}$, the same relations also hold for the effective values, or

$$E_t = \sqrt{E_r^2 + E_s^2} = I\sqrt{R^2 + 4\pi^2 f^2 L^2}.$$

There is, however, a further justification for the assumption of the sine-wave shape for the curves of E.M.F. and current. Provided that each half wave is identical with its successor except in sign, which is the case in practice, then for the actual periodic curve may be substituted the *equivalent sine curve*, being that curve which has the same value for the square root of its mean square as the distorted or unsymmetrical curve; the indefinite phase-relationship between the curves is now rendered definite, and the R.M.S. values, whether of E.M.F. or current, may be compounded as vectors. For all calculations of effective power the conclusions which may be drawn from the equivalent sine-curves are then correct,[1] although no conclusions can be drawn as to the maximum or average values. In fig. 39

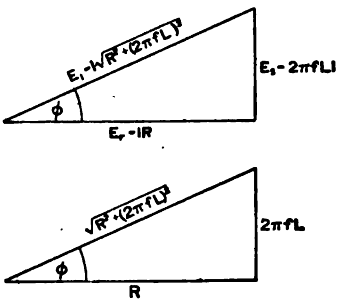


FIG. 41.

---

[1] See Rhodes, *Alternating Currents*, chap. ix. § 54; Steinmetz, *Alternating Current Phenomena*, chap. x. § 77; and *Theoretical Elements of Electrical Engineering*, part i. § 18.

the real curves of fig. 35 have been replaced by their equivalent sine curves, so that $E_e = \frac{3190}{\sqrt{2}} = 2255$, $I_e = \frac{17 \cdot 1}{\sqrt{2}} = 12 \cdot 1$, as before, and the apparent power is 27·3 kilovolt-ampères, the definite angle of lag of the current behind the E.M.F. is $\phi = 18°$, and the power factor is $\cos \phi = \frac{26}{27 \cdot 3} = 0 \cdot 95$. Thus although the quantities, reactance and impedance, imply a sine wave in virtue of the way in which they have been derived, they may legitimately be extended to cases where the E.M.F. and current do not follow a sine law, so far as any calculations as to power are concerned.

Given any one of the three E.M.F.'s with a knowledge of the R and equivalent L of the circuit, it is easy to determine the values of the other two, *e.g.* if the effective value of the impressed E.M.F. E_e is given and is represented by O E (fig. 42), on O E describe the semicircle O M E on that side of O E which lies towards the direction of rotation of O E. Take a point M on this semicircle, such that O M bears to M E the same ratio as the reactance to the resistance, or as $2\pi f L : R$. Through O draw O R parallel to M E and of the same length. Produce M O and make O S equal to O M. Then on the same scale as that to which O E represents the effective value of the impressed E.M.F., the effective value of the active E.M.F. or the E.M.F. consumed by the resistance will be represented by O R, the effective E.M.F. consumed by the reactance by O M, and the effective counter E.M.F. of self-inductance by O S, the two latter being necessarily exactly opposed to each other, the one ahead of the current by 90° and the other lagging behind it by the same angle.

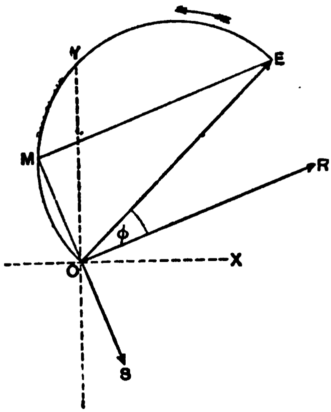


FIG. 42.

CHAPTER VII

CLASSIFICATION OF DYNAMOS

§ 1. **The two structural portions of the dynamo.**—The variety of different forms which the dynamo may take is well-nigh endless, yet in all there may be traced the two structural portions which the dual nature of the machine requires. It will be advantageous here to repeat and enlarge our first description of these two portions.

There is, first, the *field-magnet*, the function of which is simply to serve as the path for the lines of induction constituting the field, and which is therefore composed entirely of iron. It may usually be divided into three parts, which can be distinguished in fig. 2, viz. (1) the magnet 'cores' or limbs (*d d*), on which the exciting coils (*e e*) are wound or placed, and which lead the lines up to or away from the pole-pieces; (2) the 'yoke' (*y*) which joins together the limbs of the magnet; and (3) the pole-pieces themselves (N, S). These latter by reason of their particular shape cause the lines to issue forth into and pass through the air-gap in such directions and with such density as will best adapt them to be cut by the inductors.

Secondly, there is the *armature* (*a*), which consists of a number of inductors, almost invariably of copper, which are systematically arranged and connected together in a definite and particular order; in the dynamo of fig. 2 they form portions of a large number of loops or turns of wire, each of which is entirely insulated from the structure upon which they are wound, and also from every other turn, save where the end of one loop is electrically joined to the beginning of the next. In the inductors, by reason of their movement relatively to the field, E.M.F.'s are generated, and by their grouping are added together in series or placed in parallel, or in general brought into practical use at certain definite points or 'terminals' (A D) to which the external circuit is applied. When we leave the sphere of theoretical diagrams such as those in Chapter IV., it is evident that the inductors must be supported in some way, and in most cases they are in practice arranged or wound upon a mass of iron, which itself forms part of the magnetic circuit, and is traversed by the lines of induction; it is this structure of iron to which in the first instance was given the name of the armature or 'keeper' of the magnet or magnets employed to produce the field. Thus, in the ordinary two-pole dynamo the cylindrical armature of iron, *a* (fig. 2), may be likened to the 'keeper' placed between the two poles of a simple horseshoe magnet; but from its intimate connection with the electrical conductors which it supports, the term has now been extended to cover the system of inductors, the iron portion itself being called the 'core' of the armature. Finally, the central support on which the conductors are wound or laid is now termed the 'armature core,' whether it be made of iron or porcelain or any other substance, magnetic or non-magnetic.

§ 2. **Rotary motion of the inductor, and its two kinds.**—In the diagrams of Chapter IV. the principle of electromagnetic induction was mostly exhibited by means of a straight inductor drawn across a magnetic field in a straight line, as, *e.g.*, in fig. 14. It is, however, evident that such a process, being simple rectilinear motion in one and the same direction, cannot practically be maintained for any length of time, for the simple reason that the pole-pieces would have to be of great length in order to provide the indefinitely long magnetic field. But our definition of the dynamo demands 'continuous relative motion,' and hence, when the inductor has been moved to one end of the magnetic field, we are under the necessity of reversing its direction of movement. We thus pass to *oscillatory or reciprocating motion*, first in one direction, and then back again in the opposite direction. Such a motion would be obtained by attaching the inductor directly to the piston of a steam engine; it would give an E.M.F. alternating in direction, and if the field were of uniform density, the value of the E.M.F. would vary as the speed of movement, reaching a maximum during the middle of the stroke, and reversing at either end (compare A B, fig. 24). But the arrangement is not convenient,[1] since it is not easy to secure a speed high enough to give an appreciable number of volts with any practicable length of inductor and density of field in the air-gap. No such difficulty meets us when we have recourse to the *rotary motion* of fig. 24, and hence in all practical dynamos, without exception, it is by rotation that the 'continuous relative motion' of field and conductor is secured, and an E.M.F. continuously generated. Reverting to the case of a straight inductor rotated in an air-gap formed in a single horseshoe magnetic circuit by a pair of opposing pole-pieces, it will be seen from fig. 43 that

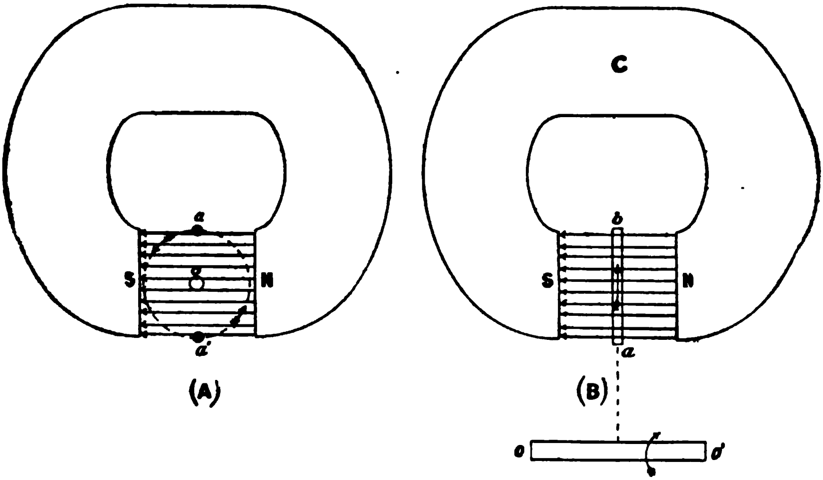


FIG. 43.

there are two distinct ways in which the cutting process can be carried out. In case A, as the inductor shown in section at *a* is rotated round the axis *o* to *a'*, it moves downwards across, or cuts, all the lines passing from the N. to the S. pole, and by further rotation from *a'* to *a* it cuts the same lines again, but in the opposite direction, *i.e.* upwards; consequently the E.M.F. induced in it is, during the first half of a revolution, in one direction along its length (downwards into the plane of the paper), and then during the second half in the opposite direction along its length (upwards out of the plane of the paper); in other words, it is the seat of an alternating E.M.F. whose direction undergoes reversal twice every revolution. But in case B, if the conductor is rigidly attached to the shaft *oo'*, and is thus continuously swung round, when an E.M.F. is induced in it by its passage past the poles, this E.M.F. is always in the same direction along its length (in the diagram from *a* to *b* or radially outwards).

---

[1] Its only successful use has been in the 'oscillator' devised by Tesla (*Electr. Eng.*, vol. xiii. p. 83) for the production of small currents at high frequencies.

Another radical distinction between the two cases of fig. 43 is connected with the position of the length of the inductor relatively to the axis round which it is rotated. In case A the length of the inductor is parallel to the axis of rotation; in case B it is at right angles to the axis of rotation, and from this difference of position great differences of structure arise.

§ 3. **Bases for classification of dynamos.**—In the classification of dynamos several bases of division present themselves, each of which has its advantages in throwing light upon the affinity of one machine to another. It might be thought that the distinction between dynamos yielding an E.M.F. and current always in the same direction round the external circuit, and those yielding an alternating E.M.F. and current in the external circuit would be clearly marked; yet it will be found that of two so-called 'continuous-current' dynamos giving a current unidirected in the external circuit, the one may be entirely distinct from the other in its whole nature and structure. As a matter of fact, dynamos which primarily and in themselves give a current continuously in the same direction are few and comparatively unimportant—at least up to the present time.

Two questions have, however, been suggested by fig. 43, which must be asked and answered when considering any form of dynamo machine; they are as follows:—

(1) Is the E.M.F. produced in each inductor as it rotates always in the same direction along its length, or does it alternate by reason of its cutting the same lines twice over in each revolution in alternate directions?

(2) Is the length of the inductors parallel or at right angles to the axis of rotation?

By the above two criteria any dynamo can be placed in its proper position and rank. In the following pages it is therefore proposed to make use of three bases of division, applying each in turn so far as it is helpful to the due understanding of the different types of dynamos: the first of these is the question whether each inductor is the seat of an alternating E.M.F. or not; the second is the position of the length of the inductor, and the direction of the field with reference to the axis of rotation (for by this we may to a large extent determine other points, such as the form of field-magnet required, and even the structure of the armature core), while third and last is the question whether the external E.M.F. and current are alternating or unidirected. In any dynamo either the field or the inductors may be rotated, or both in opposite directions; it is merely a matter of convenience to be decided by the relative adaptability of field-magnet or armature to withstand the centrifugal force and the various strains due to rotation.

§ 4. **Dynamos divided into (I) heteropolar and (II) homopolar.**—At the outset, therefore, dynamos may be divided into Class I., those in which each inductor cuts the same lines twice over, in opposite directions, as they issue from a N. or enter into a S. pole, and by reason of its so cutting the lines has an alternating E.M.F. induced in it, henceforth called *heteropolar dynamos*; and Class II., those in which each inductor, when cutting lines, is always cutting them in the same direction, and therefore the E.M.F. induced in it is always in the same direction along its length, or *homopolar* dynamos.

If the idea underlying the two names hetero- and homo-polar, be once clearly grasped, they will be found useful as distinguishing two radically different kinds of dynamos. In Class I. the fields are of *different* sign, as they correspond to N. and S. poles occurring in alternate succession, and there must be at least two fields traversed by the inductors. In Class II. there may be one or more fields, but they are always of the *same* sign. The distinction is in fact the same as that between 'unipolar' and bi-polar dynamos. In the ordinary two-pole dynamo of fig. 2 one and the same group of lines existing between a pair of poles is cut twice by each inductor in each revolution, as in fig. 43, A. Similarly, in the multipolar dynamo properly so called, in which there are several magnetic circuits or groups of lines, the same fact holds good with respect to any pair of poles, viz. that each set of lines belonging to a distinct magnetic circuit is cut twice by each inductor in opposite directions in each revolution. In the so-called 'unipolar' dynamo there is a single field just as in the bipolar dynamo, and it must from the nature of the flux in an imperfect magnetic circuit exist between a pair of poles; but this single field is only cut once in each revolution, and the cutting always takes place in the same direction, so that there is no reversal of the direction of the E.M.F. induced along the length of the inductor: thus, the B arrangement of fig. 43 would form a unipolar dynamo. In the unipolar machine, since the lines are always cut in the same direction, it might be said that they are cut as, *e.g.*, they emerge from a N. pole, and thus the necessary presence somewhere of the other pole might be left out of sight; the need of this second pole is not at once apparent, except in theory, since it is not wanted to concentrate and collect the lines in order for them to be cut a second time in the same revolution. Such a view, however inaccurate, would be represented diagrammatically by fig. 44, where N is the N. pole of a bar magnet, the other pole of which is not in view (compare with fig. 43 B); and from it arose the term 'unipolar.' There is, however, nothing to prevent the field of the unipolar dynamo from being divided into a number of sharply separated groups of lines each corresponding to a polar projection of the same sign; it may thus become in a sense multipolar, but at the same time the inductors never cut the same lines twice, and it therefore never loses its distinctive 'homopolarity.'

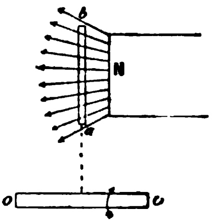


FIG. 44.—Conventional unipolar dynamo.

§ 5. (I) **Bipolar dynamo** (A) **with inductors parallel to axis of rotation.**—Having premised thus much, we pass to an examination of dynamos of Class I., and in the first place (A), those in which the inductor's length is *parallel to the axis oj rotation.*

Through the middle of a rectangular gap formed in a simple horseshoe magnet by two pole-pieces having plane faces (fig. 45), let a shaft be placed which is capable of rotation, and at some distance radially from it let there be a wire, *a d*, supported at either end in a position parallel to the axis of rotation. Then, when the shaft is rotated, *a d* will cut the lines of the field and act as an inductor of an alternating E.M.F. If the ends of the inductor are electrically connected with two collecting rings fixed on the shaft and insulated from it, against which springs, *b b*, are pressed so as to form a rubbing contact, we have an *alternator*, yielding an alternating difference of potential at the two stationary rubbers or brushes to which an external circuit, R_e, can be applied. Assuming for the present that the field swept through by the inductor is of uniform density, and that all the lines pass straight across from one pole to the other, the curve of instantaneous induced E.M.F., if plotted for one revolution forming a complete period, would be a sine curve. (*Cp.* fig. 24.)

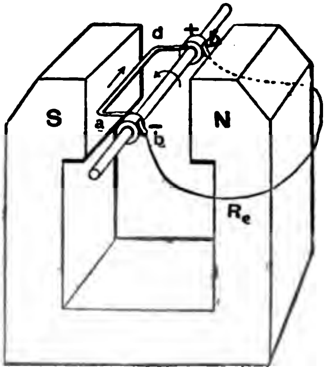


FIG. 45.

§ 6. **Function of iron armature core.**—But the air-gap in which the magnetic field that is to be cut exists should be as short as possible in the direction of the lines, in order that the ampère-turns of exciting power required to produce the field may not be very large; for economy in the exciting power means economy either in the first cost of the machine or in its cost of working, or both. The first step would obviously be to shape the inner surface of the pole-pieces so that they follow the curve traced out by the rotating inductor, that is, instead of having pole-pieces with parallel faces, to bore out a cylindrical gap within which the inductor may rotate, allowing only such room or 'clearance' as is required for mechanical safety and durable working (fig. 46). But this will not entirely have the desired effect of strengthening the field cut by the inductor, inasmuch as it will still remain weak except at the edges, *a b*, *c d*, and there the field will be most dense outside the limits of the area swept through by the rotating inductor; lines will, in fact, leap across in considerable numbers from edge to edge where the distance is least, being lost as far as any useful purpose is concerned, and therefore to be reckoned as mere 'leakage.' Further, the relative directions of the movement and of the lines are not continuously at right angles, which is one of the conditions at which we should aim in order to obtain the maximum E.M.F. A complete remedy is found if, instead of removing the whole portion of the magnetic circuit which is bored out, an iron cylinder is retained on the surface of which the inductor can be placed (fig. 47). The exciting power required to produce a given number of useful lines within the sweep of the inductor will be immensely reduced, since the path of the lines is now more nearly a closed magnetic circuit of iron, like a horseshoe magnet, with its armature or 'keeper' between its poles; two short air-gaps are alone left, instead of one of great length, and the density under the pole-pieces will far exceed that of any straying lines between the pole-tips. Besides these two functions, viz., of reducing the exciting ampère-turns to a minimum for a given number of lines by shortening the length of the air-path, and of concentrating and guiding the lines in the required direction, the cylinder will also serve as a supporting structure on which the inductor and its connections can be secured. If the armature is the part that is to be rotated, then when the iron core is used as a support for the inductors, it must itself be rotated with them; although on any other ground it is entirely unnecessary. The problem of rotating the inductors only round a stationary iron core has possessed great attractions for inventors; the difficulties inherent to it

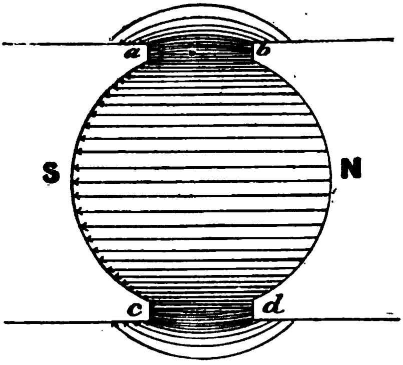


FIG. 46.

have, however, proved too great, and the mechanical simplicity of rotating both core and winding has led to its universal adoption. Apart from certain secondary and minor effects which need not here be considered, the rotating of the iron has no effect on the field of lines; these latter remain stationary, passing into and through the core just as if it were at rest. By the insertion of the iron core, the distribution of the lines in the field as cut by the inductor is essentially altered; they now enter the core much more radially, and are distributed over the arc embraced by the pole-piece almost uniformly, the result being a considerable modification in the shape of the curve of instantaneous E.M.F. If their distribution were perfectly uniform, and their direction at all points exactly radial, it would consist of a series of detached rectangles as in fig. 47 (contrast fig. 24); but in actual practice the corners are rounded off by the existence of a weak field or 'fringe' at the edge of the pole-piece, which is not truly radial, and the curve of induced E.M.F. for one period or complete revolution is flat-topped with rounded corners, or intermediate between a pair of sine curves and a pair of rectangles (fig. 48).

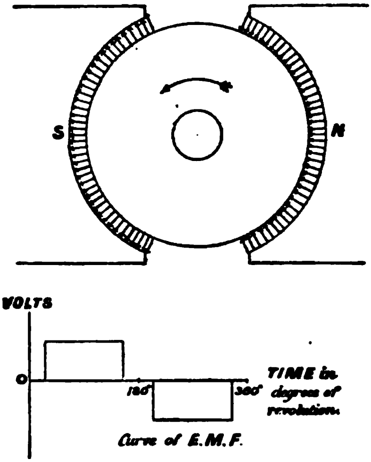


FIG. 47.

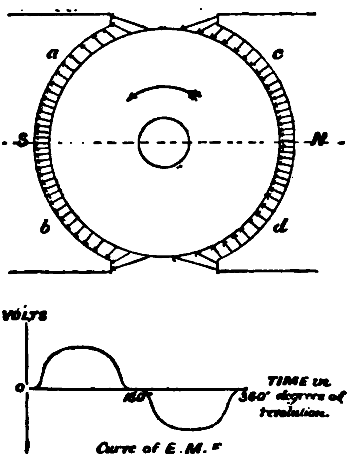


FIG. 48.

But the E.M.F. generated by one single inductor of reasonable length thus rotating in a field of high density, and at as high a speed as is practicable, is very small, not amounting to more than a few volts at the most; hence it is usual in most commercial dynamos to have several such inductors and to add up their E.M.F.'s by connecting them in series.

§ 7. (1) **Ring winding.**—Thus, in order to generate a considerable E.M.F., a number of inductors have to be rotated in an interferric space, which must be as small as is consistent with mechanical safety, and which should be approximately of uniform length at all points of a revolution. Evidently, therefore, in any dynamo, both the polar surfaces which the lines leave and enter, and the moving system of inductors, must, as above, in some way form portions of an approximately cylindrical or spherical structure whose axis coincides with the axis of rotation; and according to the position of the length of the inductor relatively to this cylindrical structure the direction of the magnet field with reference to the axis of rotation will be fixed. In our first type of dynamo, imagine a number of inductors arranged symmetrically round the same armature core (fig. 49) parallel to our original inductor, which was placed on the surface of a cylinder along its length, the direction of the lines being perpendicular to the axis of rotation. Then it will be seen that, with either direction of rotation, at any moment each of the inductors moving under the N. pole has an E.M.F. induced along its length, in the opposite direction (as viewed by an observer at either end) from the E.M.F. induced in each of the inductors moving under the S. pole; *e.g.* with a counter-clockwise direction of rotation as shown, the rule of the hand tells us that the E.M.F. will be directed away from the observers in all the inductors (crossed) under the pole-piece to the left of the neutral diameter, and towards the observer in the inductors (marked with a dot) under the pole-piece to the right.[1]

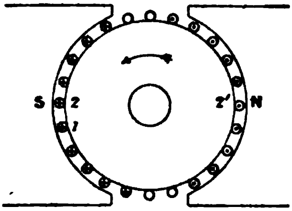


FIG. 49.—Inductors parallel to axis of rotation.

---

[1] The marking of the direction of currents or E.M.F.'s by crosses and dots

This fundamental fact must be the guide to any method by which the inductors can be connected together electrically in a useful manner.

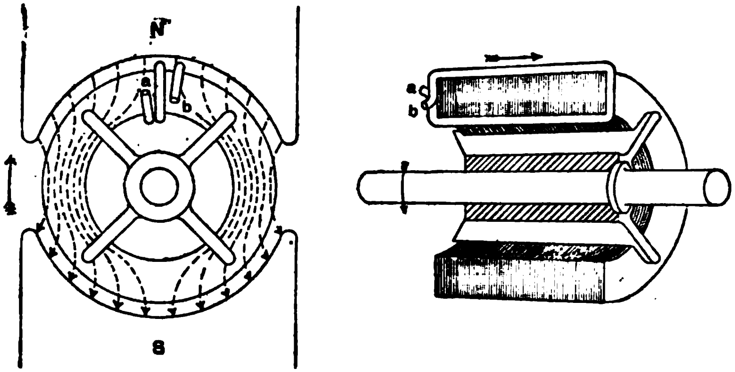
Two methods of adding together the inductive action of a pair or more of inductors at once present themselves; by the first, inductor 1 of fig. 49 is connected in series with another inductor, 2, which is next to it on the surface of the core or lies immediately above it, and under the same pole-piece; by the second it is connected with another inductor, such as 2′, which is situated nearly diametrically opposite on the other side of the core.

(1) The first or *ring* method was employed by Pacinotti in 1860 and described by him in 1865, but is also frequently called by the name of the French electrician Gramme, who reintroduced it in 1870. If the iron core, which served to guide the lines through the space between the pole-pieces, be supported at some distance radially from the shaft by means of an open hub or spider of some non-magnetic metal, such as gun-metal, a connecting piece of wire can be brought through the inside space between the core and the shaft, passing between the arms of the hub; by it the near end of inductor 1 (fig. 49) can be connected to the further end of 2, the inductor next to it on the surface of the core, and the whole arrangement is shown in the ring-wound armature of fig. 50, I. Without the iron core the lines of induction would go straight across the gap from pole to pole; the inner connector would therefore be cutting lines as well as, and in the same direction as, the outer inductor, and an E.M.F. would be produced in each. These two E.M.F.'s thus acting round the loop would oppose and tend to neutralise each other, so that the net E.M.F. would be merely the difference between the two. But with the iron core interposed, the lines take a more or less curved path through the mass of the iron, as shown by dotted lines in the end view of the ring armature; and if the core be supported by a non-magnetic hub, very few lines will leak across the internal central space; consequently, there is little or no

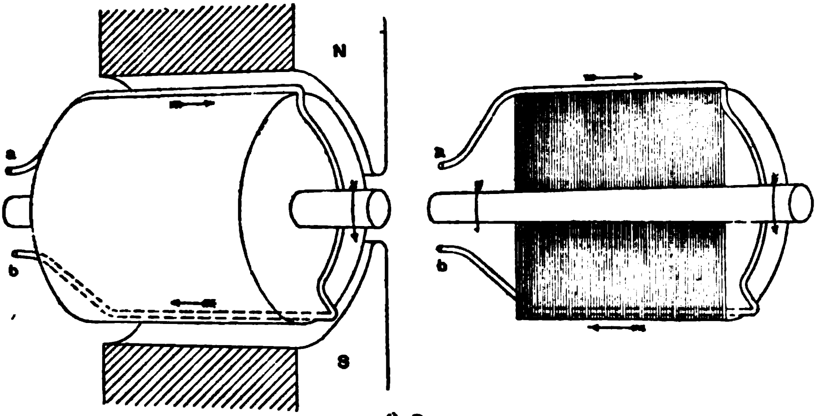
---

according as they are away from or towards one's point of view, is a useful convention, employed by Prof. S. P. Thompson: it is easy to recall if we consider the cross to represent an end view of the feathers of a receding arrow, and the dot to represent its advancing point.

E.M.F. induced in the inner wire, which thus serves purely as a conducting connector to sum up the E.M.F.'s produced in the two external inductors, and does not itself cut the lines of the field. If any E.M.F. is induced in the inner wire, it is simply harmful, but with proper methods of construction and right proportioning of the iron in the armature core it becomes so very small that it may be neglected. It will be seen that this method



I. Ring



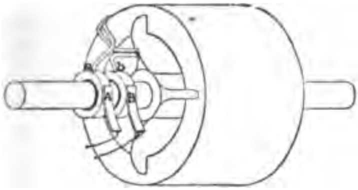
II. Drum

FIG. 50.—I. Ring-wound armature. II. Drum-wound armature. Two inductors in series.

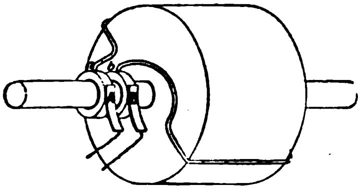
really amounts to threading the same wire through the central hole, so as to form a 'loop,' and then winding a second loop of the same wire continuous and side by side with the first. A coil of two loops is thus formed, containing two active inductors on the outside of the core, whose E.M.F.'s are added together in series, and the core becomes a 'ring' over-wound with a coil, whence the name for this winding arises. Further, it is evident that the process of winding the wire round the core may be continued so as to form more than two loops; for the present, however, we confine ourselves to indicating the method by which two inductors can be joined in series by the formation of two loops on a ring core.

§ 8. (2) **Drum winding.** — The second or *drum* method is identified with the name of 'Siemens.' In its complete form it was first introduced in 1871 by Von Hefner Alteneck as a modification of the original Siemens shuttle-wound armature. It is even simpler than the first method, and consists in joining the further end of one inductor by a connecting piece of wire passing across the end of the core to the further end of another inductor situated nearly diametrically opposite to the first inductor and under the opposite pole-piece (fig. 50, II.). The E.M.F.'s induced in the two inductor wires will assist each other round the loop thus formed, being in opposite directions along their length as viewed from either end. The core now becomes, as it were, a cylindrical 'drum,' which may be solid right down to the shaft; as before, two inductors have been placed in series, and precisely the same amount of the surface of the armature core has been covered up and rendered useful; and, therefore, one loop of the drum winding is the exact equivalent of the two loops of the ring winding. To both the ring and the drum the iron core is essential, since without it almost all the lines would pass from the one pole to the adjacent edge of the other without being cut by the armature conductors at all: but it is doubly essential to the ring, in order that it may shield the internal conductors from any inductive action. By bringing the wire across the near end of the drum, *i.e.* by the addition of a second connector, the loops may be multiplied, the end of one loop forming the starting-point of the next.

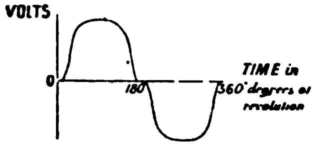
§ 9. **Simple alternator and unidirected-current armatures.**—If the curve of E.M.F. induced by either the ring or drum arrangement of fig. 50 be plotted, then with the same field and speed of rotation the ordinates will have twice the height that they have in fig. 48; and if the free ends of the loops *a b* be connected to two insulated collecting rings mounted on the shaft, we have the simple bipolar 'ring' and 'drum' *alternators* of fig. 51. This figure also shows the curve of E.M.F. acting during one revolution at the brushes A and B, which lead to the external circuit, the angle through which the loops have rotated being reckoned from a position midway between the pole-pieces as a starting-point.



Ring wound armature
Two loops



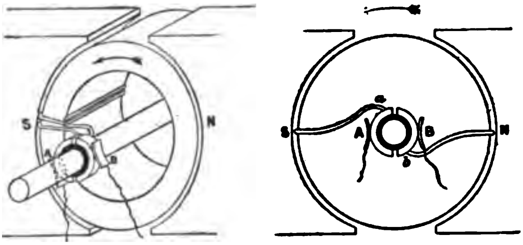
Drum wound armature
One loop



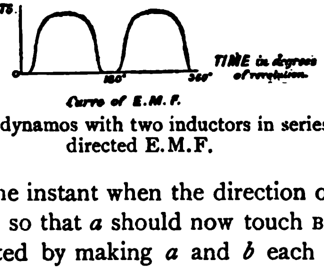
Curve of E.M.F.

FIG. 51.—Alternator armatures with two inductors in series.

For a variety of purposes an alternating E.M.F. is inconvenient or positively useless; yet, from the nature of the dynamos under Class I., the E.M.F. induced in each inductor is necessarily alternating, since it is produced by first cutting lines in one direction and then in the opposite, by continuous rotation, and therefore any current flowing under that E.M.F. must necessarily be alternating in the armature conductors themselves, whatever it may be in the external circuit. It remains to be inquired how the alternating E.M.F. of the armature loops can be 'commuted' or changed to an E.M.F. always acting in one direction in the external circuit. Let us consider the loop or loops of fig. 51: when in the course of rotation they arrive at a central position under the two pole-pieces, the E.M.F. is a maximum, and collecting ring *a* is, say, positive; later, when the loops have turned through 90°, or have reached a position in the gap between the poles, no E.M.F. is being generated, since no lines are being cut, and they have reached the position of reversal. Immediately after passing this point, ring *a* will be negative, ring *b* positive. What is required, therefore, in order that the E.M.F. at the brushes may act always in the same direction round the external circuit, is that the rings in contact with the springs or brushes should be automatically reversed at the instant when the direction of the E.M.F. in the loop is reversed, so that *a* should now touch B, and *b* touch A. This is easily effected by making *a* and *b* each half of one and the same split ring, the two halves being separated from each other by air or other insulating material, and, further, by so setting the brushes that they pass over respectively from *a* to *b*, and from *b* to *a* at the instant of reversal (fig. 52). This device is the simplest form of *commutator*, by which each brush always remains either positive or negative, as the case may be, and therefore the current flowing in the external circuit is *unidirected*, although in the loops of the armature itself it is alternating in direction.



Ring-wound armature Side View. *Drum wound armature. End View.*

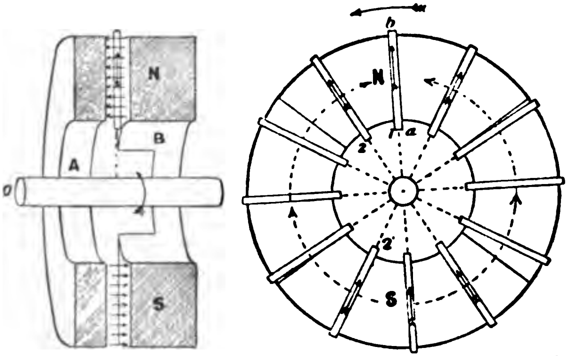


Curve of E.M.F.

FIG. 52.—Bipolar dynamos with two inductors in series giving a unidirected E.M.F.

We have now obtained a ring and a drum unidirected-current machine, and the curve of E.M.F. at the terminals or brushes will be entirely above the horizontal line (fig. 52). Given, therefore, two open ends of a loop or loops *a* and *b*, the same armature yields either an alternating or a unidirected current, according as they are attached to a pair of collecting rings or to one split-ring commutator.

§ 10. (I.) **Bipolar dynamo (B) with inductors at right angles to axis of rotation.**—So far the rotating inductor has been one whose length is parallel to the axis of rotation; we now pass to the second or (B) group of Class I., *i.e.* bipolar dynamos, in which the length of the inductors is *at right angles to the axis of rotation*, as in fig. 43, B. For a dynamo to belong to Class I., it is necessary that the inductor should cut the same lines twice in different directions, which cannot be done so long as the arrangement is that of fig. 43, B, and so long as reciprocating or oscillatory motion is impossible. Nothing, however, prevents us from breaking the magnetic circuit at another point, C (fig. 43), and bringing this second air-gap into the circular path of the inductor. In fig. 53 let B be a circular ring of iron with two pole-pieces, N and S, projecting from one side of it; lines of induction entering at S will divide and pass round B in two groups by either branch of the magnet (as shown by dotted lines) and reunite in the N pole-piece,



View from O after removal of A.

FIG. 53.—Inductors at right angles to axis of rotation.

whence they emerge into the air-gap. If a second iron ring, A, is arranged on the other side of the inductor, the lines will enter into it and pass in a similar manner round the two halves of A, and reunite to emerge opposite to the S pole-piece, thus completing their tour of the magnetic circuit. The inductor *a b* will now cut the same lines twice in each revolution in different directions at a different portion of their path, and the E.M.F. induced in it will be first radially outwards from the centre, and then inwards towards the centre, according as it cuts the upper or the lower group. With this arrangement it is easy to make the air-gap short by bringing the two portions of the magnetic circuit close together, leaving only sufficient room for the clear rotation of the inductor. But the same necessity exists, as before, of connecting two or more such inductors together in series in order to add up their E.M.F.'s. Since the direction of the magnetic field in the air-gap is parallel to the axis of rotation, the several inductors must be arranged somewhat as the spokes of a wheel radiating outwards from the centre, and the poles must be curved to a circular shape in order to follow the path of a line passing through the centre of the length of the inductors. In all the inductors passing through one field, the E.M.F. will be directed radially outwards, in all those passing through the other field, radially inwards, and this must be our guide in connecting them together. As in the case of Group A of Class I., two methods present themselves which are exactly analogous to the ring and drum windings and form the third and fourth types of armatures: by the one, inductor 1 is joined in series with its next neighbour, 2; by the other it is joined to the inductor 2′, which is nearly opposite.

§ 11. (3) **Discoidal or flat-ring winding.**—By the flat- or *discoidal-ring* method a connecting piece of wire is passed round and through A, so as to join the outer end of *a b* (fig. 53) with the inner end of the inductor by its side. The result is an arrangement (fig. 54) exactly analogous to the ring armature of fig. 50, and the iron A, which now becomes an armature core, and is rotated with the inductors, serves exactly the same purposes, viz., guidance of the lines in the required path and mechanical support to the armature winding. The only change introduced has been the flattening out of the cylindrical ring core until its radial depth is greater than its length parallel to the axis of rotation, and it assumes a discoidal form; the polar surfaces are then presented to the ends of the cylinder instead of to its periphery. But now a second magnet, M′ (shown dotted, fig. 54), can be presented to the other face of the flat-ring core, the poles of this second magnet being exactly opposite to those of the first and of the same sign. The lines will thus enter the core from both sides alike, and will pass in opposite directions round the armature, to leave it where the second set of poles is presented to its surface. By this means both sides of the winding become active, and the inactive connectors are reduced to the short lengths of wire at the top and bottom of the core. The whole arrangement may be evolved out of a ring armature, which, besides the external poles presented to its outer surface, has also a pair of internal poles arranged exactly opposite to the external poles, a N. facing a N., and a S. opposite to a S., so as to render the internal portion of the connecting wire active (fig. 55). The entire loop of the flat-ring machine, although containing two inductive portions, should

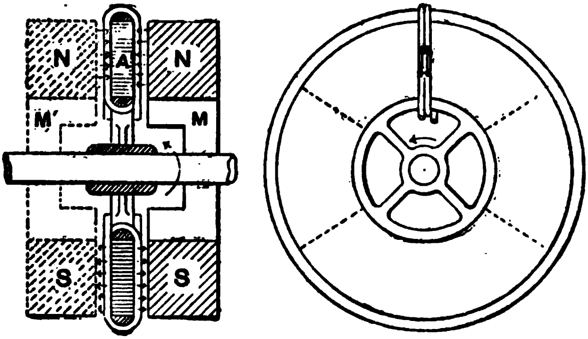


FIG. 54.—Discoidal-ring armature with two inductors in series.

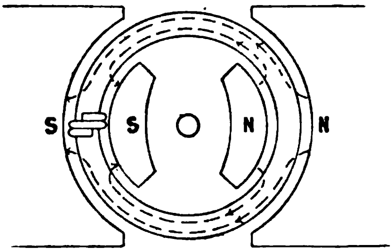


FIG. 55.—Ring armature with external and internal poles.

therefore be regarded as equivalent to a single loop of the ring, the lines of one field being simply divided into two portions, one on either side of the core.

By bringing the ends of the winding to a pair of collecting rings, similar to those of fig. 51, the dynamo becomes a discoidal-ring alternator.

§ 12. (4) **Disc-winding.**—The fourth, or true *disc* method of connecting two radial inductors in series, will be to join the outer end of one inductor by a connecting piece passing round the periphery to the outer end of another inductor, situated nearly diametrically opposite, under the second pair of poles (fig. 56), the

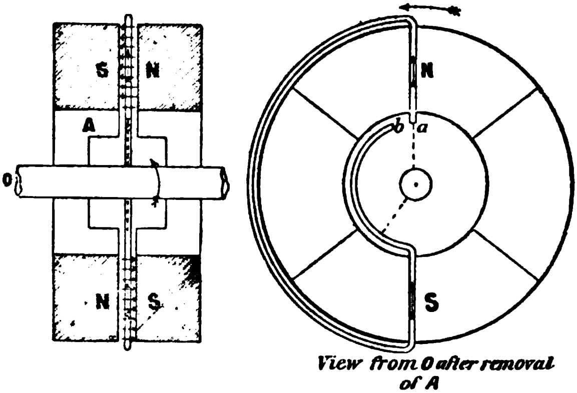


FIG. 56.—Disc armature with two inductors in series.

loop being completed, if required, by a second connector, joining the inner ends of two inductors, so as to render a series of continuous loops possible. The iron portion, A, of fig. 53 now forms no portion of the armature, and remains stationary: it simply serves to complete the magnetic circuit, which is cut at two points, and is exactly analogous to B. Opposite poles are of opposite sign, and the lines pass straight across each air-gap from one pole-piece to the other. In this, which may be called the true disc machine, it will be seen that the inductors are actually in the central plane of rotation, whereas in the previous flat or discoidal-ring machine they are arranged in a plane to the side of and parallel with the central plane of rotation. The peculiarity of the true disc winding is that it is indifferent whether the loop be wound on an iron core or not. A supporting core may be necessary, but it need not be of iron so as to guide the lines in the right direction through the air-gap, since the two opposite poles can be brought close together, only allowing sufficient room for the coils to pass between; and hence, if an iron core is present, it can only fulfil its first function of reducing to a minimum the exciting power required to produce a certain field. Just as the discoidal-ring armature can be evolved out of the simple-ring armature with two pairs of poles external and internal, so this second method is in reality drum winding adapted to a disc. If we replace the core inside the drum winding with a pair of internal N. and S. poles, respectively facing the S. and N. poles of the field, and then imagine one end of the armature to be gradually enlarged, and the other reduced in size, it would take the form of a truncated cone, which would become flatter and flatter; finally, the winding would be turned into a flat disc, and the external and internal poles will be converted into two pairs of poles arranged on either side of the disc.

By the addition of a split-ring commutator, to which the ends of the loops of the discoidal and true disc winding are attached, we obtain unidirected-current discoidal and disc machines.

The principles on which the dynamos of Class I. are based have now been diagrammatically exhibited in the case of bipolar dynamos; the further examination of their multipolar forms will be found to present no special difficulty, and will be postponed for subsequent treatment.

§ 13. **(II.) Homopolar dynamos with one inductor.**—We now pass to dynamos of Class II., or those in which each inductor, when cutting lines of induction, is always, at any moment during the revolution, cutting them in the same direction, and further never cuts any line twice in one revolution; hence the E.M.F. generated in each inductor always has the same direction along its length, or is never reversed. As before, we may subdivide the class into the two groups of (A), those in which the inductors' active lengths are parallel to the axis of rotation; and (B) those in which they are at right angles to the axis of rotation.

Consider a single inductor rotated between two curved pole-pieces, as in fig. 57; the E.M.F. generated in these two cases will be always in the same direction along the inductor's length, and it will last so long as the inductor is moving through the field. Now if in both cases the pole-pieces are made tubular, there will be no break in the E.M.F., which will be maintained continuously at a constant value (fig. 58), and if the circuit be completed by joining the ends of the inductor by rubbing contacts to an external resistance, we have a homopolar dynamo giving a unidirected E.M.F. and current, both externally and within the armature itself. In the second or B case, where the inductor's length is at right angles to the axis of rotation, it is simpler to mount the field magnet on the shaft as shown, thus making it the rotating portion, and allowing the armature to remain stationary; if the direction of rotation be the same as in previous cases, the E.M.F. in the inductor will be directed radially inwards.

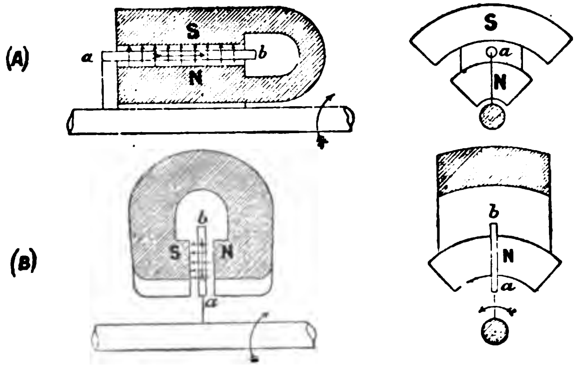


FIG. 57.—Homopolar dynamos.

Several such inductors can be arranged symmetrically round the axis of rotation, as in fig. 59, and these can be placed in parallel, so as to carry a larger current, but without any increase of E.M.F., until finally, when the whole available space is occupied, we arrive at a solid tubular inductor, or a solid circular disc of metal. These may be resolved mentally into a number of contiguous strips in electrical contact with each other, but their action is the same as if there were only one inductor.

§ 14. (II.) **Homopolar dynamos (1) with ring armature winding.**—But, as stated before, the E.M.F. which can be thus produced is very small, and in order to increase this, two or more inductors must be connected in series. In order to adapt the ring armature to this class of machine, it is evident that when an inductor on the external periphery of the core has cut all the lines as, *e.g.*, they enter the core on the right-hand side (fig. 60), the direction of E.M.F. will be reversed in it unless it avoids the lines as they issue from the core: this will be effected if the other pole on the opposite side of the core, instead of being external, as usual, be placed inside the ring, provided that the dimensions of the ring are such that the lines will not leak across inside in a direction unsuitable to our purpose. In fig. 60 the new internal position of the second pole is shown, together with the new path of the lines of flux. The armature core must now be supported from one end only, while the curved horseshoe magnet, shown in section at M, bends round at the other end, one pole embracing the outside and the other projecting into the interior of the ring.

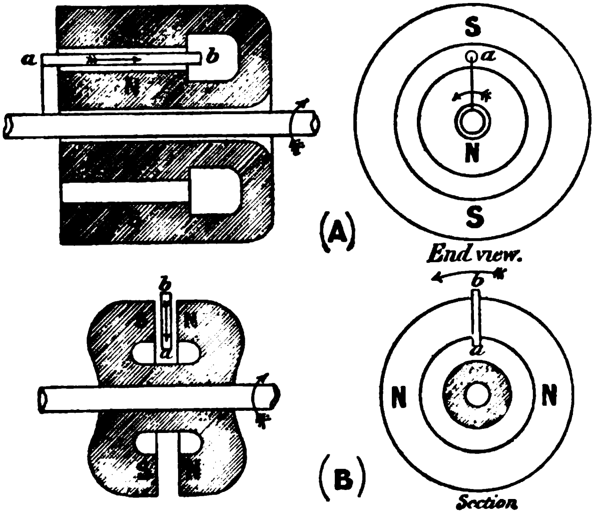


FIG. 58.—Homopolar continuous-current dynamos.

But at once it is evident that the inner wires of the loops, which previously formed inactive connecting pieces coupling together the external inductors, will themselves become active inductors when the loops pass the inner pole-piece, and that they will be coupled together by the conductors on the outside of the ring. Hence the same conductors will serve first as inductors and then as connectors; the E.M.F. generated in each conductor will be always in the same direction along its length, but round the loops the E.M.F. will alternate. We have thus obtained a loop in which the E.M.F. alternates, and from it, by attaching the ends to a pair of collecting rings, a homopolar ring alternator.

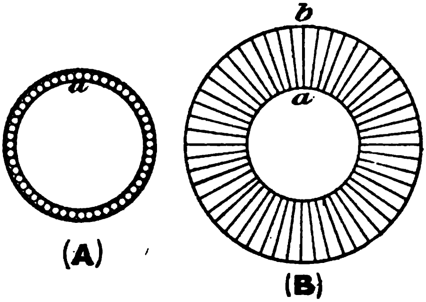


Fig. 59.—Homopolar continuous-current armatures.

§ 15. (II.) **Homopolar dynamos (2) with discoidal-ring armature winding.**—In the case of the discoidal-ring armature, with the loop wound round and through the core, the same idea can be applied as in the case of the simple ring armature; for if the S. pole be shifted to the opposite side of the ring from the N. pole (fig. 61), opposite sides of the loop will alternately act as inductors, and the E.M.F. will alternate round the loop. The

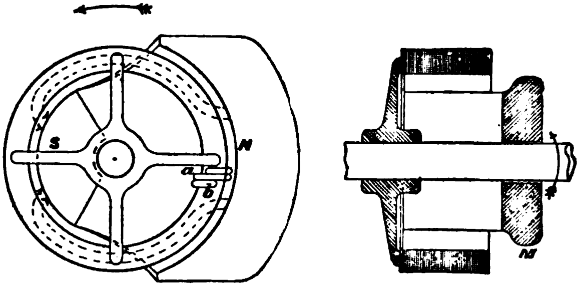


Fig. 60.—Ring homopolar dynamo with two inductors in series.

two poles in this arrangement must not overlap each other, and the lines must pass in a sloping direction across the core.

In ring and discoidal homopolar alternators, therefore, the two poles are arranged either outside and inside the core, or one on either side of it; hence we are not mechanically prevented from making each field extend over more than half the circumference of the mean path described by the rotating inductors; but this is not electrically permissible any more than in the case of bipolar armatures, since were it done there would be no distinction always present between the wires which are inductors for the time being and those which are mere connectors. If this distinction is not maintained, and both sides of the loop are allowed to be moving under the pole-pieces simultaneously, the E.M.F.'s generated in the two sides act in opposition to each other, or, as it is termed, act 'differentially,' the net effective E.M.F. being simply the difference between the two. Any such differential action is therefore detrimental to the proper working of the machine.

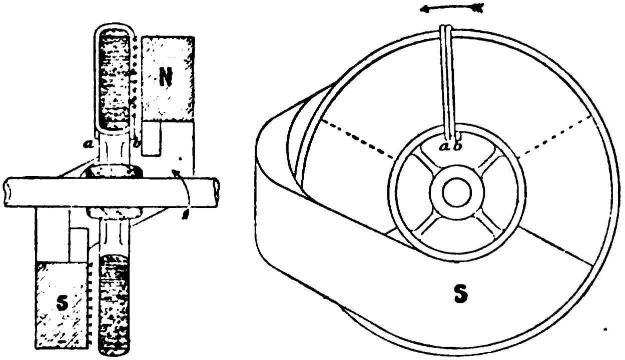


FIG. 61.—Discoidal-ring homopolar dynamo with two inductors in series.

§ 16. (II.) **Homopolar dynamos (3) with drum armature winding.**—In fig. 62 is shown a drum wound homopolar machine with a single magnetic circuit. In this type, the magnet is usually the revolving element, and in the simple form chosen for illustration, there is what may be described as a moving joint in the magnetic circuit at the S. pole. The magnet is excited by the coil E, and it has one pole with a semicircular face between which and the stationary armature core there exists a magnetic field The remaining half of the internal circle of the armature core is a neutral space through which few or no lines pass. The single loop shown must have a width at least equal to the width of the field, and also not greater than the width of the neutral space in order to avoid differential action. Thus as the field rotates, one side of the loop acts as inductor for one half of the revolution, and then during the other half of the revolution acts as a simple connector. The lines of the flux are as it were taken out of the end of the drum, and so are not cut a second time by the wires of the armature. The second break in the magnetic circuit at s forms a uniform field with as short a length of air-gap as can be conveniently used in practice.

§ 17. (II.) **Homopolar dynamos (4) with disc armature winding.**—Similarly in fig. 63, which represents a homopolar disc alternator with rotating field magnet and stationary armature, while the leading edge moves from 1 to 3 no lines traverse *cd*, and therefore no E.M.F. is induced in it; but when by further rotation the leading edge moves from 3 to 1, *ab* does not in turn cut any lines itself, and consequently the E.M.F. is induced only in *cd*. Hence the E.M.F. is alternating in direction round the loop as one side or the other forms the active inductor; but in the conductor, which is for the time being cutting lines or acting as inductor, it is always in the same direction along its length. It is evident from both figs. 62 and 63 that the field must not under

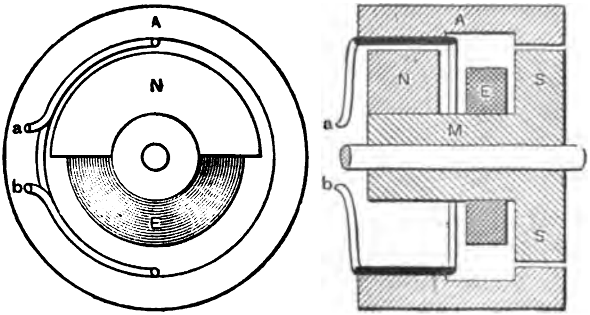


FIG. 62.—Homopolar drum alternator.

any circumstances extend over more than half the circumference of the mean circle described by the inductor, the width of the loop being also equal to half the circumference, since otherwise the two sides of the loop will both be cutting lines in the same direction, and therefore acting differentially.

The distinctive feature of the ring, drum and disc loops, when applied to homopolar machines, is that they are a device for subsequently joining two active inductors in series; they do not themselves do so. In this they resemble the loop of the ring bipolar armature, but may be distinguished by the fact that in the latter the connecting wires never become inductors, whereas in homopolar rings first one side of the loop and then the other is active. Thus in the homopolar armature there is never more than one active inductor at any one time in each loop. Even in disc homopolar machines the loop does not itself join two inductors in series, though it forms a starting-point by which it can be done; and hence the action of disc machines, however similar their winding in appearance, is entirely different according as they are hetero- or homopolar.

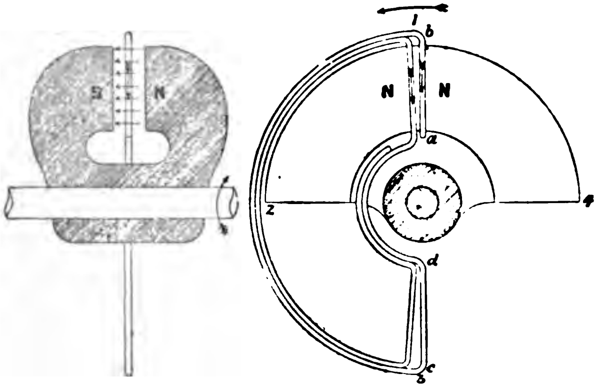


FIG. 63.—Disc homopolar dynamo with two inductors in series.

By connecting a split-ring commutator to the ends of the loops the homopolar machine can be reconverted into a dynamo giving a unidirected E.M.F. and current in the external circuit.

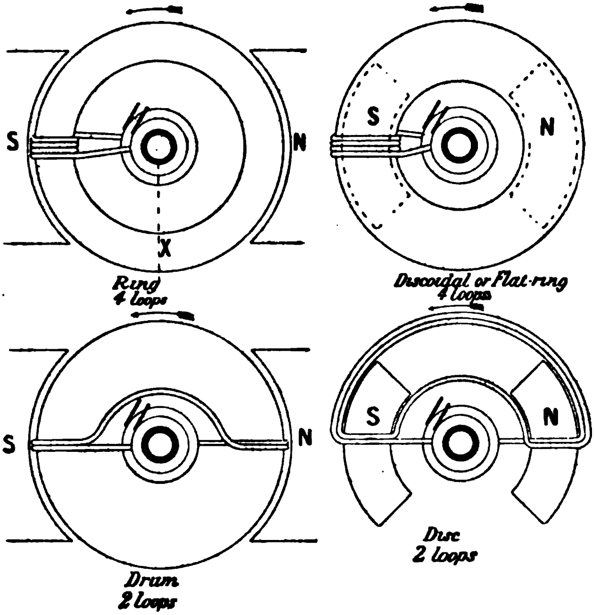
CHAPTER VIII

HETEROPOLAR ALTERNATORS

§ 1. Formation of a coil with uniform distribution on a smooth armature.—The previous chapter has shown the methods by which, in the bipolar dynamo, two inductors are connected together in series, so as to form a pair of loops in the case of the ring and discoidal-ring armatures, or a single loop in the case of the drum and true disc armatures; the next step will be the *formation of a coil*, or coils, of many such loops, and their several modes of connection in order to still further increase the total E.M.F. of the machine.

From the nature of the loops described in Chapter VII. it is evident that they can be multiplied by simply winding more of the wire continuously on to the core after exactly the same fashion as the original loop, since the end of one loop will serve as the starting-point for the next. Two cases now arise; either the surface of the armature core is *smooth*, or the armature is *toothed* and the winding is embedded in slots placed close to the outer edge of the armature core. Let us consider the smooth-surface armature in the first place. If the several loops are wound on the top of each other in the case of the ring, discoidal-ring or drum armatures, or by the side of each other in the case of the true-disc, each loop will produce exactly the same E.M.F. as the first original loop, provided the density of field be kept the same; but if the winding be thus piled up in two or more layers, the length of the air-gap is necessarily increased, and, therefore, with a given exciting power, the density of field is greatly reduced. Hence it is advantageous to wind the loops side by side in ring and drum, or on the top of each other in disc smooth-surface armatures, so as to permit of the retention of the same air-gap as for a single loop.

Fig. 64 shows the loops thus wound for equivalent armatures of the ring, discoidal, drum, and disc types, in all of which four inductors are joined together in series. As before, the two free ends of the loops are connected to two collecting rings, one of which in the diagrams is shown of large diameter in order to be visible behind the other; the whole then forms a bipolar alternator, from which current is taken by means of the brushes.



Ring
4 loops

Discoidal or Flat-ring
4 loops

Drum
2 loops

Disc
2 loops

FIG. 64.—Bipolar armatures with four inductors in series

§ 2. **Formation of a coil with concentrated or grouped distribution on toothed armature.**—But when the loops are wound as shown in fig. 64, they have a certain width, and do not therefore all occupy identically the same position relatively to the polar surfaces at any point of a revolution. It follows that the E.M.F. induced in the different loops forming the coil need not at the same moment be always alike, either in direction or amount;

in other words, they differ in phase, and this renders it necessary to carefully consider the effect of the width of a coil.

Next, if the armature be slotted and the additional loops are wound in the same slot as the original loop, the winding may be called '*concentrated*' in contrast to the *uniform distribution* above described, and each new loop will add the same E.M.F. as that produced by the first loop. If, however, this statement is to hold true in practice, the dimensions of the slot in order to receive the numerous wires must not be increased above certain normal limits. Should the slots become too deep or too wide, it will be necessary to divide the wires between two or more slots placed at some distance apart, and in this case the E.M.F. produced by the inductors in one slot will not be in phase with that of the inductors in the second or other slots. The slots being spaced out cover in fact a certain proportion of the pitch, and a *grouped distribution* is obtained which falls midway between a concentrated winding and the uniform distribution of the smooth-surface armature. In the last named, a large number of phases, each differing little from its neighbour as we pass from one side of the coil to the other, are in effect compounded together. In the grouped winding of the slotted armature, a small number of sharply distinct phases are compounded, but as the slots between which the side of a coil is divided are increased in number, the distinction between the uniform and grouped distributions gradually vanishes. In all cases the effect of adding in series two or more E.M.F.'s differing in phase must be a reduction of the total as compared with the product of the E.M.F. of one inductor multiplied by the number in series.

§ 3. **The ratio of pole-width to pitch.**—The problem before us is therefore the best proportioning of the widths of field and coil, so as to make the most advantageous use of an armature of given dimensions running at a given speed. In the investigation of this problem, the three widths of the coil, pole, and gap between the poles must be measured along the mean circular path traversed by the rotating coil or magnetic field as the case may be. Thus in the discoidal or disc armature of fig. 64 the actual dimensions of the three will vary according as the circle along the circumference of which they are measured is taken at the outer or inner edge of the armature core or at its centre. It is, however, sufficient to consider the widths as measured on the circle passing through the centre of the length of the inductors and giving the mean path along which the line-cutting takes place. Such a circle is called the *mean pitch line*, the pitch in the case of the hetero-polar alternators now to be discussed being the distance measured along this line between the centres of a pair of neighbouring poles of opposite sign, just as in a toothed wheel the pitch is the distance measured along the pitch-line from the centre of one tooth to the centre of the next. If the ring armature of fig. 64 be imagined to be cut across along the line marked X and flattened out, as in fig. 65, A A is the pitch-line and *a b* the pitch. The widths of each field of magnetic lines, of the interpolar gap between two fields, and of the coil, are then all measured along the pitch-line, and are expressed in terms of the pitch.

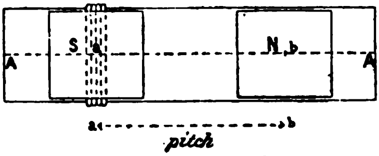


FIG. 65.—Development of armature showing pitch-line.

Since the N. and S. poles of the magnet system are in the same line round the armature, it is evident that there would be excessive leakage of the lines across from the edge of a pole into the adjoining edge of the next pole if the width of the pole-face were made equal to the pitch. The efficiency of the magnet would be thereby considerably reduced, and further, as will be more evident from Chap. XXIII., the inherent regulation of the machine would be bad, or, in other words, the drop of volts on an inductive load would be excessive. On the other hand, if the ratio of the pole-width to the pitch be made small, the flux of each field will be small, and the exciting coils on the poles will be comparatively uneconomical in weight of wire or in exciting watts. *Cæteris paribus*, the larger the area of a magnet-core the more cheaply is the flux obtained, since, *e.g.*, with a round core the area is proportional to the square of the diameter, while the length of an exciting turn is only proportional to the diameter. Evidently, then, it is not advisable for the ratio of the pole-width to the pitch to approach either of its two theoretical limits, viz., unity and zero.

§ 4. The ratio of coil-width to pitch in the ring armature. Differential action.—Turning to the ratio of the coil-width to the pitch, let the case of the ring and discoidal-ring armatures be first considered. In all cases the E.M.F. generated in each inductor as it moves under a N. pole will be in the opposite direction along its length to that which is induced in it when, as its rotation is continued, it passes under a S. pole; and when loops are wound on to a ring, so as to form a continuous coil, and it is sought to add together the E.M.F.'s generated in their active sides, these E.M.F.'s must be in the same direction along the lengths of contiguous inductors. Hence whenever the loops of one and the same coil are moving at the same time under two poles of opposite sign, the E.M.F. generated in the portion of the coil which is moving under a N. pole, will be in the opposite direction *round the coil* to that which is generated in the portion moving under a S. pole, and they will tend to neutralise each other; in other words, the two portions of the coil will act *differentially*, the resultant E.M.F. being only the difference between the sums of the E.M.F.'s generated respectively in the two portions.

First let the entire core be over-wound throughout with loops forming one continuous coil (fig. 66); in this extreme case it will be seen from the diagram that, whatever the width of the field, the single coil must always be under both poles at the same time, and consequently there will always be differential action between inductors on opposite sides of the core; further, the E.M.F. produced on either side being at any moment exactly equal, the resultant E.M.F. is nil. If, however, instead of forming one single coil, the winding be divided into two separate portions, as in the ring of fig. 67, where one of the two portions, B, is shown dotted, the action of B will be exactly similar to the action of the other portion, A, both in phase and intensity. It will suffice, therefore, in the first place, to consider the action of one half of the ring, A, apart from the other, B; having determined the proper width of coil A, we shall also have determined the proper width of coil B, and it will remain to be seen how the two coils A and B can afterwards be coupled together in order to produce a useful result.

Starting, therefore, with coil A of width equal to the pitch, we obtain therewith a maximum number of inductors, but accompanied with the disadvantages of a maximum amount of heating of the armature and a maximum inductance which in its turn affects the regulation of the machine. But, more than this, it is seen from fig. 67 that the outer loops of the coil are not so effective as the more central turns, and this in proportion to the width of the pole. Thus if the pole-ratio be nearly equal to unity, as in fig. 67, immediately that the coil A moves from the position shown, the outside loops at end *b* will be cutting lines emerging from the N. pole, while those at end *a* are still cutting lines entering into the S. pole. When the coil is exactly midway between the poles, as in fig. 68, no E.M.F. at all will be obtained from the coil as a whole, since the outside loops at *b* are cutting lines issuing from the N. pole at the same rate as those at *a* are cutting lines entering into the S. pole, and therefore their E.M.F.'s are exactly equal and opposite. A little later, however, the loops at *b* will more than counterbalance those at *a*, until once again the coil is central under one pole, and the maximum E.M.F. is produced. Hence, when the coil is entering or leaving a field, all the loops are not equally active; and there will be but little loss of E.M.F. if some of the loops at each end of a coil are removed and its width reduced.

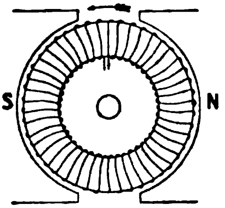
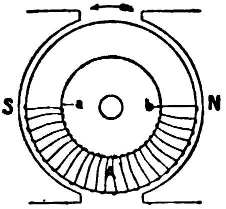
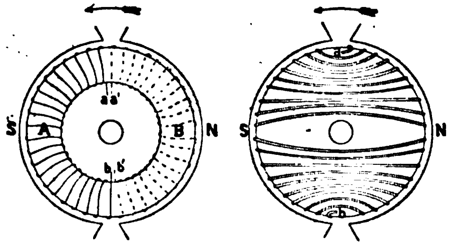


Fig. 66.



Position of greatest differential effect

Fig. 68.

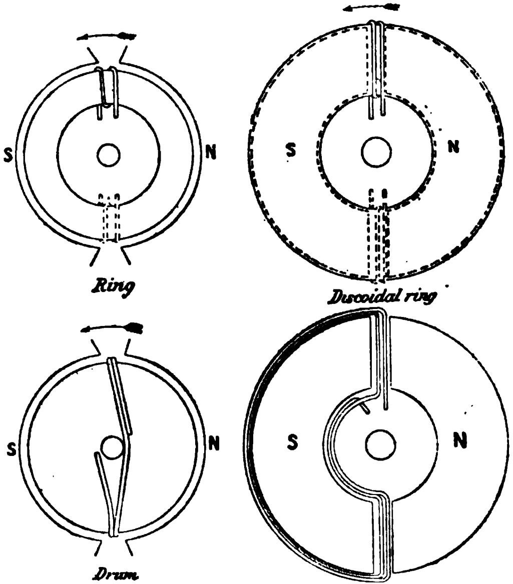


Ring *Drum*

Bipolar alternators with Maximum width of field & coils.

Fig. 67.

Inspection of fig. 68 shows that the time during which there is direct differential action depends on the amount by which the width of the coil exceeds the width of the neutral gap between the fields, and will therefore be almost avoided if the coil be reduced to a width equal to the gap. Owing to the spreading out of the lines from the edges of the poles, there will always be a certain amount of differential action so long as the coil has any appreciable width, but though, strictly speaking, it only vanishes when the coil is concentrated into a single line, its effect is of small importance when the width of coil does not exceed the width of the interpolar gap. Thus fig. 69 shows a ring arrangement in which the width of the poles is nearly equal to the pitch, but differential action is practically avoided by making the coil very narrow. When, however, this is so, there cannot at best be more than one or two loops, and therefore the total E.M.F. produced by the coil will be very low with any practicable density of field. Further, the



Bipolar Alternators with minimum width of coil & maximum width of field.

FIG. 69.

greater portion of the available space round the armature core on which loops could be wound will not be utilised, although, when moving centrally under one or other pole, the coil might usefully be wider.

Just as it was not found advisable to employ too large a ratio for the pole-width, so it is now found to be advisable to limit the coil-width to something less than equality with the pitch, and the above arguments are equally applicable to the case of grouped distribution when more than one slot per pole is employed. In modern alternators both field and coils are concentrated to different degrees; but the general rule to which most single-phase smooth-surface machines approximate may be considered to be a width of winding equal to the width of field, and each equal to half the pitch.

§ 5. **Connections for two ring coils in series or in parallel.**—We have now, therefore, discussed the proportions which should exist between the pitch and a coil wound on one half of the armature ring in order that it may give its due E.M.F. But a second coil may be wound on the other half of the armature core, diametrically opposite to the first, since all that has been said about the first coil will hold good equally for the second. If both the coils are of width equal to half the pitch, as shown in fig. 70, it will be seen that half the total surface of the armature core is covered with winding, and this is divided between two distinct coils. The second coil will give a useful E.M.F. exactly similar to that of the first coil, both in phase and intensity; and, as has been hinted, it can be usefully connected with the first, either in series or in parallel. To do so, it must be remembered that if the E.M.F.'s generated in the inductors forming parts of the coil A (fig. 67) are up out of the plane of the paper, the E.M.F.'s generated in B will be down through the paper. Each coil may be wound right- or left-handedly; just as a right-handed screw advances onwards when rotated right-handedly, *i.e.* away from the end at which it is turned in a clockwise direction, so a right-handed loop or coil if imagined to be screwed right-handedly round its core would advance onwards. Thence it follows for ring-winding that if we take any inductor on the *exterior* of the ring above the horizontal diameter and trace it *towards* our point of view and thence onwards, it will lead us on to another inductor or loop further towards our *right* if the coil be *right*-handed, or further to our left if the coil be left-handed; *e.g.* in fig. 67 both coils are wound left-handedly. Hence, if the two coils are each composed of one or any uneven number of layers, and are to be joined in series, and (1) they are both wound in the same direction, the end of one coil must be taken across to the further end of the second coil (fig. 70), the other two ends being attached to the two collecting rings; or (2) if they are wound in opposite directions the contiguous ends of the coils are joined together. If they are to be placed in parallel, the connections are exactly the reverse. Having regard to the rise of potential in each coil, and ease of connection, it will be seen, from fig. 70, that when the coils are in one or any uneven number of layers, for series connection, a direction of winding alternately right- and left-handed is the more convenient arrangement; while for parallel connection a similar direction of winding in the two coils is the better.

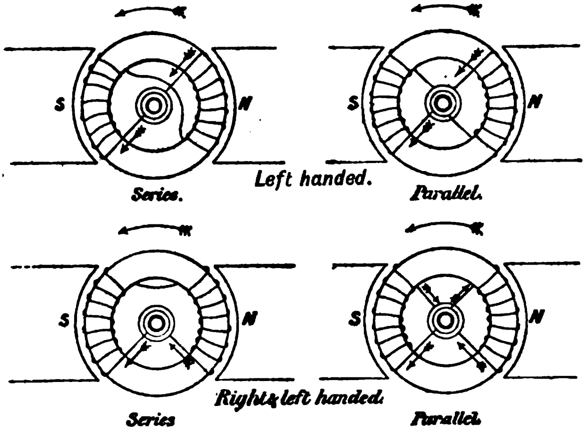
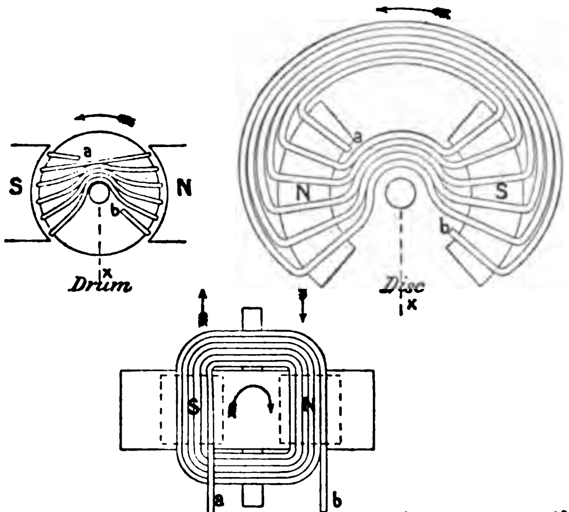


FIG. 70.—Series and parallel connections for ring alternator.

§ 6. **Ratio of coil-width to pitch in drum and disc armatures.**—On passing to bipolar alternators whose armatures are wound on the drum or true disc method, exactly the same considerations limit both the width of the pole and the width of the coil as in the case of ring winding. The pole must be made appreciably less in width than the pitch, in order to avoid excessive leakage between adjacent pole-edges; on the other hand, if very narrow, the cost of the copper in the exciting coils or the energy absorbed in the excitation will be unduly high as compared with the total flux produced by them.

From the nature of the winding, the maximum width which a coil can have is twice the pitch, each side having a width equal to the pitch, as is shown in the diagram of a drum armature in fig. 67. Such an arrangement is the exact equivalent of the ring-wound armature shown at its side, wound with two coils each having a width equal to the pitch, and connected together in series; there is again the same objection that if the field have any appreciable width, differential action is set up as soon as the coil is rotated away from the position shown in fig. 67. In the drum and disc, as opposed to ring or discoidal, armatures, while the opposite sides of each loop must always be moving under poles of opposite sign, *one side* of a coil must never be moving at the same time under two poles of opposite sign, if there is to be no differential action.

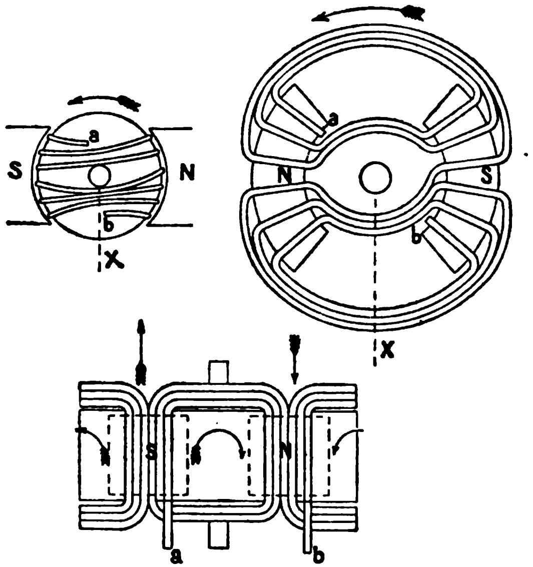
The time during which such differential action lasts can be reduced by decreasing either the width of the winding on each side of the coil or the width of the field, or both. If there is never to be any differential action at all, and the width of field be retained at its maximum, the side of the coil must have no appreciable width, and the inner and outer loops of the coil must be taken off, until only one or two remain, and the width of the coil is equal to the pitch (fig. 69). If, on the other hand, the maximum number of inductors is still retained, the width of the field must be reduced to a mere line in order to avoid all differential action. Considerations of heating, efficiency, or regulation would lead to a width of coil and a number of inductors less than the maximum possible, so that again we are led to compromise in the widths both of winding and of field. If the inner loop which has the minimum width is less in width than the pole-face, opposite sides of a loop will never be moving under the same pole; further, if the width of the winding in one side of a coil is less than the width of the gap between two fields, one side of a coil will never be moving under two poles of opposite sign. Under these conditions, which are represented in the drum and disc armatures of fig. 71, direct differential action will be avoided. The second of the two conditions determines the advantageous width of winding in the bipolar drum or disc alternator when wound with a single coil, as in fig. 71; but this single coil may equally well be regarded as divided into two separate coils, each giving an E.M.F. exactly similar in phase and intensity, and connected in series. If the drum or disc armature of fig. 71 be cut down to the spindle



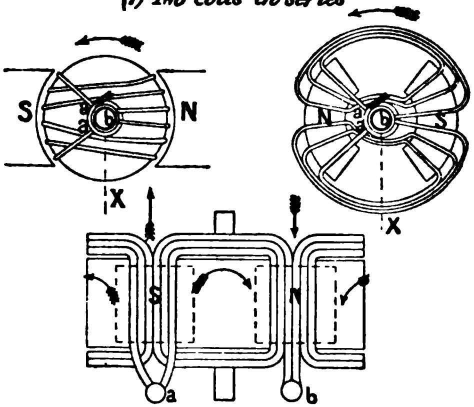
Bipolar alternator armatures with winding in one coil

FIG. 71.

along the line x o and flattened out, then if the inactive end-connectors are entirely on one side of the spindle, as shown, the development of the flattened-out armature shows in plan one single coil containing two groups of inductors with E.M.F.'s in opposite directions, as viewed from either end (fig. 71). If, however, the single coil be divided into two halves by taking one half of the connectors round on either side of the shaft, as in fig. 72, the same two groups of inductors are divided into two separate coils in which the current circulates in opposite directions



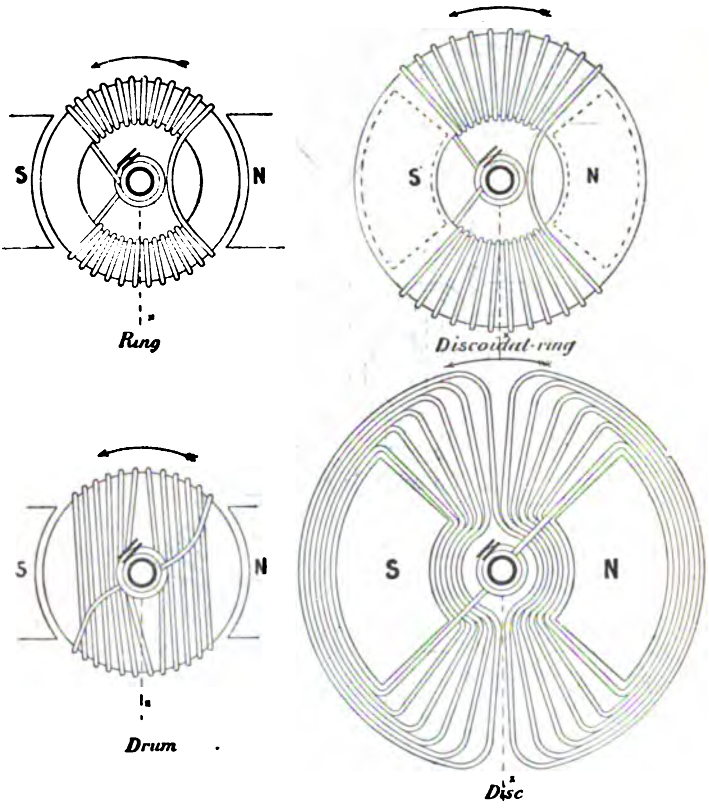
(1) Two coils in series



(2) Two coils in parallel

FIG. 72.—Series and parallel connections for bipolar drum and disc alternator armatures with winding divided into two coils.

without in any way affecting the total E.M.F. When so divided the depth of the connecting pieces, which join together the inductors, is halved, and consequently space is economised, as is seen by comparing the bipolar drum and disc armatures of figs. 71 and 72; and this advantage becomes even more marked in the case of multipolar drum and disc alternators. Further, the two separate coils can now be connected in parallel equally as well as in series; when they are to be connected in parallel it is advan-



Ring

Discoidal-ring

Drum

Disc

FIG. 73.—Bipolar alternators with both width of winding and width of fields equal to half the pitch.

tageous to wind them in opposite directions, as shown in fig. 72 (2), since then the adjacent wires of the two coils are always at the same potential. When thus divided the width of each of the two coils is equal to the pitch; in other words, the outer loop spans the diameter of the armature, and the width of each side of each coil is equal to half the gap between the fields.

We thus arrive at two coils for a bipolar drum or disc, as for a ring, alternator, *i.e.* as many coils as there are poles in the case of drum, or as there are fields in the case of disc alternators; further, the proportions which are found in practice to be most advantageous in a single-phase alternator are in general width of inner loop equal to width of field, and both equal to half the pitch, the outer loop having a width equal to the pitch. Or to express the same proportions in a way more suitable to the divided coil, the width of a group of inductors forming the adjacent sides of two coils is equal to half the pitch. This leads to the same result as for the ring-wound armatures, one half of the armature core being covered with winding. Fig. 73 represents diagrammatically equivalent armatures of the four types, each with its coils in the position of reversal of direction of the E.M.F., and all avoiding any direct differential action; since there are the same number of inductors similarly disposed in all the machines, their curves of impressed E.M.F. will be analogous, and in all will be approximately sinoidal in shape.

§ 7. **Multipolar alternators. Periodicity.**—When entering or leaving a magnetic field, all the turns are not equally active, but only gradually come into play, the inductive effect of each inductor, as it enters the field, being added on to the E.M.F. produced in those inductors which are already cutting the lines of the pole under which they are moving. Owing to this action the E.M.F. of each coil as a whole rises gradually to a maximum when centrally situated under a pole, and then gradually falls to zero, to rise again to a maximum in the opposite direction. The curve of impressed E.M.F. for a whole coil during one complete revolution has therefore an undulatory shape very similar to the sine curve of fig. 24; thus in fig. 74 each of the small curves represents the E.M.F. induced in one turn of the coil, and by the addition of these small curves the large curve is obtained which closely resembles a sine curve, although its component curves are much flatter and show no marked peak.

Thus in single-phase alternators the inductors are concentrated into coils sharply separated off from one another by intervening spaces of core on which there is no winding. Although less marked in the case of polyphase-machines, this characteristic is still present in all cases, and will be found to distinguish the alternator not only in action but even in appearance from the dynamos to be described later which give a unidirected current.

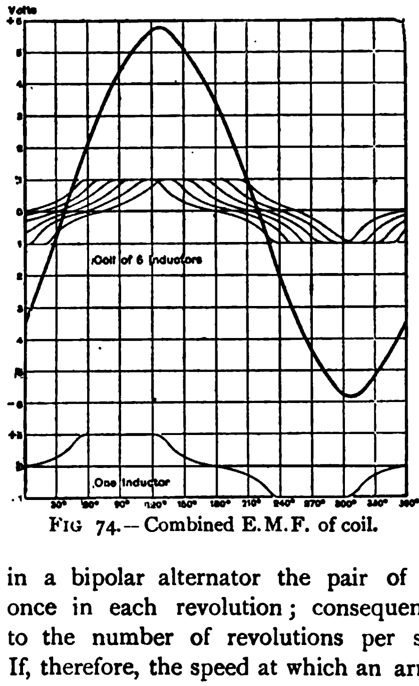


FIG 74.—Combined E.M.F. of coil.

In most commercial alternators it is desired to produce a large number of alternations per second, or a high 'periodicity,' ranging from 40 to 140 complete periods per second, and, further, a high E.M.F., such as of 1000 or 2000 volts. Each complete period or undulation of the E.M.F., as shown in fig. 74, is produced by the passage of a coil through two magnetic fields in opposite directions, and in a bipolar alternator the pair of fields is passed through once in each revolution; consequently its periodicity is equal to the number of revolutions per second (Chap. IV. § 15). If, therefore, the speed at which an armature of sufficient size to give the required output can be safely or conveniently driven is not sufficiently high to give the desired periodicity, it is necessary while retaining the same width of winding on the armature relatively to the width of field to make the coils pass through more than one pair of fields in each revolution. In other words, the alternator must be multipolar. If we suppose the bipolar alternators of fig. 73 to be divided along the line o x by a cut passing through the armature core down to the spindle, and then to be opened out, so as to form a portion of a large cylinder with a flatter surface or a larger disc, we obtain the results shown by full lines in the left half of fig. 75. From

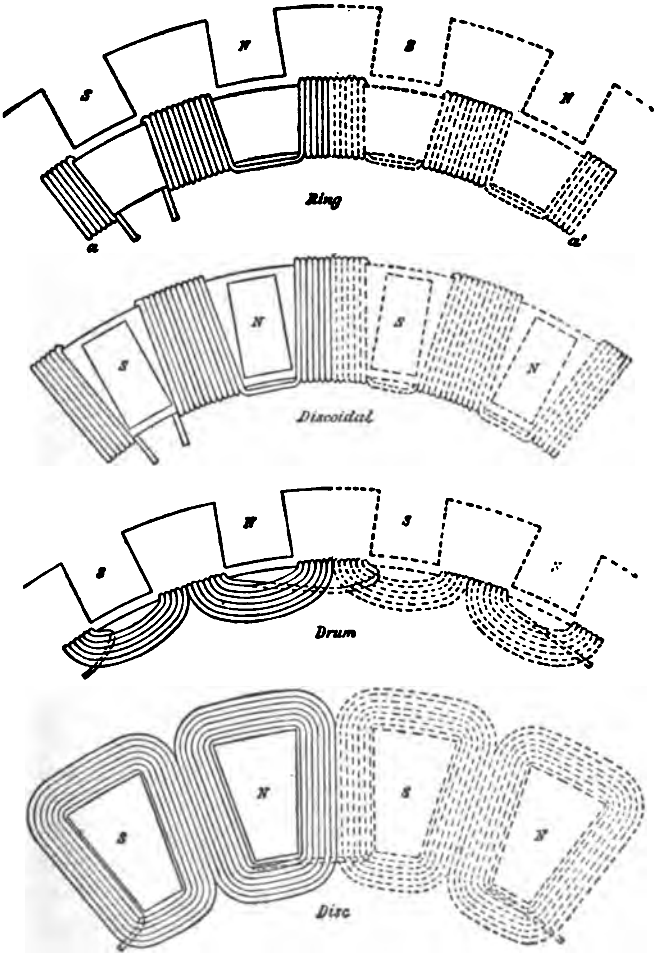


FIG. 75.—Portion of armatures of multipolar alternators.

this figure it is evident that the pair of fields to the left can be repeated indefinitely any number of times, if they are arranged symmetrically round the enlarged armature core, so as to form a complete circle, a N. pole and a S. pole succeeding each other alternately all round; and the dotted portion to the right-hand side of fig. 75 shows a pair of fields thus repeated. By thus multiplying the number of pairs of fields through which the coils must pass in each revolution, the number of complete undulations of the E.M.F. of the coils shown in full lines is increased correspondingly. If p = the number of pairs of fields, each pair is passed once every revolution, so that the number of periods in one revolution is p, and if N = the number of revolutions per minute, the periodicity or frequency of the multipolar alternator is $f=\frac{p\text{N}}{60}$.

In the case of ring and drum alternators, p is also the number of pairs of poles; but in the case of discoidal-ring and disc machines the number of pairs of fields is equal to the number of pairs of poles counted on one side only of the armature, the opposite poles forming a portion only of the same fields.

Hence, by increasing the number of pairs of fields, the periodicity of the *multipolar alternator* for a given number of revolutions per minute can be raised to any required figure. A very usual periodicity is 50, or, as it is symbolically expressed, 50 ∿: the larger the output and size of the alternator, the slower the speed at which it is desirable to drive it; consequently the number of poles required to give a periodicity, or 'frequency,' of 50, will range from 8 poles with 750 revolutions per minute to 30 poles with 200 revolutions per minute, or double these numbers for a periodicity of 100.

But it is not only the pairs of fields which can be multiplied: the coils can also be repeated for every pair of fields, and fig. 75 shows by dotted lines a second pair repeated and connected in series with the original pair. We thus obtain in the multipolar alternator several pairs of fields, and as many coils as there are fields, both being arranged symmetrically in circles about a common centre or axis; all the coils will occupy at any moment exactly the same position relatively to a magnetic field, and the phase and intensity of the alternating E.M.F. induced in each will be identical. Hence they can be coupled together in series, so as to form a continuous winding, or can be divided into two or more parallels, according as we desire a high E.M.F. or a large current; and the total E.M.F. of the machine at any moment is equal to the E.M.F. induced by any one coil multiplied by the number of coils that are in series. Thus by increasing the number of coils that are in series, the E.M.F. of the alternator is increased, just as the periodicity is increased by increasing the number of pairs of fields. The presence, therefore, of several pairs of poles and coils introduces nothing new in the theory of the action, and the multipolar alternator may be simply regarded as made up of several bipolar alternators. The principle deciding relative widths of fields and of winding in each coil remains the same, and with the proportions generally adopted in single-phase alternators approximately one-half of the armature core is covered with winding.

§ 8. **General description of single-phase multipolar alternators, ring, drum, discoidal and disc.**—The methods of connecting up the coils in series or parallel are the same as those above described for bipolar alternators; if the entire winding be in series, the full difference of potential of the machine exists between the adjacent coils which form the beginning and the end; hence in the cases of drum and disc machines, it is sometimes advantageous to divide the winding into two halves in parallel, so that each coil then carries only half the total armature current. When this is done, it is best, as already explained, to retain the same direction of winding at the junction of the two halves in the case of ring and discoidal-ring armatures, but to reverse it in the case of drum or disc armatures. In all cases it is indifferent whether the armature or the field magnets rotate. In the former case the ends of the armature winding are brought to two *collecting rings*, which rotate with the armature spindle, and upon which rest the stationary brushes forming the terminals of the external circuit. In the latter case stationary terminals are attached to the armature, but collecting rings have to be applied to the system of rotating magnets, into and out of which the exciting current for the field winding flows by means of brushes or rubbing contacts.

The form of the magnetic circuit and of the magnets which suits each of the different types of armatures remains to be considered. In the cases both of the ring and drum-wound armatures, the armature may be internal or external to the poles, these latter being in the one case united by a common circular yoke-ring external to the whole (fig. 76), or in the other by an internal ring, so as to form a kind of star (as in fig. 77). Fig. 76 shows diagrammatically a ring-wound armature, surrounded by twelve external poles, pointing radially inwards, and each wound with an exciting coil, *e e*; one-half of the machine is shown in section in the end view, and one quarter in the side view, the paths of the magnetic induction being there marked by dotted lines. It will be seen that the total flux of lines forming one field passes entire through each magnet core, and bifurcates on reaching the armature core or the yoke-

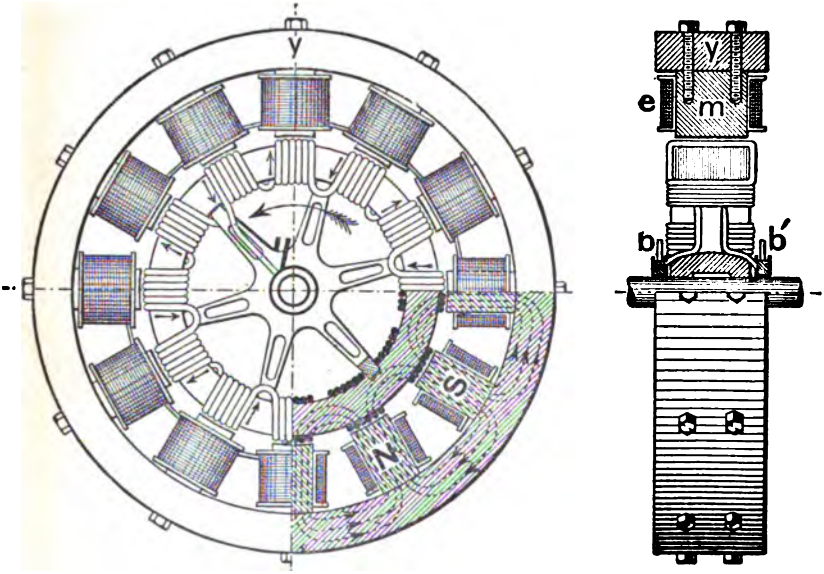
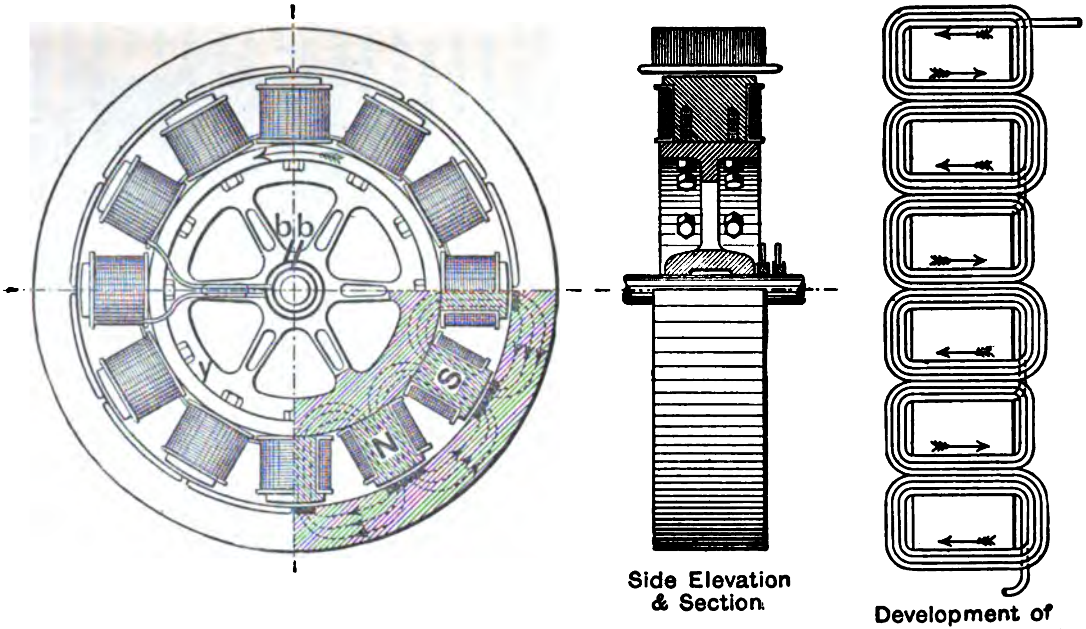


FIG. 76.—Diagram of single-phase ring multipolar alternator.



Side Elevation & Section

Development of half of Armature

FIG. 77.—Diagram of single-phase drum multipolar alternator.

ring, *y*. In practice the system of magnets would be supported from the bed-plate, which is omitted in the diagram for the sake of clearness. The twelve armature coils are connected to form a single series; the inductors of each coil are shown as disposed in one layer only of six turns, although in most cases there would be a large number of turns, possibly in two layers. The current is collected by the brushes *b b′*, bearing on the two collecting rings, which are placed on opposite sides of the armature. Fig. 77 shows an equivalent drum-wound alternator with internal poles, which rotate while the armature remains stationary. Current is supplied to the exciting coils by the brushes and collecting rings, while the alternating E.M.F. and current is obtained from the stationary terminals of the armature. At the right hand is shown an internal view of one half of the armature winding after removal of the magnets. Just as the ring armature can be adapted to suit a revolving internal magnet, so also the drum method of winding may equally be applied to the outside of a cylindrical iron core, which revolves within an external set of magnets. In the case of the discoidal-ring and disc machines (figs. 78 and 79) the magnets *m m* form two crowns, being united by two circular yoke-rings, *y y*, one on either side of the armature. In the former or flat-ring machine, poles of the same sign are presented exactly opposite to each other on either side of the armature core, and the lines which enter into it from a pair of N. poles flow in either direction through the armature, issuing forth where a neighbouring pair of S. poles is presented; in the disc alternator, on the contrary, opposite poles are of opposite sign, and the lines flow straight across the air-gap from one pole to another. In figs. 78 and 79 the left-hand lower quarter of the side view shows the armature with the nearer field-magnets and yoke-ring removed, and the right-hand lower quarter shows the further system of poles, the armature itself being removed. In both the winding is arranged in series; the magnet cores on which the exciting coils are wound are oval, while the pole-pieces have a trapezoidal shape, the width of field being equal to the width of winding in a coil when measured on the pitch-line or mean circumference of the armature.

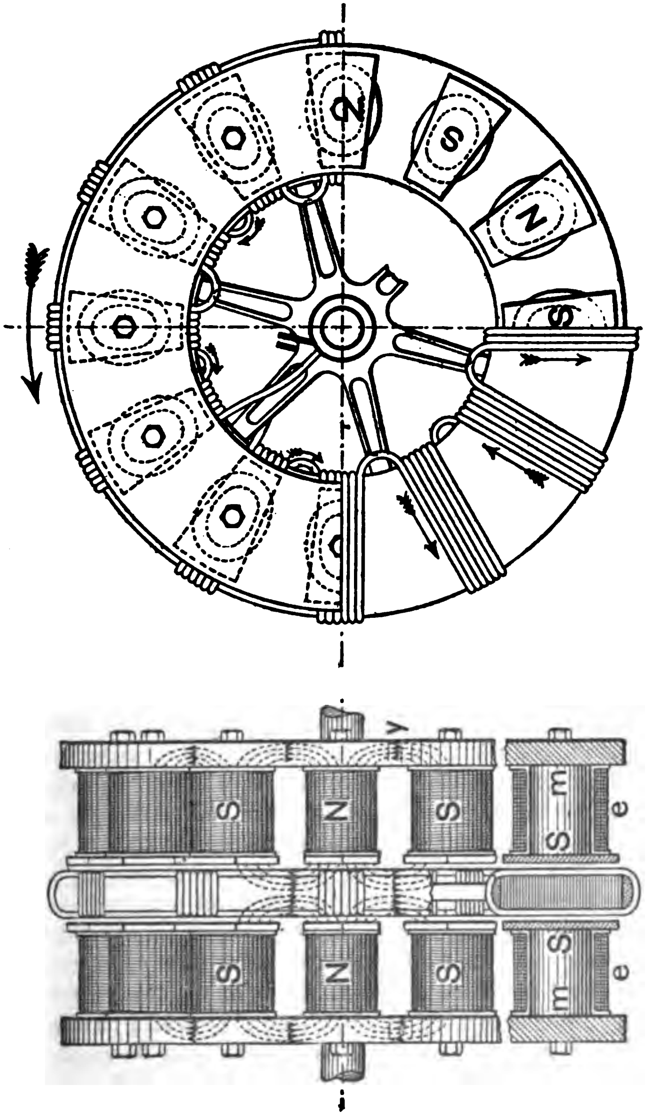


FIG. 78.—Diagram of single-phase discoidal-ring multipolar alternator.

In the disc alternator (fig. 79) the winding is analogous to that of the drum armature of fig. 77, except that twelve inductors are shown in each coil. The coils being arranged so as to form a complete circular disc, their shape, as shown in fig. 79, is more or less trapezoidal or pear-shaped, being broadest on the external periphery of the armature, and gradually tapering inwards along their radial depth. The polar surfaces are shown of a shape corresponding to the inside loop of a coil; as in the other alternator diagrams, the width of field is equal to the width of winding when measured on the pitch-line, and both are equal to half the pitch.

If the pole-pieces have a rectangular or trapezoidal shape in the case of discoidal rings and discs, the whole length of each inductor does not enter or leave the field at once and instantaneously; consequently it does not begin or cease to cut lines simultaneously throughout its entire active length, but the action is at first confined to one or other end (fig. 80). The effect of this is to render the rise and fall of the induced E.M.F. both in each inductor and in the coil, as a whole, more gradual than it would otherwise be, the curve of induced E.M.F. being therefore rounded off into a gentle sweep throughout.

In general, the exact shape of the curves of E.M.F. and current admits of very great variation with different dispositions of iron and copper in field magnet and armature. (See Chap. XXII.)

§ 9. **Polyphase alternators.**—In the single-phase alternators so far described, it has been stated that, roughly speaking, only one half of the armature core is covered with winding; it is therefore possible to wind an entirely distinct set of coils forming a second armature circuit in the vacant spaces between the coils of the first circuit. The second set of coils may be used to feed an entirely separate external circuit, possibly at a different pressure; or if the two windings are identical as regards number of inductors and size of wire, they may be inter-connected to form a *quarter-phase alternator*. The phase of the E.M.F.'s in the two circuits will differ by a quarter of a period, or 90°, the one reaching its maximum when the other is at zero. This process may be carried still further if the width of winding in each coil be reduced to less than half the pitch; *e.g.* if in a ring-machine the width of winding be

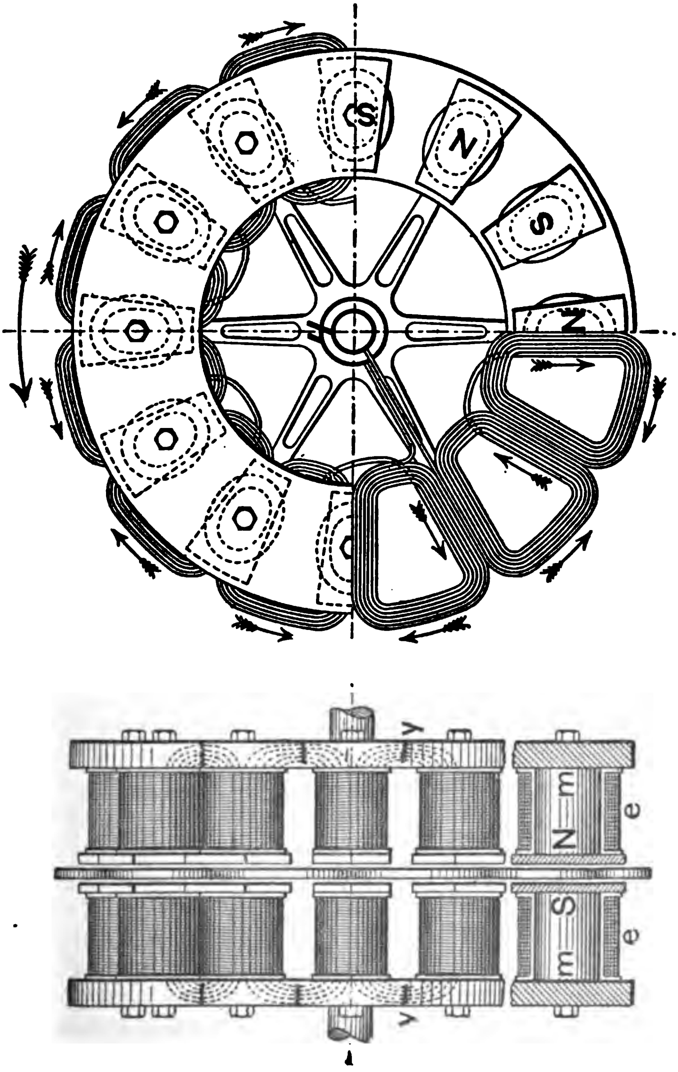


FIG. 79.—Diagram of single-phase disc multipolar alternator.

one-third of the pitch, or if in a drum machine the width of each side of the divided coil be one-sixth of the pitch, three distinct armature circuits can be wound on the same core, giving curves of E.M.F. differing in phase by 120°, and a *three-phase alternator* is obtained. The process of subdivision may be carried still further if desired, but in practice the largest number of phases which has been used is six. Since 1890, such alternators have come into prominence in connection with the use of alternating-current motors with rotating magnetic fields, and have been called in general *polyphase* machines. It will suffice here to call attention to the possibility of winding alternator armatures with sets of coils so spaced that the E.M.F. induced in each set differs in phase from that induced in any other set by a certain fraction of a period; the subject will again be referred to in Chap. XXII.

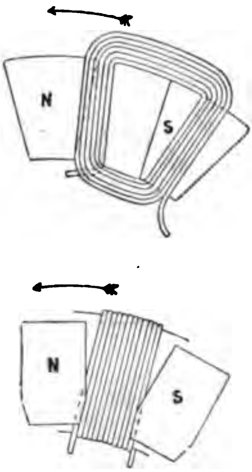


FIG. 80.

§ 10. **The E.M.F. equation of the heteropolar alternator.**—The fundamental equation of the E.M.F. induced in the armature of a heteropolar alternator can now be given a more fully developed form. Let z_a = the total number of C.G.S. lines which issue from any one pole and pass into the armature, or which emerge from the armature and enter into a pole of opposite sign. In the ring, drum, and discoidal multipolar armatures the lines forming a single field in the air-gap bifurcate within the armature, half leaving by a neighbouring pole on one side and half by a neighbouring pole on the other side, while in the disc machine they pass straight through the armature without division; but in every case z_a is the total flux corresponding to one pole which passes into and through the armature. Since each inductor cuts these lines first as they enter the armature core in one direction and then a second time as they make their exit, the total number of lines cut in one revolution by any one inductor is $2z_a . p$, where

p is the number of pairs of poles. As explained in § 7 of this chapter, in the case of disc and discoidal machines p is rather the number of pairs of fields, or the pairs of poles must be reckoned only on one side of the armature; further, in the discoidal machine, any pair of similar poles facing each other on opposite sides of the armature is to be considered as forming one joint polar surface from which z_a lines enter the armature core.

Now the time in seconds taken to perform one complete revolution is $\frac{60}{N}$, where N is the number of revolutions per minute. The average E.M.F. induced in each inductor in one revolution being proportional to the total number of lines cut divided by this time is therefore $2z_a \cdot \frac{pN}{60} \cdot 10^{-8}$ volts. Let t be the number of inductors in series in each coil; *i.e.* in the ring coil t is also the number of turns in the coil, and in the drum coil divided into two halves, as is the more usual case (§ 6), is equal to the number of wires or twice the number of turns in the half coil; while if the drum coil is undivided, t is the number of wires in one side or the number of turns in the large coil. Thus in all cases t corresponds to the number of inductors in one group, and each such group corresponds to one pole. If the t inductors are so closely adjacent that any difference of phase between them may be neglected, each will simultaneously generate an E.M.F. of the same value and in the same direction round the coil, so that the average E.M.F. of the whole coil is $2z_a \cdot \frac{pN}{60} \cdot t$. Such a case would be practically realised in a toothed or tunnelled armature when the inductors of one coil are all wound in the same slot or hole. If, however, each ring coil or each side of a drum coil is distributed between two or more slots, or in the case of a smooth-surface armature if the winding of the coil measured on the pitch-line has appreciable width, there must be a certain amount of differential action as the coil passes across the neutral line between two poles; especially will this be the case if the width of the winding exceeds the width of the gap between two neighbouring poles of opposite sign. A distinction therefore exists between the *gross average E.M.F.* of the coil on the supposition that there is no differential action at all within the coil, and the ***net average E.M.F.*** which is the arithmetical mean of all the instantaneous values of a half-wave of the actual E.M.F. Thus the gross average E.M.F. of the coil, on the supposition that each inductor always assists every other, is the same quantity as before, viz. $2Z_a \cdot \frac{pN}{60} \cdot t$, while the actual net average is less in proportion to the amount by which the E.M.F. of some inductors is at times neutralised by that of others. The effect of differential action on the average E.M.F. may therefore be discounted if the gross value is multiplied by a certain *width coefficient*, k''. The net average E.M.F. of the coil is then

$$k'' \cdot 2Z_a \cdot \frac{pN}{60} \cdot t \times 10^{-8} \text{ volts},$$

where k'' is the ratio of the net to the gross E.M.F. The value of k'' will depend upon the ratio of the coil- and pole-widths to the pole-pitch, and while it may approach to unity it must always be less if the coil has any width.

The shape of the instantaneous E.M.F. curve of the coil still has to be taken into account in order to find its effective E.M.F. as given by the square root of the mean square; for with the same average value the E.M.F. curve may be given any form that we please by so shaping the pole-pieces as to give the required variation in density of the lines in the air-gap where they are cut by the inductors. Let k' = the ratio which the square root of the mean square bears to the mean ordinate of the E.M.F. curve, its value being also dependent upon the relative widths of coil and pole; then the effective E.M.F. of the coil is

$$\begin{aligned} e_e &= k' \times \text{ the net average E.M.F.} \\ &= k' \cdot k'' \cdot 2Z_a \cdot \frac{pN}{60} \cdot t \times 10^{-8} \text{ volts}, \end{aligned}$$

and k' is known as the *form factor*, since it varies with the shape or form of the E.M.F. curve.

In any one phase there are $2p$ coils each containing t inductors, and these may be divided into q parallels where q may be one or two or any whole number of which p is a multiple. Hence the effective E.M.F. of the one phase is $E_e = \frac{2p}{q} \cdot e_e$. Finally, if m be the number of separate phases into which the armature winding is divided (§ 9), and τ be the total number of inductors counted all round the armature periphery, the number of inductors in one coil is $t = \frac{\tau}{m \cdot 2p}$; therefore the effective E.M.F. induced in one phase of the armature is

$$E_a = \frac{2p}{q} \cdot k' \cdot k'' \cdot 2Z_a \cdot \frac{pN}{60} \cdot \frac{\tau}{m \cdot 2p} \times 10^{-8} \text{ volts.}$$

Uniting the two factors k' and k'' into one joint coefficient K,[1] we have

$$E_a = \frac{K}{mq} \cdot 2Z_a \cdot \frac{pN}{60} \cdot \tau \times 10^{-8} \text{ volts} \quad . \quad . \quad . \quad . \quad (11)$$

Here $\frac{pN}{60}$ is the frequency of the machine, and $\frac{\tau}{mq}$ is really the number of inductors in series in one phase. The numerical value of the constant K will vary with the shape of the poles as affecting the distribution of the lines and also with the type of machine, yet in the main it depends upon the *relative* widths of the coils and fields as compared with the pitch; certain general cases may therefore be taken which will serve as guides to practice, and in Chap. XXII. the values for such cases will be tabulated. It need only here be remarked that if the curve of E.M.F. be sinusoidal, as is often assumed for approximate treatment of alternating problems, the form factor being the ratio of the effective to the net average value is $k' = \frac{\pi}{2\sqrt{2}} = 1{\cdot}11$; since, as follows from Chap. VI. § 17, the square root of the mean square, or the effective value of the sine $= \frac{1}{\sqrt{2}}$, while the average value of the sine is $\frac{2}{\pi}$.

---

[1] In many books, the numerical constant 2 is likewise thrown into the combined coefficient, which then becomes for a sine curve 2·22.

If the E.M.F. equation of the alternator in its fully developed form be now compared with the fundamental equation (1), it will be seen that each of the three variables still finds its appropriate equivalent; instead of the density B_g there now appears the total flux of one field Z_w, for the simple length of the one inductor is substituted the total number of inductors τ, and for the velocity of movement the number of revolutions.

§ 11. **The mechanical force in an alternator.**—The mechanical-force equation of the alternator does not admit of any simple expression. Since the magnetic pull on any inductor or group of inductors involves the term B_g, it is evident that its instantaneous amount depends on the shape of the curve of B_g. But, further, the current c is always varying, so that the mechanical effect would require to be derived by considering the product of the instantaneous value of the current in all the inductors and the density of field in which they are situated. The curve obtained by plotting this product relatively to time for a complete period would evidently be of an undulatory character, and the maximum drag in a single-phase alternator would be greatly in excess of the average value. In consequence, the mechanical strength of the alternator armature or the support of its windings must be given ample margin of safety to withstand the maximum force to which it is subjected. The necessity for ample strength is, however, even greater than the above considerations would indicate. The effect of armature reaction in the alternator has been treated in Chap. VI. on the basis of two superposed fields, the one of which is the main field due to the field excitation, while the other is self-induced by the armature current. If now the external circuit has no inductance, and if the two component fields of the alternator were strictly confined to one and the same magnetic circuit or system of circuits, the resultant curve of the field-density would correspond precisely in phase with the current curve. Under these circumstances, the direction of the tangential drag on the armature would be always opposed to the direction of rotation, and would merely fluctuate in amount during the period. But as a matter of fact the magnetic circuit of the self-induced flux is not entirely the same as that of the main flux. The case then becomes analogous to fig. 35, and the curve of flux-density in the main field corresponds to the impressed curve of E.M.F. E_t; the product of the instantaneous values of this with the current values for the same instants when plotted will now give certain areas of negative work as in fig. 36. The current in the armature inductors lags behind the induced E.M.F. and behind the B_g curve; the result is that the direction of the pull on the inductor changes twice in each period, and for short intervals the alternator is driven forwards as a motor, or gives out mechanical energy instead of absorbing it. If the external circuit itself has inductance, an entirely separate magnetic circuit (or circuits) is evidently added, and there is still less correspondence between the phase of the main field and that of the current. In other words, the angle of lag is increased, and the forward impulses on the machine become more powerful and last longer as compared with the backward drags, although they must always be less in amount so long as the machine continues to act as a generator. Thus the mechanical action on an alternator, partly from its own inductance and still more from any inductance in its external circuit, is of a 'racking' nature which subjects it to strains of greater severity than are found in the equivalent continuous-current dynamo.

CHAPTER IX

HOMOPOLAR ALTERNATORS

§1. Difficulties in homopolar continuous-current machine.—While the bipolar dynamo may be said to be naturally an alternator, and only indirectly and artificially a continuous-current machine, it may with equal truth be said that the homopolar dynamo naturally gives a unidirected E.M.F., and is artificially made to yield an alternating E.M.F. At first sight it might be supposed that a continuous-current machine which gives a perfectly constant and steady current without any need for continual starting, stopping, and reversing of it in the inductors, and which, therefore, requires no commutator, with all its attendant troubles of sparking at the brushes, must possess considerable advantages over the bipolar continuous-current machine; it was, too, the earliest form of dynamo and motor invented, since it dates from the years 1823 and 1831, when a disc homopolar motor and dynamo were respectively made by Barlow and Faraday. Yet practical experience shows that heteropolar machines have been and are in possession of the entire field of continuous-current work, and that there are inherent difficulties in the homopolar dynamo which have up to the present baffled all the ingenuity of inventors.

The chief of these is the small value of the E.M.F. which can be procured with the machine in its simplest form, even with a very high speed of rotation. It has, in fact, but one inductor, and therefore yields at best only from two to five volts. The remedy would be to connect several inductors in series, so as to add up their E.M.F.; yet if the reader refers back to Chap. VII. it will be seen that the methods of connecting inductors in series as there described only result in forming loops, round which the E.M.F. and current alternate, the characteristic feature of the dynamo being thereby entirely lost. If we take the rotating armatures of fig. 59, with a number of insulated inductors arranged either in a tube or in a disc, and join the end of one inductor on to the opposite end of another by means of a connecting wire fixed to the two inductors and rotating with them, it will be found that these connecting pieces, when so fixed, must necessarily cut the lines of flux at exactly the same rate as the inductors proper, and in the same direction; they thus become inductors themselves, producing an E.M.F. which is in the reverse direction round the loop to that given by the inductors proper, and the total E.M.F. of the combination is nil. Hence if the armature rotates as we have supposed, the connecting pieces must be stationary, so that they can be taken through the iron of the field-magnet without cutting its lines, and the current must pass into and out of them by some kind of rubbing or sliding contact. There is, in fact, no method of *winding*, properly so called, which is applicable to the homopolar armature, so long as it has to give a unidirected E.M.F. All that can be done is to join one inductor on to another through the agency of rubbing contacts, and the multiplication of these at once introduces such difficulties as to render the whole arrangement unworkable. In practice, the utmost that has been done may more properly be described as the placing of two distinct machines in series, the two being mounted on the same spindle; yet even then the twofold collection of a very large current by means of brushes or mercury troughs presents great difficulties.

§ 2. **Homopolar ring and discoidal-ring alternators.**—It will suffice, therefore, to pass immediately to *homopolar alternators*, in which one or other of the definite methods of homopolar winding described in Chap. VII. §§ 14–17, has been adopted. As in the case of bipolar armatures, it will be necessary, when we deal with coils instead of simple loops, to determine the best widths of winding and field, and the same principles must be our guide.

In homopolar machines with one or more fields separated and marked off by a neutral space or spaces through which very few lines of flux pass, the general definition of the pitch requires to be varied in order to meet all cases. It is now the distance measured along the pitch-line between the centre of a field and the centre of the gap or space between it and the next field. In the ring and discoidal machines there are in fact two circles or 'crowns' of poles and two pitch-lines, one on the outside and one on the inside of the ring armature, or one on either side of the discoidal-ring. In these cases, however, the two pitch-lines may be supposed to be transferred to a single circle passing round through the centre of the armature core. Since a pole is opposite to each neutral space, the pitch may then be equally well expressed as the distance measured along the single pitch-line between the centre of a pole and the centre of the next pole, and the definition becomes identical with that of the heteropolar alternator. The only difference is that now such two poles are either outside and inside the armature core in the case of the ring, or on opposite sides in the discoidal machine. While in the heteropolar alternator it is impossible for the width of pole to exceed the pitch, this is possible in the homopolar type, yet is not permissible for electrical reasons.

As shown in Chap. VII. § 15, the maximum permissible width of field, even with only one loop, is not more than the pitch, since otherwise differential action reduces the time during which any resultant E.M.F. is produced. The field can, however, be as wide as the pitch without the leakage being in any way detrimental, since the two sets of poles of opposite sign are well separated from each other, and on this account the ratio $\frac{\text{width of pole}}{\text{pitch}}$ usually approaches unity more nearly than in the corresponding machines described in Chap. VIII. The coil-width will have values much the same as in the heteropolar machine, since if the ratio $\frac{\text{width of coil}}{\text{pitch}}$ approaches unity, there will be considerable differential action, and the outer loops are not well placed for the production of E.M.F. If direct differential action is to be entirely avoided, the outer loops at the sides or edges of one and the same coil must never be moving at the same time past two poles of opposite sign, whether these are outside and inside, as in the ring form, or on each side of the armature, as in the discoidal form; otherwise opposite sides of the same coil are under inductive action at the same time, and the wires which for the time being are connectors will also act as inductors of a back E.M.F., and so neutralise the effect of the inductors proper. Hence the width of the coil must be not more than the width of the neutral space between two fields, where very few lines enter or leave the armature core. If the armature cores of figs. 60 and 61 be thus wound with a pair of coils, each of width equal to half the pitch, and the fields be correspondingly reduced in width, we obtain a dynamo which, when cut through to the shaft and opened out, is shown diagrammatically in the full-line portion of the upper diagram in fig. 81. Differential action is then avoided, yet if the width of the poles, as will often be the case, exceeds half the pitch, the coils may need to be wider than the neutral gap, and the above rule cannot be strictly adhered to. The path of the lines in fig. 81 is shown dotted, with the poles outside and inside the core, as they would be in the case of the ring form. If the armature were discoidal, it will be understood that the poles would then face opposite sides of the core, and the lines would pass aslant across the armature. As in the case of bipolar alternators, the whole can be repeated a second or any number of times, as shown by the dotted half of fig. 81, and thus is obtained a homopolar alternator with two or more pairs of fields, the armature coils being similarly multiplied.

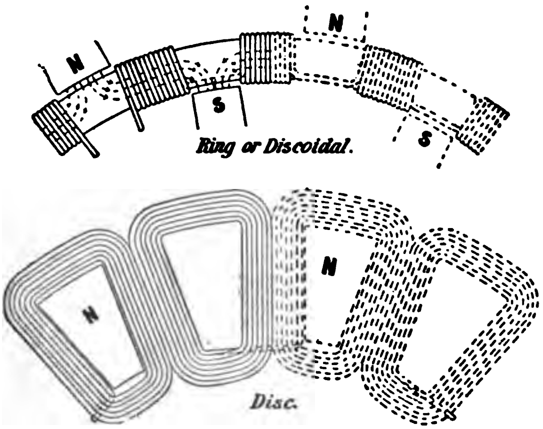


FIG. 81.—Armatures of homopolar alternators.

In practice there are usually more than one pair of fields and pair of coils, as shown in figs. 82, 83, the number of coils being equal to the number of fields or poles. If p = the number of pairs of fields

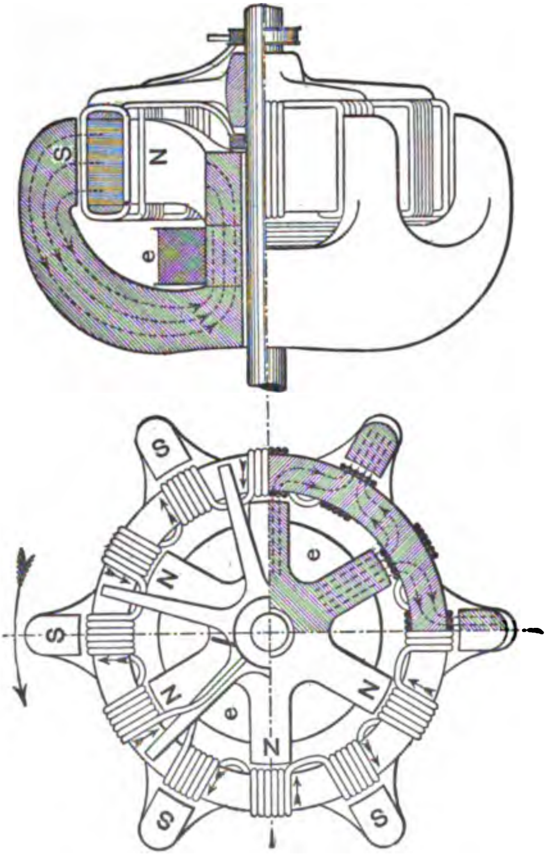


FIG. 82.—Diagram of single-phase ring homopolar alternator.

where the lines enter or leave the armature core, the periodicity is the same as in the case of heteropolar alternators, viz. $f = \frac{pN}{60}$.

In the case, however, of the discoidal-ring homopolar alternator, since half the poles of the corresponding heteropolar alternator are

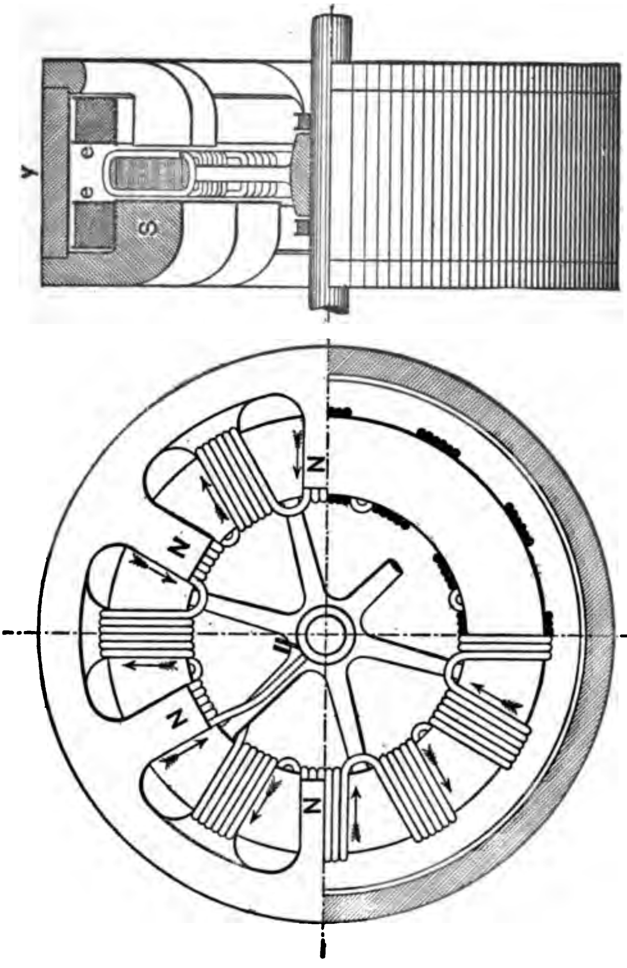


FIG. 83.—Diagram of a single-phase homopolar discoidal-ring alternator.

absent, p is equal to the number of pairs of fields or poles counted on both sides of the armature.

Each type has its own appropriate form of magnetic circuit. In the ring homopolar alternator all the external poles are of the same sign, and so also all the internal poles; hence they are formed by branches out of a magnet-core common to them all, and placed on one or other side of the armature (fig. 82); upon the magnet-core is wound the single exciting coil shown at *e*.

In the discoidal form all the poles on the one side are N., and all those on the other are S., and their common yoke may be either external or internal to the armature. If the latter rotates, it is most convenient to place the yoke on the outside, as was shown in fig. 61, and the exciting coil, *e*, is either suspended from the external yoke-ring, or is divided into two halves, one on either side of the armature, and supported by the poles. Fig. 83 shows an external yoke-ring, *y*; in the upper half of the side view the exciting coil is supposed to be removed in order to show the curved poles on the opposite side.

§ 3. **Homopolar drum and disc alternators.**—When we pass to the drum and disc homopolar forms derived from figs. 62 and 63, the pitch is necessarily defined as the distance along the pitch-line between the centre of one field and the centre of the adjacent interpolar gap. When there are several loops wound side by side or several slots between which each side of the coil is divided so that it has appreciable width, there will be differential action unless the width of the field be reduced to equality with the inner loop and the width of the outer loop be not greater than the width of the gap. Various proportions of coil and pole are therefore possible, and again their relative advantages cannot be decided by any mathematical law, but only as a matter of practical compromise. Thus in the single-phase alternator the width of the inner loop might be equal to half the pitch, and the width of the outer loop equal to one and a half times the pitch, so that half the armature is covered with winding. But any such single coil may be more conveniently divided into two, so that the end-connecting portions have less depth, as in the case of the heteropolar drum and disc alternator (Chap. VIII. § 6). Divided coils are therefore illustrated in the simple drum alternator of fig. 84, which also shows by dotted lines how the whole may be repeated with a second field. We thus arrive at *twice* as many coils as there are fields, the coils being virtually arranged in pairs, and each pair corresponding to one field instead of two fields as in the heteropolar machine. The width of the outer loop of each of the two coils is now equal to the pitch, and the coil-ratio may be best expressed as in the heteropolar case in terms of the width of the winding or the width of one sheaf of inductors forming the adjacent sides of two coils; the only difference from the heteropolar formula is then in the definition of the pitch necessitated by the homopolar arrangement. The fields are produced by polar projections from one central revolving iron mass. The exciting coil

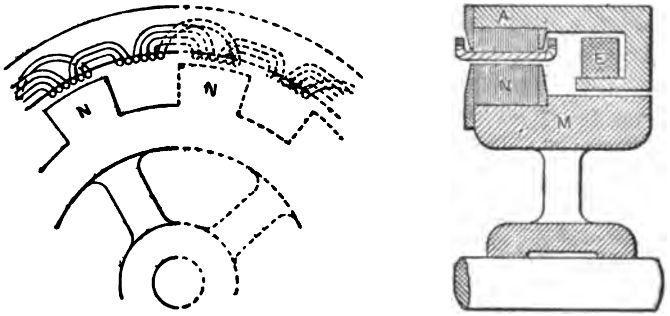
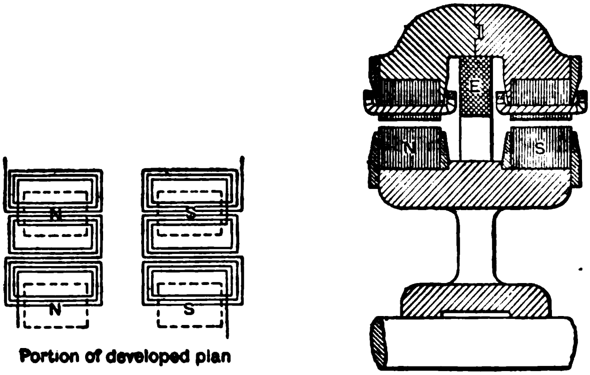


FIG. 84.—Drum inductor alternator with divided coils.



Portion of developed plan

FIG. 85.—Drum inductor alternator with double armature.

is not rotated but is supported from the stationary armature ring, so that there is no revolving copper at all. The rotating iron mass may be likened to a number of 'keepers,' and is sometimes known as the 'inductor,' whence the class of homopolar drum alternators has received the name of '*inductor' generators.*

From fig. 84 is easily derived what is known as a 'double-armature' inductor alternator. Instead of the simple continuous air-gap shown on the right-hand side of fig. 84, the second gap in the magnetic circuit may be broken up into separate poles and utilised by a second armature similar to the first (fig. 85). If the poles on one side of the machine are abreast of those on the other side, and the coils on the two sides are staggered relatively to one another, the machine will give two phases. Or the poles may be themselves staggered, when we may again consider that there is only one pitch-line, a pole being in each case opposite to a neutral interpolar gap; and from this another arrangement, shown in fig. 86, may be obtained which is suitable for a single-phase machine. The armature conductors are now taken straight through both halves of the armature core in tunnels or slots, and each straight portion consists of an inductor and a connector end to end. Finally, the two armatures may be arranged one inside and one outside the circle of revolving 'inductors' or 'keepers' (fig. 87); these latter are bolted to the side of the fly-wheel of the driving engine. In all cases of double armatures, the windings are preferably connected in series so as to avoid any liability to slight differences in the curve of E.M.F. or of current on the two sides.

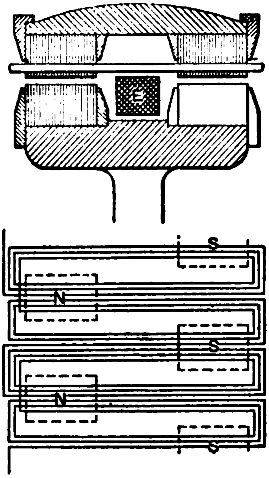


FIG. 86. — Drum inductor alternator with staggered poles.

Fig. 88 shows a disc homopolar dynamo evolved out of fig. 63 with revolving magnet and stationary armature; the several fields are again formed by dividing one magnetic circuit into a number of curved branches, each terminating in a pole-face. All the poles on one side of the armature are N., and all those on the other side S.; those on either side of the armature are shown united on the outside by a thin web of iron, pierced with holes and with deep recesses intervening between the projecting poles. The lines of induction thus form separate groups, between which intervene neutral spaces, clearly marking off the several fields. The single exciting coil which is shown at *e* need not of necessity rotate with the magnet, but might be attached to the armature, in which case no collecting rings for the exciting current would be required; it is, however, more advantageous for other reasons to rotate it with the magnet. When the armature is rotated, the yoke or undivided portion of the magnetic circuit may be made external to the armature in the shape of a ring, similar to that shown in fig. 83, but with the opposite poles directly facing each other. Since the poles on each side are of the same sign, there is no tendency for lines to leak across from one pole to a neighbouring pole on the same side of the armature, as in the case of a heteropolar disc armature, and the corresponding economy in the production of the fields to a large extent compensates for the fact that at any one time only half of the winding is active.

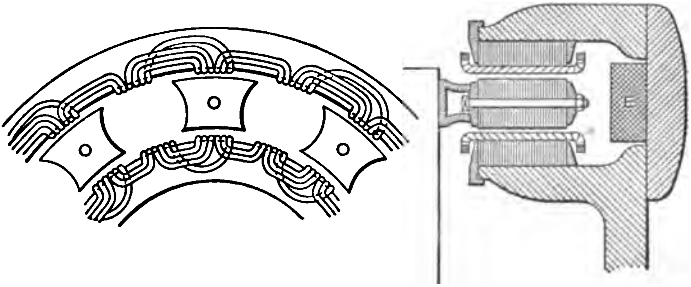


FIG. 87.—Drum inductor alternator with double armature.

Since the passage of one field past a pair of coils gives one complete double wave of E.M.F., the number of periods per second, or the 'periodicity' of a homopolar drum or disc alternator, is $\frac{pN}{60}$, where p is either the number of pairs of coils or the number of fields on one side of the armature. Thus in the drum

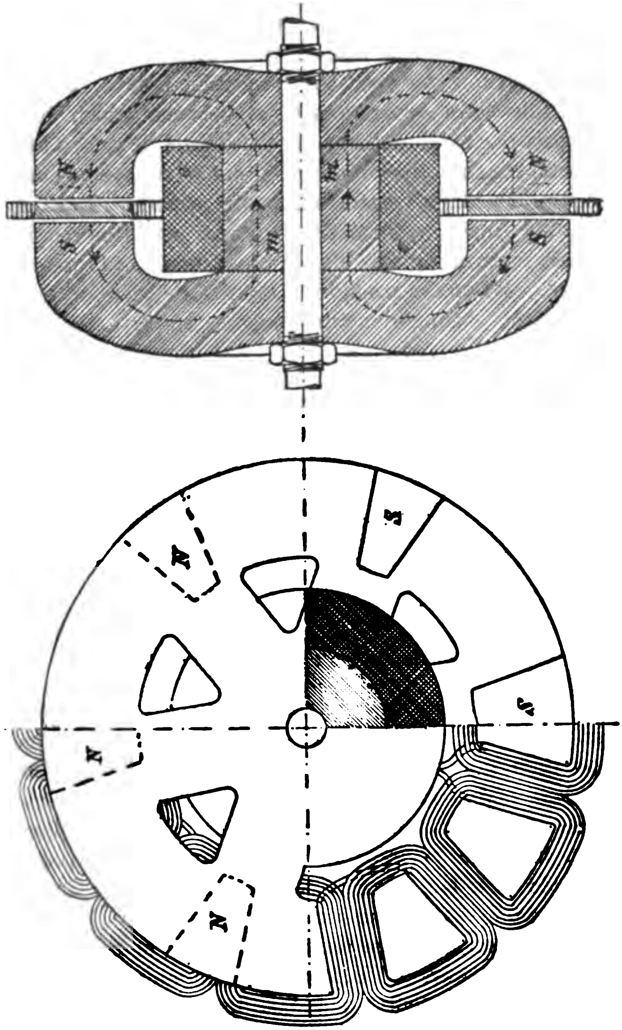
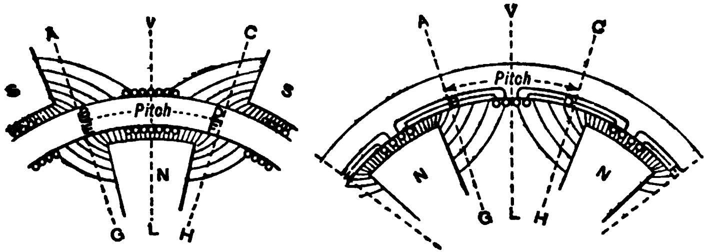


FIG. 88.—Diagram of a single-phase homopolar disc alternator.

or disc homopolar machine, p is not, as before, the number of pairs of fields, and this change in its definition is the counterpart of the change in the definition of the pitch for the homopolar drum or disc armature as given in § 3. It is therefore the same as the periodicity of a heteropolar disc alternator with an equal number of armature coils, although in the latter there are twice as many fields as in the homopolar machine.

A characteristic feature of all homopolar alternators is that the poles form branches of one and the same magnetic circuit, and therefore only require one exciting coil wound on that portion of the circuit where all the lines unite to flow in a common stream. Each inductor thus cuts the lines from the numerous subdivisions of one common pole, as the term 'homopolar' emphasises when applied to an alternator with many pairs of fields, and it is on this account that we have spoken of the pairs of fields rather than of pairs of poles. Although not confined to homopolar alternators. the single exciting coil is always employed in them.



I. Ring II. Drum

FIG. 89.—Armature fluxes in inductor alternator.

§ 4. **Useless lines entering the armature.**—In all homopolar alternators, one point must be especially insisted on. If a line V L be drawn symmetrically between two neighbouring poles of like sign, any flux which enters or flows through the armature in the interpolar gap up to the limit of half the pitch on either side of the symmetrical line, *i.e.* between B D (fig. 89) is not only useless but is positively detrimental, since it causes a back E.M.F. by cutting the wires which should act purely as connectors. In the case of a heteropolar alternator, there can be no differential action so long as the coils are very narrow or are wound each in a single slot; but in the homopolar machine the differential action above described is present whatever the width of the winding. The back lines are often called 'leakage,' but the name is not strictly appropriate, since they do enter the armature although not in a useful manner. Their number must therefore be deducted from the useful lines which enter immediately from a pole-face. Thus in fig. 89, if Z'_a be the total number that pass into the armature from the N. pole within the limits of the pitch, *i.e.* between EF in the ring or in the case of the drum between either EB or ED, and z_a be the number of back lines between BD, the number which must be reckoned as forming one field and corresponding to the Z_a of the E.M.F. equation is $Z_a = Z'_a - z_a$. After thus discounting the effect of the direct differential action peculiar to the homopolar alternator, we are left with the ordinary differential action due to the coil having width; this can again be taken into account by a width coefficient k'', and the only difference between the homopolar and heteropolar machines with similar coil- and pole-ratios will lie in the slight difference of the densities of the fringes between the edges of the poles and the lines AG, CH, (fig. 89), when the width of the pole is not equal to the pitch. Thus with but little modification the values of K for different ratios of pole- and coil-width in the heteropolar alternator are equally applicable to the analogous homopolar machine.

§ 5. **The E.M.F. equation of the homopolar alternator.** —The equation for the effective E.M.F. induced in one phase of a homopolar ring or discoidal alternator has then exactly the same form as in the similarly wound heteropolar machine, viz.:

$$E_a = \frac{K}{mq} \cdot 2(Z'_a - z_a)\frac{pN}{60} \cdot \tau \times 10^{-8} \text{ volts};$$

p being reckoned as explained in § 2. All that has been done in the case of the simple ring is to change one-half of the poles from the outer to the inner periphery, and as this change is at the same time accompanied by the change of the cutting side of the loops during one half of each period, the two changes neutralise one another, and the E.M.F. has the same direction and value round the loops as before. The ring or discoidal armatures of fig. 81 will therefore give a curve of E.M.F. analogous to the curves of the corresponding armatures of fig. 75, and the average and effective E.M.F.'s will bear the same relation to each other in the two cases. τ is to be reckoned as before, *i.e.* as the total number of loops or of inductors on the outside of the armature; for then each loop cuts $2z_a p$ lines in a revolution which was the basis of the equation. If we reckon τ as equal to twice the number of loops or the total number of wires that at different times act as inductors, then each wire only cuts $z_a p$ lines in one revolution, so that the product of τ and $z_a p$ still gives the same numerical result. It is, however, simplest to make $\tau =$ the total number of loops, so that the formulæ of both heteropolar and homopolar machines remain analogous. In the case of the discoidal ring, similarly wound, and in the one case placed in a heteropolar field and in the other case placed in a homopolar field, it might seem at first sight as if the homopolar machine would only give half the voltage of the corresponding heteropolar alternator, since the total area of pole-face is halved. And this is to a certain extent true, but the difference is largely counterbalanced by the fact that the pole in the homopolar machine may be given a greater width as compared with the pitch than in the heteropolar without excessive leakage, and also by the fact that owing to the field not being reversed in the armature core a higher flux-density can be employed.

Thus for the same exciting power $z'_a - z_a$ need not be very greatly different from the z_a of the heteropolar alternator of the same general dimensions.

In the case of the drum homopolar machine with single armature, p has already been defined as the number of fields, or of polar projections, or as the number of pairs of coils, and in the case of the disc, as the number of polar projections on one side of the armature. If, then, τ be reckoned as the total number of wires which at different times act as inductors, and has therefore the same meaning as in the corresponding heteropolar machines, each wire in one revolution only cuts $p\ (z'_a - z_a)$ lines. The average E.M.F. is therefore, with the same pole-width and the same net flux in the air-gap, only half that of the similarly wound heteropolar armature, and so also is the effective E.M.F. The equation for the effective E.M.F. in one phase of the drum or disc homopolar is therefore

$$E_a = \frac{K}{mq} \cdot (z'_a - z_a) \cdot \frac{pN}{60} \cdot \tau \times 10^{-8} \text{ volts} \quad . \quad . \quad . \qquad (12)$$

and this may be regarded as the especial equation of the homopolar alternator, since the ring forms are seldom used. But as in the case of the ring, the pole-face is usually of greater width as compared with the pitch in the homopolar than in the heteropolar form, and a higher flux-density is permissible, so that $z'_a - z_a$ of the former may not be so very different from the $2z_a$ of the latter, and the effective E.M.F.'s are more nearly equal for the same values of p, τ, and N. The disc armatures of figs. 88 and 79 will then give analogous curves, and the height of the ordinates will not be far different. In the case of the combined double armature of fig. 86, if τ be reckoned as the number of straight wires or bars passing from end to end across both armatures and $z'_a - z_a$ be reckoned on one side only, we again return to the original equation

$$E_a = \frac{K}{mq} \cdot 2(z'_a - z_a) \frac{pN}{60} \cdot \tau \times 10^{-8} \text{volts} \quad . \quad . \quad . \quad (13)$$

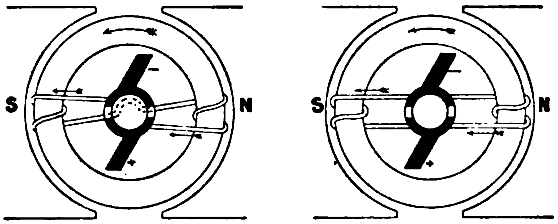
and the same holds for the two separate armatures of figs. 85 and 87 if these are in series and τ be the number of inductors in one armature.

CHAPTER X

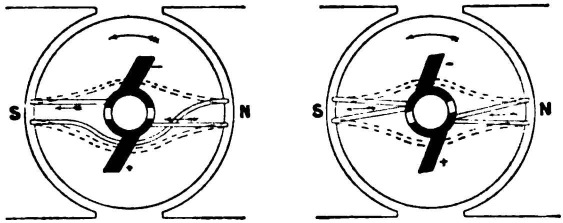
OPEN-COIL ARMATURES

§ 1. **Open and closed coil armatures.**—In the alternators discussed in the two preceding chapters, whether hetero- or homopolar, the current due to the induced E.M.F. is alternating in direction in the external circuit, just as is the current in each armature loop and in each armature coil formed of loops. Returning to our first class of dynamos, viz., those which are heteropolar, we now enter upon a consideration of the second division, consisting of those dynamos which yield a unidirected, or, as it is often called, a direct current in the external circuit, and in which, therefore, since their terminal E.M.F. must be unidirected, some form of commutator developed out of the split ring of fig. 52 must be present. This second division is, however, itself divisible into two sub-groups, viz., those with *open-coil* armatures and those with *closed-coil* armatures; in neither does the E.M.F. at the terminals alternate, but in the former it still pulsates or fluctuates in value during each revolution, while in the latter it has a practically constant value. 'Open-coil' armatures therefore occupy an intermediate position between alternators and 'closed-coil' machines. Their use is almost entirely confined to arc lighting, for which they have been found peculiarly suited; the pulsations which still remain in their E.M.F. curve, and which cause the current in the external circuit to fluctuate, although to a still smaller extent, are not disadvantageous to arc lamps, while their method of construction lends itself to the generation of a high E.M.F. and constant current, as required by arc lamps when run in series.

§ 2. **Two coils side by side.**—The previous discussion of alternators will have familiarised the reader with the principle that, in the case of ring or discoidal armatures, with one coil wound as in fig. 52 on one half of the ring, and placed in a bipolar field, a second coil can be wound on the portion of the ring diametrically opposite, and in this second coil an E.M.F. will be induced exactly similar to the E.M.F. of the first coil; it can therefore be connected either in series or in parallel with the first, the same split-ring being made use of without any modification (fig. 90). Similarly, in the case of drums, a second coil can be wound on by the side of the first and connected with it in series or in parallel, exactly the same proportion of the entire surface of the armature core being covered as in the ring. So, too, in the case of the disc armature, a second coil symmetrical with the first, but on the opposite half of the disc, introduces no fresh element to our theory. When the two coils are connected in series they virtually form one continuous coil: this is most evident in the case of the drum (fig. 90), but in reality the two opposite coils of the ring and disc may equally be regarded as one coil divided for practical reasons into two portions.



Series. *Ring* *Parallel.*



Series. *Drum* *Parallel.*

FIG. 90.

§ 3. **Two coils at right angles to each other. Levelling of the E.M.F. curve.**—But whether in series or parallel, the two coils thus symmetrically placed do not alter the character of the curve of E.M.F. as given in fig. 52, and do not, therefore, assist us in the work of reducing the pulsation of the E.M.F. to a comparatively small amount. An entirely new method must be tried, viz., the arrangement of coils on the armature in such positions that they are not diametrically opposite to each other, and so are not symmetrical in every respect.

Reverting to the simple coil and split-ring of fig. 52, let us arrange a second coil on the armature at right angles to the first, and attach its ends to a second split-ring, the angular position of which relatively to the first differs by a corresponding interval of 90° (fig. 91). The armature shaft on which both split-rings would be mounted and the connecting wires which join the ends of the coils to the commutator sectors are omitted for the sake of clearness, the connections being simply indicated by numerals; at the right hand is shown the corresponding drum armature, wound with two loops 90° apart. Let us suppose the insulating gaps between the two halves of each split-ring to be small, and the rubbing surface of the two pairs of brushes or contact springs, *a a'*, *b b'*, to be

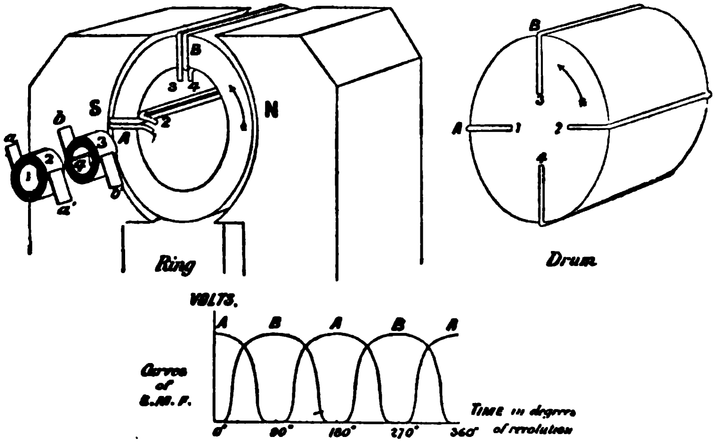


FIG. 91.

narrow. Now, since the second coil B is 90° behind the other, it will come into inductive action when the first coil is going out of action, and will be in the position of greatest action when the other is entirely inactive. If we plot along the same horizontal axis of time the curves of the terminal E.M.F.'s on the two pairs of brushes for one revolution, the maximum E.M.F. or crest of the wave due to one coil will coincide with the hollow of the wave of the other; thus, in fig. 91, curve A A gives the value at any moment of the E.M.F. at the brushes *a a′*, while B B is the corresponding curve for coil B. It is evident, therefore, that if the crests of the second wave were used to fill up the hollows of the first, the fluctuations of the joint curve of E.M.F. would be very largely decreased. Further, if we arrange three coils round the armature at angles of 120° from each other (fig. 92), we again see that if their curves were in some way amalgamated the pulsations might be still further decreased. In the class of machine now before us, therefore, a comparatively small number of coils is required, so arranged relatively to one another that when one ceases to be active another takes up the action, this change being continually and successively repeated at the proper moments in each revolution. Further, since in practice the edges of the pole-pieces are not brought into very close proximity to each other on account of the excessive leakage of lines which would ensue, the portion of each half of a revolution in which a comparatively narrow coil cuts few lines or none, and therefore yields little or no E.M.F., is of appreciable length. Hence, whenever a coil is inactive, it may be of advantage to cut it entirely out of circuit, since if it formed part of the circuit a portion of the E.M.F. of the coil in action would be spent in driving the current through its useless resistance. Or, again, it may, while comparatively inactive, be thrown into parallel with another coil, also comparatively inactive. An incidental advantage so gained will be that the electrical resistance of the armature is during that time reduced, but the main object of such parallel connection will be to prepare a coil for the

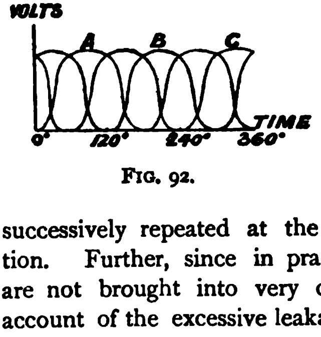


FIG. 92.

rapid changes which it undergoes from action to inaction, and *vice versâ*.

Any armature in which a coil is either left open-ended, or has its connections with the other coils rearranged during any portion of a revolution, is an *open-coil* armature, the exact meaning of which term will be more apparent later, when it can be contrasted with 'closed-coil' armatures.

§ 4. **Combination of the two split-rings into one commutator.**—Starting with two coils wound on a bipolar armature at right angles, as in fig. 91, let us increase the width of the gaps in the split-rings until each commutator sector has an angular width of something less than 90°, while the angular width of contact of each brush is still kept small; then in the position shown in fig. 93, since the brushes *a a'* are in the middle of their sectors, they are yielding an E.M.F. while coil B is not in contact with its brushes, but shortly after there will be an E.M.F. at *b b'*, and none at *a a'*. Hence, one pair of brushes of greater length along the axis of rotation might be substituted, which should press on both split-rings alike, as *a b*, *a' b'*, in fig. 93, and the whole is represented in fig. 94 by the four commutator sectors, each having an angular width of nearly 90°, and connected to one end of a coil. The pairs of coils are shown diagrammatically by the coiled lines A A', B B', each of which may represent either a simple coil on a ring armature, or a pair of such coils wound on opposite sides of the ring, or the corresponding forms which these take in the drum, discoidal, and disc machines, as shown in fig. 90 and explained in § 2.

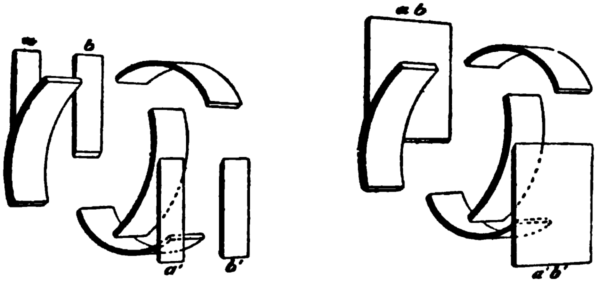


FIG. 93.

§ 5. **Action of the open-coil armature.**—Let *m m* be the line of maximum field, where the lines of induction are most dense, and *n n* be the neutral line, where there is no field at all; the reason why these are not drawn exactly horizontal and vertical will be explained in Chap. XVII. It is important to observe that the brushes are now placed near to or on the line of maximum field, and, as shown, coil B B′ has just come into action, and is supplying E. M. F. and current to the external circuit, while coil A A′, which is just passing into the position of least action, is cut out of the circuit. The width of each commutator sector being nearly 90°, each of the two coils will in turn be in contact with the brushes for nearly a quarter of a revolution, but as the gaps of insulating material are not bridged over by the brushes, the circuit will be broken four times in each revolution. If now the curve of E.M.F. acting at the brushes on the external circuit be plotted for a revolution, starting from a point where the brushes are on the gap, just in front of the position shown in the diagram, it will be entirely above the horizontal line, and the fluctuations in its value are very much reduced; yet it will be discontinuous, being divided by four breaks corresponding to the passage of the insulating gaps under the

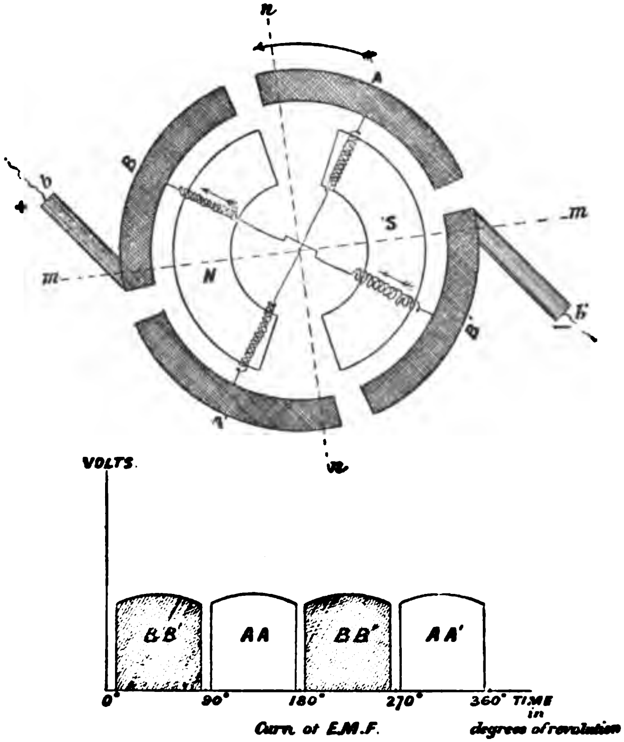
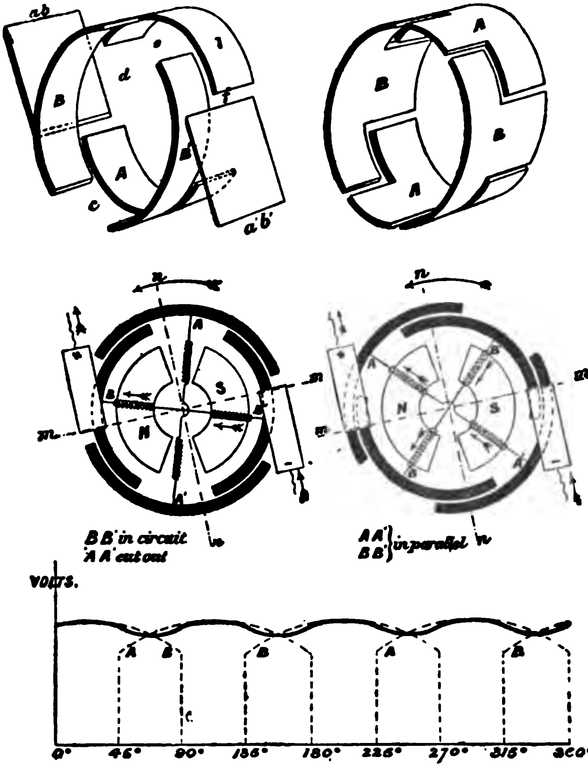


FIG. 94.

brushes. At these moments the brushes cease to press on one pair of sectors and have not yet begun to press on the other pair. The arrangement is therefore totally unworkable, owing to the fact that the circuit of the current is continually being broken, which would produce destructive sparking on the commutators. And, further, when a coil is just cut out of circuit, as A A′ in



Curve of E.M.F

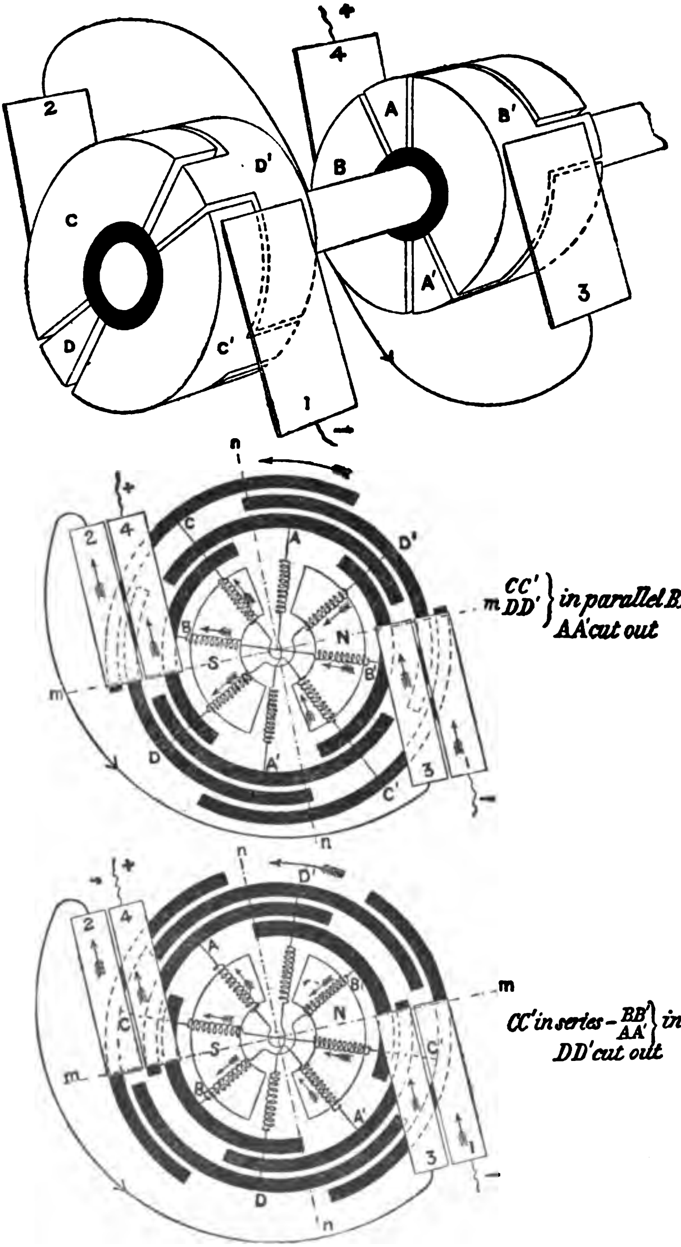
FIG. 95.

fig. 94, it is only half-way between the line of maximum field and the neutral line, and is, therefore, still generating some E.M.F. which might be utilised. To remedy these defects it is only necessary to retain the split-rings as shown in fig. 91, that is, with the neighbouring sectors overlapping each other and with one pair of broad brushes *a b*, *a′ b′*, pressing on both split-rings ; each brush before leaving one sector will now touch another belonging to the other split-ring (fig. 95). Below the commutator there shown is a diagrammatic representation of two positions of the armature, the brushes being supposed to touch along the line *m m*; it will be seen that the time during which a coil is entirely cut out of circuit is lessened, and during the time so gained the two coils are placed in parallel as shown at the right hand. If each pair of sectors overlap the other by an angle of 45°, and each revolution be divided into eight equal parts, then for the first eighth the two coils are in parallel, for the next only one is supplying E.M.F., and the other is open-circuited; for the third eighth the two coils are again in parallel, after which for the next eighth the second coil is alone supplying E.M.F., the whole being repeated during the second half of the revolution in the same order. The curve of E.M.F. is now no longer discontinuous, but is a continuous undulatory line without any breaks; it is obtained by plotting the E.M.F. of each coil, when acting separately, and the ordinate, which is common to the two separate dotted curves (fig. 95); the intervening periods of 45° are then filled up approximately by curves representing intermediate values between the E.M.F.'s of the coils when in parallel: these will fall and rise rather more suddenly than the curve of the coil which has the higher E.M.F. Four times in each revolution the two coils are in parallel, and during these periods the E.M.F. in one is increasing as it approaches the position of best action, and in the other is decreasing as it recedes from that position; the two are theoretically equally active only for an instant when they are equidistant from the line *m m*. It might be thought that this inequality between the E.M.F.'s of the two coils at the moment when they are put in parallel, and just before they part company, would cause large local currents and waste of power, since the higher E.M.F. of the one coil naturally tends to drive a back current through the other of which the E.M.F. is lower. Were the coils free from inductance, this would be the case, and the net E.M.F. acting at the brushes during the periods of parallelism would be simply that of the coil which at the moment was less active, the excess E.M.F. of the more active coil being expended in producing an internal and useless back current. Owing, however, to the inductance of the two coils, the current in them cannot be suddenly reversed, increased, or started; and consequently the curve of E.M.F. at the brushes falls and rises gradually, taking intermediate values between the E.M.F.'s of the stronger and weaker coils. Thus when first thrown into parallel, the resultant E.M.F. of the weaker coil is higher than its induced E.M.F. owing to the self-induced E.M.F. called into existence by the current which is started in it, while the resultant E.M.F. of the stronger coil is less than its induced E.M.F. by reason of the increase of current through it. Further, when a coil is about to be cut out of circuit, the inequality of E.M.F. is of advantage, since it stops the current flowing in it much more quickly than would otherwise be the case; for the greater E.M.F. of the other coil tends to drive a current in the reverse direction through the leaving coil, and so prepares it to be cut out of circuit without an excessive amount of sparking.

Each coil has its own split-ring, and the winding of the armature does not form a closed circuit in itself, as will be found to be the case in 'closed-coil' armatures. The two overlapping split-rings may be conveniently united, so as to form a single mechanical structure, by making the neighbouring sectors of one split-ring project into the gaps left between the two sectors of the other split-ring while still being insulated from them. Thus in fig. 95, if *a* projects into a gap *e* between B and B′, and A′ into the other gap *c*, while B and B′ similarly project into the corresponding gaps *d* and *f* between A and A′, we get the joint commutator shown at the right-hand side. Owing to the high voltage which this class of machine usually gives, air forms the most suitable insulator between the adjacent sectors, since there is always a certain amount of sparking present, which would otherwise damage the surface of the commutator; hence the two split-rings, though projecting into each other, are still divided by comparatively narrow air-gaps. The whole is mounted on the shaft of the armature, and held in place by a thick ring and washers of insulating material. (*Cp.* fig. 96.)

§ 6. **Two pairs of coils 45° apart.**—The method above described may be pushed a step further by winding on the armature more pairs of coils intermediate between the first pair; the pulsations of the E.M.F. are thus still further reduced, while more of the available space round the armature is rendered useful. The first pair of coils were 90° apart, and in every case a pair of coils at right angles to each other must be taken and connected to a joint commutator exactly similar to the joint commutator of the first pair. Thus fig. 96 shows diagrammatically an armature of four pairs of coils, two pairs at right angles to each other being connected to each of the two joint commutators; one joint commutator is set an eighth of a revolution, or 45°, in advance of the other, just as any two neighbouring coils are separated by an angle of 45°; the arrangement is really equivalent to two separate machines in series with each other, the two pairs of brushes being connected in series by joining together one brush of one commutator with that brush of the other commutator which is on the opposite side, the other two brushes leading to the terminals of the machine. The perspective view of the two commutators corresponds with the first diagrammatic representation of the armature and coils. In the inner commutator, in the position there shown, coil A A′ is cut out of circuit, being in the position of no action, and coil B B′ is alone supplying E.M.F. to the pair of brushes 3, 4; in the outer commutator both coils, C C′, D D′, are in parallel, and supplying E.M.F. to the other pair of brushes 1, 2; brush 2 is connected with brush 3, so that the current flows through coils C C′ and D D′ in parallel, and thence through coil B B′. The lower diagram represents the same after the armature has revolved through an angle of 45°, when D D′ is cut out, B B′ and A A′ are in parallel, and C C′ in series. This arrangement of one coil cut out, two in parallel, and the third in series with the two, is continually repeated as each coil arrives at the positions of neutral gap, intermediate action, and best action.

Similarly, if six pairs of coils are used, there must be three joint commuta-



CC', DD' } in parallel BB' in series
AA' cut out

CC' in series – $\frac{BB'}{AA'}$ } in parallel
DD' cut out

FIG. 96.

tors, each arranged one-twelfth of a revolution, or 30°, in advance of or behind its next neighbour.

§ 7. **Three coils 120° apart.**—We have now to return to the case of three coils arranged at angles of 120° with each other, represented diagrammatically in fig. 97, and in this case one end of each coil must be brought to a common junction, as shown at the centre. The other free ends are joined to three commutator sectors, each having an angular width of rather less than 120° and insulated from each other. Evidently when in the position shown in fig. 97, since coil C is in the neutral line, no E.M.F. is being generated in it, while in coils A and B the E.M.F. is respectively radially outwards and radially inwards; if, therefore, the brushes are set on the line of maximum action a current will flow through A and B; but shortly after, when the insulating gap comes under brush *b′*, the circuit will be broken, and a similar break of the circuit will occur six times in each revolution—thrice at brush *b* and thrice at brush *b′*. In this case, therefore, while keeping the insulating gaps small, we require to widen the arc of contact of the brushes until each spans or bridges over the insulating gaps (fig. 98). The effect of this is that at the instant when a coil is about to be cut out, it is placed in parallel with a second coil, since the two come under the same brush; the current, therefore, at this moment flows through the two in parallel, and so through the third, which is in series with them. The sequence of the changes which take place in a half-revolution is as follows: Starting from the position shown in fig. 98, in which B and C are in parallel, and A in series with them, B is cut out, leaving A and C in series; then A and B in parallel, C in series with them; A cut out, B and C in series; A and C in parallel, B in series with them; lastly, C cut out, leaving A and B in series. There is now no break in the circuit, and the E.M.F. curve will be continuous with only slight fluctuations as the various changes are passed through. These are best followed by cutting out a small disc of paper on which are represented the three coils and commutator sectors, and rotating it round a pin on another sheet of paper on which are drawn the two stationary brushes. Each coil is first in parallel, then cut out, again in parallel, then in series, and lastly in parallel again before it is again cut out. The time during which two coils are in parallel depends on the relative angular widths of the air-gaps in the commutator and of the brushes: if they are nearly equal the two coils will only be in parallel for a short time, but if, while keeping the width of the air-gap small, the width of the brushes be increased, this time will likewise be increased. The

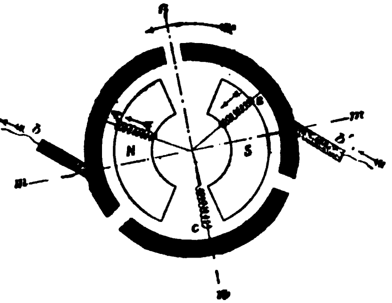


FIG. 97.

effect of different widths of the brush surface requires to be carefully considered.

§ 8. **Effect of different brush widths.**—If the brush surface be only just wider than the gaps, as in fig. 98, the time when the two coils are in parallel will be a mere instant; and, further, the two coils are then at equal angles from the line of maximum action, *m m′*, so that their E.M.F.'s are precisely alike. Hence we may practically disregard the moments of parallelism and say that during the whole revolution the E.M.F. of the machine is that due to two coils in series, the one approaching and the other receding from the line of maximum action; each coil is under one brush for nearly a third of a revolution at a time, and is cut out of circuit for a sixth of a revolution. The joint curve obtained by compounding together the curves of the three coils in the required manner is shown in fig. 98, and is composed of six crests and hollows. But now let the width of the brushes be greater, and the time of parallelism longer; then the E.M.F.'s of the two coils in parallel are not always equal, and there is a tendency for the one of higher E.M.F. to drive a back current through the one of lower E.M.F. by means of the path of the brush surface. The effect is exactly the same as was previously described under the first type of open-coil machine; when first thrown into parallel, the weaker coil by reason of its self-induction does not admit of a large back current being instantly passed through it, while when about to be cut out of parallel, the inequality of E.M.F. is positively of advantage, since it serves to stop or even reverse the current at the moment when the circuit of the coil is to be broken. Next, let the width of the brush be just 60° (fig. 99), *i.e.* half a commutator sector plus half an air-gap. This entirely alters the series of changes, for now the coils are open-ended for a moment only, and except for these brief moments, there are *always* two coils in parallel and in series with the third. Each coil is under one brush for half a revolution, and this time is divisible into three equal parts: for a third of it, it is in parallel with the coil in front of it; for the next third, it is in series with the other two which are in parallel; and for the last third it is in parallel with the coil behind it. It will

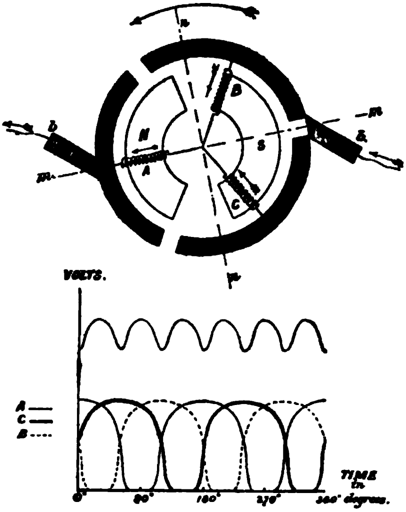


FIG. 98.

be seen that the most active coil is in series only for a sixth of a revolution at a time, and that when it is only 30° from the line of maximum action, it is thrown into parallel with a less active coil, which is itself practically on the line of no action. But this latter coil has itself had a reverse current started in it by the higher E.M.F. of the coil from which it has just parted company; for a coil is only withdrawn from parallelism when its partner is 30° from maximum action, and it is itself on the neutral line, and therefore it is already prepared to be put in parallel with the third coil, its self-induction sufficing to keep up the flow of current until its E.M.F. is equal to that of its new partner. Shortly after, its E.M.F. becomes higher than that of the coil with which it has just been put in parallel; consequently in its turn it prepares this latter coil to receive a reverse current prior to its leaving one brush and coming under the other. By such mutual action and reaction, the total E.M.F. of the machine is kept fairly constant, and there are not the violent fluctuations which might perhaps be expected from so small a number of coils so continually rearranged as to their connections. The crests and hollows of the E.M.F. curve are still the same as when the brushes were only slightly wider than the gap. Yet the average E.M.F. throughout a revolution is not quite so great as that which is obtained when the width of brush is less, and the difference of E.M.F. of the two coils which are placed in parallel is smaller.

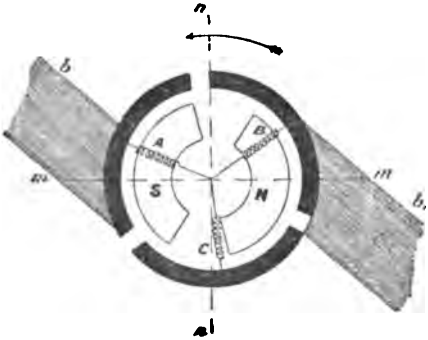
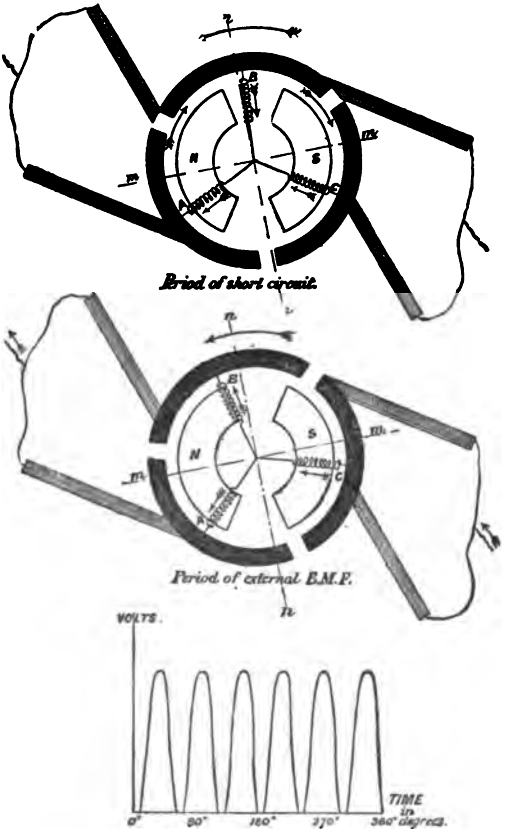


FIG. 99.

Next let the width of the brush be more than 60° (fig. 100); then the two brushes can both rest on the same sector at once, and the whole machine is for the time being short-circuited on itself; the sector which is under both brushes at the same time affords a path of almost negligible resistance to a local current, due to the combined E.M.F.'s of the two coils attached to the other two sectors, neither of them being far from its position of maximum action; consequently the E.M.F. of the machine is for the time virtually abstracted from the external circuit. Such short-circuiting will occur six times in each revolution, and the actual length of time during which each short-circuit lasts will depend on how much the brush width exceeds 60°. Further, the time during which each coil acts separately before it is thrown into parallel with a coil of weaker E.M.F. is proportionately reduced; for example, if the width of brush were as much as 80°, and the air-gaps are 10°, as in fig. 100,



Period of short circuit.

Period of external E.M.F.

Curve of E.M.F. at the brushes.

FIG. 100.

each coil would be in series for only 25° on either side of the line of maximum action, or for only about one-seventh of a revolution at a time. The total result is, therefore, that the average E.M.F. which the machine supplies to its terminals throughout a revolution is very largely reduced; the actual height of the crests is reduced, but chiefly owing to the six periods of short-circuit, each lasting for 10°, the total area of the curve of E.M.F. acting at the brushes is much diminished. The local short-circuit current does not produce the heating that might be anticipated; in fact, in a machine running at 800 revolutions per minute it has to be started 4800 times per minute, and each time the self-induction of the coils prevents it from growing to any great strength.

Evidently, therefore, if an arc of 60° were chosen as the normal width of brush contact, and an easy means were provided for altering this width, the voltage of the machine could be regulated within certain limits by increasing or decreasing it accordingly as we require a lower or greater E.M.F. In the Thomson-Houston arc-light dynamo the alteration of the effective brush-width is carried out in practice by dividing each brush into two, set at a certain angle apart (fig. 100), which angle can be varied according to the requirements of the moment; the two brushes are electrically connected together, so as to form in reality one brush spanning a considerable arc of commutator surface.

Two forms of 'open-coil' armatures have now been described,[1] and beyond these no other 'open-coil' arrangement has ever been successfully introduced and embodied in a practical machine.

---

[1] In the second edition of the present book may be found a fuller description of the Brush and Thomson-Houston machines; this is now omitted owing to the machines being less in use than formerly.

CHAPTER XI

CLOSED-COIL ARMATURES

§ 1. **Closed-coil continuous-current dynamos.**—The third and last group of heteropolar dynamos still remains for consideration. Like the machines of the last chapter, they give a unidirected E.M.F., and therefore presuppose a commutator; but they differ in that this E.M.F. is for a uniform speed of rotation constant in value during the whole period of a revolution; the current, therefore, which such a constant E.M.F. causes to flow through a circuit of fixed resistance is itself also constant and steady, without fluctuations or pulsations. Hence dynamos of this type are also called *continuous-current* machines, the E.M.F. and current which they yield being not only continuous in direction, but continuous also in value.

The purposes to which such machines can be applied are so numerous, and so important, that they are used more extensively than any other class of dynamos; whether it be for incandescent lighting, charging accumulators, motor driving and transmission of energy over moderate distances, or electro-deposition of metals, they are equally suitable for all alike, while the voltages for which they are made vary from 5 to 3000 volts. Their characteristic features will be found to be intimately connected with the fact expressed by their other name of *closed-coil* dynamos; they are so called because the winding of their armatures forms an endless coil closed upon itself, and, in contrast to the previous sub-group, no portion of the armature-winding is ever open-circuited, or has its connections with the rest rearranged.

§ 2. **Necessity for short-circuiting a coil.**—Since the E.M.F. of machines belonging to the present group is to be steady and free from fluctuations, it is reasonable to recur to and extend the method which was found so effective in the case of open-coil machines, viz., that of placing two or more coils on the armature in positions which are not symmetrically opposite to each other. We have, therefore, first to determine how the E.M.F.'s of such coils can be combined together, their connection with the external circuit being still reversed at the right moment when the direction of the E.M.F. induced in them changes. Further, and most important, this must be done, not only without opening or breaking the entire circuit, but also without opening or leaving open-ended any coil. An entirely new device must now be adopted, by which, whenever a coil is in the position of reversal of E.M.F. between two adjacent poles, it is *short-circuited* on itself. To effect this, it is only necessary to make each of the two brushes of fig. 52 touch both sectors of the split-ring simultaneously at the moment of reversal; the coil is then closed upon itself for a short time through the interposition of the brushes, and this allows the current previously flowing round the coil in the one direction to die away, and a current in the reverse direction round it to be started by the reversed E.M.F. as it approaches the other pole. The brushes must be so set that the short-circuiting takes place approximately at the position of zero E.M.F., and the curve of E.M.F. of the loop, as given in fig. 52, is thus practically unaltered.

§ 3. **Two coils at right angles.**—Starting, then, with a pair of coils at right angles to each other, and connected with a pair of split-rings, as in fig. 91, instead of widening the insulating gaps (as was done in the case of the first sub-group of direct-current machines, viz., those with open-coil armatures), we must keep the gaps small, give the brushes a sufficiently wide contact surface, and set them in such a position and at such an angle to the split-rings that they bridge over the insulating material between the sectors whenever a coil is in the position of reversal. Having done this for both pairs of brushes, let us connect one brush of one split-ring with the brush on the opposite side of the other ring; thus fig. 91 will take the form shown in fig. 101, which represents a two-coil ring, and by its side the corresponding drum-wound armature; as before, for the sake of clearness, the actual connections of the coils to the sectors are omitted, and merely indicated by corresponding numerals. Brush *a′* is joined to brush *b*, and the remaining pair of brushes form the terminals to which the external circuit is applied; as shown in the figure, coil B is in the position of reversal, and, therefore, is short-circuited through its brushes; the current flows into *a*, through coil A, which is alone supplying E.M.F., and out by *a′* into *b* and *b′*. But a moment later, when the armature has moved further round, both coils will be in action, and neither will be short-circuited by the brushes; the current due to the E.M.F. induced in both will now flow through coil A, and thence through coil B, leaving the armature by brush *b′*. As the armature continues to rotate, coil A will in turn be short-circuited by its brushes when it reaches the second position of reversal, and coil B will alone supply E.M.F. to the external circuit; finally, both coils will come again into action, until B is again short-circuited at the end of half a revolution.

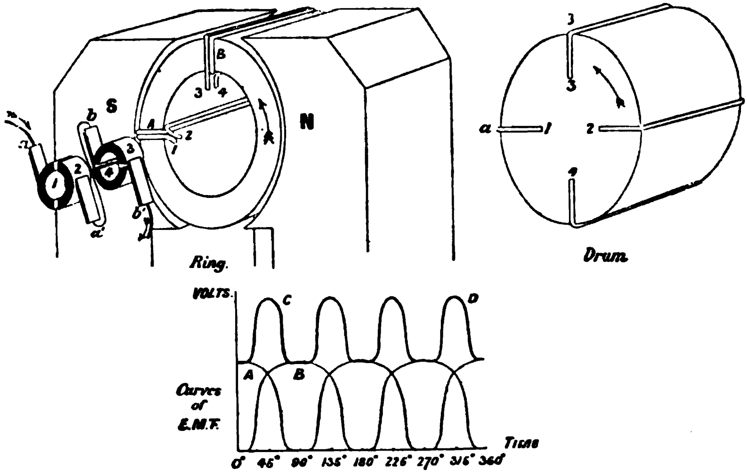


FIG. 101.

The armature circuit is thus never broken, and no coil is ever open-circuited; each gives a curve of E.M.F. as shown in fig. 91, but the important result has been obtained that the two coils are now in series, and the curve of the total E.M.F. acting at the brushes *a* and *b′* will be given by adding together simultaneous ordinates of the two separate curves and plotting their sums as a third curve. This has been done in the lower part of fig. 101, the curve C D showing the effect of adding together in series the E.M.F.'s generated at each instant in the two coils. At once it will be seen that the fluctuations in the third curve of joint E.M.F., though still marked, are very much reduced in value from what they are when each coil acts separately. Expressed as a percentage, the fluctuation, which is roughly 100 per cent. on either side of a mean value in the case of a single coil, is reduced to about 30 per cent. in the case of two coils in series.[1]

§ 4. **Four coils 45° apart.**—A further step towards greater constancy of E.M.F. would be made by arranging still more coils at successive small angles in front of each other, so as to come into and out of action successively, and fig. 102 shows the effect of superadding the E.M.F.'s of four coils arranged with angles of 45° between neighbouring pairs, there being only eight small undulations in a revolution. The fluctuation is now reduced to only 10 per cent. on either side of the average value throughout a revolution.

If such third and fourth coils be added, their split-rings must be so arranged that the angles between the positions of the diametric lines through the insulating gaps of each commutator correspond with the relative angular position of the coils on the armature (fig. 102); and, as before, brushes on opposite sides of the commutators must be joined together in order to sum up the E.M.F.'s of the coils. Evidently the levelling process may be continued until the total E.M.F. is practically continuous and uniform, provided that there be a sufficient number of small separate coils regularly spaced round the armature. More coils must therefore be wound on, and similarly joined together, until finally there is a number of coils and split-rings, all arranged with a regular angular advance on one another until an angle of 180° has been passed through. There will then always be a nearly constant number of inductors actively cutting the lines of the field, and placed in series so that the average E.M.F. due to them as a whole will be constant. Not only does the continued sequence of one coil after another level the curve, but from the fact that each coil now contains few inductors, and is one of a large number, the effect upon the whole, as each is short-circuited, is enormously reduced. Such a set of symmetrically-arranged coils might be made to cover the whole of the armature core; if composed of drum-wound loops they would embrace the whole of the core, and if of ring-wound loops, then for each coil on the one half of the ring a corresponding coil may be wound diametrically opposite on the other half of the core, which will give an E.M.F. similar in phase and intensity, and which may therefore be joined either in

---

[1] The curve C D should be contrasted with the open-coil curve of fig. 95. The general effect of both open-coil and closed-coil arrangement in reducing the fluctuation is alike, but there is a difference in the two methods of obtaining it, and consequently in the shapes of the two curves. In that of fig. 95 a coil was first open-circuited, next thrown into parallel, and then acted separately. Now, the two coils are either in series or one of them is short-circuited; as the E.M.F. induced in one coil is diminishing in value, that of the other coil is rising, so that at 45°, 135°, 225°, and 315° the two are equal, and the total E.M.F. then reaches a maximum, while it never falls below the value of the maximum E.M.F. given by one coil when in the position of best action.

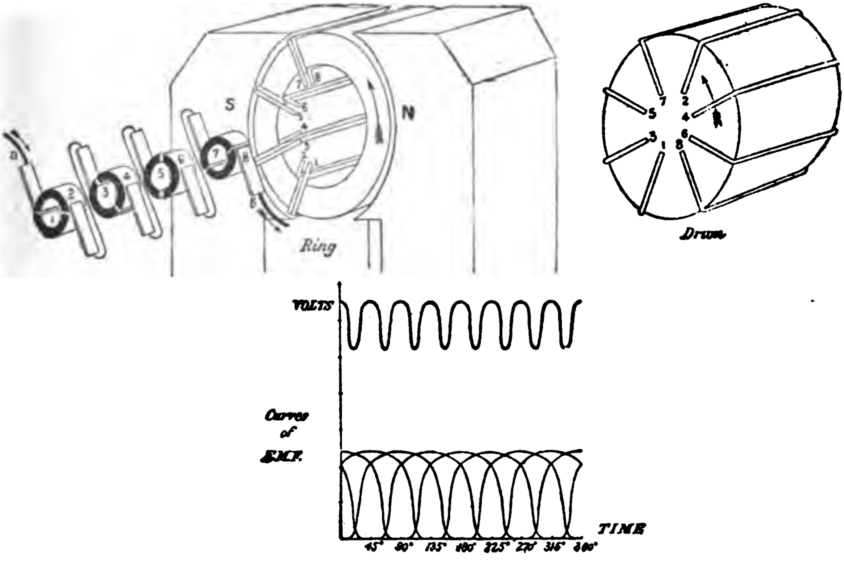


FIG. 102.

series or in parallel with the opposite loop, as in fig. 90. Thus the whole of the armature surface would be utilised for winding, and, with a large number of coils, the fluctuations of the E.M.F. would be reduced to an inconsiderable amount.

§ 5. **Combination of split-rings to form one commutator.**—But this would necessitate a correspondingly large number of split-rings and brushes in a row, which would be at once cumbrous and expensive. Hence a simple device is adopted, which results in the division of the armature winding into two equal and similar halves in parallel. In fig. 101 the effect of joining brush *a′* across to *b* is virtually to connect one end of coil A with the beginning of the next coil, B. Similarly, in fig. 102, the end of one coil is electrically connected to the beginning of the next, as 2 to 3, 4 to 5, 6 to 7. If, therefore, the four split-rings of fig. 102 be concentrated into a tube or cylinder divided into four pieces forming portions of geometrical sectors (whence their name) and insulated from each other, upon which one pair of brushes rests, the effect at the moment chosen for illustration will be unaltered. An end view of the new arrangement is shown by the full line portion of fig. 103. Pairs of sectors in fig. 102 are replaced by single sectors as indicated by the numerals, and the loops are diagrammatically represented by coiled lines external to the commutator. But immediately after the moment illustrated, as the rotation is continued, brush *b′* will leave the sectors, unless the whole arrangement is repeated with other coils so as to make the joint commutator symmetrical, as shown in dotted lines. If this be done, the coils as they pass from between the brushes on the one side are continually replaced by other coils from the other side; it will, however, be seen that the possibility of having a second set of coils joined to the dotted sectors must now be considered.

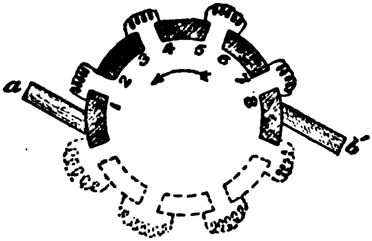


FIG. 103.

§ 6. **Division of the winding into two halves in parallel.** —First let us repeat the arrangement of fig. 101, so as to obtain four coils symmetrically placed round the armature, and each with its ends attached to its own split-ring commutator (fig. 104); as before, the commutators are shown in perspective, and the connections of the coils are marked by the numerals 1, 2, 3, etc. As in fig. 101, a current can be collected from coils A and B by applying the external circuit to brushes *a* and *b*; similarly, a current can be collected from coils A′ and B′ by applying the external circuit to *a*′ and *b*′. If, therefore, we join all the brushes consecutively on opposite sides of the commutators, and finally join brush *b*′ to brush *a*, both currents can be collected by applying the external circuit to brushes *a* and *b* as before. We now have a dynamo giving an E.M.F. curve identical with that of fig. 101, but carrying twice the current; the two halves of the armature winding are in parallel, and the current entering at *a* divides within the armature into two equal portions, one flowing through coils A and B, and the other through B′ and A′, the two reuniting to leave at *b*. If we now replace the separate brushes and separate commutators by one cylindrical structure divided into four sectors insulated from each other, and upon which one pair of brushes rests, the effect is to convert the whole winding of the armature into a

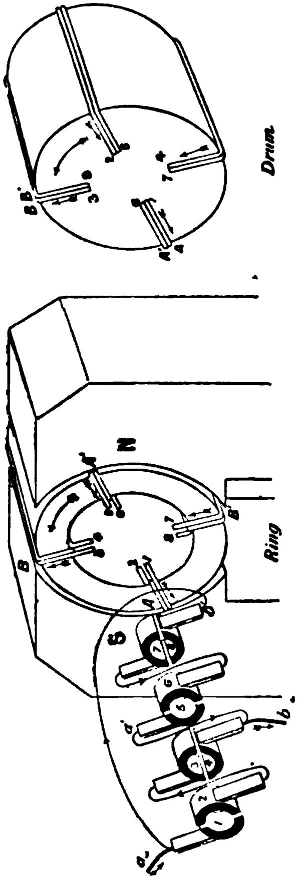


Fig. 104.

closed spirally-wound coil, joined at intervals to commutator sectors. The end of one coil is consecutively joined to the beginning of the next, as the numerals show, until the end of the last is joined to the beginning of the first. Fig. 105 shows the joint commutator in perspective, and below are shown the connections of the coils; at the right of the diagram the brushes are shown in the act of short-circuiting coils, A and A′. The whole is now symmetrical on both sides of the brushes, and rotation can be indefinitely continued without, at any time, breaking the circuit or leaving any coil open-circuited. As soon as any coil passes away from one pole-piece, it is short-circuited under a brush, and passes over into the other half of the winding. The complete closed-coil armature is now seen to justify its name, not only because a coil is never opened, but also because it is itself a closed helix, formed of a number of continuously wound loops; starting from any point, the winding can be traced right round the armature, without any breaks, until the starting-point is again reached. In sharp contrast to open-coil machines, the brushes must be set at the opposite ends of a diameter corresponding, not with the line of maximum field, but with the neutral line of zero field.

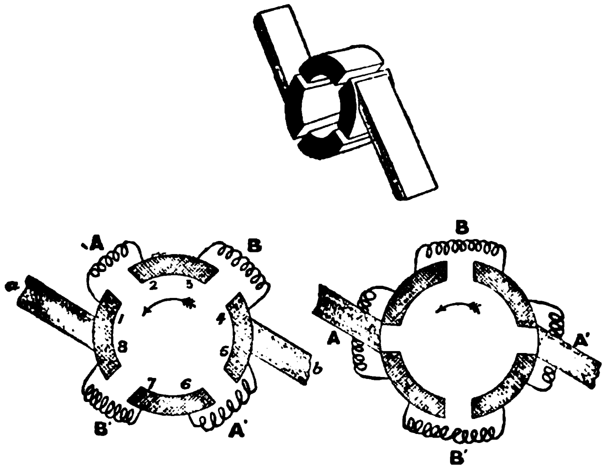
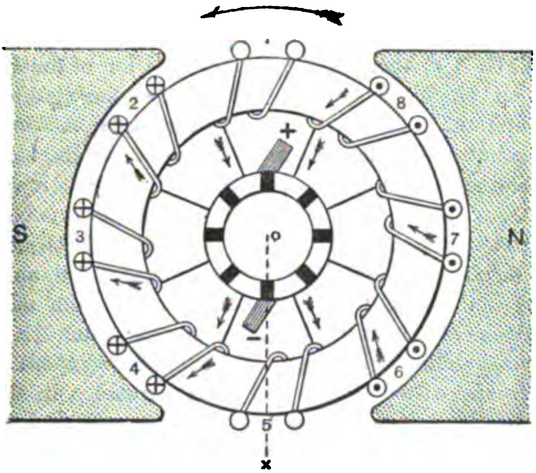


Fig. 105.

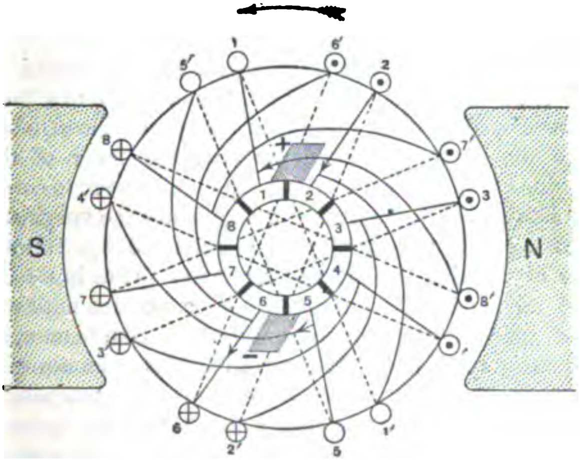
§ 7. **Eight-coil bipolar armatures.**—Having once arrived at an armature winding divisible into two halves in parallel, it is easy to pass from two coils in each half to three or four, or any larger number, in each half; all that is necessary is to introduce more sectors into the commutator, each forming a connection between the end of one coil and the beginning of another.

Thus, with eight coils, we get the complete commutator of fig. 103 with eight sectors. In fig. 106 is shown a ring-wound armature with an eight-part commutator, and also the analogous drum-wound armature, each with the same number of inductors, occupying the same amount of available space on the core; in the latter each loop is indicated by the same numerals, as 1, 1′, the end connections at the further end of the drum being shown dotted. Both armatures are wound left-handedly (the curved connections at the front of the drum being assumed to lie behind the straight ones), and the current enters and leaves by corresponding brushes; further, the direction of the current along the length of the inductors is shown by marking those in which it flows towards an observer at one end with a dot, those in which it flows away being crossed, while the short-circuited inductors are left white. Such armatures with eight coils will give a curve of E.M.F. similar to that given by the four coils of fig. 102, but will again carry twice the current.

To still further decrease the fluctuations of the E.M.F., more coils or 'sections' can similarly be wound on to the armature, with a correspondingly increased number of commutator sectors. The amount of fluctuation with a given number of sectors depends upon the shape of the curve of E.M.F. yielded by each section of the winding, and so upon the ratio of the polar arc to the pitch. The greater this is, the less the fluctuation, and if the pole-width amount to about 0·75 of the pitch, as in practice is usually the case, any number of sectors more than thirty may be regarded as giving a sensibly constant E.M.F. at the brushes. With a large number of commutator parts, the simple tubing divided as shown in fig. 105 has to be replaced by a built-up structure, consisting of a number of wedge-shaped bars or sectors of hard-drawn copper or gun-metal, placed side by side, but completely insulated from each other by intervening strips of mica, so as to form a smooth cylinder upon which the brushes rest, the whole being insulated from the sleeve and collars by which the sectors are held tightly together.



I. Ring.



II. Drum

FIG. 106.—Eight-coil bipolar armatures. (Left-handed.)

§ 8. **The hand of the armature system.**—It must here be remarked that in the continuous-current dynamo apart from the hand of the winding of its several loops or coils as defined in Chap. VIII. § 5, there is the further question of the *hand of the system*[1] on which the coils or sections are connected together at the commutator. The analogy of the screw is in fact extended to the system of connections between the coils, and the hands of the system and of the coils are not necessarily the same when, as is usually the case, the commutator is a separate structure such as has been described and is placed at one end of the armature. In the ring machine, if we stand at the commutator end, and, starting from any one sector, trace the winding *away* from our point of view *through the interior of the ring*, we pass round the sectors in a *clockwise* direction if the armature system be *right-handed*, or in a counter-clockwise direction if it be left-handed. In the case of a drum armature, if we follow any sector away from our point of view at the commutator end, we come to the two connectors leading to the inductors, and of these the one which lies *nearest to the armature core* must always be taken. If we then follow this, and trace the winding on to the next sector, we pass round the commutator in a *clockwise* direction if the system be *right-handed*, or in a counter-clockwise direction if it be left-handed. When the winding is developed on a flat surface and a core is imagined to be thrust through the loops, the condition that the inner of the two connections from the sector must be followed is seen to correspond with following the lower arm of a loop below the imaginary core, and this again is the counterpart of the condition "through the interior of the ring" in the ring armature. Thus in the drum of fig. 106 the hand of the system cannot be decided until we know which of the two sets of connections from the commutator, the one shown straight and the other curved, happens to lie nearest to the core; when it is assumed that the curved connectors pass nearest inwards to the body of the core, it follows that the system of the armature as a whole is left-handed. If the above rules are carried out with either the ring or drum armature, it will be found that, whatever the actual hand of the individual coils, we are virtually describing on the top of them, as it were, an imaginary helix, either right- or left-handed as the case may be, and it is on this account that the term 'hand' is metaphorically extended to the system of interconnections, and their sense defined by the foregoing convention.

---

[1] See *Electrician*, vol. xxxix. p. 652.

§ 9. **The nature of the closed-coil armature.**—The fundamental principle which formed the starting-point to a discussion of the armature winding of bipolar dynamos was that in all the inductors under, say, the N. pole the E.M.F.'s induced along their lengths were in the opposite direction to the E.M.F.'s in the inductors under the S. pole; an inspection of fig. 106 shows that, by both methods of winding, the small E.M.F.'s induced simultaneously in the several inductors are added together in each half of the winding, so that it is their sum which causes the current to flow through the circuit, entering and leaving at the brushes. Every bipolar closed-coil armature overwound with inductors may be divided diametrically into two halves by a line corresponding to the diameter of commutation, or, in other words, by a line at right angles to the general direction of the lines of induction as they cross from the one pole-piece to the other; along the outside of the half which is on the one side of the dividing line, say under the N. pole, there is a sheet of current flowing in all the inductors in the one direction, as viewed by an observer at either end; while along the outside of the other half, the sheet of current flows in the opposite direction. The brushes remaining stationary on the revolving commutator form the contact surfaces, by means of which the current is collected from both halves of the winding, and passed into the external circuit; while further, by short-circuiting the coils which are in the position of zero E.M.F., they serve also to 'commute' the current, or enable a coil to pass from the one half of the armature over to the other, where the current round it is in the reverse direction. The commutator sectors which are at any moment situated between the brushes serve merely as junctions between the end of one loop or coil of many loops and the beginning of the next, but their true function is called out as rotation continues, and they pass successively under the brushes.

§ 10. **The E.M.F. equation of the continuous-current dynamo.**—With a sufficient number of inductors, and of commutator sectors, since the several inductors which form one circuit or path from brush to brush are spaced out uniformly round the armature, the sum total of the E.M.F.'s due to their different positions will be equal to the average E.M.F. of one inductor multiplied by the number of inductors in series; for each value of the single rotating inductor as it passes from point to point of the field, will be reproduced in some one inductor which at the moment is situated at the same spot. Thus with a given speed of rotation the total E.M.F. is independent of the distribution of the flux-density round the polar arc, and further remains constant throughout a whole revolution, its instantaneous and average values being identical. The general equation for the electromotive force of a continuous-current dynamo is therefore easily established as follows. As in Chapter VIII., § 10, in any armature, whether bi- or multipolar, the number of lines cut in one revolution by each inductor is $2Z_a \cdot p$, and the average E.M.F. of one inductor is $2Z_a \cdot \frac{pN}{60} \times 10^{-8}$ volts. If τ as before = the total number of inductors counted all round the surface of the armature, whether it be ring, or drum, or disc, and q = the number of parallel circuits or paths from brush to brush into which the armature winding is divided, the number of inductors in series in one circuit is $\frac{\tau}{q}$. Hence the internal E.M.F. induced in the armature of the continuous-current dynamo is in every case without exception

$$E_a = \frac{2}{q} \cdot Z_a \cdot \frac{pN}{60} \cdot \tau \times 10^{-8} \text{ volts} \quad . \quad . \quad . \quad . \quad (14)$$

This may be compared with the similar expression for the E.M.F. of an alternator (equation 11). The term $\frac{pN}{60}$ is the periodicity of the current in the inductors before it is commuted, and its value in the continuous-current dynamo will be found to be of some importance as bearing on the loss by hysteresis in the core and by eddy-currents in the armature as a whole. There can be no division of the winding into different phases, so that the m of the alternating E.M.F. equation disappears. The constant K is also absent, since there is no differential action[1] and no distinction between the average and effective E.M.F.'s; in fact, the E.M.F. of the alternator was really expressed in terms of that which would be given by a continuous-current dynamo similarly wound, multiplied by a constant K which took into account the shape of the curve of E.M.F., and it was only on this account that the average E.M.F. of the alternator possessed any interest. The only remaining difference between the two equations lies in the term q. While in the alternator, there may be and usually is only one circuit through the armature per phase, or $q=1$, in the closed-coil armature it has been shown that there can never be *less* than two circuits; further, if there are *more* than two, the number must be some multiple of 2, so that an uneven number is impossible. In the simple bipolar dynamo there are two parallel paths, and this is also the number of poles, $2p$, so that

$$E_a = Z_a \cdot \frac{N}{60} \cdot \tau \times 10^{-8} \text{ volts.}$$

§ 11. **The multipolar parallel-connected ring armature.**—The *spirally-wound ring* of fig. 106, forming a single closed helix, is in itself unaffected by the number of poles; by merely placing $2p$ sets of brushes on the surface of the commutator at equal distances apart, the winding is at once divided into as many equal and symmetrical paths through the armature, and q becomes $=2p$. Or by the same method as was applied to alternator armatures in Chap. VIII. § 7, let the spirally-wound ring of fig. 106 be cut radially along the line X O, and opened out; then the whole can be repeated a second or any number of times, as in fig. 107, where the dotted lines show a repetition of the bipolar armature, the full and dotted lines together forming a four-pole ring dynamo. A multipolar armature is thus obtained having as many parallel circuits and as many points of collection of the current as there are poles. Since $q=2p$, the expression for the E.M.F. of such a *multipolar parallel-connected* armature is exactly the same as for the bipolar machine, viz.:

$$E_a = Z_a \cdot \frac{N}{60} \cdot \tau \times 10^{-8} \text{ volts} \quad . \quad . \quad . \quad . \quad (15)$$

---

[1] Except in a minor degree, as will appear in Chap. XVII.

Just as a bipolar machine is in reality two dynamos in parallel, each giving the same E.M.F. but together capable of carrying twice the current that either could carry by itself, so the four-pole machine is in reality two bipolar machines on a common core, together capable of carrying four times the current that each quarter of the winding could carry separately. And as in the two-pole dynamo there are two neutral points where the coils are commuted, and two corresponding points on the commutator where the current is collected by the brushes, so for the parallel-connected armature placed in a four-pole field there are four such points. To collect the current opposite brushes *a* and *c*, *b* and *d* are joined together to form a pair of terminals to which the external circuit is applied. If the increased number of points of collection is regarded as a disadvantage, they may again be reduced to two if all commutator sectors which are situated $\frac{360^\circ}{p}$ apart are joined by *cross-connections*. The sectors which are at any moment at the same potential are thereby joined together, and two sets of brushes are alone required. Thus in a four-pole machine each commutator bar must be connected to that which is diametrically opposite, and there is a choice between two positions for the brushes at right angles to one another. In

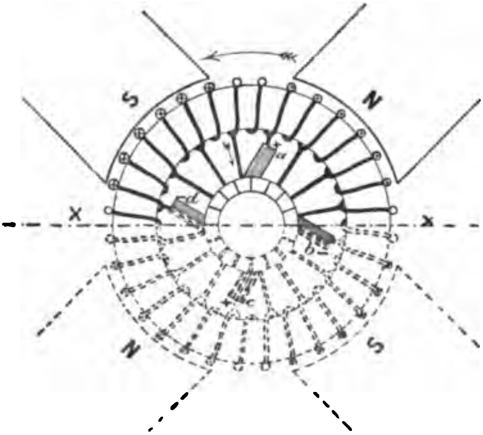


FIG. 107.—Parallel-connected multipolar ring armature.

a six-pole machine each cross-connection must unite three sectors situated 120° apart, and there is a further choice of the relative angular position for the brushes, since they may be either 60° or 180° apart; in an eight-pole machine each cross-connection must join four sectors 90° apart, and the brushes may be either 45° or 135° apart. Thus in general the angle between the brushes may be $\frac{180°}{p}$, or any uneven multiple of this angle. Fig. 108 illustrates the cross-connections of a four-pole and a six-pole machine, arranged within the commutator in two planes and formed by curved strips of copper of double evolute shape, as shown to the right of the diagram; in order to spread the connections quite evenly round the shaft, it is advisable even in the four-pole machine to employ two connections for every commutator sector passing on opposite sides of the shaft, unless half the number of commutator sectors or $\frac{N_2}{2}$ is an uneven number. When cross-connected, the commutator must be p times longer than before in order to provide sufficient surface of contact to collect the current by the two sets of brushes; further, the number of commutator sectors N_2 must be divisible by p.

§ 12. **Rise of E.M.F. in the ring armature.**—In the ring armature there is at any moment a gradual rise of potential as we pass along the winding from a negative brush to a positive, and an equal fall of potential when we pass onwards from a positive to a negative brush. In fig. 109 the ring of fig. 106 is supposed to have been cut along its length at the point marked x, and to have been opened out flat. Arrowheads on the lines representing the external inductors indicate the direction of the rise of potential or of the current in each inductor, each active inductor is assumed to be giving two volts, and those which are short-circuited by the brushes are marked by little circles. The difference of potential between adjacent wires is only that due to the added voltage of one inductor if the winding be in one layer, or to one section if there be several layers;[1] and as this is but a few volts even in high-tension machines giving as much as 3000 volts at their terminals, the insulation of the wires from one another presents little difficulty. In this lies a distinguishing feature of the ring armature which separates it from the drum, wherein the full difference of potential of the machine exists between two pairs of neighbouring inductors, as is in fact shown by the drum of fig.

---

[1] If N_2 = number of sections, and E_b is the voltage at the brushes, the difference of potential between adjacent wires cannot exceed $\frac{2p \cdot E_b}{N_2}$.

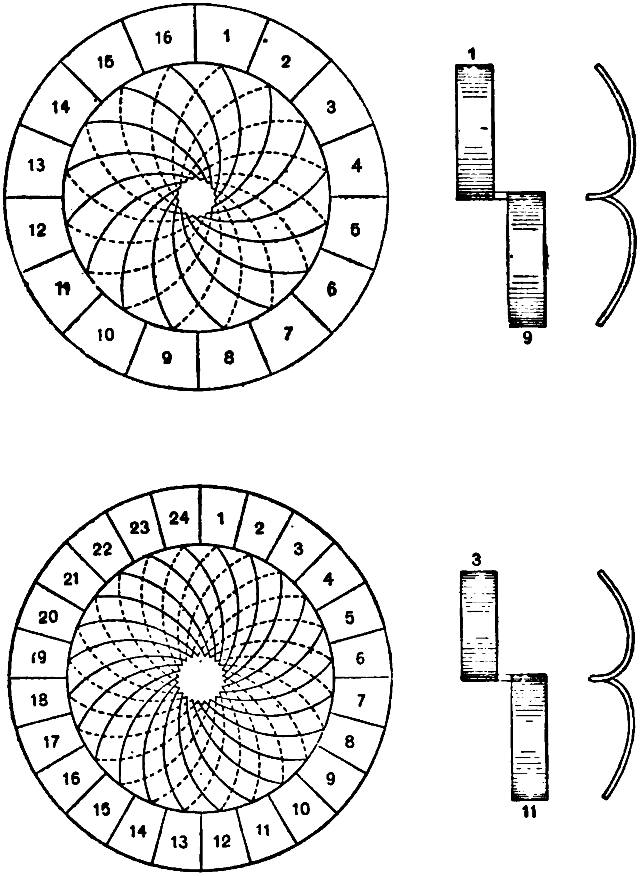


FIG. 108.—Cross-connected commutators.

106; for there it is seen that the two short-circuited loops, one of which is under the positive brush and the other under the negative, are contiguous to each other, *i.e.* loops 1, 1′, and 5, 5′; while, further, between the short-circuited loops and the neighbouring loops on either side the full pressure of the whole E.M.F. of the machine exists.

The rise or fall of the potential difference between adjacent commutator sectors reproduces that between the inductors, but occurs by successive and approximately equal steps. Each sector has its potential raised above that of the sector behind it by the voltage due to the section of winding that lies between the two. The average difference of potential between two adjacent sectors is thus $\frac{2p \cdot E_b}{N_2}$. In ordinary dynamos for incandescent lighting this difference may be taken as ranging from two to five volts; *e.g.* in a bipolar dynamo giving 100 volts with a 50-part commutator, four volts are generated per sector. It should be observed that in the ring there can be as many sectors as there are inductors or loops, but that in the drum-wound armature there can never be less than two inductors in a section, the number of loops being always half the number of inductors.

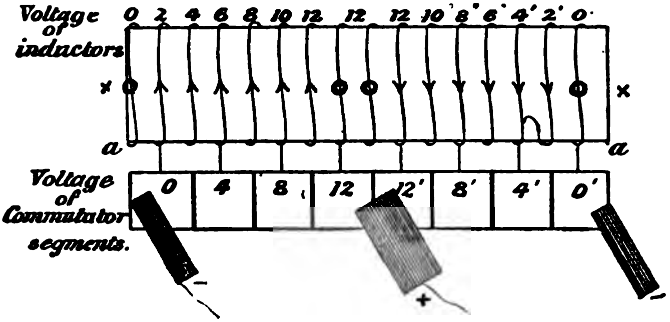


FIG. 109.—Development of ring armature.

If each section of the spirally-wound ring consists of a single loop, the hand of the loop is necessarily the hand of the system on which they must be connected; if each section is composed of several loops in one layer, this should again be the case in order to avoid crossing of the commutator connecting wires, and to cause the rise of potential round the winding to follow that round the commutator (fig. 109). If the several loops are arranged in more than one layer and the coil has a width of more than one turn, the winding should be begun at or near the centre of the total length of wire in the coil, so as to bring both the free ends to the outside layer, and in this case the hand of the system should be the same as the hand of the bottom layer.

If following the direction of rotation we pass along the general course of the lines from a N. to a S. pole, a negative brush is crossed in the interpolar gap if the armature be right-handed, or a positive brush if it be left-handed.

§ 13. **The multipolar series-connected ring armature.**— In the multipolar ring armature of fig. 107, each pole corresponded to one circuit through the armature, the total number of circuits being equal to the number of poles; it is, however, also possible to add together the inductive effect of two or more poles, so that the E.M.F.'s produced by half the total number of inductors are summed up in series. The armature is thus divided into two halves in parallel, whatever the number of poles, and we obtain the *wave-wound series-connected ring*, in which $q=2$. The winding is then no longer a simple spiral, although it is still closed on itself, and its description as "wave-wound" will be clearer after the analogous drum winding has been considered. The method necessitates the connection in series either of $2p$ coils similarly situated one under each pole (compare fig. 90), or of p coils similarly situated in fields of like polarity. The first of the alternatives alluded to above involves a complicated system of end connections, and further brings into contiguity coils at widely different potentials; the great advantage of the ring is thereby lost, and the method is not used in practice on this account. Of the second alternative there are two variations. By the first (fig. 110), due to Prof. Perry, there are as many commutator sectors as there are coils, and each forms the junction of two coils situated nearly but not quite $\frac{360°}{p}$ apart, and so in fields of like polarity. In any ring armature the finished winding may be divided into N_1 sections which are spaced at equal distances and which may be marked with consecutive numbers up to N_1, that direction round the armature being followed which agrees with the hand of the winding; each section may consist of only one turn, in which case $N_1 = \tau$, or of t turns wound either side by side or on top of each other so as to form a separate coil or little group of wires, which may be regarded as an element of the winding; the total number of such elements is then $N_1 = \frac{\tau}{t}$.

Let the *pitch of the winding* be defined as the number of such small elements that must be passed through in joining one element to another, and let it be indicated by y. To find its value, take any coil x as a starting-point, and reckoning it as one element, count the number of elements round the armature which must be passed through until we reach the position of the element to the beginning of which the end of coil x has to be joined; thus in the simple spirally-wound bi-polar or multipolar ring of fig. 106 and 107, $y = 1$, since the *end* of coil x has to be joined to the *beginning* of the $(x + 1)$th coil. When the same process is applied

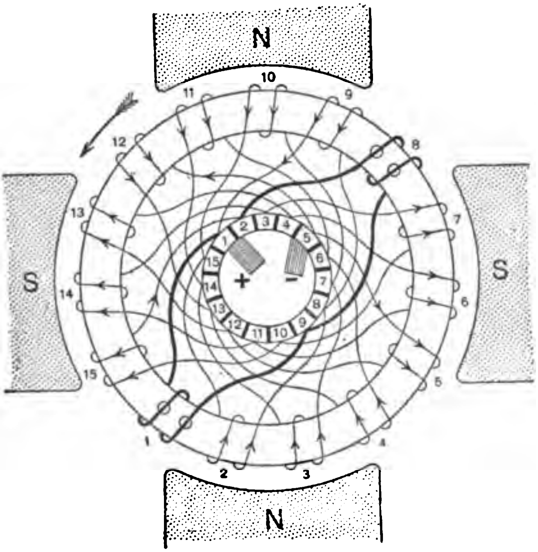


FIG. 110.—Wave-wound series-connected ring (1st method).

to the multipolar wave-wound series-connected ring of fig. 110 the pitch is found to be $y=\frac{N_1-1}{p}$, in our diagram $=\frac{15-1}{2}=7$; the coils being wound left-handedly are numbered in a counter-clockwise direction, and the end of coil 1 is joined to the beginning of coil $1+7=8$, the end of coil 8 to the beginning of coil $8+7=15$, the end of coil 15 to the beginning of coil $15+7=7$, and so on.[1]

If p be even, N_1 must be uneven, but if p be uneven, N_1 may be either odd or even. The difference of potential between neighbouring wires cannot exceed $\frac{2(p+1)E_b}{N_1}$ or $\frac{2(p+1)E_b}{N_2}$, since $N_1=N_2$; *i.e.* it is equal to the volts induced in one section multiplied by the number of pairs of poles plus one; *e.g.* in a four-pole machine it is equal to three times the average volts of one section. The average difference of potential between adjacent sectors is equal to the voltage of one section multiplied by the number of pairs of poles, which gives the same formula as before, viz. $\frac{2p \cdot E_b}{N_2}$. The number of coils which are simultaneously short-circuited in series at each brush is equal to the number of pairs of poles; thus in the position shown in the diagram, of a four-pole machine, coils 1 and 8 are short-circuited at the positive brush, and an instant later coils 12 and 4 will be short-circuited at the negative brush. There is always one more element in one branch of the armature than in the other, but neither this fact nor the fact that the sections are not short-circuited simultaneously at the two brushes is detrimental in practice when the number of sections is considerable.

By the second variation (fig. 111), due to Prof. Arnold, there are p times as many commutator sectors as there are armature sections, and the pitch when the coils are numbered in accordance with the direction of their hand as required above[2] is $y=\frac{N_1+1}{p}$, or in fig. 111, $\frac{15+1}{2}=8$. The difference of potential between adjacent wires cannot exceed that due to one coil, or $\frac{2E_b}{N_1}$, so that the winding is specially suited to high-potential machines, and the average difference of potential between neighbouring sectors is only $\frac{1}{p}$ times that of the preceding case. Only one coil is short-circuited at each brush in succession; thus the short-circuiting of coil 1 in fig. 111 will be followed immediately by the short-circuiting of coil 12 at the positive brush, then by that of coil 8 at the negative brush, next by that of 4 at the positive brush, and so on. If the number of pairs of poles be even, *e.g.*, with a four-pole machine when $p = 2$, $\frac{N_2}{2} = N_1$ must be an uneven number; and the winding is best suited to four poles, since with six or more poles the cross-connections become complicated. With a six-pole machine there must be three times as many commutator sectors as there are coils, one attached to

---

[1] The pitch might also be made $\frac{N_1+1}{p}$, but this leads to unnecessary crossing of the connections, and is not therefore to be recommended.

[2] Thus the longer of the two possible pitches is in this case to be used, since the shorter leads to an irregular distribution of potential around the commutator.

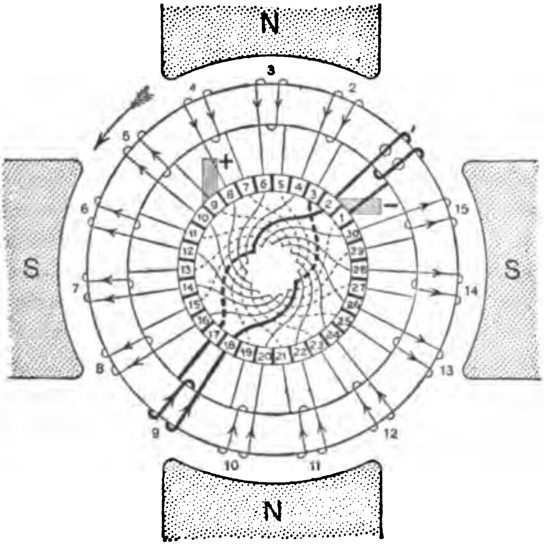


FIG. 111.—Series-connected multipolar ring armature (2nd method).

each end of a coil, and an additional one at their side to which only a cross-connection is joined.

In each of the above windings, since $q=2$, the formula for the E.M.F. of the multipolar wave-wound series-connected armature is

$$E_a = p \cdot z_a \cdot \frac{N}{60} \cdot \tau \times 10^{-8} \text{ volts} \quad . \quad . \quad . \quad . \quad (16)$$

As regards the position of the brushes, there is in each case the same choice as has been before described in connection with the cross-connected parallel-wound armature. If required there may be as many sets of brushes as there are poles; *e.g.*, in fig. 111 a second negative brush may be set on sectors 17 and 18, and a second positive brush on sector 25. In this case more coils are simultaneously short-circuited but no further change is introduced, and the current still remains divided between two parallel paths. Finally the hand of the armature system in the wave-wound multipolar as in the spirally-wound ring is the same as the hand of the coils; *e.g.*, in figs. 110 and 111 the commutator sectors are not traversed in succession but the winding dips into the commutator at intervals, yet the direction in which we pass through the sectors is counter-clockwise in the left-handed armature.

§ 14. **Drum armature winding and its two forms.**—Passing to the drum armature, we see from fig. 69 that in order to avoid direct differential action of the two sides of a loop, its width must exceed the width of the pole-face or polar arc; so long, however, as this is exceeded and the loop has therefore but little E.M.F. induced in it when in the position of commutation, it is not necessary that the width of the loop should be precisely equal to the pitch of the poles. In fact, if the number of commutator sectors is even and the winding is disposed in one layer, a symmetrical winding cannot be obtained, unless the width of the loop falls short of equality to the pitch by at least the width of one inductor or inducing side of a coil composed of several inductors. The finished drum winding may be divided into N_1 inducing sides or elements; if each section consists of a single loop, and this is very usually the case, $N_1 = \tau$, or if there are t loops in each section the group of wires forming the side of one coil is to be taken as one element and treated as if it were a single wire, so that N_1 then $= \frac{T}{t}$. If a number of elements, each consisting for the sake of simplicity of one inductor, be distributed at equal distances round a bipolar armature core as in fig. 112 and a loop be started from sector 1, the second side of the loop may be formed either by inductor 8 or 10. When the first loop has been formed on one or other side of the diametric line, *e.g.*, as in fig. 112 (i), let the elements be numbered consecutively from No. 1 in the direction of the end-connection furthest from the commutator; the *pitch* can then be defined as the number of elements of winding inclusive of No. 1 which are passed through in forming the loop. As opposed to the ring armature, each of the N_1 elements has now a front and a rear end, so that there must be two connections which may not have the same pitch, and these must be distinguished as y_F and y_B, the latter being the one furthest from the commutator. In order that each element may be successively traversed, each of the pitches must be an uneven number. In our case with an even number of commutator sectors and the winding disposed in one layer, the pitch at the rear end of the armature or y_B must not exceed $\frac{T}{2} - 1$, and is shown with this maximum value, viz., $\frac{16}{2} - 1 = 7$. Thus the loop spans a chord slightly less than the diameter *e.g.*, - 18, or is in effect wound on one side of the core. Two

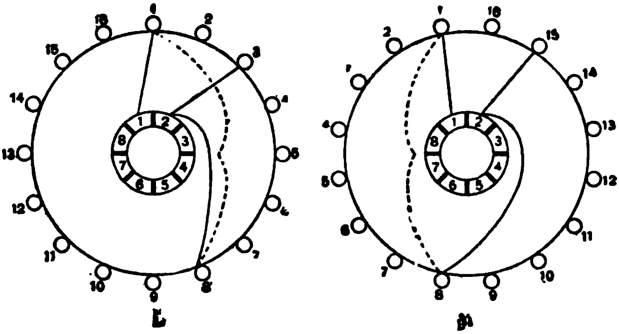


FIG. 112.—Element of lap- and wave-wound drum armatures.

distinct methods now present themselves, by which the loop may be completed and joined on to the next loop.

§ 15. **(I) Lap-winding.**—By the first or *lap-winding* (fig. 112, i) the end of loop 1–8 is formed by bringing the wire backwards and joining it to a commutator sector, whence it starts off again to a third inductor (No. 3) on the same half of the core as the first loop and lying within its two sides, *i.e.*, the new loop starts with the inductor next but one to the first inductor. To whichever side of the diameter the two inductors of the first loop fall, it is evident that the rear end-connector should pass round the shaft on the same side, since this gives the least length of copper; and the direction in which the back end of the loop is formed gives the positive direction for numbering. It is evident also that the front connections to the commutator and onwards to the second loop should also pass round the shaft on the same side. Thus the two pitches y_B and y_F have opposite signs, being in opposite directions, and the front pitch must be less than the rear pitch; in our case they differ by two, y_B being 7 and $y_F = -5$. Thus the winding works continuously forwards and backwards round the armature until it closes on itself, after all the inductors have been traversed. Fig. 113 repeats exactly the same left-handed drum as that of fig. 106, but with the inductors numbered in succession round the armature instead of by loops; further, the end-connections at the far end, instead of being shown passing straight across from side to side, are now curved as in practice would be the case. At the bottom is given the development of the winding, when the armature is supposed to be cut at the point x and opened out after the same manner as the ring armature in fig. 109; the letters *a*, *b*, *c*, etc., mark corresponding portions of the end-connectors, where they would join if the armature were again bent up into a cylinder. The developed winding takes the form of a number of overlapping loops (two of which are marked by heavy lines), whence its name of 'lap-winding' originates. The rise of volts round the two divisions of the armature and also on the commutator sectors is marked on the supposition of two volts being generated per inductor; it should be contrasted with that of the ring armature in fig. 109. The end-connectors are seen in the development to form two layers, the lower passing underneath the straight prolongations of the bars at the commutator end; if we pass from a commutator sector inwards up to the

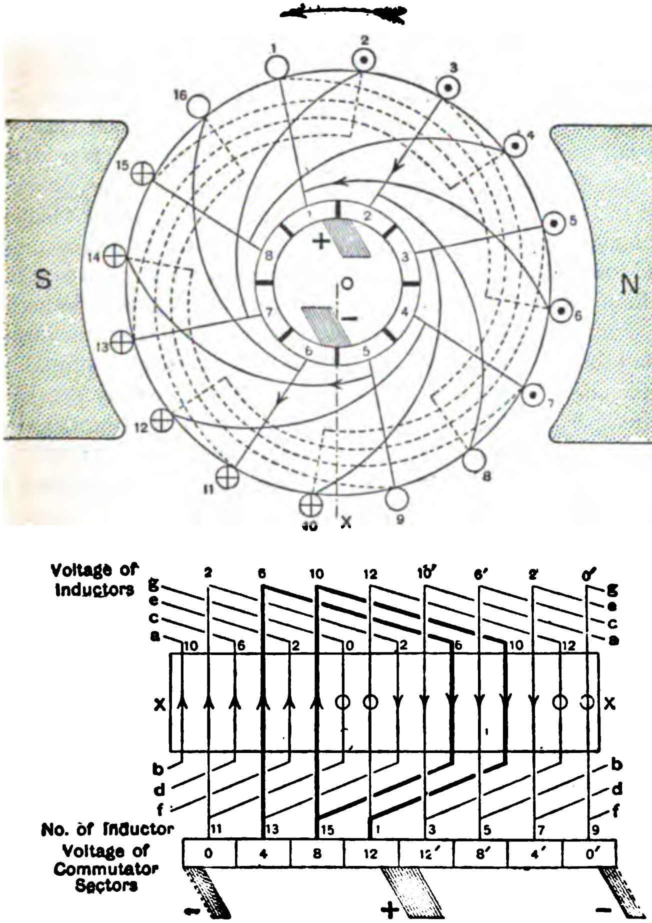


FIG. 113.—Lap-wound drum (left-handed).

core by the lower arm of a loop and thence trace the winding onwards, we pass round the commutator in a counter-clockwise direction, so that the system is left-handed. Thus if the winding is composed of bars, every other one of which projects beyond the intermediate ones at the commutator end, the rule for finding the hand of the system may be expressed more simply by saying that the winding must be traced from a sector down the end-connector leading to a short bar and not down a long bar. But the commutator connections may equally well spring from the centre of the connection between the two bars, in which case we must fall back on the general rule that the lower of the two layers at the commutator end must be followed, so that we proceed down the arm which forms part of the inner set of spirals next to the armature core. The corresponding right-handed arrangement would join inductor 1 in fig. 112 (i) with inductor 10, and end the first loop with sector 8, the second loop beginning with inductor 15. For the purpose of comparison, fig. 114 showing the right-handed form is added with the direction of numbering altered to suit the altered direction of the rear pitch as is strictly speaking required; it will be seen that with the same field polarities, the current now enters at the top brush and leaves at the bottom.

If the bars which project at the commutator end also project at the further end so that they become alternately long and short, as shown in figs. 113 and 114, the loops themselves then have a 'hand,' and if so the hand of the system should also be that of the separate loops. This is the most usual case, the bars or the sides of the loops being of different lengths, but in the bar-wound armature it is not necessarily the case. *E.g.*, in fig. 114, the sloping connectors at the back end may be imagined to be brought up to the top above the straight ends, the relative position of the two layers being thus reversed, and the connectors being made to slope at the back end in the opposite direction. The bars would now all be of the same length, so that at each end of the completed armature alternate bars would project, but the same bar would not project at both ends. The loops themselves now have no hand, although the direction of the current would be unaffected; from which it is evident that it is the relative position of the two layers at the commutator end only which determines the hand of the system.

If the winding at the commutator end consists of alternately long and short ends, and the commutator is attached immediately to the end of the long straight bars, it has what may be called its natural position, the position of the short-circuited loops in the

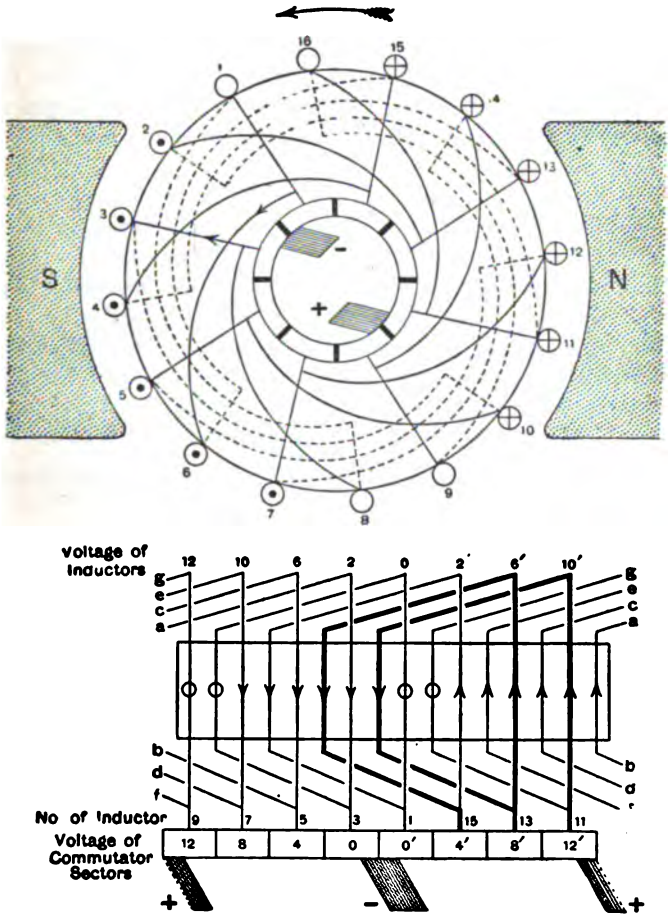


FIG. 114.--Lap-wound drum (right-handed).

gaps between the poles coinciding with the line of the brushes on the commutator; and when the direction of the wires in the finished armature is not easily traced, it is convenient to remember that in such cases, as in the ring, if we pass from a N. pole to a S. pole in the direction of the lines and also in the direction of rotation, a negative brush or point of collection is passed through in the interpolar gap if the system be right-handed, or a positive brush if it be left-handed. When the commutator is attached to the centre of the connectors, the brushes are shifted to positions under the poles, and in general if the sectors are in any way displaced relatively to the armature winding by twisting the connections between the two, the position of the brushes on the commutator may be altered, the apparent line of commutation no longer coinciding with the real line, which must always fall within an interpolar gap.

From figs. 113 and 114 is easily derived the ***multipolar lap-wound parallel-connected drum.*** Thus the dotted lines of fig. 115 show the armature of fig. 113 repeated a second time after cutting at the point x and opening out, so that it becomes a four-pole machine. The only difference from the bipolar form is that in the multipolar the pitch at the rear end is an odd number not exceeding $\frac{\tau}{2p}$, the maximum value with an even number of sectors per pair of poles being $\frac{\tau}{2p} - 1$, and the front pitch being also correspondingly shorter, or $\frac{\tau}{2p} - 3$. Thus in a four-pole field the loops, instead of spanning approximately half the circumference as in the two-pole field, will now span approximately a quarter of the circumference, and in a six-pole field approximately a sixth of the circumference.

§ 16. (II.) **Wave-winding.**—By the second method, or *wave-winding* of the drum, the end of the first loop of fig. 112 (ii) is taken to a commutator sector on the opposite side of the diametric line to the first loop, and thence the second loop starts with an inductor lying outside the first loop and next but one to the original inductor. Since the two end-connectors of any loop pass round the shaft on opposite sides their direction is continuously forwards, and y_R and y_F have the same sign. In our case $y_R + y_F = \tau - 2 = 14$, and this is divided equally, y_R and y_F being each $= 7$. Thus the rear end of bar 1 is connected to the rear end of bar

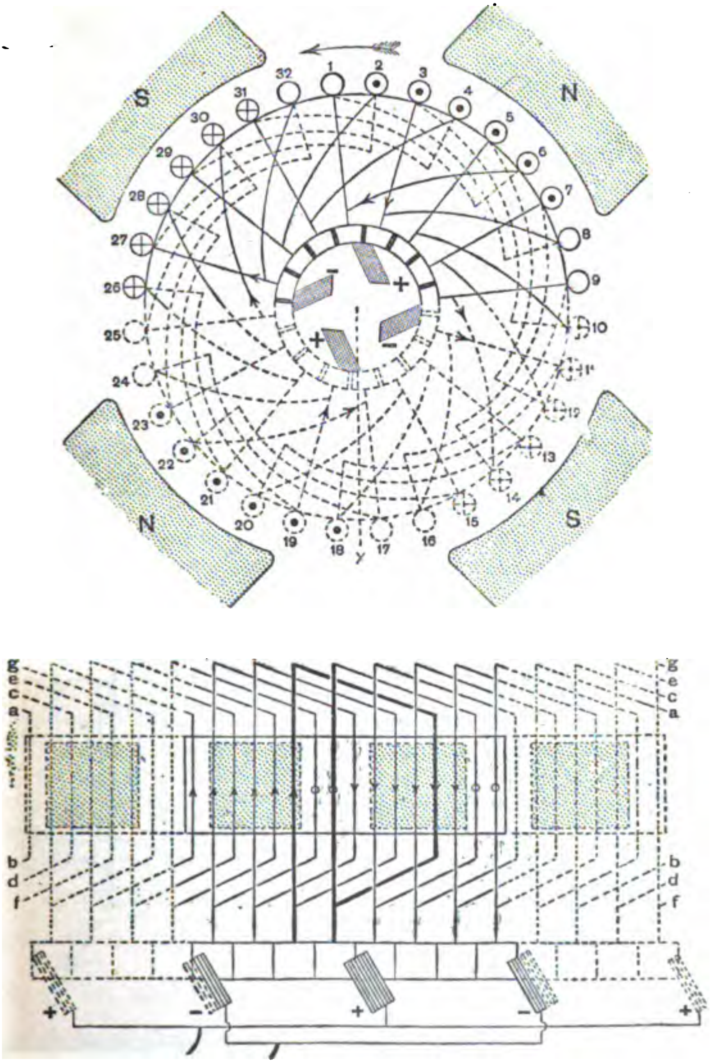


FIG. 115.—Multipolar lap-wound parallel-connected drum (left-handed).

(1 + 7) = 8, and the front end of bar 8 to the end of bar (8 + 7) = 15. When the winding is followed out in the developed plan

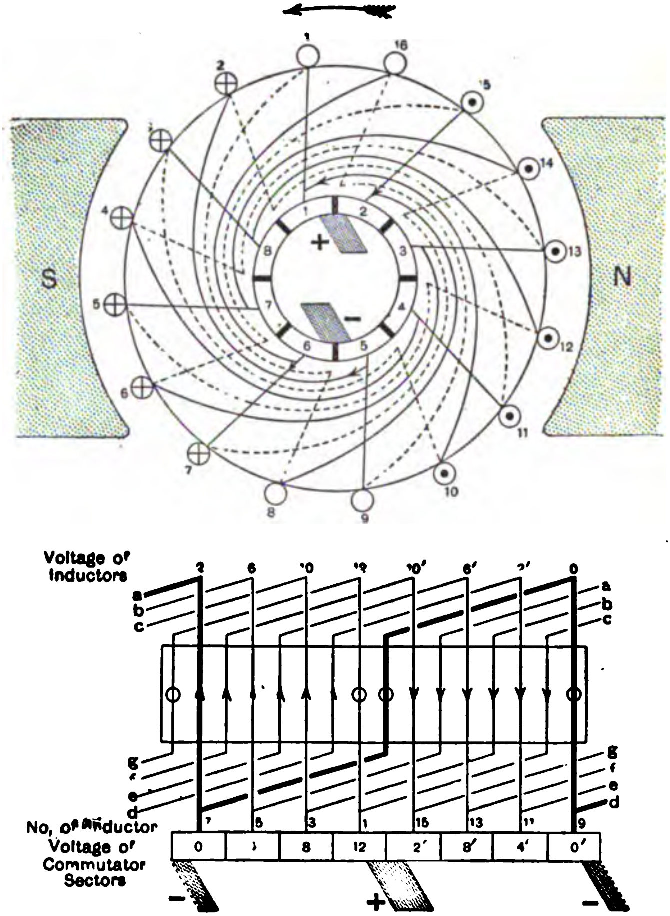


FIG. 116.—Wave-wound drum (left-handed).

(fig. 116), it is seen to work continuously forwards in a zigzag 'wave' round and round the armature. If the bipolar armature is cut open and repeated, the *multipolar wave-wound parallel-con-*

nected armature is obtained (fig. 117) with as many paths as there are poles.

In the two-pole form, after one tour of the armature the winding returns to the second inductor behind the starting-point, but now in the multipolar form after traversing as many inductors as there are poles, the winding has lost out of the 360° p times as much distance as before, or returns, *e.g.*, in a four-pole machine to the fourth inductor behind the starting-point. Thus in fig. 117, starting from say inductor 4 under a S. pole, the front end connection joins it to an inductor 11 nearly but not quite 90° ahead of it and under a N. pole; thence the circuit passes in succession to a third and fourth inductor nearly 180° and 270° ahead of the starting-point (18 and 25), and finally begins again with a fifth inductor, 32, which is the fourth behind the original starting-point. A similar tour of the armature is again made, until the winding closes on itself. The average pitch or half the sum of the pitches is therefore $\frac{y_B + y_F}{2} = \frac{\tau}{2p} - 1$. A peculiarity of the winding is that the loops are now short-circuited between two brushes of the same sign. The length of copper in each loop of the wave-wound armature is somewhat greater than in the corresponding lap-wound drum, but the difference is slight if the width of the bars is small.

The wave principle can, however, be applied to multipolar machines in the same way as in the wave-wound ring, so as to give a *series-connected wave-wound multipolar armature*, in which there are only two parallel paths and two points of collection of the current. In this form the winding resembles the bipolar wave-wound drum, in as much as, whatever the number of poles, the winding, after having once traversed the armature, always returns to the second inductor in front of or behind the starting-point. The average of the front and rear pitches must now be $\frac{\tau \pm 2}{2p}$, or in fig. 118, with a total number of 30 inductors, $= \frac{30 - 2}{4} = 7$, and the rear and front pitches are made each equal to 7. Only two sets of brushes are now necessary, and for four poles there is the choice of position at which they are set, viz. the two pairs of right angles.

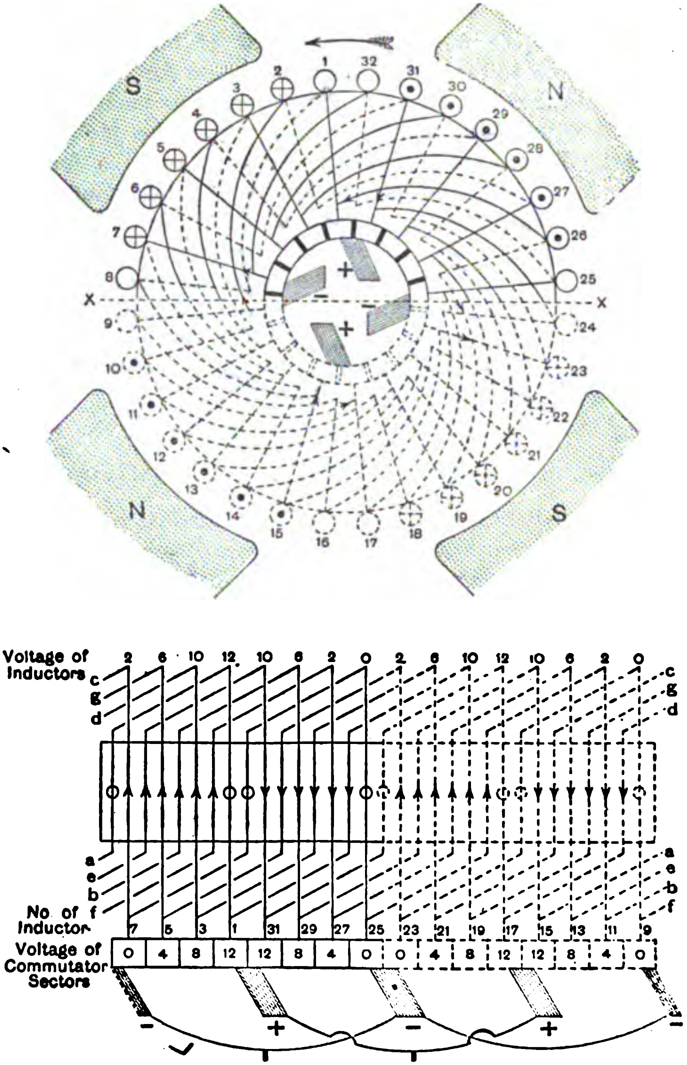


FIG. 117.—Multipolar wave-wound parallel-connected drum (left-handed).

With six or a greater number of poles there is a choice as to the angular distance at which they may be set as fixed by the

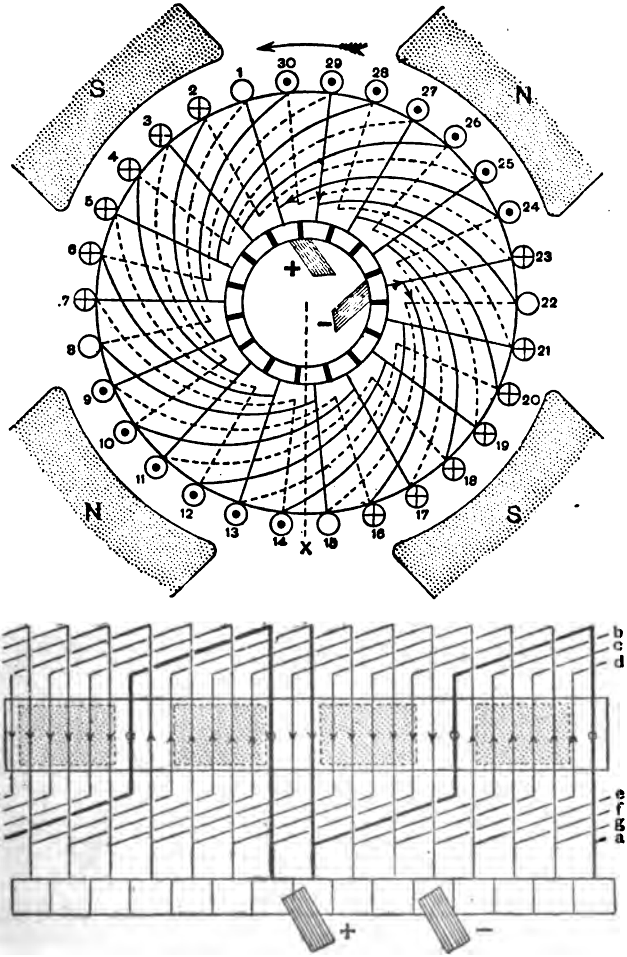


Fig. 118.—Series-connected wave-wound drum with two sets of brushes.

position of any pair of poles of opposite sign. Thus in a six-pole machine they may be set at 60° or 180°, and in an eight-pole machine at 45° or 135°. Or if necessary there may be as many sets of brushes as there are poles (fig. 119), but the armature winding

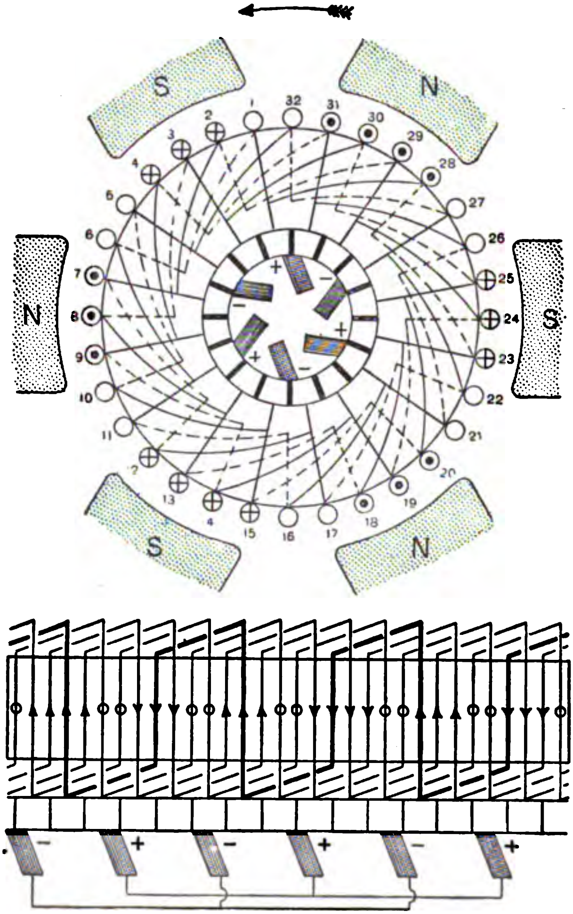


FIG. 119.—Series-connected multipolar drum with as many sets of brushes as poles.

is not thereby divided into more parallels, and the current in each conductor still remains half the total armature current. The average voltage between adjacent commutator sectors remains the same as in the parallel-connected armature, viz. $\frac{2p \cdot E_b}{N_2}$. The equations of the E.M.F. for the parallel-connected and series-connected drum armatures are identical with those for the corresponding ring machines, viz.:

$$E_a = z_a \cdot \frac{N}{60} \cdot \tau \times 10^{-8} \text{ volts} \tag{15}$$

$$\text{and } E_a = p \cdot z_a \cdot \frac{N}{60} \cdot \tau \times 10^{-8} \text{ volts} \tag{16}$$

the latter being p times the former.

Thus the chief advantage of the series-connected armature, whether ring or drum, is that for a given voltage the number of inductors is only $\frac{1}{p}$ of the number in the parallel-connected armature. Hence the space lost by the insulation of the wires is only $\frac{1}{p}$ of that lost in the parallel-connected armature, and the time taken to wind is less. On the other hand, owing to the smaller number of commutator parts in the series-connected armature, it may be difficult to keep the average difference of potential between neighbouring sectors within reasonable limits, and on this account it is better suited to fairly low speeds and small machines.

§ 17. **E.M.F.'s and resistance of parallel paths of armature must be equal.**—Owing to the fact that the circuits of the armature winding of any closed-coil continuous-current machine are in parallel, they may be likened to similar batteries joined in parallel so as to supply current in equal degree to one and the same external circuit (fig. 120). For this to be the case it is necessary that the E.M.F.'s and resistance of the batteries should be alike, but it is even more important in the case of the dynamo that the E.M.F.'s induced in each branch of the winding and their resistances should be exactly equal to each other. For consider the consequences if in the one half of a bipolar armature there is generated an E.M.F. greater by one volt than that which is generated in the other half. First suppose that the machine is running 'on open circuit,' that is, without any brushes to collect the current; then since the winding whether ring or drum forms an endless helix closed on itself, there would virtually be a constant E.M.F. of one volt driving a useless current round the armature through the resistance of the entire winding in series. Such a local current must absorb a corresponding amount of power, and this evil may assume considerable proportions if we remember that the resistance of all the armature coils in series may not be more than the hundredth part of an ohm, in which case a current of 100 ampères is produced, and power is wasted at the rate of 100 watts. Next let the brushes be 'down' in contact with the commutator, and suppose that current is being supplied to an external circuit (fig. 120); then if the negative brush be regarded as at zero potential, the potential of the commutator sectors upon which the positive brush rests must be equal to the internal E.M.F of that half of the armature which has the highest E.M.F. minus the loss of volts corresponding to the passage of the current through the resistance of that half. *E.g.*, let E_1 be the internal E.M.F. induced in the one half of the winding, and c_1 be the current flowing through its resistance r_1; then so long as $E_1 - c_1r_1$ is greater than E_2, the second half of the winding will not supply any current to the external circuit, and, further, will have a wasted current driven in a reverse direction through it; while the first half of the winding will carry both the useful and the useless currents, these together making up c_1. But if the resistance of the external circuit be decreased, and so C_e, the external current,

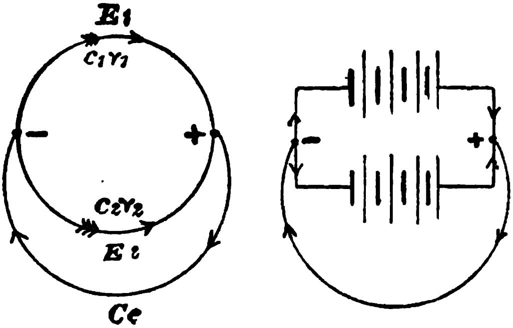


FIG. 120.

be increased, the loss of volts over r_1 will be increased, until after a certain point is reached $E_1 - c_1 r_1 = E_2$; then $c_1 = C_e$, the second half of the winding being entirely useless. If the external useful current be further increased, $E_1 - c_1 r_1$ will become less than E_2, and the useful current will now divide through the two halves in such proportions that $E_1 - c_1 r_1 = E_2 - c_2 r_2$, either value being the potential of the commutator sector E_b on which the positive brush rests. Hence, if $r_1 = r_2$,

$$c_1 - c_2 = \frac{E_1 - E_2}{r_1};$$

in other words, the current divides in unequal proportions through the two halves of the winding, the current through r_1 being greater than that through r_2 by the fixed amount necessary to lower E_1 to a terminal voltage equal to E_2. If the inequality of E.M.F. be great, an excessive current will be continually flowing through one half of the winding, while the other is carrying less than that which it is designed to pass without overheating.

The above is the result of inequality in the E.M.F.'s induced in two halves, which have the same resistance. Next let their internal E.M.F.'s be equal, but not their resistances, r_2 being greater than r_1. Then on open circuit there is no loss; but when the external circuit is closed, since the terminal voltage E_b of either half must be the same,

$$E_b = E - c_1 r_1 = E - c_2 r_2;$$

whence

$$\frac{c_1}{c_2} = \frac{r_2}{r_1};$$

and therefore the current divides unequally in the inverse ratio of the two resistances. As the armature rotates, this division of the current will adjust itself to the changing pair of resistances, and hence the current in each half will fluctuate, which is again undesirable.

In the case of a drum bipolar armature overwound with loops, since each loop passes simultaneously through the fields on both sides of the armature, it is unimportant, so far as equality of E.M.F. of the two halves is concerned, whether as many lines are cut by each inductor when moving under the one pole as when moving under the other; and therefore one field might, owing to a lateral leakage of lines from the core, be weaker than the other without prejudicial effect. But in the case of the ring armature the number of lines entering it on the one face must be the same as the number leaving it on the other, if two similar halves of the winding are to produce equal E.M.F.'s. In practice, however, this condition is usually fulfilled, and need not, therefore, be further considered.

When we pass to multipolar dynamos, it is of equal importance to both the ring and drum types that the E.M.F.'s of the several parallel branches should be exactly equal in order to avoid an unequal distribution of the current in them and consequent loss of power. On this score the wave-wound forms have a great advantage which is not shared by the simple spirally-wound ring and the lap-wound drum. In the former the two circuits from brush to brush consist of inductors, influenced by all the poles in the drum and by all the poles of like sign in the ring; hence if for any reason, such as eccentricity of the armature in the bore, or unequal permeance of the magnetic circuits, the inductive action of one pole is not equal to that of any other, this will have no effect upon the equality of the volts produced by the two halves of the winding. In the multipolar parallel-connected ring or lap-wound drum, on the other hand, it is as a matter of fact extremely difficult to secure absolute equality of the E.M.F.'s of the different branches. In order, then, to equalise any difference of pressure that there may be before the current reaches the brushes, a set of cross-connections may be added either to the commutator or to the winding proper at either end of the armature. Such connections are similar to those shown in fig. 108; taking say six equidistant points in each branch corresponding to one pole, at each point all bars or sectors situated $\frac{360°}{p}$ apart are united together by stout copper strips. Such *equalising connections* in no way prevent the evil, but they prevent the excess current of any stronger branch from flowing through the brushes, and so interfering with the commutation. This they do in virtue of the fact that they practically short-circuit the alternative path offered by the connecting lead between the brushes of the same sign, and the difference of the currents is therefore shunted through the auxiliary bye-pass without having to pass through the brushes.

§ 18. **Necessity for symmetry of each and every section of the winding.**—It is not, however, sufficient merely that the E.M.F.'s of the parallel branches should be equal. Given fields of equal strength and similar distribution on either side of the armature, it is evident that, whether it be ring- or drum-wound, since it rotates, the actual coils which are in any one parallel branch are at any moment continually and progressively changing. Hence in order to secure not only equality of internal E.M.F. but also of internal resistance in the parallel paths each loop should be exactly similar to every other loop in all respects, or at least all the sections of the winding as attached to the commutator sectors must be alike. They must therefore consist of the same number of turns or loops, with the same resistance and wound similarly; they must all be symmetrical relatively to the axis radially and circumferentially. If wound in more than one layer, the length of wire in each coil must be the same in order that all the coils may have the same resistance. In ring-wound armatures this is easily secured, since each coil, whether in one or many layers, is wound exactly similarly to every other; but in drum-wound armatures, in which there are two or more layers of winding, precautions must be taken to ensure an equal length and resistance in each and every coil. (Chap. XIII. § 23.)

There still remains another property besides their resistance in which the separate sections should be precisely alike, and this is their inductance. During the rapid change of the direction of the current in the short-circuited coils the inductance which has already been described in connection with alternating currents also comes into play, and, as will be explained in Chap. XVIII., upon this depends to a great extent the exact position at which the brushes should be set so as to short-circuit the coils at the right moment. Since the brushes can only be adjusted to suit the average coil, it is very important that the divergence of any section from the average in both resistance and inductance should be as small as possible.

Next, in ring or drum the brushes must be carefully set at the proper interval, so as to divide the commutator sectors, and therefore the armature coils, into equal circuits in parallel.

§ 19. **Necessity for magnetic symmetry with ring armatures.**—In the case of rings a further condition must be fulfilled, and this is that the field should be symmetrical. It has so far only been assumed that the number of lines cut by the inductors on either side of a bipolar armature is the same; but, more than that, if, as in fig. 48, the armature is divided into two halves by the dotted line S N, passing through the centre of the fields on either side of the armature, the distribution of the field in the upper half must be similar to the distribution in the lower half. If the density of the field, *e.g.*, at the lower corners, be greater than the density at the upper corners, the exact position in which a coil ought to be short-circuited by the brush will not be the same at the top as it is at the bottom; hence if both brushes are set so as to short-circuit the coils at their proper moments the armature will no longer be electrically balanced, since the portion of the winding between the brushes on the one side will not be similar to the portion between them on the other side, and therefore their resistances, if not their E.M.F.'s, will be unequal. Hence for ring armatures in especial not only must the winding be electrically symmetrical, but the field must also be magnetically symmetrical.

To sum up,—the condition of a successful continuous-current armature with a commutator is perfect symmetry, by reason of which not only the several parallel circuits but also the separate sections are as nearly as possible similar in both resistance and inductance.

§ 20. **Winding formulæ: (I.) Ring armatures.**—The results obtained above for the chief systems of armature winding may now be expressed in a more generalised way[1] and further expanded to include certain other cases. Throughout let $a = \frac{q}{2}$, or half the number of parallel branches into which the armature winding is divided.

---

[1] Based on the exhaustive treatment of the subject in Part I. of Prof. Arnold's *Die Ankerwicklungen und Ankerkonstruktionen* (3rd edit.), to which every writer on armature windings must be indebted.

(1.) *Ring armatures.*

(i.) *Spiral winding.* The elements or coils being numbered in that direction round the armature which agrees with the hand on which they are to be connected together at the commutator, and y being reckoned as positive when in this direction, the formula for the pitch of the spirally-wound ring is

$$y = \frac{a}{p}.$$

The end of the (x)th coil is always joined to the beginning of the ($x+y$)th coil.

According to the value of a, two subdivisions are obtained.

(a.) $a=p$. In the simple bi- or multipolar parallel-connected ring as described in sect. 11 there are as many parallel branches in the armature as there are poles, so that $a=p$, and therefore

$$y = 1.$$

The E.M.F. is $E_a = z_a \cdot \frac{N}{60} \cdot \tau \times 10^{-8}$ volts.

(b.) $a=np$, where n is a whole number greater than 1. If at a given speed an output consisting of a few volts and a large number of ampères has to be obtained, the simple spirally-wound ring may yield too small a number of inductors and sectors for satisfactory working. In such cases recourse may be had to *multiplex parallel-connected spiral winding*, in which $a=np$, and n is some whole number greater than 1. The number of parallel branches is now $q=2np$, *i.e.* n times more than before and the armature inductors and sectors are proportionally increased. The pitch is now

$$y = n$$

and the E.M.F. is $E_a = \frac{1}{n} \cdot z_a \cdot \frac{N}{60} \cdot \tau \times 10^{-8}$ volts or $\frac{1}{n}$th of what it would be if it were simplex.

Two variations are now possible. If the greatest common factor of y and N_1 is 1, the winding forms one *singly re-entrant* closed coil, *i.e.*, when it is traversed throughout from any starting-point it is found to make n passages round the armature and then to close on itself, although when the brushes are placed on the commutator it is divided into $2np$ parallels. If the greatest common factor of y and N_1 is two or more, there results two or

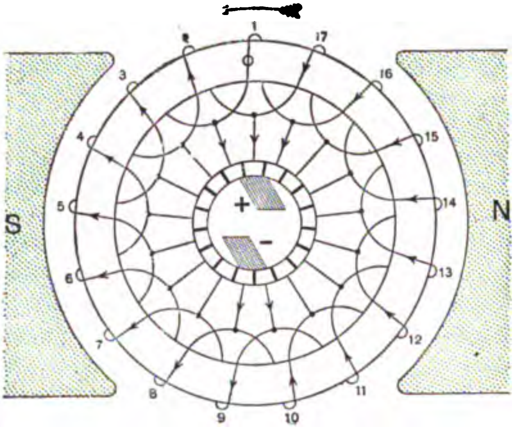


Fig. 121A.—Duplex parallel-connected ring, singly re-entrant.

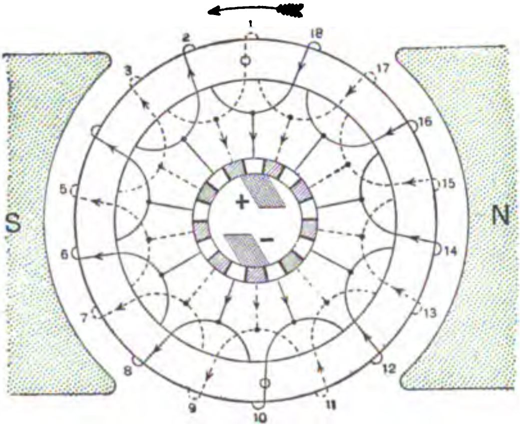


Fig. 121B.—Duplex parallel-connected ring, independent.

more independent windings, each separately re-entrant on itself. In fig. 121 (A) $y=2$ and $N_1=17$, and, their greatest common factor being 1, the winding is singly re-entrant. In fig. 121 (B) $y=2$, but N_1, = 18, so that their greatest common factor is 2, and we have two *entirely independent* windings. Both the above are instances of duplex winding, and in each case the brushes must have sufficient width of contact to cover more than one commutator sector with its insulation on either side so that neither of the multiple circuits may ever be entirely broken, and so that each coil may successively be short-circuited for a sufficient length of time. With a triplex winding the width of the brushes must exceed the width of two commutator sectors, and so on. The sectors are interleaved just as the coils are, and each coil is changed from one parallel path to another every time that it passes a brush, but the peculiarity of the first form which renders it preferable is that if any one coil be carefully traced during a revolution it will be found to pass gradually through each one of the $2np$ parallel paths; hence if there be any inequality in the resistance or inductance of the several coils, its effect is distributed, and is not confined to any one independent winding.[1]

(ii.) *Wave-winding.* The general formula for the wave-wound multipolar ring is

$$y = \frac{N_1 \pm a}{p}$$

which again yields two forms according to the value of a.

(a.) $a = 1$. There are then two armature branches whatever the number of poles, and we have the *series-connected multipolar ring*, for which

$$y = \frac{N_1 \pm 1}{p}$$

the shorter pitch being most suitable for Prof. Perry's and the longer for Prof. Arnold's method. The E.M.F. is now

$$E_a = p \cdot z_a \cdot \frac{N}{60} \cdot \tau \times 10^{-8} \text{ volts.}$$

(b.) $a > 1$ and may be $\gtrless p$ or $= p$. We thus get the *multiplex wave-wound series-connected ring*, in which the number of parallel paths in the armature, though always a multiple of two, is independent of the number of poles. The greatest common factor of y and N_1 gives the number of closed circuits that are obtained; thus if the greatest common factor is 1, or the numbers y and N_1 are prime to each other, the winding forms one singly re-entrant system. Both Prof. Perry's and Prof. Arnold's methods can be used to form these multiplex windings, the shorter of the two possible pitches or

---

[1] *Vide* Mather and Platt's Brit. Pat. 768, 1884, and Nebel's Brit. Pat. Nos. 4824, 1890 and 20840, 1891.

$$y=\frac{N_1-a}{p}$$

being preferable for the former and the longer or

$$y=\frac{N_1+a}{p}$$

for the latter. It may be observed that the above formulæ give strictly the best results, and the algebraic signs and direction of numbering the coils should be rigorously adhered to in reckoning the pitches, but the cross-connections of the second method with multiplex windings may sometimes with advantage be taken round in the negative direction or against the pitch when this is the shorter path between the two points which have to be connected.

The E.M.F. is now

$$\frac{p}{a} \cdot Z_a \cdot \frac{N}{60} \cdot \tau \times 10^{-8} \text{ volts} \quad . \quad . \quad . \quad (17),$$

and if $a=p$, becomes again the same as for the parallel-connected spirally-wound armature, viz.,

$$Z_a \cdot \frac{N}{60} \cdot \tau \times 10^{-8} \text{ volts.}$$

This leads us to a further interesting point. In fig. 122 is shown a multiplex winding with two wide brushes, but it has been mentioned that in the wave-wound ring, if it be desired to collect a larger current than can be conveniently done by two sets of brushes, other sets may be applied to the remaining neutral points. If then the two wide brushes of fig. 122 are replaced by four sets of narrower brushes as shown in the lower part of the figure, it will be found that no change will have been introduced save that a coil will be short-circuited between opposite brushes of like polarity instead of at one brush only, and that only two

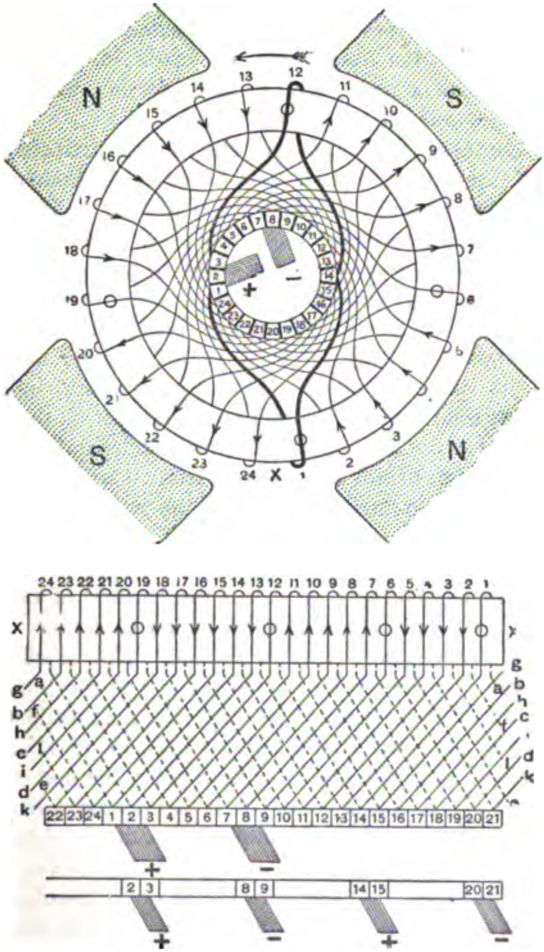


Fig. 122.—Duplex wave-wound series-connected ring.

coils are simultaneously short-circuited instead of four. Such a winding may therefore be regarded indifferently as a parallel-connected wave-wound machine or as a multiplex series-connected wave-wound armature with all the possible sets of brushes in use. The difference between the wave-wound parallel-connected armature and the spirally-wound parallel-connected armature then lies in the fact that the former can be made to give *any* whole number of pairs of paths, *e.g.*, 4, 6, 8, 10, etc., while the latter is limited to a multiple of the pairs of poles.

§ 21. **General formula for the ring armature.**—The nature of the resultant pitch may be analysed from a different point of view[1] which is of interest as rendering the expression for the wave-wound armature more intelligible and as providing a basis for a universal formula. In order to pass through all the elements from any given starting-point, one or more tours of the armature must be made, the number being determined by the class of winding employed. If K = the number of tours that have to be made round the armature for each independent winding, the total number in order to account for all the inductors is $K\,m$, where m is the number of entirely independent windings. K is itself the sum of two component parts, $A+B$; A is the number of tours due to the principal motion of the wave-wound armature whereby we pass in long strides from one element to another, and one tour is made when p elements have been traversed; the second component B may be called the 'creep' of the winding upon which depends the number of tours that must be made before the starting-point is again reached. Now it will be found that in every case the resultant pitch is equal to the total number of tours that must be made to traverse each independent winding multiplied by the number of such windings, or $y=K\,m$. In the spirally-wound ring whether bi- or multipolar, the first component motion is non-existent, or $A=0$; K is therefore simply the creep, B, and this is equal to one. In other words, after one tour of the armature the starting-point is again touched at. In the multiplex parallel-connected ring, if the winding is singly re-entrant, $m=1$, and $y=n=K$, or the number of tours is n. If the windings are independent, $K=1$ for each winding, but m tours are necessary, each being made from a different starting-point in one of the m independent windings, and the total number of tours or the pitch is $y = n = m$. In the wave-wound armature the number of tours given by the principal motion in each independent winding is $\frac{N_1}{mp}$, while $\frac{a}{mp}$ is the number of tours due solely to the creep. Their sum or difference is the actual number of times that the armature must be circled round before the starting-point is again reached. Thus in fig. 111 the principal motion would take us $7\frac{1}{2}$ times round the armature, but at each tour a certain amount is in this case gained which at the end amounts to half a tour, so that finally the number of tours is 8 and this is equal to the pitch, y. The general formula for the ring is therefore

---

[1] Due to W. B. Burnie, *Electrician*, vol. xlvii. p. 220.

$$y = m\left(\frac{N_1}{mp} \pm \frac{a}{mp}\right)$$
$$= \frac{N_1}{p} \pm \frac{a}{p}.$$

of which the first term corresponding to the principal motion is zero in the spirally-wound type.

§ 22. **Winding formulæ, contd.: (II.) Drum armatures.** —As already explained, N_1 in the drum armature is most conveniently taken as the number of inducing sides of coils, or twice the number of separate coils, and as each of the N_1 elements has a rear and a front end, there are two component pitches y_R and y_F, each of which must be a whole uneven number. The direction of the rear pitch or y_R is to be reckoned as positive, and half the *algebraic* sum of the two pitches is the resultant or average pitch y. In order that every element may be once passed through, y_R and y_F must have no common factor greater than 1, and if the winding is to be singly re-entrant the greatest common factor of y and $\frac{N_1}{2}$ must not exceed 1.

(i.) *Lap-winding.*

y_R being forwards or positive, y_F is backwards or negative, and their *algebraic* sum $y_R + y_F$ must be divisible by 2. The general formulæ for the two component pitches are therefore

$$y_R = \frac{N_1 - b}{2p} + \frac{a}{p}\,;\; y_F = -\left(\frac{N_1 - b}{2p} - \frac{a}{p}\right)$$

so that y_R is the longer of the two.

The resultant pitch reckoned in terms of the N_1 elements is

$$y_R + y_F = \frac{2a}{p}$$

or in terms of the coils is

$$\frac{y_R + y_F}{2} = \frac{a}{p}.$$

This latter expression is identical with that for the spirally-wound ring and the two windings being strictly analogous lead to similar results. If the *coils* are successively numbered *in the positive direction* as defined by the direction of y_R, the end of the (x)th coil is joined to the beginning of the ($x+y$)th coil (cp. fig. 106); or if the *sides* of the coils are numbered also in the same direction up to the number N_1, the far end of the (x)th element is joined to the far end of the ($x+y_R$)th element, and the front end of the ($x+y_R$)th element to the front end of the ($x+y_R+y_F$)th element (cp. fig. 113). The slope of the connectors at each end of the armature is seen to be in opposite directions. Following the same course as in the case of the spirally-wound ring we find that a may be equal to p or any multiple of p.

(*a*.) $a=p$.

In the simple bi- or multipolar parallel-connected drum with lap-winding $a=p$ and hence $y=1$ or $y_R+y_F=2$.

The resultant pitch is in the positive direction and corresponds to one coil; or in terms of the N_1 elements the distance traversed in joining one element to another situated in the same field corresponds to two winding spaces, and the end of one loop or coil is joined to the beginning of the loop or coil which lies next but one to the side from which we started. N_1 may be any even number with the proviso that in slotted armatures it must be a multiple of the number of slots.

The function of b in the expressions for y_R and y_F is in the first place to make (N_1-b) divisible by $2p$, *i.e.*, to round off $\frac{N_1}{2p}$ to a whole number, since y_R and y_F must themselves be whole numbers. In a two-pole dynamo N_1 is always divisible by $2p$ or 2, and therefore b can be equal to 0. This, however, makes y_R exceed the pole-pitch, in other words the rear end connector goes the longer way round the shaft, and if we suppose it to be changed over, we obtain a wave winding which is not now under discussion. Hence b must be some quantity other than 0, so that y_B may not exceed the pole-pitch or $\frac{N_1}{2p}$, and in a two-pole dynamo with an uneven number of sectors must not be less than 2, or with an even number of sectors must not be less than 4. In a multipolar dynamo $\frac{N_1}{2p}$ is not necessarily a whole number, and the special function of b is then called out. Usually, however, the number of inductors or of coils is immediately divisible by $2p$, so that the general expressions for the pitches of the bipolar or multipolar parallel-connected drums when these have their maximum length is when $\frac{N_1}{2p}$ is an even number

$$y_B = \frac{N_1}{2p} - 1$$

$$y_F = -\left(\frac{N_1}{2p} - 3\right)$$

or if $\frac{N_1}{2p}$ is an uneven number

$$y_B = \frac{N_1}{2p}$$

$$y_F = -\left(\frac{N_1}{2p} - 2\right).$$

The latter expressions show that with an uneven number of sectors per pair of poles the rear end-connector is equal to the pole-pitch or, *e.g.*, in a two-pole dynamo joins diametrically opposite bars. The actual copper strip or wire must however pass round the shaft on one side or the other, and if the drum is lap-wound, whichever side it takes, the front end-connector is also disposed on the same side.

The quantity b has, however, another function, since it enables us at will to further shorten the pitches below the maxima values which have been given above. It has been explained that the lap-wound loop of the two-pole dynamo is in effect chord-wound even when the pitches are as long as is advisable. So long, however, as y_B exceeds the polar arc, it may be made shorter than the pole-pitch so as to approximate more nearly to the polar arc, especially if this latter be small and the number of inductors large. The front pitch can also be proportionately reduced, so that both end-connections are shortened and the winding has less copper and less resistance. But the chief effect is that the two loops which are simultaneously short-circuited at the two brushes no longer lie side by side, but are separated by a small zone of inductors carrying the full current in opposite directions. In fig. 123, with 24 inductors, the rear-pitch is 7, and the front 5, and between a pair of short-circuited inductors there are 4 wires whose magnetic effect is practically nil. It will subsequently be seen that what is called the demagnetising effect of the armature back ampère-turns is thereby reduced, although this advantage is obtained at the expense of the short-circuited loops being rather far from the neutral line of symmetry between the poles. Suffice it here to say that when by an increase of b the pitches are shortened so as to produce this effect, the method is known as *chord-winding* proper, and this term is extended to cover the analogous arrangements in multipolar lap- or wave-wound armatures.

In a toothed armature if the winding is composed of N_1 bars arranged two in a slot and in two layers, the rear and front pitches may be conveniently expressed in terms of the slots; thus

$$y^1_R = \frac{y_R - 1}{2} \qquad y^1_F = -\frac{y_F - 1}{2}.$$

Since in toothed armatures it is advisable to spread the bars which are simultaneously short-circuited over as many slots as possible, a moderate amount of chord-winding has more in its favour than in the corresponding smooth-surface armature.

(*b*.) $a = np$, where n is a whole number greater than 1.

As in the case of the spirally-wound ring, the second variation gives the *multiplex parallel-connected drum* in which the total number of armature paths is $q = 2np$. The resultant pitch is now

$$y = n \text{ or } y_R + y_F = 2n$$

and the component pitches are

$$y_R = \frac{N_1 - b}{2p} + n, \qquad y_F = -\left(\frac{N_1 - b}{2p} - n\right).$$

y_R should not exceed the pole-pitch, and if required it may be made shorter so as to produce the chord-winding effect, so long as it is not shorter than the polar arc. The G.C.F. of $\frac{y_R + y_F}{2}$[1]

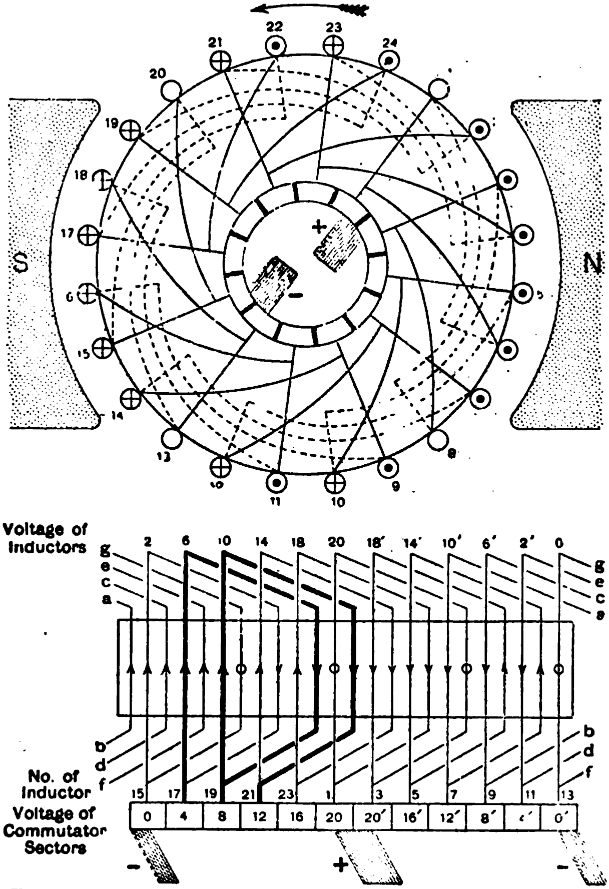


FIG. 123.—Chord-winding of lap-wound parallel-connected drum.

---

[1] Throughout the present section it must be carefully borne in mind that y_F is negative, and its arithmetical value must be subtracted from y_R to find the resultant pitch.

or y and $\frac{N_1}{2}$ is the number of times that the winding closes, so that if it is to form a single closed-coil which is to be recommended for exactly the same reason as in the ring, the greatest common factor of y and $\frac{N_1}{2}$ should be 1.

The E.M.F. is in case (*a.*)

$$E_a = Z_a \cdot \frac{N}{60} \cdot \tau \times 10^{-8}$$

and in case (*b.*)

$$= \frac{1}{n} \cdot Z_a \cdot \frac{N}{60} \cdot \tau \times 10^{-8} \text{ volts.}$$

(ii.) *Wave-winding.*

Since the direction of the two component pitches is continuously forwards, their sign is the same, and either their algebraic or their arithmetical sum must be divisible by two, or $\frac{y_R + y_F}{2} = y$, the average pitch, is some whole number.

The general formulæ for the component pitches are

$$y_R + y_F = \frac{N_1 \pm 2a}{p}$$

whence

$$\frac{y_R + y_F}{2} = y = \frac{N_1 \pm 2a}{2p}$$

which should be compared with the formula for the wave-wound ring, the two being exactly analogous. As before, if the coils are numbered successively in the positive direction round the armature, the end of the (x)th coil is joined to the beginning of the $(x+y)$th coil, or if the inducing sides of the coils are numbered up to N_1, the rear end of the (x)th element is joined to the rear end of $(x+y_R)$th element, and the front end of the $(x+y_R)$th element is joined to the $(x+y_R+y_F)$th element, but now as opposed to lap-winding, y_F is positive.

The division of the average pitch into the two component pitches is not laid down by the formula, and some choice is permissible. They may in some cases be equal, and each equal to y, which is an advantage as making the two sets of end-connections interchangeable. But when the desired number of elements N_1 is such that an equal division would lead to y_B and y_F being even numbers and an impossible winding, y_B may be made slightly greater than the pole-pitch $\frac{N_1}{2p}$ and y_F slightly less so as to secure the necessary average pitch. The slope of the connectors at either end of the armature is seen in the development to be parallel or aslant across the armature core.

According to the value of a two forms are obtained.

(*a*.) $a = 1$.

This gives the *series-connected multipolar drum*, for which

$$y_B + y_F = \frac{N_1 \pm 2}{p}$$

and

$$y = \frac{N_1 \pm 2}{2p} \text{ or } N_1 = 2py \pm 2.$$

y is always half the sum of two uneven numbers y_B and y_F, and its greatest common factor with $\frac{N_1}{2}$ is 1, so that the winding is singly re-entrant. The possible number of elements goes up by steps of four in a 4- or 8-pole machine, and by steps alternately of four and two in a 6-pole machine, and of four and six in a 10-pole dynamo. Thus the number of elements is rather limited, and even this limited number is only obtained by using in some cases unequal pitches, y_B being made slightly greater than the pole-pitch; on this account the complete formula with the alternative of plus or minus two is adopted.

The two-pole wave-wound dynamo also comes under this head, since in it $a = 1$ and $y_B + y_F = N_1 - 2$. With an even number of sectors, the two pitches can be equal to one another, each being $\frac{N_1}{2} - 1$. With an uneven number of sectors, $y_B = \frac{N_1}{2}$ and $y_F = \frac{N_1}{2} - 2$, or exactly the same values as for the similar lap-wound drum, so that the two methods coincide, save that now the front connector advances in the same direction as the rear connector, and this difference at once comes out when the armature is developed and multiplied to form a multipolar machine. When the two-pole dynamo is opened out and repeated it never gives a series-connected multipolar, but always a parallel-connected though wave-wound multipolar which will be dealt with under the next head. Or *vice versâ*, N_1 = say 34, or 30 are possible in a four-pole series-connected armature, but neither can be reduced down to the two-pole form which requires $N_1 = 16$.

If $y_R = y_F$, any two elements which are simultaneously undergoing short-circuit in the same interpolar gap lie side by side; this is also the case if the component pitches only differ by two (as is possible if the number of sectors is not divisible by the number of poles) and the rear pitch is the longer when the shorter of the two possible average pitches is taken or is the shorter with the larger of the two possible average pitches. But the same effect of reducing the armature back ampère-turns as is produced by chord-winding in the lap-wound drum can be produced if the two pitches are made to differ by more than the above amounts; a pair of elements which are short-circuited in one interpolar gap are then separated by two or more elements carrying the full current in opposite directions.[1]

In a toothed armature in which the wires are arranged in two layers and with two elements per slot, the pitches in the series-connected wave-wound drum must be

$$y^1_R = \frac{y_R \mp 1}{2} \qquad y^1_F = \frac{y_F \pm 1}{2},$$

---

[1] This effect begins if with a number of sectors not divisible by the number of poles the difference of pitch is 2, the rear pitch being made the shorter if the average pitch has its lower value or the longer if the average pitch has its greater value. With a number of sectors divisible by the number of poles and the longer average pitch, a difference of 4 will produce the same effect if y_R is the longer, as a difference of 8 if y_R be the shorter; and *vice versâ*, with the shorter average pitch a difference of 4 with y_R shorter than y_F will give the same effect as a difference of 8 with y_R the longer of the two. It is not practically convenient for the two sets of end-connections to differ in length more than is necessary; hence if the shorter of the two possible average pitches is taken with a given number of elements, it is evidently best to make y_R shorter than y_F, but if the longer pitch is used, the greatest effect is obtained by making y_R the longer of the two. Thus in the bipolar armature wherein the shorter average pitch should always be chosen, y_R should be shorter than y_F; the same also holds for the multipolar armature if the number of elements permits of the shorter average pitch being taken, which is the preferable case.

i.e., if the rear pitch is diminished by one, the front pitch must be increased by one, or *vice versâ*, before halving.

(*b.*) $a > 1$, and may be either $\gtrless p$, or $=p$. This gives the *multiplex wave-wound series-connected drum*, in which as in the analogous ring the number of armature paths is independent of the number of poles, although always a multiple of two. If the winding is to form a single re-entrant coil, the greatest common factor of y and $\frac{N_1}{2}$ must be 1.

In both the above cases when multipolar as many sets of brushes can be used as there are poles. In case (*b*) if there are as many sets of brushes as poles, and $a=p$, the winding may then be equally well regarded as a parallel-connected wave-wound drum, and the brushes need not be so wide. Thus fig. 117 shows a multipolar parallel-connected armature which may also be regarded as a multiplex series-connected drum. In such a case the shorter of the two possible values of y for a given number of elements should always be taken and the formula becomes

$$y_R + y_F = \frac{N_1}{p} - 2\ ;$$

the two component pitches can always be equal, and each $= \frac{N_1}{2p} - 1$.

The E.M.F.'s of the wave-wound drums have the same expression as for rings, viz.,

$$(a)\ E_a = p\ .\ Z_a\ .\ \frac{N}{60}\ .\ \tau \times 10^{-8} \text{ volts,}$$

$$(b)\qquad = \frac{p}{a}\ .\ Z_a\ .\ \frac{N}{60}\ .\ \tau \times 10^{-8} \text{ volts.}$$

§ 23. **General formula for the drum armature.**—The general formula of the drum may now be constructed strictly after the analogy of that for the ring in § 21. In the lap-wound armature, the creep for each independent winding is $\frac{a}{mp}$, and the total number of tours that must be made of the armature to include all the windings is $\frac{a}{p}$, which is then the average pitch

$y = \frac{1}{2}(y_a + y_r)$. In the wave-wound armature one tour of the armature is made when $2p$ elements have been passed through; the creep for each independent winding is the same as before, viz., $\pm\frac{a}{mp}$, so that the resultant number of tours for each independent winding is

$$\left(\frac{N_1}{2pm} \pm \frac{a}{mp}\right)$$

or for all the windings is

$$m\left(\frac{N_1}{2pm} \pm \frac{a}{mp}\right).$$

The general formula for the resultant pitch $y = \frac{1}{2}(v_a + y_r)$ is thus always the total number of tours that must be made of the armature or

$$y = \frac{N_1}{2p} \pm \frac{a}{p}.$$

It will be seen that the only difference from that for the ring is that the principal motion of the wave-wound drum corresponds to $\frac{N_1}{2p}$ elements instead of to $\frac{N_1}{p}$ in the ring, since in the ring successive elements are situated under poles of like sign instead of under poles of alternate sign as in the drum. In both cases, if $a = 1$ the armature is simplex series-connected; if $a = p$, simplex parallel-connected; if $a = np$, multiplex parallel-connected and for any other value of a is multiplex series-connected.

§ 24. **Relative advantages of series- and parallel-connected armatures with as many sets of brushes as poles.**—In the two cases of a series-connected and a parallel-connected armature, whether ring or drum, it has been shown that the number of paths may often be alike, as *e.g.*, if it be found convenient to employ a four-pole field with four armature paths. Under such circumstances considerable divergence of opinion exists as to which method is to be preferred. Since in the wave-wound machine each circuit from brush to brush is subjected to the inductive influence of every magnetic circuit, slight differences of pole-strength are averaged, and the total E.M.F. of each branch must be closely alike. On this score the wave-wound armature has the advantage, yet is itself open to a different objection. When more than two sets of brushes are used (and this will usually be necessary in order to keep the length of the commutator within reasonable limits), then if there be any slight difference in the contact resistances of the brushes, and such there will inevitably be, the current does not divide equally between the sets of brushes which are of the same sign; the division of the current will in fact continually alter, and trouble from sparking is thereby caused, since at one moment perhaps one set of brushes may be carrying practically all the current. On the other hand in the spiral ring or lap-wound drum, as will be apparent from Chapter XVII., the armature reaction sets an automatic check upon any very unequal division of the current—a check which is absent in the wave-wound armature. The same considerations also apply to the choice which may often be open to the designer between a parallel-connected lap-wound and a wave-wound series-connected armature, the latter being the less expensive on account of its smaller number of inductors. On the whole the balance seems to be in favour of the lap-wound armature, with the addition of equalising connections[1] if the number of poles is large or any special reason exists for anticipating unequal pole-strengths.

---

[1] For a fuller discussion of equalising connections, see Arnold, *E.T.Z.* March 13 and 20, 1902, abstracted in *Electr. Eng.* June 13, 1902.

CHAPTER XII

THE MAGNETIC PROPERTIES OF IRON

§ 1. **The conditions affecting the permeability of iron.**—The magnetic properties of iron have already been alluded to in Chapter III., but it is now necessary to consider the subject more fully, and at the outset to recur to the experiment of wrapping a number of insulated turns of copper wire uniformly round a ring (fig. 11), and passing a current through the wire. The number of lines of induction that flow through the interior of the magnetised helix is then measured by means of an exploring or test coil and the deflections which it produces in a ballistic galvanometer.[1] Attention has already been called to the primarily important fact that with a given magnetising current the number of lines flowing through the magnetised ring is very much greater if it enclose an iron core than if its coils encircle merely air, wood, or other non-magnetic material. But, further, if we make the same experiment upon two rings of iron of the same dimensions but of different quality, *e.g.*, one of soft annealed wrought iron, and the other of cast iron, it will be found that the same magnetising current will not produce nearly so many lines in the cast iron as in the wrought iron, from which it follows that the permeability of the latter is greatly superior to that of cast iron. The physical and chemical qualities of the substance do, in fact, exert a powerful effect upon its magnetic properties, *e.g.*, if steel be alloyed with 12 per cent. of manganese, it becomes almost non-magnetic.

---

[1] For a description of the exact method the reader is referred to Ewing's *Magnetic Induction in Iron and other Metals*, 3rd ed., chapter iii. ; Vignoles, 'Researches in Electro-magnetic Induction' in the *Electrician*, May 15 and 22, 1891 ; Hopkinson, 'Magnetisation of Iron,' *Phil. Trans.*, 1885 ; and Du Bois, *The Magnetic Circuit*, chaps. x. and xi.

Next, if we take a ring of any magnetic material and vary its magnetising current, the number of lines flowing through it will also vary, but not by any means in proportion to the strength of the current—a very striking fact which sharply divides off iron from air; for the permeability of air is strictly constant, and the number of lines through a ring of air is simply proportional to the magnetising ampère-turns. Roughly speaking, the number of lines induced in iron will for a small number of ampère-turns be almost proportional to varying strengths of current, but after a certain flux-density is reached each successive increase in the strength of the magnetising current will produce less and less effect; the permeability of the iron, therefore, falls off, or, as it is popularly termed, the iron becomes *saturated*, and presents a greatly increased reluctance to further induction of lines.

Lastly, if we raise the magnetising current to a certain value, and then gradually reduce it to zero, the lines produced in a ring of air also vanish entirely; if, however, there be an iron core to the ring there are still some lines passing through it, even when the magnetising current has been reduced to nothing; in other words, the iron 'retains' a certain amount of magnetism, so that zero magnetising current does not necessarily imply zero lines, and we are confronted with the problem of a flux of lines without any corresponding magneto-motive force. In more general terms if the current be raised to some high value, and be then gradually reduced to a smaller value, the number of lines flowing through the iron is not the same as when the current had the same smaller value, but was being increased in strength; it is, in fact, greater when the current is being decreased than when it is being increased, although the actual value of the current strength may be the same. Again we find that no such phenomenon takes place in the case of air or other non-magnetic substance.

It is evident, therefore, that the permeability, μ, of any material depends, not merely upon whether it is magnetic or non magnetic, but if it be the former, upon three conditions. These may be summed up as follows: (1) Its *physical and chemical state*, *e.g.*, whether it be wrought iron or cast iron, annealed or hardened; or whether it be alloyed with other substances, and what is the percentage in which these are present. So far the permeability is analogous to the electric conductivity of metals which are similarly affected by their physical state or by their purity. But the permeability even of a definite chemical substance in a definite physical state is not a constant quality; it also depends on (2) *the value of the induction.* Regarded from this point of view, the permeability is a function of B, the flux-density of the lines per square centimetre or $\mu = f(B)$. Herein it differs decisively from the analogous property of electrical conductivity, for in the case of the electric circuit the resistivity of the material composing it is not a function of the current, but is independent of the amount of current flowing through it, save in so far as this indirectly affects the temperature of the conductor. Although, therefore, the analogy between magnetic permeability and electric conductivity is perfect in the case of air and non-magnetic bodies, it is far from holding in the case of iron. While the permeability of iron may be 2000, or even more, with an induction $B = 5000$, yet when the flux-density has been raised by special methods to the extremely high value of 45,350, the value of μ is reduced to less than 2. From this it will be seen that although air, gun-metal, zinc, &c., may be regarded as magnetic insulators relatively to iron when the latter is weakly magnetised, yet when the iron is 'saturated' the difference in their permeabilities becomes extremely reduced; for values of B in wrought and cast iron such as are ordinarily reached in practice, their permeabilities may be said to stand to that of air in the ratio of about 200 or 100 to 1, but for higher values it continually decreases, until finally for intense saturation it is reduced to about double that of air. Next, the permeability depends upon (3) the *previous magnetic history* of the metal, *i.e.*, upon whether it has been previously subjected to a larger or a smaller induction. The iron has as it were a magnetic memory by reason of which the number of lines passing through it differs in the two cases of an ascending or a descending induction. Lastly the permeability of iron is affected by *temperature*, but as this effect is hardly noticeable within the range of temperatures that is met with in the ordinary working of dynamos, it may for our present purpose be dismissed as negligible. Such being the complex nature of the permeability $\mu = \frac{B}{H}$, it is necessary to examine carefully the whole process of the magnetisation of iron, and represent it graphically by *curves of induction* or flux-density, connecting together corresponding values of B and H.

§ 2. **Ascending and descending curves of induction.**— From the known number of turns, T, and ampères, A, flowing in a magnetising helix, we can obtain $H = \frac{0{\cdot}4\pi AT}{l}$, *i.e.*, the total magneto-motive force acting round the ring of fig. 11 divided by the length in centimetres over which that force is distributed, or 1·257 times the ampère-turns per centimetre length. It may here be recalled that this quantity H in C.G.S. units—in other words, the *magnetising intensity* of the field in which the iron is placed or, as it is sometimes called in the present connection the 'magnetising force,' to which the iron is subjected—is also the fall of magnetic potential which takes place over a centimetre length of the substance forming the core of the helix when B lines flow through each square centimetre of its cross-section. By varying A and measuring the corresponding $Z = Ba$ (a being the area of the iron in square centimetres) a number of corresponding values of H and B[1] are obtained, and these when plotted as abscissæ and ordinates respectively are joined to form a curve of flux-density. Let us first take a soft annealed wrought-iron ring; let this be previously unmagnetised, and therefore in an entirely neutral state, or else let it be carefully demagnetised by means of a gradually decreasing alternate current passed through the magnetising helix; let us then determine its *ascending* curve, the current being increased in strength step by step, and the flux-density being separately determined for each step. At the outset, for very small magnetising intensities, the value of B rises at a certain slow rate almost proportionately to the increase of the intensity; when, however, the intensity has reached a value of about 1 or 2 C.G.S. units the rate of rise of the induction changes very rapidly to a much increased value; at this new rate the induction again continues to rise almost proportionately to the increase of the magnetising intensity, but when the latter is raised to a value of from 5 to 10, although the induction B continues to increase, its rate of rise falls off, and becomes gradually less and less rapid. The curve thus obtained

---

[1] For certain corrections in practice, *see* Niethammer, *E.T.Z.*, 1898, p. 688.

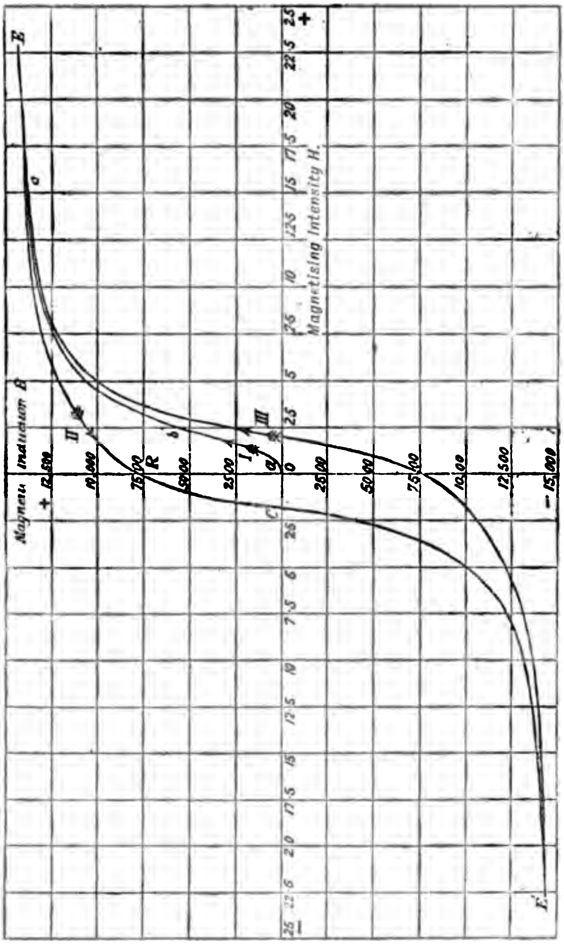


FIG. 124.

for B and H is that marked O E in fig. 124; since the iron was at the outset unmagnetised, it starts from the origin, and rises with the increasing magnetic intensity, as shown by the ascending arrow. It is divisible approximately into three different portions, marked *a*, *b*, *c*; for a short distance from the origin it is but slightly inclined to the horizontal axis, along which the magnetising intensity is reckoned; it then passes by a rapid bend into a long part, *b*, of great steepness, and approximately straight; gradually, however, the curve bends over, forming a rounded corner, or 'knee,' and finally continues to rise in a slightly curved line, *c*, inclined at a small angle to the horizontal axis.

After having reached the point E on the *ascending* curve of flux-density, let us gradually reduce the strength of the magnetising current, and so of the magnetic intensity. Again measuring values of the induction at different stages, we are thence enabled to trace the curve of *descending* induction, and this, as has been already stated, is by no means identical with the ascending curve. It is marked II. with a downward arrow in fig. 124, and is considerably higher than the ascending curve, so that when the magnetising intensity is zero it cuts the vertical axis at some point R, considerably above the horizontal axis. The height O R measures the number of lines still passing per square centimetre of the iron when the magnetising intensity has been gradually reduced to zero, or the *residual magnetic induction* of the iron; its exact amount depends upon the value to which the induction in the iron has been previously raised, and from which, as a starting-point, the reduction began, while the proportion which it bears to the maximum induction reached is an indication of the *retentiveness* of the iron. After reaching point R on the downward curve a negative magnetic intensity is required, or, in other words, the direction of the intensity H must be actually reversed by reversing the magnetising current in order to reduce B to zero. The amount of this negative intensity, which has to be applied in order to reduce B exactly to zero after it has been previously raised to a high value, is called the *coercive intensity* of the iron; it is measured by the length of the line O C in fig. 124.

§ 3. **Saturation a relative term.**—If, after reaching the point E on the ascending curve, the magnetising intensity, instead of being reduced, had been still further increased, the curve would become flatter and flatter, but it never becomes truly horizontal; although the iron may now be said to be 'saturated,' the induction

B never ceases to increase when the magnetising intensity is increased, so that there is no definite limiting value of B beyond which it cannot be raised; the permeability of the iron is enormously reduced, but even when the peculiar action of the iron in increasing the magnetic flux is almost imperceptible, more lines can always be propelled through the helix, just as if it were a simple solenoid enclosing merely air and without any core of iron. *Saturation* is, in fact, so far as lines of induction are concerned, a relative term, with no particular numerical significance. The iron is commonly said to be 'saturated' when the curve of flux-density has begun to bend over at the rounded knee between *b* and *c*, but the exact appearance of the curve and position of the bend largely depend on the respective scales to which B and H are plotted. Fig. 125 shows the curves of flux-density of three different kinds of iron, the ascending curve being shown full and the descending curve dotted: in these the induction and magnetising intensity (the latter shown in the upper of the two scales along the horizontal axis) have been carried to much higher values than in fig. 124. The scale of H has been altered to obtain a convenient size of diagram, and it will be seen that now the bend of the curve for annealed wrought iron appears to be at B = 14,000 instead of at B = 11,000, as in fig. 124.

§ 4. **Physical and chemical conditions as bearing on magnetic properties of iron.**—The permeability of iron is largely affected by the presence of impurities, intermixed or in combination with it, such as phosphorus, sulphur, tungsten, or manganese, and also by the closely-connected physical quality of 'hardness.' The exact influence of any one ingredient it is most difficult to determine, since its absence or presence may affect not only the chemical composition but also the melting-point, the processes by which the metal must be treated during manufacture, and the interaction of the other impurities. It may, however, be said that the most important ingredient on all these scores is *carbon*, its presence in a combined state being detrimental to the permeability.[1] *Wrought iron* when rolled into thin sheets or drawn into wires may usually be relied upon as being soft, owing to the processes of manufacture through which it has to be carried,

---

[1] H. F. Parshall, "Magnetic Data of Iron and Steel," *Proc. C.E.*, vol. cxxvi., 1896.

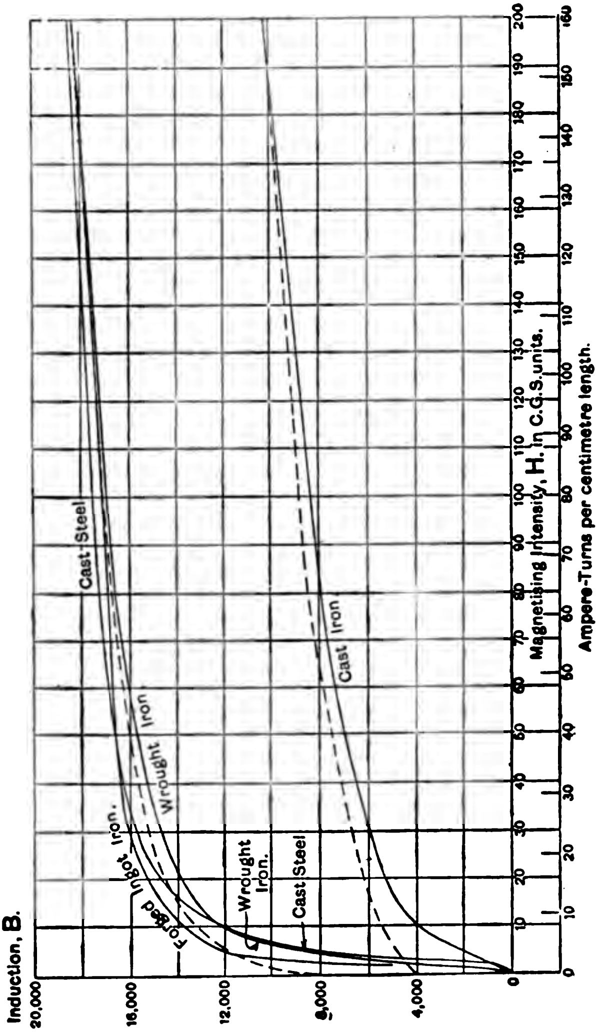


FIG. 125.

and, further, in this form it admits of a very thorough annealing.

When, however, such sheets are stamped or punched out into the shapes suitable for dynamo purposes, it is usual to subject the stampings to a further *reannealing*, since by the process of stamping they become, to a certain extent, hardened. The annealing process should be carried out in air-tight closed boxes or chambers, since mere annealing in a fire open to the air reduces the permeability. In *cast iron* the effects of impurities and of hardness are again very marked. The iron used for castings should be specially soft and pure, and all hardening or chilling of it after casting should be avoided by allowing it to cool as slowly as possible, or in other words by annealing it. The total amount of carbon present in a combined form or as graphite may vary from 3 to 4½ per cent., but should be low in order to reduce the percentage of combined carbon, if possible to less than 0·5 per cent. Approximately, it may be said that, for ordinary values of H between 40 and 80, the permeability of cast iron is less than half that of forged iron or cast steel; but different samples of cast iron show much more divergence among themselves in permeability than would be found in as many samples of wrought iron. In *steel castings* combined carbon is again objectionable, and should not be present to a greater extent than 0·2 per cent., the temperature at which it is cast being correspondingly high. An alloy of steel with 5 per cent. of nickel has been used with success to combine a permeability as high as that of good cast steel with great mechanical strength after forging.

The amount of 'retentiveness' and of 'coercive intensity' likewise varies greatly in different qualities of iron, and is affected by their purity and hardness. It will be seen from fig. 125 that when a high magnetising intensity is gradually reduced to zero, the residual magnetic induction which persists in annealed wrought iron is about 7000 or 8000, while for cast iron it is about 4000. The retentiveness of soft annealed wrought iron is greater than that of any other material, since as much as 80 or 90 per cent. of the induction may be retained, but this amount is much reduced if the iron circuit be incomplete. Its coercive intensity, is, however, very small, and the residual flux is quickly reduced by a feeble demagnetising intensity, or by mechanical vibration, jarring, or tapping. Hard iron and steel, although retaining less magnetic induction than soft iron, keep it much more strongly, and therefore permanent magnets, which are required to maintain their magnetic properties, in spite of mechanical shocks or demagnetising influences, are made of steel, the most suitable alloy being tungsten steel, of which the coercive intensity is as much as 50 C.G.S. units. Were it not for the fact that a high flux-density cannot be obtained with hard steel owing to its low permeability, the field-magnets of dynamos would be made of steel permanently magnetised, and requiring no exciting current. In default of a material which is at once permeable and retentive, and possessing considerable coercive intensity, we are compelled to employ soft iron or mild steel: these are permeable, but require the exciting current to be continuously maintained round them in order that they may not lose their magnetism entirely by reason of the mechanical vibration to which the dynamo is subject when running.

§ 5. **B, H curves of iron and steel.**—While the curves of fig. 125 serve for a general comparison of the permeabilities of various materials, they need to be supplemented by further curves showing more in detail the relative merits of such materials as are in everyday use in the commercial manufacture of dynamos. Putting cast iron on one side, these may be divided into the three main groups of iron or steel forgings, steel castings, and sheet iron or steel stampings, the first two being used in field magnets, and the last in armature cores.

As a material for dynamo magnets, the wrought iron which was for long the favourite material has been almost entirely superseded by forged ingot iron or steel castings. The latter for moderately high inductions above B = 12,000 such as are in practice required, are superior to even the best wrought iron, such as annealed Lowmoor or Swedish. The three materials of fig. 126 have this in common with wrought-iron, that they are all either forged under the hammer or press, or are rolled into bars. But while wrought iron consists of puddled balls or scrap-iron pieces welded together by hammering, its fibrous structure still bearing witness to its method of manufacture, the *forged ingot irons* are homogeneous in their nature, having been at the outset thoroughly fused together in the furnace. While chemically they may be ranked as mild steels with a small percentage of carbon, the term 'ingot iron' and its more expressive German equivalent 'flusseisen' indicate that they have been melted before they reach the hammer or the rolls. Fig. 127 gives the B, H curves of four favourable specimens of the second group, viz. *steel castings*; it also illustrates the fact that a low initial permeability does not necessarily imply any inferiority at high inductions. The curves cross one another, and they are further remarkable from the facts that the crossing-point in each case is practically coincident with B = 15,000, and that the relative position of the curves becomes exactly reversed. On the whole there is little to choose between the permeabilities of the two groups, the forgings being slightly more permeable between B = 13,000 and B = 16,000, and the castings more permeable at still higher inductions. There is indeed but little difference in the chemical composition of the two groups; both are as nearly as possible pure iron with only such

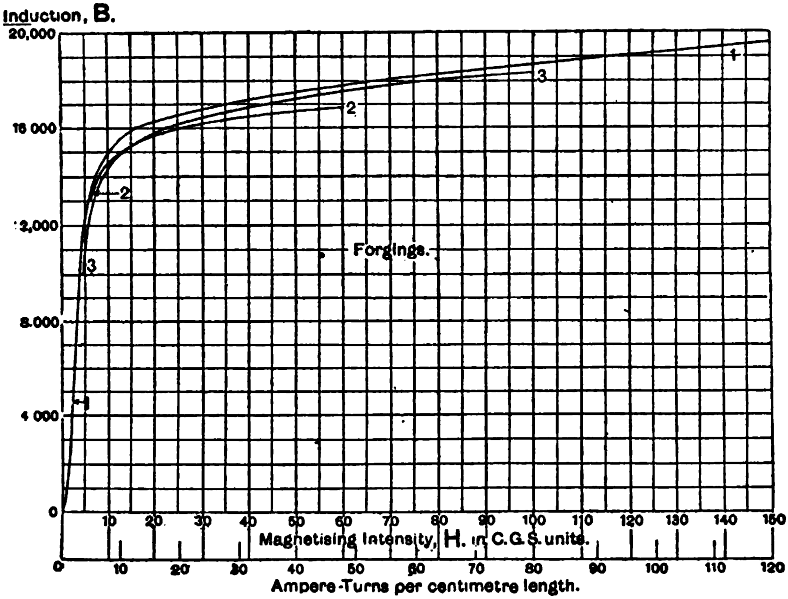


FIG. 126.—Flux-density curves of forgings.

admixture of carbon or other substances (silicon, phosphorus and manganese) as may enable them to be conveniently worked during the process of manufacture. In fact, the total impurities present including carbon may not exceed 0·3 per cent., the remaining 99·7 per cent. being pure iron. The material of curve 1 in fig. 126 may be credited with the highest permeability over an extended range; at low inductions it is slightly superior to wrought iron, and it is only surpassed by an exceptionally good cast steel, when the induction is pressed beyond B = 18,000. Its curve is nearly identical with that of an almost perfectly pure specimen of iron prepared specially for laboratory purposes.

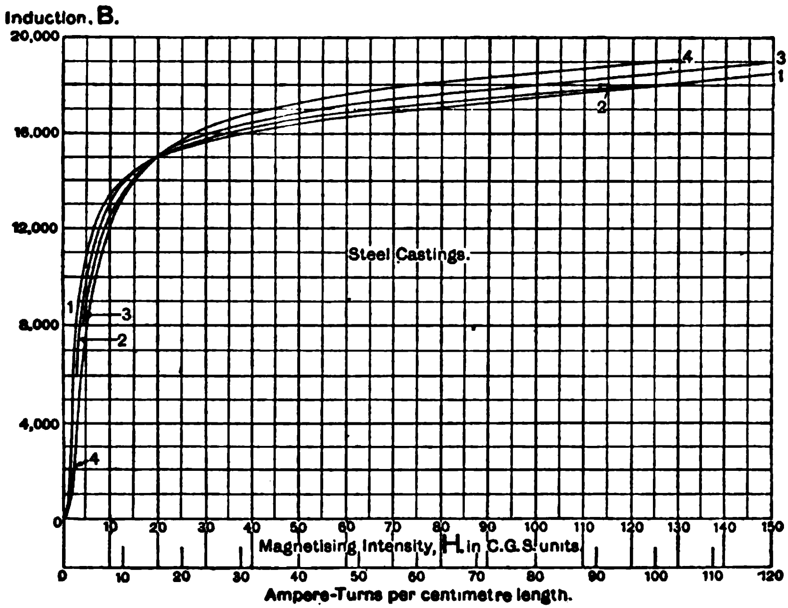


FIG. 127.—Flux-density curves for steel-castings.

For the core plates of armatures, the material in general use, although still often called 'charcoal-iron,' is no longer now manufactured after the technical sense of the term; it is in reality

Swedish iron or mild steel of very nearly the same nature as that of the larger masses of ingot iron or steel forgings. From the similarity of their chemical analysis, *armature stampings* (fig. 128) may be expected to have a permeability similar to that of the same iron or steel when tested in bulk, and such is in fact the case. The only difference is that owing to its repeated mechanical treatment in the rolling-mill and its subsequent annealing, the sheet-metal has a permeability at low inductions rather higher than that of group II., but this falls off at higher inductions. For B = 15,000, H should in no case exceed 30. In the teeth of slotted armature cores, the induction may often reach the high figure of B = 20,000, and in order to embrace even higher values fig. 129 is added. Lastly in fig. 130 is given a permeability curve which may be taken as representative of good iron or steel, such as is commonly used in dynamo work. It shows how marked is the

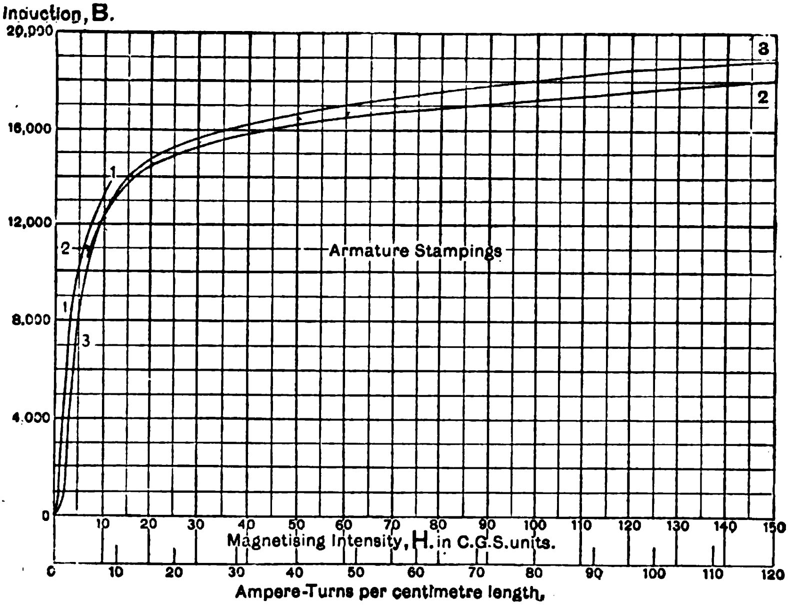


FIG. 128.—Flux-density curves for stampings.

alteration of μ for different values of B, the maximum value 2450 being reached in this particular case at $B=6000$.[1]

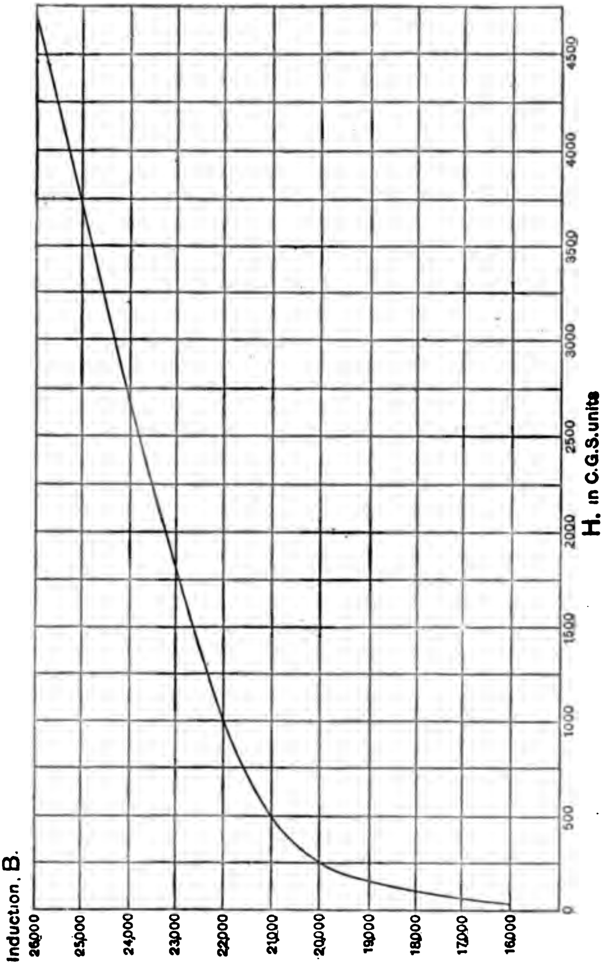


FIG. 129.—B-H curve for high inductions in stampings.

---

[1] For various methods of comparing or measuring permeabilities, some of them suitable for practical use in the workshop, see Du Bois, *The Magnetic Circuit*, chap. xi. ; Prof. Ewing, "The Magnetic Testing of Iron and Steel,"

It will now be fully evident that the relation between B and H or the permeability of a given sample of iron, even with a definite magnetising intensity, cannot be absolutely specified, and may take any one of a certain range of values according to the previous treatment to which it has been subjected. Since, however, the effect of hysteresis is very slight when the iron is strongly magnetised or 'saturated,' and since in dynamo field-magnets the iron is usually magnetised to a fairly high induction of over 14,000 in wrought iron or 6,500 in cast iron, the value of B for a given magnetising intensity will differ little, however the latter has been

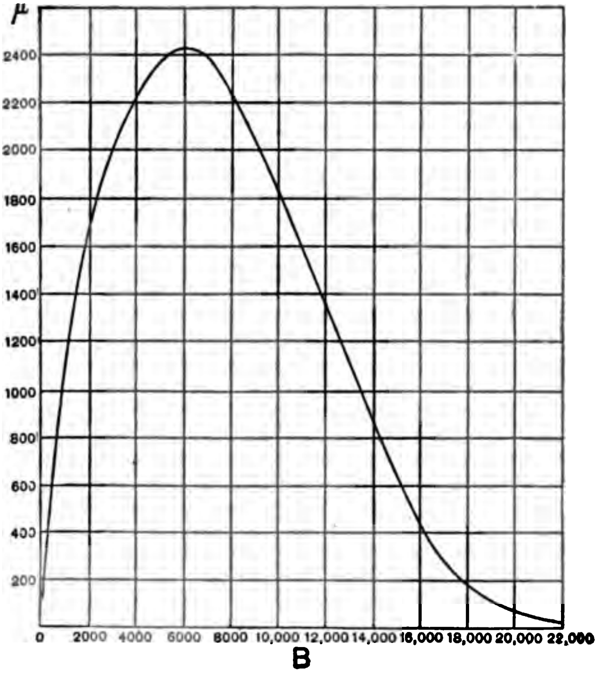


FIG. 130.—Permeability curve.

Proc. C.E., May 1896, and *Magnetic Induction in Iron and other Metals*, 3rd edit. chap. xii.; Lamb and Walker, *Journal Inst. Electr. Eng.*, vol. xxx. p. 930; Drysdale, *Journal Inst. Electr. Eng.*, vol. xxxi. p. 283.

arrived at—that is, whether the magnetising intensity has been decreased down to or increased up to that value. (See Chapter XV., § 3.)

§ 6. **Magnetic hysteresis.**—Reverting to the downward curve of fig. 124, it is seen that the changes in the magnetic state of the iron are not coincident with the changes in the strength of the magnetising intensity, but lag behind them: this is most forcibly exemplified by the residual magnetic induction which persists when the positive magnetising intensity has been reduced to zero, and by the fact that the induction only becomes zero after the magnetising intensity has reached a definite negative value. The physical fact here described is known shortly as the *magnetic hysteresis*[1] of the iron. In considering this phenomenon it should be carefully noted that it is not a lagging behind in point of *time*, since the actual time taken for the changes in the value of the magnetising intensity (provided they be not extremely rapid) is immaterial. The magnetic hysteresis depends merely on the order of succession of the different current strengths, and even long intervals of waiting, during which the magnetising intensity is kept constant, do not obliterate the distinction between ascending and descending curves. What is really implied by the term 'magnetic hysteresis' is that if the magnetising intensity is reduced from a stronger to some weaker value, the rate at which the magnetic induction becomes reduced with reference to the magnetising intensity, or the slope of the downward curve, is less than the rate at which it increased when the magnetising intensity was raised from the weaker up to the stronger value; thus, if at point R the magnetising intensity be again reapplied in a positive direction, the rate at which the induction is recovered is again less than the rate at which it was lost for a change of magnetising intensity within the same limits.

Let us now continue the descending curve from the point C onwards by increasing the negative magnetising intensity, and so reversing the direction of the induction. After reaching some point E′, let us gradually reduce the negative magnetising intensity, reverse it, and increase its strength in the former positive direction. A new ascending curve marked III. is thus traced, which again shows hysteresis: it differs in shape from the previous ascending curve, which started with the iron in a neutral or 'virgin' state, but is analogous to the descending curve II. By carrying the new ascending curve up far enough, it will eventually cut curve II., say at E, and we thus arrive at the same point whence we started to trace the descending curve. A complete *loop* has therefore been described, and the two curves Nos. II. and III. enclose a certain area depending upon the extent to which they diverge from one another between the points E and E′ The magnetising intensity has been taken through a cycle of changes in direction and value, eventually returning to the same point as that from which the cycle began, and the iron has similarly been taken from a positive to a negative induction and back again. It is not, however, necessary that the magnetising intensity should be actually reversed in order that the curve of induction may describe a complete loop; it is sufficient to partially withdraw the magnetising intensity and then reapply it. Thus, at any point on the ascending curve the gradual increase of the magnetising current might be suspended, and it might be first reduced and then again increased; a small loop would then be traced on the induction curve inside the last one.[1]

---

[1] From Greek ὑστερέω, to lag behind.

§ 7. **Dissipation of energy in heat by magnetic hysteresis.**—Now it can be shown that the area of any complete loop formed by taking iron through a complete cycle of magnetic induction represents a certain amount of energy which must be spent per unit of volume in performing this cycle; *i.e.*, it represents on the C.G.S. system a definite number of ergs of work done in taking each cubic centimetre of iron through the cycle of induction traced by the loop.

The principle from which this follows may be established in the following way. Taking the ascending B, H curve for a closed ring of iron magnetised by an exciting coil of T turns carrying A ampères, let the induction be raised from B_1 to B_2 (fig. 131) in time t. All the lines are linked with all the turns T, and the increase of the former from B_1a to B_2a (where a is the area of cross-section of the iron in square centimetres) must have produced in the exciting coil a back E.M.F. of average value $a \cdot \frac{B_1 - B_2}{t} \cdot T$. Or if the points p_1 and p_2 are taken so close together that the infinitely small increase dB in the infinitely small time dt is considered, the time-rate of increase of the linkages of lines is $\frac{dB}{dt}$, and the back E.M.F. in the exciting coil is $-a \cdot \frac{dB}{dt} \cdot T$. The source of the magnetising current must accordingly do work in overcoming this back E.M.F. at a rate equal to the

---

[1] *Vide* Prof. Ewing, *Magnetic Induction in Iron and other Metals*, 3rd ed., chap. v. pp. 94-96.

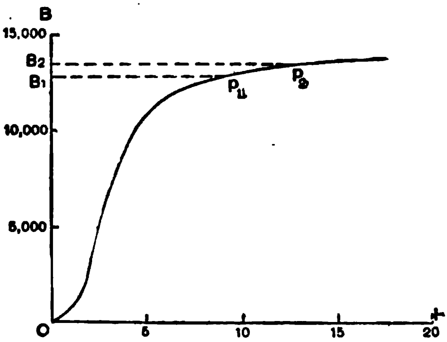


FIG. 131.

product of the current and the back E.M.F., or $a \cdot \frac{dB}{dt} \cdot T \cdot \frac{A}{10}$ ergs per second, where $\frac{A}{10}$ is the value of the magnetising current in C.G.S. units which may be regarded as constant during the infinitely short time dt. Hence, the total work done during the time dt by the current in producing the change of induction dB is $a \cdot \frac{dB}{dt} \cdot T \cdot \frac{A}{10} \cdot dt = a \cdot \frac{AT}{10} \cdot dB$ ergs. But $\frac{AT}{10} = \frac{Hl}{4\pi}$ so that the work done or energy expended on the magnetic field is $a \cdot l \cdot \frac{H}{4\pi} \cdot dB$. Further, al is equal to the total volume of iron in cubic centimetres, and as each cubic centimetre may be considered as having a proportionate amount of work done on it, the work expended per cubic centimetre is $\frac{1}{4\pi} \cdot \text{H}d\text{B}$, or $\frac{1}{4\pi}$ of the small area $p_1\text{B}_1\text{B}_2p_2$ if one unit of length along the horizontal represents one C.G.S. unit of magnetic intensity, and along the vertical represents one C.G.S. unit of induction. If we extend the same process and raise the induction from any value B_1 to any other value B_2, the work expended per cubic centimetre is equal to $\frac{1}{4\pi}\int_{\text{B}_1}^{\text{B}_2} \text{H}d\text{B}$; thus, for example, if in fig. 132, which repeats a portion of fig. 124, the induction is raised from zero up to E, a total amount of work is expended in each cubic centimetre of $\frac{1}{4\pi}\int_{0}^{\text{E}} \text{H}d\text{B}$, which is equal

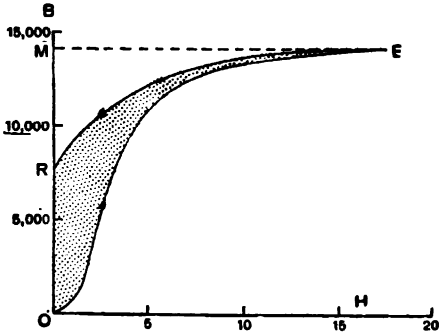


FIG. 132.

to $\frac{1}{4\pi}$ of the area OEMO. The maintenance of the magnetic field at E or any other constant value involves no expenditure of energy; the passage of the exciting current C in the magnet bobbins of resistance R does indeed involve a continuous expenditure of energy at the rate of C^2R watts, but as this is dissipated entirely in heating the copper wires of the coils, it is not to be debited directly to the magnetic field, and as we have said is only necessary owing to the lack of retentiveness in soft iron or steel. Next, let the magnetising current be lowered from the value corresponding to E down to zero; the decrease of the lines now causes a forward E.M.F. assisting the current in the exciting coil, and work is thereby done in virtue of the energy stored in the field. The amount so recovered is, however, only equal to $\frac{1}{4\pi}$ of the area EMR since the descending curve does not follow the same course as the ascending curve; hence, it is less than the amount expended by $\frac{1}{4\pi}$ of the shaded area ORE. It is not, however, proved how much is irrecoverable until a complete cycle of induction has been traced, and the iron has been brought back to the same state as at starting. Such a complete cycle is given in fig. 124, from which it is clear that the total amount lost per cubic centimetre of iron is equal to $\frac{1}{4\pi}$ of the area of the closed loop terminated by the maxima values E and E′ of the induction. The energy that is thus irrecoverably lost in any cyclic process is dissipated throughout the iron in heat. Thus the creation of a magnetic field in iron as opposed to air is not a perfectly reversible process, since some portion of the energy expended is not stored but entirely lost in heating the iron. This loss is to be sharply distinguished from the loss by eddy-currents which must to a small extent be present even in finely laminated iron. In a given mass the latter loss is proportional to the square of the periodicity or of the speed, while the former is proportional simply to the number of cycles per second or to the speed, and by this difference the two losses may be separated out (Chapter XIX. § 13). Eddy-currents may be practically eliminated by very carefully subdividing the iron (Chap. XIII. § 1), or by causing the induction to change very slowly. But no subdivision or lamination of the iron will eliminate the loss by hysteresis; nor is it reduced, however slowly the cycle be performed, as is shown by the expression $a \cdot \frac{dB}{dt} \cdot T \cdot \frac{A}{10}\, dt$, from which dt disappears. Further, the loss by hysteresis does not react magnetically upon the field, while eddy-currents, however minute, do by reason of their M.M.F. affect the field and cause a screening of the iron against induction. Indeed, the only deviation from the law that the hysteresis loss is proportional to the periodicity or speed is due to the screening action of eddy-currents which may still be present if the thickness of the iron laminations be appreciable.

§ 8. **Hysteresis loss in alternating field.**—The effect of hysteresis is most marked on the second or steep part of the curve (*b*, fig. 124), *i.e.*, if the limits within which the magnetising intensity is cyclically varied fall within or embrace that portion of the curve; cyclic changes of a strong magnetising intensity taking place entirely on the upper flat portion of the curves of fig. 125, *i.e.*, when the iron is 'saturated,' show so little hysteresis that its effect is almost negligible, and no difference is discernible in the ascending or descending curves (*vide* fig. 125). On the steep portion of the curve a cycle of changes in the strength of the magnetising intensity once performed will cause the induction curve to describe a loop, but the crossing of the descending and ascending curves may not coincide with the starting-point whence the descent was begun; hence a repetition of the same cycle of changes will not cause the loop to be exactly retraced. If, however, the magnetising intensity cycle be repeated a few times, the iron will eventually reach such a state that the same loop will be continually retraced, and the change of magnetic induction will itself become strictly cyclic and coincident with the magnetising-intensity cycle. The need for several repetitions of the magnetising cycle in order that the same loop may be exactly retraced each time is much less marked when the two limiting values of the loop are high up on the positive and negative curves, and in the drawing of fig. 124 it has been assumed that the maximum negative induction at E′ is equal to the positive induction at E, and is reached for the same value of magnetising intensity as corresponds with E on both the two ascending curves. Thus, for simplicity's sake, only three curves are shown, and the cycle of induction of curves II and III once traced is immediately capable of indefinite repetition so long as the same cycle of magnetising intensity is repeated. But even if this be not so, when the magnetising intensity is continuously varied between two fixed values in either direction, the magnetic effect soon also becomes cyclic, and the induction likewise varies between two values in each cycle. If, therefore, the loops obtained when the iron is carried from a strong positive induction to an *equally* strong negative induction, and back again to the original starting-point are determined for several different values of the maximum induction, it will be found that the area of the loops, and therefore the energy dissipated in heat in each complete cycle depends upon the nature of the material and also upon the maximum induction up to which the iron is carried. The area of a loop being approximately equal to a rectangle having a base equal to twice the coercive intensity and a height equal to twice the maximum induction, the amount of the 'loss by hysteresis' in ergs for any cycle or $\frac{1}{4\pi}$ of this area may approximately[1] be said to be equal to $\frac{\text{coercive intensity} \times \text{max. induction}}{\pi}$; thus in soft annealed iron it is very small, even for a cycle of high induction, but in certain steels it becomes very considerable, amounting to as much as 200,000 ergs per cubic centimetre per cycle in tungsten steel. Taking, however, any one substance, it has been found that the loss of energy by hysteresis in an *alternating field* is not proportional to the maximum induction, but increases more rapidly at least up to values of the induction below 16,000.

In fig. 133[2] is given a curve showing for different values of the maximum induction of the cycle the hysteresis loss of a sample of soft iron tested by Prof. F. G. Baily in an alternating field. Steinmetz has shown that over a large range, say, from $B = 1000$ to $B = 14{,}000$, the loss from the alternating cycle is very closely proportional to the 1·6th power of the maximum induction. Hence the specific loss in ergs per cubic centimetre of iron and per cycle may be expressed by the empirical formula $\eta . B^{1\cdot 6}$ where η is the *hysteretic constant* of the iron in question. Since one erg $= 10^{-7}$ joule, the specific loss in joules is

$$h = 10^{-7}\eta \, . \, B^{1\cdot 6}.$$

The value of η varies with different kinds of iron and steel between such wide limits as from 0·00138 to 0·08. It is reduced by careful annealing, but the exact effects of various processes or of chemical impurities cannot be said to be settled. For average samples of annealed sheet iron or sheet steel such as are used in the construction of armature cores, η may be taken as $= 0{\cdot}003$, while for good transformer iron it is as low as 0·002. In fig. 134 is shown the curve connecting the joules expended in a cubic centimetre in each complete cycle with the maximum induction for a hysteretic constant of $\eta = 0{\cdot}003$, and a comparison with fig. 133 will show the closeness of the correspondence of the calculated and the experimental curves. Fig. 135 gives the curve of fig. 134 as connected with the weight of the metal in lbs. (1 lb. $= 58{\cdot}5$ cm.^{3} of iron, so that 1 joule per lb. $= \frac{1}{58{\cdot}5}$ joules per c.cm.). If $v_c =$ the volume of a given mass of iron in cubic centimetres, and $f = \frac{pN}{60}$ is the number of complete cycles per second, the

---

[1] For the divergence from this approximation with high or low inductions, *see* Niethammer, *E. T. Z.*, 1898, p. 688.

[2] From *Phil. Trans.*, 1896, vol. 187, A. pp. 715–746. "The Hysteresis of Iron and Steel in a Rotating Magnetic Field."

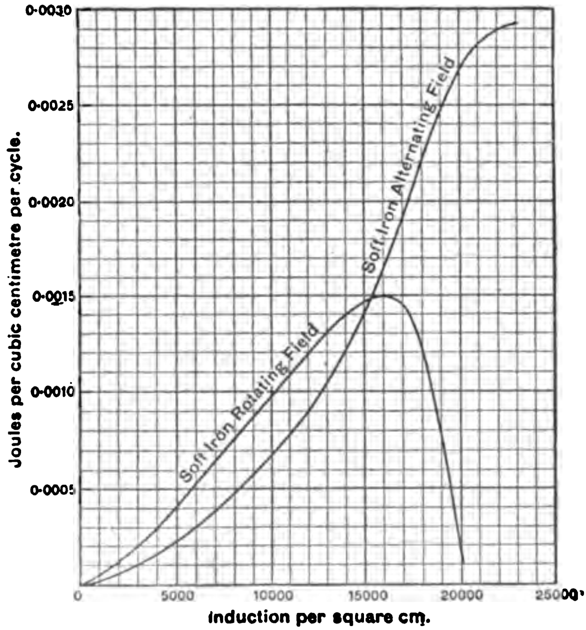


FIG. 133.—Dissipation of energy in iron through hysteresis in alternating and rotating fields.

total hysteresis loss per second or the rate at which heat is generated in the iron is

$$H_w = hfV_c = 10^{-7}\eta B^{1 \cdot 6} \, . \, \frac{pN}{60} \, . \, V_c \text{ joules per second or watts} \; . \; . \; . \quad (18)$$

and the value of $10^{-7}\eta B^{1 \cdot 6}$ or h may be read off the curve of fig. 134. The exponent of B is however not really constant over any long range, and when the induction is raised to a very high value the empirical formula of Steinmetz no longer holds even approximately true; the rate at which the curve rises falls off, as the iron approaches saturation,[1] and eventually the loss reaches a nearly constant value (fig. 133). The point of flexure occurs in good soft iron at about B = 15,000, and the curve becomes fairly flat for

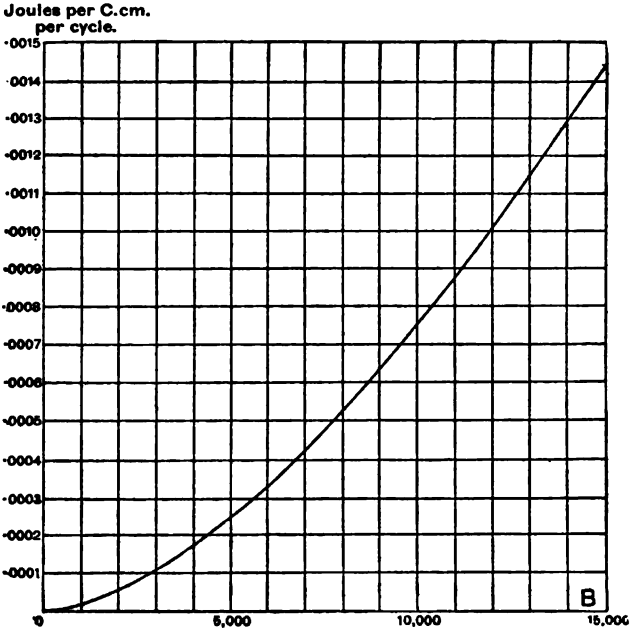


FIG. 134.—Value of h in an alternating field for $\eta = 0{\cdot}003$.

---

[1] Prof. Baily, *Electrician*, vol. xxxvi. p. 116. "The Hysteresis of Iron in an Alternating Magnetic Field."

values of B above 23,000. The tendency towards a constant value does not, however, appear to be so clearly marked in all kinds of iron as in the case of steel.[1]

§ 9. **Ewing's molecular theory of magnetism.**—At this point a brief outline may be given of the theory of magnetism which at the present day seems best able to account for the very large range of facts that have been brought to light by experimental research. The *molecular theory*, as it is termed, starts with the fundamental assumption that the molecules of a magnetic body are already and always permanent magnets endowed with polarity, and that the process of magnetisation consists in turning these small magnets so that the direction of their magnetic axes coincides more or less with the direction of the magnetising intensity due to some external cause The actual distances between the centres of the molecular magnets are supposed not to be changed (save by the effects of mechanical stress or heating), so that each magnet is only capable of rotation about its centre. Since, however (as the ascending curve of fig. 124 shows), a strong magnetising intensity is required to produce a high magnetic induction in iron, it is evident that the molecules cannot be perfectly free to turn and set themselves along the direction of the

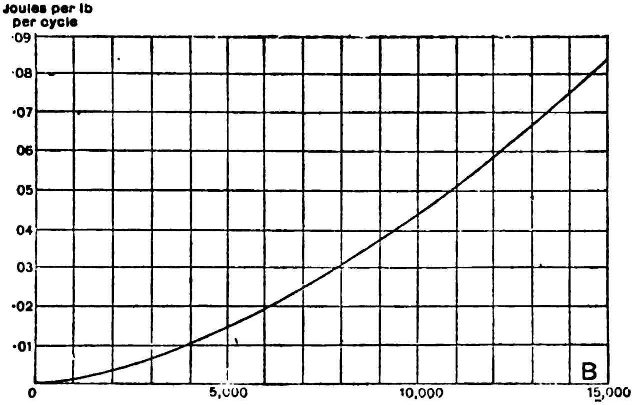


FIG. 135.—Hysteresis loss in Joules per lb. for $\eta = 0{\cdot}003$.

---

[1] Cp. Beattie and Clinker, *Electrician*, vol. xxxvii. p. 727.

magnetising intensity; for in that case, even when a weak magnetising intensity was applied, they would all swing round into line with its direction, and the iron would at once become entirely 'saturated.' They must therefore experience some restraint, and the exact nature of this restraint was for long a stumbling-block. But in 1890 the experiments of Professor Ewing led him to return to the simplest hypothesis, viz., that it is the mutual attraction and repulsion of the molecular magnets which supplies the constraining force, and hinders their immediate alignment into perfect parallelism with the magnetising intensity. By means of a model visibly representing the supposed molecular structure of a magnetic body, he was able to imitate almost all of the phenomena of magnetism, and thence to deduce the following theory.[1] The molecules of a piece of iron when in a neutral state are assumed to be arranged in groups: these are not necessarily identical in configuration, but each of them is stable, and has no external magnetic effect, the attractions and repulsions of each magnet being satisfied within its own group. For small displacements of its members the whole group remains stable; but if the members are turned through a sufficiently great angle, and the group is distorted, one or more members become unstable, and their equilibrium is liable to be upset. The result is that for a slight increase of the displacement the whole group becomes broken up, and has to be partially or wholly rearranged and reconstructed after a new plan. From the internal structure above described, it follows that when a group is subjected to a magnetising intensity which is gradually increased in strength from zero, the first effect is to produce a *stable* deflection of all the component members, except those which lie exactly along or opposite to the direction of H, and the general lines of the group are still retained. This corresponds with the initial stage of magnetisation (*a*, fig. 124) when the induction increases at a slight rate almost proportional to the increase of the magnetising intensity. But now let the value of H be increased still further; the members of the group are still further deflected, until at last one or more become *unstable*. The ties which bound them are then ruptured, and by the intermolecular attractions and repulsions they and their neighbours are constrained to entirely change their grouping, and take up some new configuration, the main lines of which agree more closely with the direction of H. In general, with a considerable number of different groups, this stage of instability will not be reached by them all simultaneously at one given value of H, but group after group will gradually become unstable and break up. This second stage, when the groups are one after another passing through an unstable condition, corresponds to the steep part of the ascending curve (*b*, fig. 124) when the induction increases at a rapid rate. If H be still further increased in strength, we have the third stage (*c*, fig. 124), in which the alignment of the molecules becomes gradually more and more perfect; as each molecule is pulled more and more into line with H, the iron becomes more and more 'saturated,' and its permeability decreases. The deflections of the molecules in their third stage are, however, again stable, as in the first stage.

---

[1] "Contributions to the Molecular Theory of Induced Magnetism," by Professor Ewing, reprinted in the *Electrician*, September 12 and 19, 1890, and now embodied with additions in the same author's *Magnetic Induction in Iron and other Metals*, chap. xi.

But now, if after stage 2 the force H is gradually removed, the majority of the groups, having swung over to a new stable condition, retain their new configuration; hence, if the force be reduced to zero, there is still a considerable amount of residual magnetic induction, and it will require the magnetising intensity to be actually reversed to produce a condition of instability in the new groupings, and so cause them to be in their turn upset and replaced by fresh configurations. Thus the gradual removal of the magnetising intensity does not lead to the exact and complete repetition backwards of what happened when the magnetisation was being increased; in other words, the movements of the molecules are not reversible without qualification. Given, however, a sufficient number of groups composing the body, if the magnetising intensity be removed and reapplied, then, unless it be very weak, there are, in general, some groups which pass through a condition of instability. Especially will this be the case if the piece of iron be not perfectly homogeneous, and if, therefore, the lines of the different groups or chains are differently inclined at different places. This exactly corresponds with the observed facts, that hysteresis is always present in all cyclic changes of H, but that its effect increases rapidly for changes extending over the second stage of the curve of induction. The approach to a steady value of the hysteresis loss in the third stage may also be expected: thus in fig. 124 if the iron were in the strict sense of the term saturated at point E, the area of the closed curve should give the final maximum of the hysteresis, any increase of the induction beyond E simply carrying the molecules with it in complete alignment without the passage through fresh configurations.

The phenomenon of hysteresis is thus amply accounted for. It occurs whenever on the ascending curve a molecule is deflected from one stable position to another through a position of instability; since then, on the descending curve, the new position will persist until the change in the magnetising intensity becomes so marked as to cause it to again pass through a position of instability. Further, whenever a molecule passes through a position of instability, energy is dissipated in the form of heat; its equilibrium being upset, it acquires kinetic energy in falling over towards a new position of equilibrium, about which it then oscillates and causes its neighbours to oscillate. How the oscillations are damped out and converted into heat is not yet precisely known; the damping cannot be due to mechanical friction, and is more probably due to some form of molecular eddy currents. That mechanical vibration should lessen the residual magnetic induction is easily explicable on the molecular hypothesis, since it will cause changes in the distances between the molecular centres. During the swinging thus set up as the magnets recede from each other, their stability is reduced so that they respond more easily to change of magnetising intensity, and hysteresis is lessened.

§ 10. **Hysteresis in dynamo armatures.**—All that is required in any material for dynamo magnets is high permeability under strong magnetising intensities; its hysteresis and loss of energy in consequence thereof are of no interest to the designer. It is different when we come to the materials for armature cores, where both high permeability and low hysteresis are desirable. The question of the hysteresis in armatures requires therefore some further consideration.

In a heteropolar dynamo with a disc armature, such as fig. 79, if we suppose that the coils of the armature are wound on iron cores, it is evident that, as each core passes from one field to another, the magnetising intensity and induction in it are increased and reduced in the one direction, then reversed, and again increased and reduced in the other direction; hence it is taken through a complete cycle of induction, varying from a positive maximum to an equally great negative maximum, and back again. Further, exactly midway between the two fields there is zero magnetising intensity, and the molecules of the iron are for a moment free to pass through a position of instability. The case is therefore fairly analogous to curves II. and III. of fig. 124. There will be hysteresis, and the determination of the amount of energy lost by hysteresis per cycle, as obtained from the curve of fig. 134 for the maximum induction in the armature, will be applicable. Next, in the homopolar disc alternator of fig. 88 the magnetising intensity is continually varied through a cycle of changes from a positive maximum to zero. Again, therefore, there will be hysteresis in an iron core although the amount of the loss would not be so great as in the former dynamo. The induction will not itself vary from $+B$ to 0, but from a maximum $+B_2$ to some smaller value $+B_1$ in the same direction, the minimum induction $+B_1$ being partly due to the residual induction in the iron, but also to the presence of the useless lines z_a which enter the armature between neighbouring poles. The residual flux might not by itself be large owing to the magnetic circuit being imperfect, but the combined effect of the two cannot be neglected. Since the E.M.F. of the homopolar machine is itself proportional to $B_2 - B_1$, and B_2 must be higher than in the heteropolar machine to give the same E.M.F., the hysteresis loss in the former would become relatively high, although still remaining less than in the latter. Modern machines, however, with disc armatures seldom have iron cores, and need not therefore be further considered. Although in the above cases there must be some little adjustment of the molecules to the direction of the field in space which will vary somewhat, this tendency becomes much more marked when we pass to ring, discoidal-ring, and drum armatures, whether of alternators or continuous-current machines.

In these as the armature rotates, each molecule of iron must be twisted through an entire circle of 360° in each period; but this is effected by its gradual adjustment of itself to the changing direction of the lines, as it is carried round with the armature, and it is not at once apparent how far the magnetising intensity is ever removed, so as to allow it to fall back through a position of instability.

§ 11. **Hysteresis in a rotating field.**—It is, however, clear that the hysteresis loss as determined by variation of H through a cycle of values alternating from a + to a − maximum and back again is no longer applicable, and that the analogous process would be to rotate a piece of divided iron in a constant magnetic field, or conversely to rotate a constant magnetic field about a stationary iron armature. The logical deduction from Prof. Ewing's theory as applied to such a case was first pointed out by Mr. Swinburne. During the first or quasi-elastic stage corresponding to a weak field, the hysteresis loss should increase but slowly; during the second stage when the groups of little magnets are passing through irregular combinations, the loss should increase rapidly and approximately in proportion to the induction, but finally as saturation is approached the hysteresis curve should reach a maximum and then not merely remain constant as in the case of an alternating field, but bend over and fall rapidly towards zero. When the magnetising intensity is never removed and at each point in the path of the molecules is of such strength as to keep them in perfect alignment even though rotated, there would be no opportunity for them to pass through a position of instability and so to break up into new combinations. This view which was first advanced as an objection to the molecular theory has since been amply verified by direct experiment and furnishes additional evidence of the correctness of the theory. Indeed by means of an ingenious model in which a number of little magnets are rotated in a constant magnetic field we are enabled to watch the actual progress of the phenomena; the spasmodic breaking up of the initial configurations under the strain of rotation in a field of moderate strength is rendered visible, and we can trace the gradual increase of alignment, until finally the field becomes of sufficient strength to maintain the magnets pointing always in a definite direction, however quickly they are carried round.[1] In the early stages the ***hysteresis loss of iron in a rotating field*** or of iron rotating in a constant field, although of the same order as in an alternating field, is somewhat greater.[1] This may be due to the more gradual change of the rotating field and also to the smaller choice in the direction of movement of the molecules, so that some combinations offer more resistance to dissociation, and on their rupture the oscillation is greater. Thus up to B = 8000 or 10,000, the hysteresis loss in a rotating field may be some 50 per cent. higher than in an alternating field. In fig. 133 the second curve shows the hysteresis loss in a rotating field[2] for a sample of sheet iron cut from the same sheet as that for which the first curve shows the loss in an alternating field; the latter corresponds to a hysteretic constant η about 0·003, the former to a value somewhat greater than 0·004 so long as the curve is rising rapidly. But when B, the maximum induction across a section of the core, reaches the value of 15,000, the loss becomes less when reversal takes place by rotation, and at about 16,000 or 17,000 it attains a maximum; it thence decreases rapidly, and almost disappears at B = 21,000, but with a slight tendency for the curve to turn off as it approaches the zero axis. In the complete curve of fig. 136 are given the results of tests made by Messrs. Beattie and Clinker[3] on sheet iron; the initial portion up to B = 14,000 agrees roughly with a hysteretic constant of 0·003. The upper short curve of the same figure is obtained from tests made by Prof. Baily[4] on an actual dynamo armature. Of two specimens tested by Mr. Holden[5] up to B = 8000, one gave results agreeing fairly closely with the upper and the other with the lower of the curves of fig. 136, so that they may be taken as representing average cases. The exponent was found by Mr. Holden to vary in a number of samples from 1·4 to 1·7, the average being 1·5.

---

[1] The first experimental verification of this fact was communicated to the British Association by Prof. F. G. Baily in 1894 (*Electrician*, vol. xxxiii. p. 516), and

In the ring or drum homopolar alternator of the so-called inductor

---

a model illustrating it was shown by Prof. Ewing in 1895 (*Electrician*, vol. xxxiv. p. 670, and *Magnetic Induction in Iron and other Metals*, chap. xi.).

[1] Cp. Holden, *Electrician*, vol. xxxv. p. 329, and Hiecke, *E.T.Z.*, vol. xxiii. p. 142.

[2] *Phil. Trans*, 1896, vol. 187, pp. 715-746.

[3] *Electrician*, vol. xxxvii. p. 723.

[4] *Electrician*, vol. xliv. p. 323.

[5] *Electrician*, vol xxxv. p. 327.

type, the molecules are subjected only to *partial rotation*; owing to the armature core being continuous the change of direction of the molecules as they adjust themselves to the changing field will be more marked than in the case of the disc homopolar machine previously discussed, yet again the change of induction is not from B_1 to 0, but from B_{max} to B_r, where B_r is the induction due to the useless lines entering the armature from the flanks of the poles between two neighbouring pole-shoes.[1]

§ 12. **Influence of vibration and temperature on hysteresis in dynamos.**—The mechanical vibration[2] to which an armature is subjected when running only slightly reduces the loss from hysteresis, but possibly the much more rapid vibration due to the alternations of the current in an alternator armature, even when stationary, may affect the amount of the loss. Experiment shows that with increasing temperatures the hysteresis loss decreases by about 1·1% for each 20° F., so that when a dynamo is at work and thoroughly heated there may be a reduction of some 4 per cent. from the loss when cold. On the other hand the body of evidence points to the fact that after the iron has been subjected to continuous and prolonged heating at a fairly high

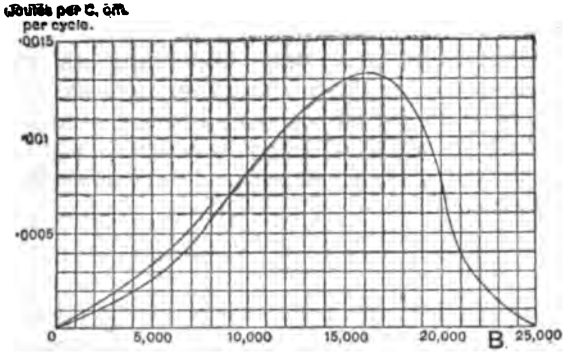


FIG. 136.—Hysteresis loss in rotating field.

---

[1] For approximate formulae for the calculation of the hysteresis in such cases, see Behn-Eschenburg, *E.T.Z.*, 1897, Jan 14th; Kapp, *Dynamo Construction*, p. 2; and Dr. Niethammer, *E.T.Z.*, 1898, p. 689.

[2] *Phil. Trans.*, loc. cit.

temperature such as 150° F., there is a permanent increase in the loss. The effect of temperature in the case of dynamos is however on the whole of small moment.

For the practical comparison in the workshop of different samples of sheet iron for armatures, the hysteresis tester of Prof. Ewing is the most convenient instrument. The specimen is rotated between the poles of a magnet free to rotate about the same centre, and the deflection of the latter is compared with that produced by standards with a known hysteresis loss,[1] but although a rotary motion is employed, the process of reversal resembles that of the alternating rather than of the rotating field. A number of specimens should be tested in order to obtain an average result, since even samples taken from the same sheet show considerable variations among themselves.

§ 13. **Hysteresis loss only important as affecting efficiency.**—In all calculations, therefore, as to the heating of armatures, or as to the efficiency of dynamos, some allowance must be made for a loss of energy by hysteresis, and in default of more decisive data, it will suffice to determine its amount from the formula

$$H_w = h \cdot \frac{pN}{60} \cdot v_e \text{ Watts,} \qquad (18)$$

the value of h being taken from the curves of fig. 133 or fig. 134 up to $B = 15,000$, and for higher values in ring or drum armatures of heteropolar machines from fig. 133 or 136. In the teeth of a slotted armature the accurate determination of the hysteresis loss would be a matter of great difficulty owing to the continuous variation of the density in the tapering tooth caused by lines passing out of or into the slot. An approximate formula on the assumption that all the lines, entering at the top of the tooth and giving an induction B_{t_1}, pass through the tooth unchanged, has been given by Dr. Niethammer[2]; it is further assumed that the loss obeys the law, $h=\eta\,B^{1.6}$, with a constant hysteretic coefficient. If then $a=\frac{k_{t_2}}{k_{t_1}}$ the ratio of the width of the tooth at the bottom to its width at the top, and if $V_t=$ the total volume of iron in all the teeth in cubic centimetres, the loss therein is

---

[1] *Journal Inst. Elec. Eng.*, vol. xxiv. p. 398, and *Proc. C.E.*, vol. cxxvi., May 1896. Cp. Prof. Fleming, *Electrical Engineer*, vol. xx. p. 366.

[2] *E.T.Z.*, 1898, p. 689. The proof is as follows. Taking any point x cm. distant from the top of the tooth, the width of the tooth at that point is $k_{t_1} - \frac{k_{t_1} - k_{t_2}}{l_t} \cdot x$, where l_t is the length of the tooth. The number of lines

$$10^{-7}\eta B_{t_1}^{1.6}\,.\,\frac{pN}{60}\,.\,V_t\,.\,f(a),\text{ watts}$$

where

$$f(a)=5\,\frac{1-a^{0.4}}{1-a^2}.$$

passing through the tooth for one cm. length along the core is $B_{t_1}\times k_{t_1}$, so that the induction at the point x is

$$\frac{B_t\,.\,k_{t_1}}{k_{t_1}-\frac{k_{t_1}-k_{t_2}}{l_t}\,.\,x}.$$

The volume of the element being

$$\left(k_{t_1}-\frac{k_{t_1}-k_{t_2}}{l_t}\,.\,x\right)dx,$$

the hysteresis loss therein is

$$10^{-7}\,.\,\eta\,.\,\frac{pN}{60}\,.\left(\frac{B_{t_1}\,.\,k_{t_1}}{k_{t_1}-\frac{k_{t_1}-k_{t_2}}{l_t}\,.\,x}\right)^{1.6}\times\left(k_{t_1}-\frac{k_{t_1}-k_{t_2}}{l_t}\,.\,x\right)dx,$$

or

$$10^{-7}\,.\,\eta\,.\,\frac{pN}{60}\,.\,(B_{t_1}\,.\,k_{t_1})^{1.6}\times\left(k_{t_1}-\frac{k_{t_1}-k_{t_2}}{l_t}\,.\,x\right)^{-0.6}dx.$$

The integral of

$$\int\left(k_{t_1}-\frac{k_{t_1}-k_{t_2}}{l_t}\,.\,x\right)^{-0.6}dx=-\frac{2.5\,l_t}{(k_{t_1}-k_{t_2})}\,.\,\left\{k_{t_1}-\left(\frac{k_{t_1}-k_{t_2}}{l_t}\right).x\right\}^{0.4}$$

and between the limits l_t and 0

$$-2.5\,\frac{l_t}{k_{t_1}-k_{t_2}}\left\{k_{t_1}^{0.4}-k_{t_2}^{0.4}\right\}$$

$$=2.5\,.\,\frac{l_t\,.\,k_{t_1}^{0.4}}{k_{t_1}-k_{t_2}}\left(1-a^{0.4}\right).$$

Now $\frac{k_{t_1}+k_{t_2}}{2}\times l_t=v_t$, the volume of one tooth per cm. length along the core, whence $l_t=\frac{v_t\times 2}{k_{t_1}+k_{t_2}}$. Substituting this value for l_t in the above equation, we have loss in one tooth per cm. length $=10^{-7}\,.\,\eta\,.\,\frac{pN}{60}\,.\,B_{t_1}^{1.6}\,.\,k_{t_1}^{2}\,.\,v_t\,.\,5\frac{1-a^{0.4}}{k_{t_1}^{2}-k_{t_2}^{2}}$, and in all the teeth of volume $V_t=10^{-7}\,.\,\eta\,.\,\frac{pN}{60}\,.\,B_{t_1}^{1.6}\,.\,V_t\,.\,5\frac{1-a^{0.4}}{1-a^2}$.

It will be found that even in the case of alternators with a periodicity of 100 the total heat generated by hysteresis is but small, and, even under conditions unfavourable to cooling, is incompetent to raise the temperature of the entire iron-mass more than a few degrees Fahrenheit. Hence the permeability of the armature stampings is of more importance than their hysteresis; the loss by hysteresis only becomes of consequence as increasing the heat due to other and more serious losses. Since it bears, so far as is known, no direct relation to the permeability, it is to the latter that attention must be chiefly directed.

CHAPTER XIII

CONTINUOUS-CURRENT ARMATURES

§ 1. **Lamination of armature core to avoid eddy-currents.**—The practical construction of armatures, and the methods by which the cores are built up and the inductors are wound on, must next claim our attention. The subject is too wide to be fully treated within our limits, but in the present chapter certain general principles will be discussed which in everyday practice govern the mechanical design of dynamos and more particularly the construction of continuous-current armatures with their commutators.

Except in the case of certain disc machines, the armature winding is invariably supported on metal cores, and these, in ring, drum, or discoidal armatures, are of iron. As mentioned in Chapter VII, the use of the metal supporting core either necessitates its rotation together with the rotating inductors, or, if the armature is stationary and the field magnet revolves, implies that its surface is swept over by the moving field of lines. From this fact there follows a most important consequence: whenever a metallic mass actively cuts lines of induction, it becomes the seat of induced E.M.F.'s, which will, unless prevented, set up electric currents; the direction of these in that part of their path where the E.M.F. is generated will be at right angles to the direction of the lines and to the direction of movement. If, therefore, the core whereon the inductors proper are wound be formed of a solid metal mass, the latter will itself act inductively, and although the E.M.F.'s induced in it may be small, yet, owing to the low resistance of the numerous metallic paths open to them, they will cause enormous *eddy-currents* to circulate round the core; the latter will then become heated and energy will be wasted equal in amount to the product of the E.M.F. and the strength of the eddy-current. In a solid ring or drum armature the eddy currents will flow in two sheets along the length of the iron core, in opposite directions under the two poles. Their intensity will be greatest near the surface, where the rate of cutting lines and the induced E.M.F. is greatest; throughout the entire length of the core, and especially at its ends, the current-sheet under one pole will pass across from one side to the other of the armature and complete its circuit under the opposite pole. Analogously, in a solid discoidal-ring armature the eddy-currents will circulate round the core, running up one side of the flat ring, across and down the other side. In all cases their direction on the surface of the core will be similar to that of the useful currents in the inductors proper, since these latter are expressly arranged to obtain the best inductive effect. So long as the metal core rotates, or is swept by the lines, it remains impossible to prevent the induction of E.M.F.'s in its surface, but it is possible to interpose in the paths which the eddy-currents would follow such very high resistances as to prevent their attaining any appreciable strength. This is effected by dividing the core up into small portions, each of which is more or less completely separated from its neighbours by a thin layer or covering of some insulating material. The directions in which the subdivision is carried out must be so chosen that the insulation obstructs the chief paths of any induced currents. In the iron-cored armatures now under consideration this can only be effectually secured if the core be divided in planes parallel to the direction of the lines and to the direction of motion; the mass of metal must be broken up at the actual seat of the E.M.F. and transversely to its direction, and the subdivision should be especially perfect at the periphery, where the E.M.F. is greatest. In short the core must be *laminated*, the direction of lamination being suited to the type of armature. In ring or drum armatures the planes of lamination should be at right angles to the axis of rotation, or to the shaft as indicated in the sections of fig. 50, while in discoidal-ring armatures the divisions should be approximately concentric with the axis of rotation (figs. 54 and 78). In both cases the lamination is at right angles to the inductors themselves, and the edges of the divisions are presented to the lines of the fields. Small

E.M.F.'s will still be induced transversely across the surface of each division, but they will be prevented from acting summationally or in series by the intervening insulation; the currents that can be set up are thus reduced, and likewise the energy that is absorbed (cp. Chap. XIX, § 14). The thinner the divisions, the less will be the amount of the eddies, and the limit to the advantage of further subdivision is only reached when questions of the cost of manufacture or of mechanical strength outweigh the advantage of a further slight gain in efficiency.

§ 2. **Discs for ring and drum armatures.**—The requirements above laid down for ring or drum armatures are met by the use of thin discs or washers, either threaded on to the arms of a central hub or keyed directly to the shaft. Each core-disc is well insulated from its neighbours on either side; hence they make little or no contact with each other except through their connection with the shaft, and the passage of eddy-currents along the surface or through the mass of the core from one disc to another is prevented. The same end is obtained, although somewhat less effectually, by using wire, wound to form a cylindrical core, the separate turns making but slight electrical contact with each other. The requirements of the discoidal-ring armature are fulfilled practically by a thin band or ribbon coiled on a supporting ring, adjacent layers being lightly insulated from each other.

Of the three magnetic materials discussed in Chapter XII it is at once evident that cast iron and cast steel are out of the question for armatures, since they do not permit of the core being sufficiently laminated. Thus, the only magnetic materials which are practicable for armature cores are soft annealed[1] wrought iron or mild steel in the form of thin discs, thin ribbon, or wire.

The use of *discs* stamped out of thin sheet-iron or steel is now almost universal for ring and drum armatures, and in good machines their thickness does not exceed 0·025″ or No. 24 (sheet-iron gauge). In general, the thickness ranges[2] between 0·020″ and 0·025″, or, say, 0·5 and 0·6 mm.; below the former

---

[1] See Chapter XII, § 4.

[2] The latter dimension is now almost universally adopted in Great Britain; its weight per superficial square foot is 1 lb.

limit, the reduction in the eddy-current loss within the core is so slight that it does not compensate for the increased cost of the discs and the greater labour in assembling them. Up to four feet in diameter, discs may be procured as complete rings, each forming one stamping. For armatures of larger size, segments are employed; these are built up with butt joints in any one layer, but so arranged that the joints of neighbouring layers do not coincide in position (figs. 142 and 143). Each disc is either coated with insulating varnish or paint, or is effectively separated from its neighbours by an interposed thin sheet of paper cut out to the required shape; or the oxide, which forms on the discs after heating, and which is practically a non-conductor, is relied on to insulate the discs. When varnish is employed, the discs are either painted by hand, or are passed between rolls dipping into a bath of the varnish; if heated and put under pressure, they will then adhere so tightly that the whole core is formed into a solid and compact mass. After allowing for slight inequalities in the thickness of the discs, and for the two coats of varnish, one on either side, and measuring in all about 0·0006″, from 90 to 92 per cent. of the total cross-section of the core may be counted on as available for the passage of the magnetic flux. The paper, when handled as separate sheets to be laid between the discs, cannot be conveniently thinner than one or two mils., or 0·025 to 0·05 mm.; the net volume of iron in the core then amounts to 88 or 90 per cent. of the gross volume. In the case of toothed armatures, some 1 or 2 per cent. greater allowance must be made for irregularities in the assembled discs, and a combination of the two methods is frequently employed; a thinner paper, less than 1 mil. in thickness, is pasted by rolls on to the sheet-iron with varnish and at the same time dried, before the discs are punched in the notching press. The teeth then remain well insulated right up to their edges, and the work of clearing the paper from the slots of the assembled core is obviated.

§ 3. **Construction of ring and drum armatures.**—We now turn to the consideration of rotating armatures, which, to a large extent, covers the same ground as continuous-current machines, since in this class, as opposed to alternators, the armature is more often the rotating portion.

In small two-pole drum armatures, the discs are usually threaded directly on the shaft, and held in position by a driving feather running along the length of the core. At the two ends of the core are cast-iron end-flanges, between which the discs are tightly compressed; one of these is first driven home against a collar on the shaft, the discs are then assembled, and after compression by hydraulic or mechanical means the second end-flange is slid on and secured in its place by a tightening-nut screwing on to the shaft (fig. 137). For a given armature diameter, the maximum radial depth of disc is thus obtained, but in larger machines, especially if multipolar, sufficient cross-section of iron is secured without using the full depth or difference between the radius of the disc and the radius of the shaft. The discs may then be pierced with holes towards their inner edge in order to lighten them (fig. 138), or they may be cut away on the inside, four projecting lugs being left, which fit exactly on to four flats milled along the length of the shaft (fig. 139). It is even more common to employ discs of lesser depth and to support them on a cast-iron hub, which is itself keyed to the shaft. Keyways are stamped on the inner periphery of the discs to fit four or more arms or ribs which project centrally from the central sleeve of the hub. The latter abuts against a collar on the shaft, and has an end-flange either cast in one with it or fastened to it by bolts. The tips of the radial arms are turned in the lathe to the correct diameter to fit the inside of the keyways, and the sides of the arms are milled to the correct width to fit the

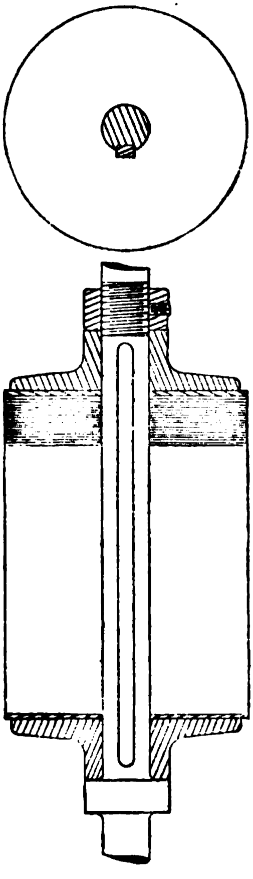
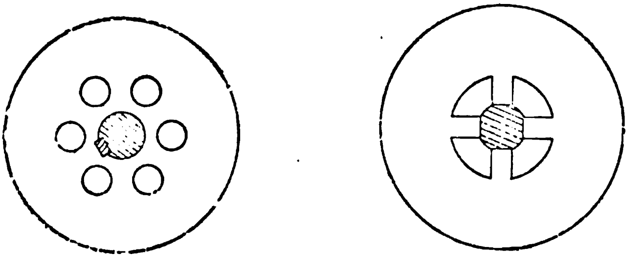


FIG. 137.—Small two-pole drum armature core.

notches; the discs are then slid over the radial arms and compressed by the second end-flange, which is screwed directly to the shaft, or, in larger machines, is fastened to the hub by bolts, making the core entirely self-contained. Fig. 140 i. shows such an armature core complete with its end-flanges, while fig. 140 ii. reproduces the actual hub without the core discs. In other cases, the discs are driven by round keys of steel, which are sunk half in the arms of the hub and half in the discs, the latter being stamped with semicircular notches on their inner edge. A third variation consists in making the hub in two precisely similar halves, each with its arms alternately long and short; the discs are assembled on each half, and the two halves are brought together on the shaft, so that a long arm in one half is exactly opposite to a short arm of the other half. Under pressure the long arms of the one half are caused to enter the key-ways in the other half, until the two sets of arms project alternately past one another, and all the discs are positively driven, those in the centre being held by at least one half of the arms. Long bolts run through the arms and are cut off to the right length after the core has been compressed to the required dimension (fig. 141).



FIGS. 138 and 139 —Drum core-discs.

When the diameter of the armature is so large that the discs must be built up out of segments, these may be held by bolts passing through holes near their inner edge, and clamping the segments to the end-flanges; the latter are conveniently made in several pieces, each being secured against centrifugal force by engaging beneath a ring on the armature hub. Each segment

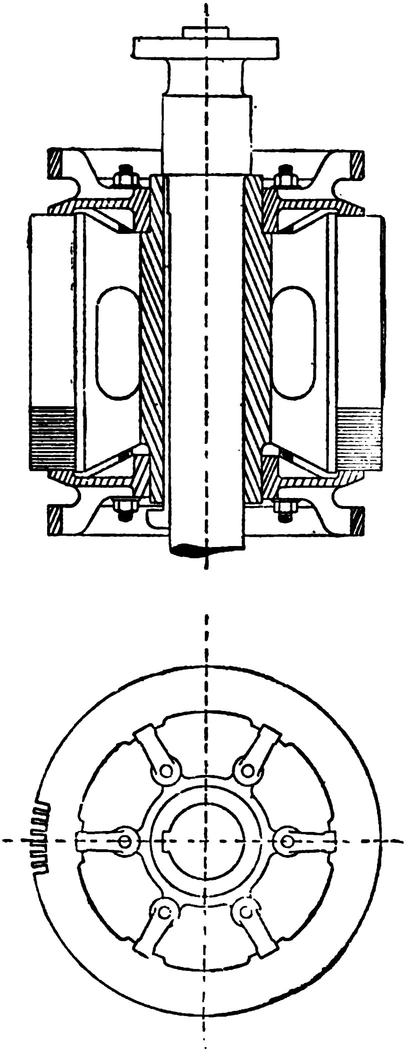


FIG. 140 i.—Multipolar drum armature core.

must be held by at least two bolts (fig. 142, cp. fig. 337). Or the segments may be dove-tailed to the arms of the hub by wedge-shaped lugs, so as to protect the discs from radial movement (fig. 143) under the action of centrifugal force.

In ring armatures an open hub with radial arms is necessary, and it must be made either wholly or in part of a non-magnetic material, usually gun-metal. Any of the constructions of hub above shown can be also adapted to a ring machine, and fig. 144 shows a ring armature with a hub entirely of gun-metal. In very large rings, the hub may be made, for the sake of cheapness, out of cast-iron, with radial arms of gun-metal bolted or keyed on to the body.

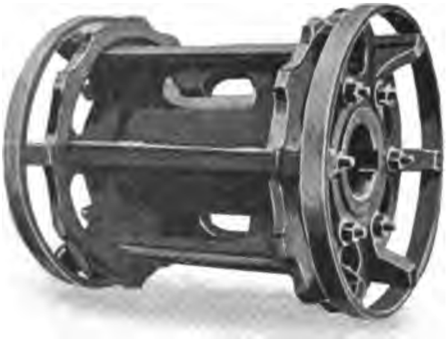


FIG. 140 ii.—Hub and end-plates of armature.

In all cases if a tightening-up nut is employed to hold the armature core in place, it is advisable to lock it by a set screw, the point of which takes into the shaft, since otherwise there is a possibility of the nut and discs slacking back through mechanical vibration.

At either end of the ring or drum armature there are often placed one or more stout iron plates (figs. 137 and 144), which serve to support the thinner discs where they are driven up against the end-flanges or end-arms of the hub. In order to avoid eddy-currents being set up in these end core-plates (by reason of lines curving round from the poles into their outer

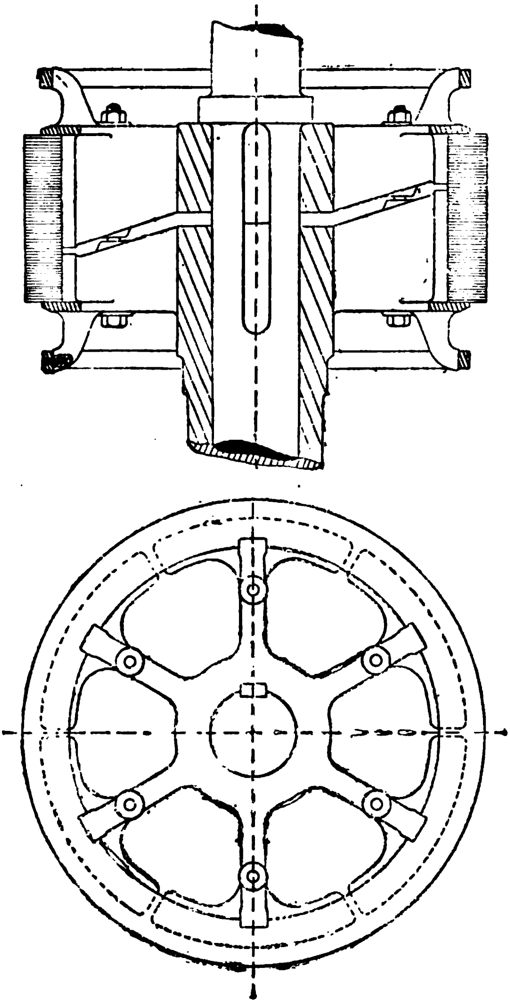
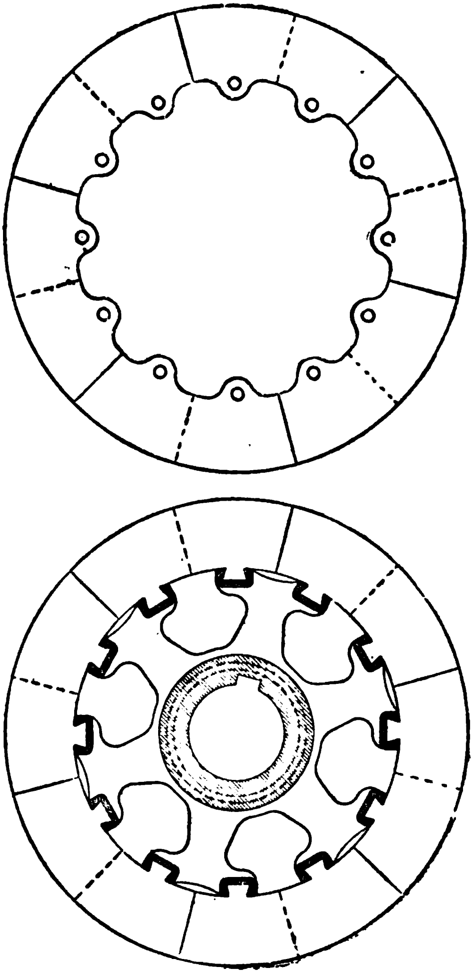


FIG. 141.—Multipolar drum armature core in section and end-view with front half of hub removed.

surfaces), the axial length of the core is frequently made slightly greater than the length of the pole-pieces parallel to the shaft.

§ 4. **Inferiority of wire cores.**—*Wire* cores are now used only for very small ring armatures: they are constructed by winding soft charcoal-iron wire either on a gun-metal flanged pulley, or between two flanges on the arms of an open spider, a cylindrical interior being in this case obtained by inserting,


FIGS. 142 and 143.—Multipolar cores with segmental discs.

during the winding, wooden supports, which are subsequently withdrawn. The iron wire is either varnished or single-cotton-covered, or it is slightly oxidised in order to obtain moderate insulation between contiguous turns. Such armatures are, however, distinctly inferior to those composed of thin discs. The interstices which exist between adjacent turns of the round wire must be reckoned as lost space, and the actual volume of iron in a core of given cross-section is much less than could be obtained by the use of discs. Further, the lamination is carried out in two planes, whereas in the ordinary ring armature it is only required in one plane, and therefore more space is wasted by the insulation than is necessary. Lastly, the iron is magnetically discontinuous in the direction which the lines take through the armature, and in consequence to reach the lower layers of iron the flux has to pass through several non-magnetic gaps. Hence the full benefit of the radial depth of iron is never obtained, the outer layers become saturated before the inner, and the total reluctance of the armature is considerably greater than even the actual sectional area of the iron would warrant.

§ 5. **Construction of discoidal-ring armatures.**—To form discoidal-ring armatures, soft iron ribbon or tape about ·025″ thick is coiled on to a supporting wheel or foundation ring. The ribbon is at the outset riveted to the wheel, and is then wound on in a lathe, being drawn from between rollers so as to put considerable tension on it; a layer of thin paper of the same width as the iron is wound on simultaneously to insulate the successive layers. In order to give mechanical rigidity to the whole, long pins are in some cases passed radially through the centre of the iron ribbon and screwed into the rim of the wheel, or the latter is furnished with a central projecting flange, on either side of which the ribbon is wound. In small armatures it is preferable to use gun-metal arms or spokes for the foundation wheel, but in the case of multipolar armatures of large internal diameter the whole may be of cast iron, the spokes being so arranged that the points which they directly connect are of similar polarity. If the poles on either side of the flat ring are connected by a common pole-piece arching over the core, lines of induction enter the ring radially from the top as well as from the two sides, and it is therefore necessary to laminate the core in two planes—at least near the outer periphery. This is, to a certain extent, effected by adopting a narrow width of iron tape, and winding it on in three or more rows of turns side by side, while in other cases rectangular or square iron wire, lightly insulated, has been used.

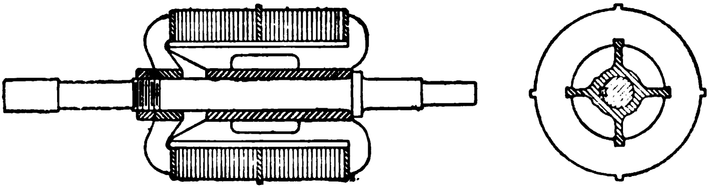


FIG. 144.—Ring armature of discs.

§ 6. **Avoidance of eddy-currents.**—All bolts used to hold the core in position and passing through the mass of the discs must either be so placed as to be cut by as few lines as possible or must be insulated from the core by fibre tubes and fibre or mica washers in order to obstruct the paths that eddy-currents generated in them would follow.[1]

In all cases, care must be taken that the end-plates, end-flanges, supporting hubs, etc., are not the seat of eddy-currents: so far as possible they must be kept without the influence of the magnetic field. In drum armatures with discs of considerable radial depth very few lines pass across from inner side to inner side of the discs through the shaft; but in ring armatures worked at a high induction, not only may a small percentage of the lines of the field cross the internal opening of the discs, but a further number of lines crossing the ring internally will be induced by the current in the armature winding (see Chap. XVII). Although the armature revolves, this internal field remains stationary; it is therefore cut by the arms and sleeve of a central hub, and the latter will become heated by eddy-currents. In order to minimise this waste it is important that the hub should be lightened so far as possible, while still maintaining sufficient mechanical strength (compare fig. 144).

§ 7. **Mechanical strength of armature hubs.**—For the calculation of the strength of the arms or spokes of a hub, the moment of the force acting at any point must be equated to the moment of resistance, *i.e.*, to the product of the modulus of the section multiplied by the safe working stress of the material of which the hub is made. If the armature makes N revs. per min. and transmits HP horses' power, the torque or *twisting moment* acting on the armature is $T_m = \frac{H.P \times 33{,}000 \times 12}{2\pi N} = 63{,}000 \frac{HP}{N}$ inch-pounds. For dynamo work this may be translated into a more convenient form. If the output of a dynamo be *kw* kilowatts, the actual power which passes into the shaft either through the pulley of a belt-driven machine or through the coupling connecting it directly to the crank-shaft of an engine, is $KW = \frac{kw}{\eta}$, where η is the efficiency, or the ratio between the useful power given out at the terminals, and the total power absorbed in obtaining it.[1] By substitution of the quantity KW, reduced to horse-power in the above equation, we obtain

---

[1] Kapp, *Dynamos, Alternators and Transformers*, 2nd. ed. p. 279.

$$\text{T}_m = 85{,}000 \frac{kw}{\eta \text{N}} \text{ inch-pounds} \quad . \quad . \quad . \quad (19)$$

The commercial efficiency of dynamos varies considerably with their type and size, but in the greater number of cases it ranges from 70 in small to 90 per cent. or more in large machines; an average value for η is therefore about 0·85. Thus the coefficient for the conversion of the horse-power input into kilowatts of output may be taken on the average as $= \frac{746}{1000} \times 0{\cdot}85 = 0{\cdot}63$ and any equation expressed in terms of horse-power must be divided by this coefficient to express it in terms of kilowatts, or $\text{T}_m = 100{,}000 \frac{kw}{\text{N}}$ inch-pounds. If w = the output in watts,

$$\text{T}_m = 100 \frac{w}{\text{N}} \text{ inch-pounds,}$$

where $\frac{w}{\text{N}}$ = the watts per rev. per min.—a quantity the importance of which in dynamo designing will appear subsequently.

If then r be the radius of the arm in inches (fig. 145), or the distance from the centre of the shaft to the tip of the arm, where the resistance to the motion is applied, and n_s is the number of arms or spokes, the force acting at the tip and tending to bend the spoke is $\text{P} = \frac{\text{T}_m}{r \times n_s}$ pounds. If any point be taken along the arm, distant l inches from its tip, the leverage is l and the *bending moment* at that point is $\text{B}_m = \frac{\text{T}_m}{r \times n_s} \times l$. Let h_s be the thickness of the arm at this point in inches, and b_s the corresponding breadth parallel to the length of the shaft, the modulus of resistance of the cross-section being thus $\frac{h^2_s \times b_s}{6}$; and let f_t = the safe working stress of the material in lbs. per square inch for tension or pressure. The bending moment for a transverse section taken at any point in the length of the arm from its tip to the nave of the hub must not exceed the moment of resistance, or

---

[1] In the case of a separately excited machine, the exciter of which is not driven from the main dynamo shaft, the power absorbed in the field should strictly be taken into account in calculating the commercial efficiency, although in our present connection such loss in the field stands outside the calculations of the strength of shaft.

$$B_m \leqq \frac{h^2_s \times b_s}{6} \times f_t.$$

The direction of the resisting force in a continuous-current

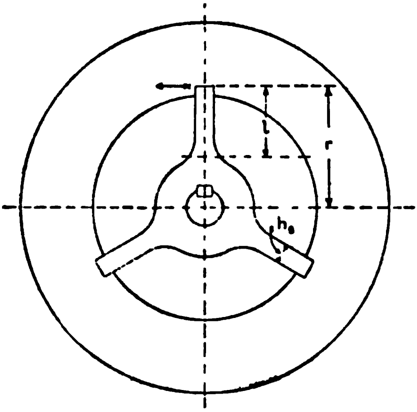


FIG. 145.

dynamo is always the same, but in a reversible motor or in an alternator it may change, so that in every case it is best to allow a high factor of safety suitable to an alternating load. For cast-iron, f_t may be taken as = 1250 and for gun-metal as 1500 to 2000 pounds per square inch for stresses of tension or pressure. The arms of the hub must also be strong enough to withstand the shearing action; this is uniform throughout the length of the arm and is equal to the drag or force acting at the tip of each arm, *i.e.*, to $P = \frac{T_m}{r \times n_s}$ transferred without change to any transverse section of the arm. Then

$$P \leqq bh \times f_s$$

where f_s or the safe working stress under shearing may be given the same values as f_b.[1]

It will be found that even in the ring hub there is no difficulty in obtaining ample strength against either bending or shearing. In the ring the axial dimension b_s should be made long and the thickness small so as not to encroach upon the internal winding space more than is needful. In all cases care must be taken that sufficient surface is given to the tips and driving edges of the arms, and that the discs are very accurately fitted or keyed to them. If this precaution be neglected, the driving stress will after several years of use end by causing the edges of the discs to cut into the arms; especially is this likely to occur in the hubs of ring armatures, which being made of gun-metal are of comparatively soft nature, with the result that as soon as the least relative movement takes place, wear of the arms begins and rapidly increases, until the whole core becomes loose on the hub.

§ 8. **Mechanical strength of armature shafts.**—Armature shafts are usually of mild steel, produced by the Bessemer or Siemens-Martin process. The determination of the diameter required at different parts of their length is largely a matter of experience, but nevertheless in the main it requires to be based on the theoretical rules appropriate to shafts which are subjected to the combined stresses of torsion and bending. The calculation of the torsional stress due to the continuous and steady transmission of horse-power through a rotating shaft is an easy matter; since, as in the preceding paragraph, the twisting moment about the axis of the shaft or $T_m = 100{,}000 \frac{kw}{N}$ inch pounds.

But torsion is by no means the only stress to which the armature shaft is subjected; even more important is the bending stress, due to several causes, chief among which is the weight of the armature itself. In ring or drum armatures the weight is more or less uniformly distributed over the length of the shaft between the bearings; in discoidal and disc armatures the weight is more nearly concentrated at the centre, although even then the axial length of the supporting hub is usually a considerable number of inches. If W be the weight of the complete armature in pounds, and l be the span in inches between the bearings on either side of the armature, then if we assume the whole of the weight to be concentrated at one point (fig. 146), the reactions at the centres of the bearings Q, R are $w_1 = \text{W}\,\frac{l_2}{l_1+l_2}$ and $w_2 = \text{W}\,\frac{l_1}{l_1+l_2}$. The bending moment diagram is drawn as follows; upon the direction of R produced, mark off a length $a\,b$ representing to some convenient scale the moment of w_1 about R, a negative or clockwise moment being measured downwards and a positive or counter clockwise moment upwards. Join $b\,c$, intersecting the direction of W at d and lastly join $d\,a$. Then $a\,b\,c$ is the moment area of w_1 and $a\,b\,d$ is the moment area of W; W$\,l_2$ being positive, its area above the horizontal line $a\,c$ is equally well given by the area of the triangle, $a\,d\,b$ lying between the same parallels, when the base $b\,a$ is measured upwards. Thus the positive moment area $a\,b\,d$ balances a portion of the negative moment area and the remaining shaded triangle $a\,d\,c$ is the bending moment diagram, its vertical ordinates being proportional to the bending moment at corresponding points of the shaft. The greatest bending moment is $\text{W}\,\frac{l_1 l_2}{l_1+l_2}$ and in the case of fig. 146 if we take $\frac{l_1}{l_2} = \frac{0{\cdot}4}{0{\cdot}6}$, it is but little different from that due to a load concentrated at the centre which gives the dotted equilateral triangle of fig. 147 and $\text{B}_m = \frac{\text{W}l}{4}$. If, however, we assume the weight to be uniformly distributed along the entire length between the bearings,

---

[1] See Unwin, *Elements of Machine Design*, part i. (edit. 1902) p. 58.

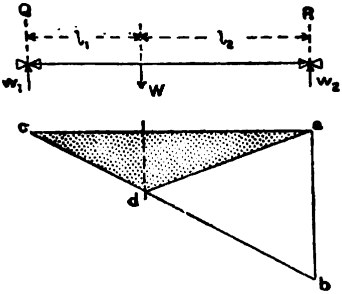


FIG. 146.—Bending-moment diagram due to weight of armature if concentrated.

the bending moment diagram is given by the dotted parabola with vertex at the centre (fig. 147), the greatest bending moment being reduced to $B_m = \frac{Wl}{8}$. Neither supposition is strictly true, and the best approximation is probably obtained if the intermediate full line of fig. 147 with $B_m = \frac{Wl}{6}$ is adopted.

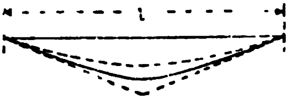


FIG. 147.—Comparison of bending moment diagrams due to weight, concentrated and distributed.

Comparison of a large number of dynamos of different sizes and types shows that the weight of the armature is largely independent of the type of field and may be expressed to a fair degree of accuracy as proportional to the ⅔rds. power of the kilowatts per rev. per min., or $W = c \cdot \left(\frac{kw}{N}\right)^{\frac{2}{3}}$ lbs. The value of the constant c ranges from 6000 to 10,000, but more often approximates to the lower limit, so that its average value may be put at 7200,[1] *i.e.*, $W = 7200 \left(\frac{kw}{N}\right)^{\frac{2}{3}}$ lbs. or $72 \left(\frac{w}{N}\right)^{\frac{2}{3}}$ lbs.

The length of the span between the bearings in average cases is of such an order that we assume it to be $l = 96 \left(\frac{kw}{N}\right)^{\frac{1}{3}}$ inches. The bending moment due simply to the weight of the armature as given by the above formula will then be

$$B_m = \frac{7{,}200 \left(\frac{kw}{N}\right)^{\frac{2}{3}} \times 96 \left(\frac{kw}{N}\right)^{\frac{1}{3}}}{6}$$

$$= 115{,}000 \frac{kw}{N} \text{ or } 72{,}500 \frac{HP}{N} \text{ inch-pounds.}$$

---

[1] In alternators in which the revolving portion is very usually the field-magnet and this is combined with, or itself forms, the fly-wheel of the driving engine, the weight will be considerably higher than in a continuous-current dynamo with revolving armature; owing to the great variations of the conditions, c may then reach values between 10,000 and 20,000, and can hardly be said to be a constant, even in alternators of the same type.

In engine-driven dynamos, where the heavy mass of the fly-wheel is supported between the inner bearing of the engine and the outer or commutator bearing of the dynamo, it will be necessary to take this further weight into account as concentrated at a certain distance from the centre of the armature. The bending moment diagram may then be obtained by adding the ordinates of the diagram due to each separate weight, or we may set off the moment of w_1 about R, *i.e.*, $ac = \mathrm{W} l_2 + \mathrm{W}_1 l_1$ downwards and join *ce* (fig. 148) intersecting the direction of the fly-wheel weight W_1 produced at *d*; measure $cb = \mathrm{W}_1 l_1$ upwards from *c*, and join *db* and *fa*; then *afde* is the combined bending-moment diagram. The fly-wheel being supposed to be three times as heavy as the armature, and at the same distance from the commutator end that the pulley bearing formerly was, it will be seen that the bending moment is increased 1½ times.

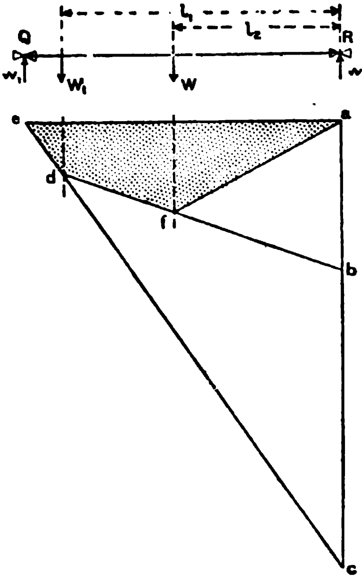


FIG. 148.—Bending-moment diagram due to weight of armature and of fly-wheel.

§ 9. **Causes increasing the bending moment on shafts.** — There are still, however, a number of causes increasing the bending moment, which in the above equations are entirely unrecognised. First amongst these may be mentioned the lateral pull caused from the pulley end by the tension of the driving belt or ropes. The pulley in belt-driven machines is usually placed outside of, but as close as possible to, one of the two bearings (cp. fig. 2); and a certain amount of bending stress must be allowed for within the span of the shaft from the one bearing to the other, over and above that due to the weight of the armature

If $T = T_2 - T_1$ is the driving force at the periphery of the pulley in lbs., or the difference in the tensions of the two sides of the belt, the total pull on the shaft at the centre of the pulley is $T_2 + T_1$. The arc embraced by the belt on the smaller driven pulley being taken as about 0·4 of its circumference and the coefficient of friction between the belt and the pulley as 0·3 (these assumptions being such as usually hold in practice[1]), $T_2 + T_1$ may be reckoned as $= 3T$. This pull causes reactions in the two bearings (fig. 149) of values

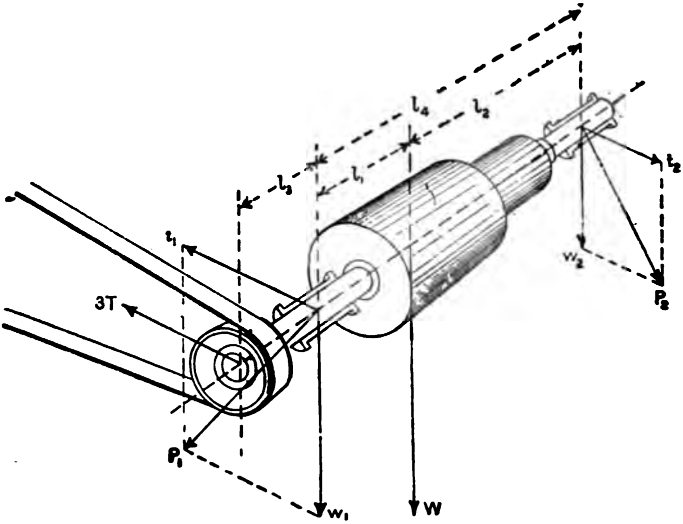


FIG. 149.—Forces acting on belt-driven dynamo.

$$t_1 = 3T\left(\frac{l_3 + l_4}{l_4}\right) ; \ t_2 = 3T \cdot \frac{l_3}{l_4}.$$

Their directions when viewed from above are as shown in fig. 150, whence the bending moment diagram due to the overhung pulley is easily drawn by setting off $ab = 3T(l_3 + l_4)$ on the direction of t_2 produced and joining bc. If the driving force and the weight were acting in the same plane, the two shaded areas acd of figs.

---

[1] See Unwin, *Elements of Machine Design*, part i. pp. 459–465 (1902 edit.).

150 and 146 could be combined by adding together their corresponding ordinates algebraically; but in general the pull of the belt acts horizontally or at right angles to the weight. In order to find the maximum bending moment the diagrams must now be combined by taking the square root of the sum of the squares of the ordinates at corresponding points. This has been done in fig. 151, and from it the increase and altered position of the maximum bending moment within the bearings due to the belt is clear. To this has next to be added the possibility that there is a considerable magnetic pull, the nature of which will be described in Chapter XIV; any such pull will tend to deflect the shaft, and will often coincide nearly in direction with the weight of the armature and increase the downward pressure by at least one-third.

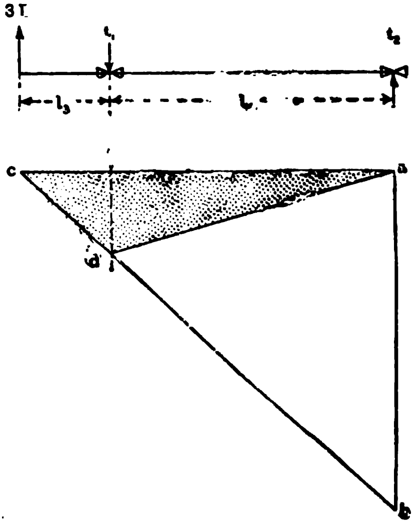


FIG. 150.—Bending-moment diagram due to belt pull.

Perhaps of even greater importance is the necessity for the utmost stiffness in the shaft. In a rapidly rotating armature the slightest want of balance when running will tend to deflect the centre-line of the shaft away from the straight axis of rotation; unbalanced centrifugal force then comes into play, and vibration, which further increases the evil, is set up. The shaft must therefore possess ample stiffness to withstand this tendency, especially when the span between the

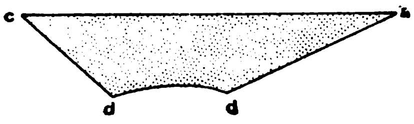


FIG. 151.—Combined bending-moment diagram due to weight of armature and belt pull.

bearings is long, and the armature is a large, heavy mass running at a high speed. Further straining actions are set up when the masses of the armature and fly-wheel are accelerated or retarded by reason of sudden changes in the armature load, such as occur, *e.g.*, in a generator supplying current to tram-car motors. The influence of all these causes is to a large extent indeterminate, but it is owing to them that an armature shaft which is strong enough to transmit the requisite horse-power torsionally, and to resist the bending due to the armature weight, may still not be stiff enough to secure freedom from vibration and withstand the working shocks and strains. Experience shows that the various influences enumerated above can only be effectually guarded against by regarding the bending moment in practice as about three times that which would be due to the simple weight of the armature—in other words by taking

$$B_m = 345{,}000 \frac{kw}{N} \text{ or } 345 \frac{w}{N} \text{ inch-pounds}$$

$$= 217{,}000 \frac{HP}{N} \text{ inch-pounds.}$$

The twisting moment being

$$T_m = 100{,}000 \frac{kw}{N} \text{ or } 100 \frac{w}{N}$$

$$= 63{,}000 \frac{HP}{N},$$

the two may be combined by the well-known rule through which the *equivalent bending moment* is obtained,[1] viz.

$$B_e = \frac{1}{2} B_m + \frac{1}{2} \sqrt{(B_m^2 + T_m^2)}$$

$$= 352{,}500 \frac{kw}{N} \text{ or } 352 \frac{w}{N} \text{ inch-pounds} \qquad (20)$$

$$= 222{,}000 \frac{HP}{N} \text{ inch-pounds.}$$

It is evident that the bending action is of far greater importance than the simple torsion, and in consequence it is in the estimate of the bending moment which is to a great extent indeterminate that the apparently large factor of safety of 3 must be introduced. The absence of the belt pull in engine-driven dynamos is roughly counterbalanced by the greater span and the share which the dynamo shaft takes in carrying the weight of the flywheel.

---

[1] Unwin, *Elements of Machine Design*, part i. p. 90.

The *moment of resistance of a circular section to bending* is $\frac{\pi}{32} f_t\, d^3 = 0{\cdot}098\, f_t\, d^3$, and equating this to the equivalent bending moment, we obtain

$$d = \sqrt[3]{\frac{B_e}{0{\cdot}098 f_t}} \quad . \quad . \quad . \quad . \quad (21)$$

where d is the minimum diameter of a solid circular shaft for a given value of f_t, and f_t is the safe working stress of the material of the shaft in pounds per square inch.

§ 10. **'Life' of a shaft.**—During rotation the twisting moment and the straining action due to change of velocity may vary within wide limits. Further, owing to the bending of the shaft, the fibres, which are at one part of a revolution extended, are in turn compressed after half a revolution. The bending stress therefore not only varies in value, but its direction in relation to the fibres of the shaft is continually and rapidly reversed every half revolution. The experiments of Wöhler have shown that under these circumstances the apparent breaking strength is very much less than for a constant load of unchanging direction, and the safe working stress must be correspondingly reduced. Later investigators[1] have shown that the apparent reduction in the breaking strength may be explained on the hypothesis that every material permits a certain amount of work to be done on it per unit volume, and that when this capacity is exhausted either by the single application of a large load or by the repeated application of a very much smaller load, the shaft is ruptured. The ultimate exhaustion of the shaft's capacity by repeated reversals of the smaller load is therefore simply a question of time. Let K = the breaking strength in lbs. per unit section for a given material under tension, and for a load once gradually applied, and let $L' =$ the elongation of the bar which was initially L inches long and of area a square inches; then since the gradually increasing stress has been acting through the length L', the total work done in extending the bar is $\propto K a L'$ inch-lbs. or per unit volume of the bar is $W' \propto \frac{KL'}{L}$, and W' may be defined as the specific capacity of the material limiting the amount of work that can be done on it before rupture. Let a similar bar be subjected to tension of value f per unit area, this value being well within the elastic limit; the elongation of the bar is $l = \frac{fL}{E}$, where E is the modulus of elasticity, and the work done in the process of stretching the bar is $cfal = cf^2a\ \frac{L}{E}$, or per unit volume, $w = \frac{cf^2}{E}$. The value of c depends upon how the load is applied, *i.e.*, whether impulsively, or by gradual increase from zero, in which case $c = \frac{1}{2}$. When the load is removed, the bar does not return to exactly its original length L, but to some length $L + l'$ very slightly greater than L; even when the force is within the limit of elasticity, there is thus a very minute permanent lengthening, and the number of inch-lbs. per unit volume which are recovered on the removal of the load and the straightening out of the deformed fibres is less than the number expended in bending the bar by some amount w'. Further it is assumed that the value of l', and so the amount of lost work, varies with the time or duration of the load in one direction, being greater the less rapidly the reversal is effected, so that $w' = c'\ t\ \frac{f^2}{E}$. The same effects take place when the bar is compressed, so that each half revolution of a rotating shaft corresponds to a certain amount of lost work. Now let R be a number of reversals of the direction of the applied stress f, such that $Rl' = L'$, or $Rw' = W'$; in other words, the accumulated elongations, each in itself of minute amount, have now reached the same total as corresponded to the initial breaking stress K. The capacity of the shaft to withstand work done on it is then exhausted, and it breaks when $R = \frac{W' \cdot E}{c'tf^2}$. If $Y =$ the number of years of continuous work, $R = Y \times 365 \times 24 \times 60 \times 2N$, the time of a reversal in hours being $\frac{1}{60 \times 2N}$ and if sufficient experimental data were to hand, the required *life* of the shaft could be calculated from an equation of the form

---

[1] Especially Herr Frederic Autenheimer. See Fischer-Hinnen, *Continuous-current Dynamos*, p. 254 ff., to whose suggestive treatment of the subject the authors are chiefly indebted for the present paragraph

$$Y = \frac{W' . E}{c' \times 365 \times 24 \times f^2}$$

the value of c' having reference to hours. It will be seen that the safe working stress must be lowered in proportion to the square root of the life which is expected from the shaft.

§ 11. **Diameter of armature shaft.**—In the absence of complete data, we must, however, fall back upon the value of f_t which is found by experience to give practical results, and for the more or less alternating load of a dynamo shaft; this ranges from 4500 to 7500 lbs. per square inch. Inserting the intermediate value of 6000 in equation (21) we have for the diameter of the shaft

$$d = \sqrt[3]{600 \frac{kw}{N}} = 8{\cdot}5 \sqrt[3]{\frac{kw}{N}} \text{ or } 0{\cdot}85 \sqrt[3]{\frac{w}{N}} \quad . \quad . \quad . \quad (22)$$

$$= 7{\cdot}25 \sqrt[3]{\frac{HP}{N}}.$$

By such equations the minimum diameter of the shaft at the centre of the armature or at any part beneath armature or commutator may be determined, and a life of some thirty years may be counted on. Both the hub of the armature and the sleeve of the commutator add to the rigidity of the shaft, and if, as is often the case, there is a gap between these two, the bending is largely concentrated thereat, so that it is of chief importance to maintain the diameter to its full value at this spot. For very small armatures, the shaft as given by the above general equations may not be sufficiently stiff, so that $\frac{3}{8}$ to $\frac{5}{8}$ in. must be added to the diameter. In every case any special circumstances of the design must be considered; thus in the case of dynamos with a very small air-gap or of large diameter with a comparatively small clearance, the magnetic pull due to inequalities in the strength of the several fields and any tendency to vibration must be specially guarded against by employing an exceptionally stiff shaft.

If a key-way is to be cut in the shaft for keying on the discs or their supporting hub, further allowance must be made; and in general to give stiffness at the centre, the shaft is usually there swelled out to a larger diameter than it has within the bearings. Any such alteration in the size of shaft requires, of course, to be effected with a fairly large radius in order to avoid opening of the fibres of the steel at the corner, where the change of diameter is made.

In the case of traction generators which are subjected to great and sudden changes of load causing large and rapid interchanges of energy between flywheel and armature, the actual dynamo shaft may be relieved of a large portion of the stress by prolonging the armature cast-iron hub to form a coupling by which it may be immediately bolted to the flywheel (fig. 276). The shaft has then merely to take the bending moment due to the weight of the armature at the outer bearing and the stress of driving is transmitted directly from the flywheel into the hub and its core-discs without passing through the key or keys by which the hub is fixed to the shaft.

Within the bearing nearest to the driving-point, the bending moment is less than at the centre of the armature, and the diameter of shaft may be correspondingly reduced. The possible reduction is, however, much greater in small than in large machines, since in the former the diameter at the centre is proportionately larger in order to give sufficient stiffness to the shaft. Thus in machines giving less than 10 watts per rev. per min. the diameter within the bearing may be only 75 per cent. of the smallest diameter within the armature, and this proportion will rise to, say, 90 per cent. in machines giving over 100 watts per rev. per min. The diameter of the shaft within the outer or commutator bearing may be still further reduced since the horse-power is absorbed within the armature and the twisting moment becomes negligible; in many cases, however, for convenience of manufacture, the same diameter of journal is retained throughout, even though within the one bearing there is a surplus of strength.

The above approximate figures require in every case to be checked by consideration of the strength of the journal. Thus in the case of the belt-driven dynamo of fig. 149 the journal next

to the pulley is subjected to a combination of a bending moment $3\text{T}.\,l_8$[1] and a twisting moment T_m.
The equivalent bending moment[2] is then

$$\text{B}_e = \frac{1}{2}(3\text{T}\,.\,l_8) + \frac{1}{2}\sqrt{(3\text{T}\,.\,l_8)^2 + (\text{T}_m)^2}.$$

Equating B_e to the moment of resistance of the circular shaft to bending, the minimum diameter of the shaft within the pulley bearing is given by the relation

$$\text{B}_e \leqq 0{\cdot}098 f_t (d')^3$$

where f_t as before may be taken as 6000 lbs. per square inch.

In the outer bearing furthest from the pulley, there is no twisting moment but the bending moment is the combined result of the weight of the armature and the pull of the belt. The load on the bearing is $\text{P}_2 = \sqrt{w_2^2 + t_2^2}$, and the bending moment[3] of this at the inside end of the shaft nearest to the armature is $\frac{1}{2}\text{P}_2 l'$. The diameter is then deduced from

$$\frac{1}{2}\text{P}_2 l' \leqq 0{\cdot}098 f_t (d')^3.$$

§ 12. **Length of journals.**—In order to ensure durability and coolness in working it is necessary in all high-speed machinery to

---

[1] The weight of the pulley itself is not here taken into account.

[2] More accurately if l' is the length of the journal, and if t_1 = the reaction of the bearing against the tension of the belt as before, the bending moment due to this reaction if distributed uniformly over the whole length of the journal $= t_1 . \frac{l'}{8}$, and this amount must be deducted from the bending moment due to the belt pull. The true bending moment at the centre of the journal neglecting the weight of the armature which may or may not act in the same plane is thus

$$\text{B}_m = 3\text{T}\,.\,l_3 - t_1\frac{l'}{8} = 3\text{T}\,.\,l_3 - 3\text{T}\,.\,\frac{l'}{8}\left(\frac{l_3 + l_4}{l_4}\right)$$
$$= 3\text{T}\left\{l_3 - \frac{l'}{8}\left(\frac{l_3 + l_4}{l_4}\right)\right\}$$

which must be substituted for $3\text{T}\,.\,l_3$ in the equation for the equivalent bending moment. Unwin, *Elements of Machine Design*, part I. § 125 (1902 edit.).

[3] Unwin, *Elements of Machine Design*, part I. § 117 (1902 edit.).

make the journals of considerable length as compared with their diameter, and in belt-driven armatures of small size running at 1000–1200 revolutions per minute, the ratio most frequently observed is $\frac{l'}{d'}=4$, where d' is the diameter of the journal. In engine driven armatures and in large machines generally, running at about 300 revolutions per minute or less, the proportionate length may be reduced, say, to 2½ diameters.

The increase of the ratio of length to diameter with higher speeds, such as is shown in the following table,

N revs. per min.	200	300	400	500	700	900	1100
$\frac{l'}{d'}$	2	2·5	3	3·25	3·5	3·75	4

really amounts to a reduction of the pressure per unit of bearing surface as the velocity of the relative movement between journal and bearing increases. The determination of the length of bearing has in fact to be decided by two considerations; on the one hand the intensity of the pressure per square inch of the projected area $d'l'$ of the bearing must not be so great as to squeeze out the lubricant and so cause the journal to seize; on the other hand the rate at which heat is generated per unit of bearing surface in the work of overcoming frictional resistance must not be greater than the rate at which it can be dissipated through the same surface by conduction without undue increase of temperature. The two conditions are closely connected, since the heating of the lubricant reduces its viscosity and renders it more liable to be squeezed out by the pressure, the film which separates the two surfaces becomes thinner, and, as soon as it is ruptured, the bearing seizes. The total load on the pulley bearing of fig. 149 is $P_1 = \sqrt[2]{w_1^2 + t_1^2}$, or if $\frac{l_1}{l_2} = \frac{4}{6}$ as before, and $\frac{l_3}{l_4} = \frac{1}{4}$, $P_1 = \sqrt{(W \times 0{\cdot}6)^2 + (3T \times 1{\cdot}25)^2}$; that on the other bearing is $P_2 = \sqrt{w_2^2 + t_2^2} = \sqrt{(W \times 0{\cdot}4)^2 + (3T + 0.25)^2}$, and the intensity of pressure in pounds per square inch is $p = \frac{P}{d'l'}$.

Since $T = 126{,}000 \frac{H \cdot P}{dN}$ pounds $= 200{,}000 \frac{kw}{dN}$ pounds where d is the diameter of the pulley in inches, or if V is the velocity of the belt in feet per minute since $T = \frac{33{,}000 \text{ HP}}{V} = \frac{52{,}000 kw}{V}$ pounds, the above equations may also be expressed as follows ;—

$$P_1 = \sqrt[2]{18{\cdot}6 \times 10^6 \left(\frac{kw}{N}\right)^{\frac{4}{3}} + 3{\cdot}8 \times 10^{10} \left(\frac{kw}{V}\right)^2}$$

$$P_2 = \sqrt[2]{8{\cdot}4 \times 10^6 \left(\frac{kw}{N}\right)^{\frac{4}{3}} + 15{\cdot}2 \times 10^8 \left(\frac{kw}{V}\right)^2}.$$

It will be found that $P_1 = 4T$ is a convenient approximation which holds very closely in all ordinary cases, the effect of the weight acting at right angles to T raising the total load from $3{\cdot}75T$ to $4T$.

In ring or drum machines, the armature core centres itself longitudinally within the pole pieces by reason of the magnetic pull to which it is subjected when displaced axially. A slight amount of end-play is not disadvantageous, since it secures a more uniform wear of the commutator surface, but it is seldom allowed to exceed $\frac{1}{16}$th of an inch to $\frac{1}{8}$th in large machines, an it is to be limited by raised shoulders on the shaft. In discoidal and disc armatures, end-play must be definitely avoided by the use, at one or other end, of a thrust or collar bearing, since otherwise there is danger of the armature striking the poles at the sides.

§ 13. **Friction of bearings.**—The horse-power that can be conveniently dissipated per square inch of bearing surface has been reckoned to range from about $h = 0{\cdot}085$ to $h = 0{\cdot}065$ according to various circumstances, chief among which is the amount of oil which is passed through the bearing and so assists in conducting away the heat. The rate at which heat is generated in the bearing is $\frac{\mu P \pi d' N}{12 \times 33{,}000} = \frac{\mu P d' N}{126{,}000}$ horse-power $= \frac{\mu P \cdot d' N}{169}$ watts, or per unit of projected surface is $\frac{\mu P d' N}{126{,}000 d' l'} = \frac{\mu}{126{,}000} \cdot \frac{PN}{l'}$ horse-power. Taking $h = 0{\cdot}07$ as the average specific rate of dissipation that can be counted on, and equating the two rates of dissipation and of generation,

$$l' = \frac{\mu}{126{,}000 \times 0{\cdot}07} \,.\, \text{PN} = \frac{\mu}{8800} \,.\, \text{PN inches.}$$

The length of the bearing should therefore depend upon the law which μ, the coefficient of friction, follows.

With a copious supply of oil well introduced between the surfaces, if the intensity of the pressure per square inch is not so great as to squeeze out the lubricant, μ, so far from being a constant as in the case of friction between solids, has been found[1] to vary nearly proportionally to the square root of the velocity and inversely as the pressure or $\mu = c \,.\, \frac{\sqrt{v}}{p}$, and further to diminish as the temperature is increased. Under these circumstances the frictional resistance $\text{R} = \mu\text{P} = c \,.\, \frac{\sqrt{v}}{p} \,.\, \text{P} = cd'l' \,.\, \sqrt{v}$, the value of c when v is in feet per min. being about 0·039. The resistance is, therefore, for a given diameter and length of bearing completely independent of the load; the work done and the heating are due to the shearing of the viscous liquid, and though the application of the load may alter the distribution of the pressure round the circumference of the journal within the bearing, the mean thickness of the lubricating film remains practically constant, and so also the work done in shearing it. The power lost in the bearing is then proportional to the bearing surface and to the 1·5th power of the velocity or $= 1{\cdot}6 \times 10^{-7}\, d'l'\, (d'\text{N})^{1{\cdot}5}$ horsepower $= 1{\cdot}2 \times 10^{-4}\, d'\, l'\, (d'\text{N})^{1{\cdot}5}$ watts. Thus a reduction in the diameter of the bearing has a greater effect in reducing the friction loss than a reduction in the length, but if d' has the minimum value required by considerations of strength, l' might be reduced until the intensity of the pressure approaches the limit when the lubricant is squeezed out, which would take place when $p =$ about 500 lbs. per square inch.

---

[1] See Unwin, *Elements of Machine Design*, part 1, Chap. VI., and G. Dettmar, "On Friction Losses in Dynamos," *E.T.Z.*, vol. xx. pp. 380 and 397. The latter gives a much higher value to c, whence the friction loss at 37·5° C. becomes $2{\cdot}1 \times 10^{-4}\, d'\, l'\, (d'\, \text{N})^{1{\cdot}5}$ watts, but this appears to give higher results than are obtained in practice with good lubrication.

The calculation of bearing surface on such a basis would, however, lead to excessive wear; hence, although the above expression may be used with some caution in estimating the friction loss of a given bearing in normal good working, and although it is true that the frictional resistance is certainly in many cases independent of the load on the bearing, it is obvious that in the design some allowance must be made for less favourable conditions, when either the oil is not so well introduced or the parallelism of the surfaces is slightly affected by deformation of the shaft due to increase of the speed or of the load. If the lubrication should for any reason become imperfect, the condition of rubbing between solids is more nearly approached and μ becomes practically a constant. On this assumption then $\frac{\mu}{8800} = \frac{1}{\beta}$ is a constant, $l' = \frac{PN}{\beta}$, and the limiting intensity of pressure must be reduced as the speed is increased. If $\mu = 0{\cdot}05$, $\beta = 176{,}000$, or if the lubrication is less perfect and $\mu = 0{\cdot}08$, $\beta = 110{,}000$, whence

$$l' \geqq \frac{PN}{176{,}000} \text{ to } \frac{PN}{110{,}000} \text{ inches,}$$

and l' may accordingly be taken on the average as $= \frac{PN}{145{,}000}$. It may finally be remarked that the exact values of two of the factors which go to make up β, viz., μ and h are largely uncertain, the determination of h in practical instances being usually only made by assuming a certain value of μ which may have been far from the truth. Such values of β however give results approximating to ordinary practice, and in them reasonable durability under the influence of wear may be regarded as having been taken into account as well as the simple heating of the bearing.

§ 14. **Pedestals and plummer-blocks.**—The pedestals which support the journals either form part of the base-plate casting or are separate plummer-blocks (fig. 152). They are usually fitted with a gun-metal 'brass' or bush lined with white-metal or other anti-friction alloy; or the 'brass' is itself a cast-iron shell, into which the white-metal is run. The bush or liner is prevented from turning either by snugs which engage in corresponding recesses in the pedestal or by a screw in the upper part of the

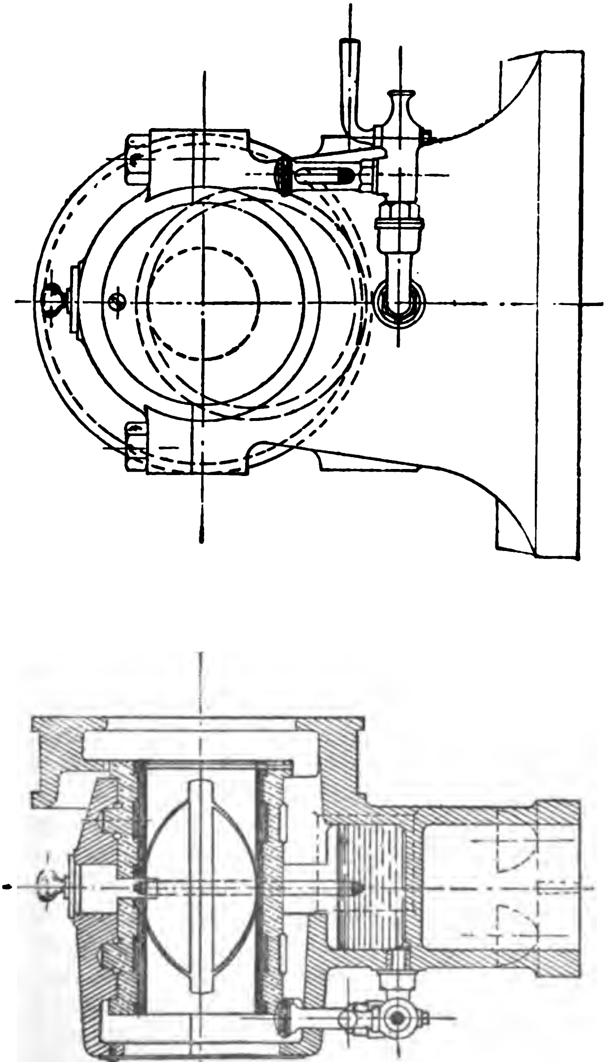


FIG. 152.—Self-oiling bearing for commutator end.

pedestal. The bush may be solid when it can be slipped over the end of the shaft, but in other cases must be split; the pedestal is then provided with a separate cap, and the arrangement has the advantage that the two halves can be closed together to take up wear, while the solid bush when appreciably worn requires to be entirely re-lined with white metal. The division of the cap from the pedestal should be approximately at right angles to the line of maximum resultant pressure upon the bearing; the joint should therefore in large dynamos driven by horizontal belting or by a horizontal steam-engine be inclined at about 45° to the vertical.

In order to facilitate the self-alignment of the bearing on the shaft, the sleeve or bush in the commutator bearing of the directly-driven dynamo is frequently so made as to allow of a certain amount of *swivelling* movement. The outside of the bush at the middle of its length forms part of a sphere and is given a correspondingly spherical seating (fig. 153), or if the pressure be small, the seating may be cylindrical when the bush may be slipped in from the end within a solid pedestal.

At each end of the bearing are hooded chambers in which the oil which exudes from the ends of the journal is caught and returned by grooves cut at the bottom of the bearing into a reservoir. At the end nearest to the armature the shaft must be provided with an *oil-thrower* to prevent the oil from creeping along the shaft on to the armature or commutator and there destroying the insulation. Either an upstanding annular ridge is formed on the shaft when it is turned, or a collar with a thin pointed edge of larger diameter is shrunk on to the shaft (figs. 152 and 153 and 186); in either case the oil is drawn to the outer edge of the ring and thence is flung off by centrifugal force and caught in the hollowed chamber within the bearing. The curvature of the hood must be such as to cause the oil to be returned on the bearing side rather than on the armature side whence it may drip on to the shaft past the oil-thrower.

In small machines the white metal is sometimes run directly into the cast-iron pedestal, an oiled mandril or the armature shaft itself being previously inserted and held centrally in place so that the metal may run round it. The whole is heated up, and passages are left to allow the heated air to be expelled. As the

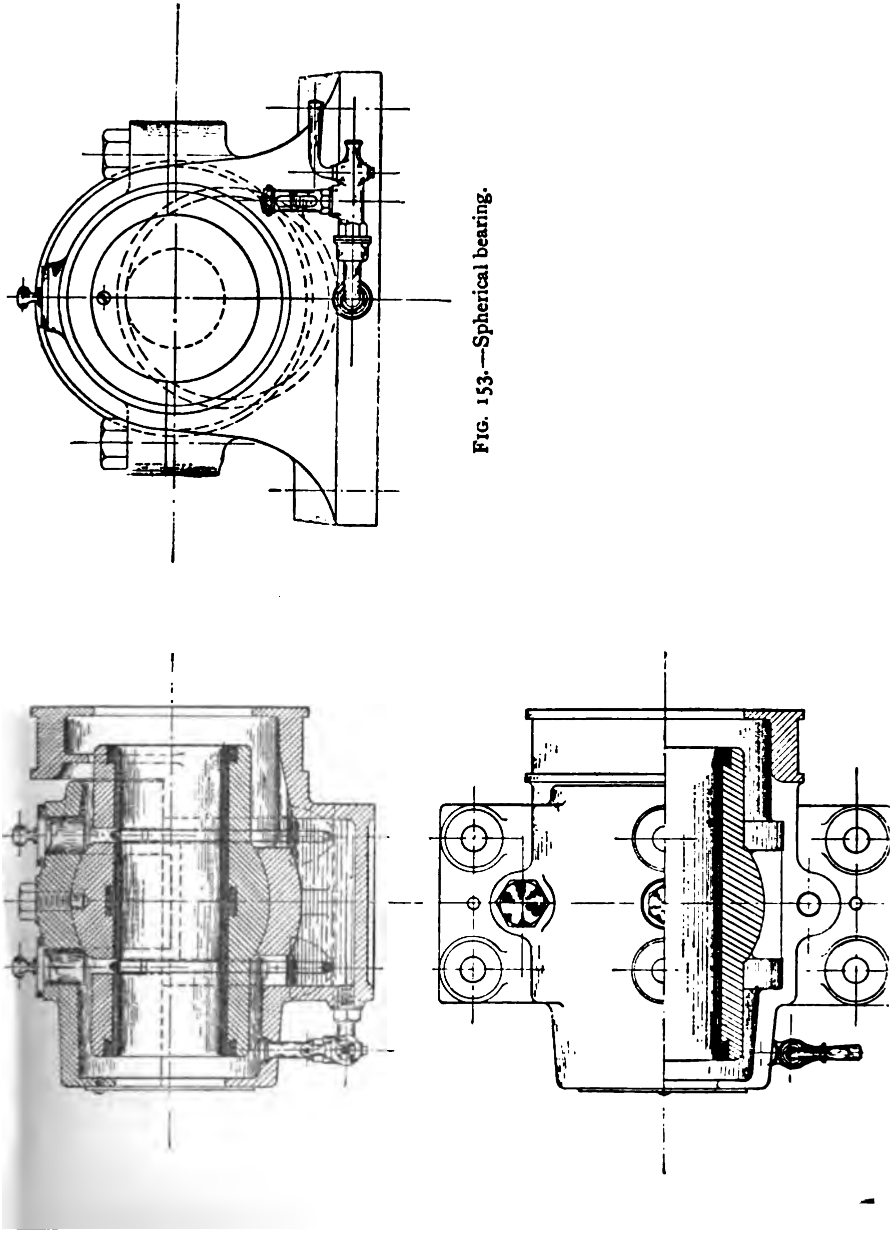


FIG. 153.—Spherical bearing.

metal cools, the mandril is turned round in guides by hand, and a hard smooth surface is obtained which does not require to be subsequently machined.

§ 15. **Lubrication of bearings.**—Lubrication is usually effected in dynamo bearings by means of brass or cast white-metal rings (figs. 152 and 153) which rest on the journal and dip into an oil reservoir formed in the hollow pedestal. Their section is slightly tapered, being broader at the base; when the shaft rotates they are carried slowly round by the friction of their contact with it, so that the journal becomes self-lubricating as soon as the armature rotates. In the upper half of the bush or liner are as many gaps as there are rings, while the bottom half remains whole. The oil is drawn up by the motion of the ring and at the top of the journal enters a recess cut longitudinally along the brass, whence it is distributed by grooves cut spirally along the inner surface. All such grooves and recesses must be sealed at their further ends so as to confine the oil, and it is important that the oil should be introduced at the point of least pressure so that it may be swept forwards by the rotating shaft; hence the grooves should follow the direction of rotation, and if the pressure between journal and bearing should prevent the entry of the oil on this side, the grooves must be made to feed a second recess at some point where the pressure is relieved. The diameter of the rings should be about one and a half times that of the journal and large enough to pass over the ends of the brass. On bearings exceeding 8 inches in length two or more rings may be employed. Chains are also sometimes employed in place of rings, but although they lift more oil, they are not so suitable, since they necessitate more of the brass being cut away and at high speeds cause frothing of the oil owing to the admixture of air. The oil should not stand so high in the reservoir as to clog the free movement of the rings, and its height is usually indicated by an oil gauge with which may be combined a draining tap for use when at intervals the oil has to be changed. The joint of the split bearing should be carefully made so as to prevent the oil working its way through it to the outside. In large machines the bearings are sometimes arranged with a water jacket through which a stream of water can be passed in the emergency of a hot bearing.

§ 16. **Insulation of armature core.**—Returning to the process of constructing an ordinary ring or drum armature, let us assume that the discs have now been built up into a cylindrical core. If the surface is to be smooth, the next step consists in turning it true in a lathe. Unless carefully done with a sharp and fine-nosed tool, this operation is apt to burr over the edges of the discs: they are thus brought into contact on the surface, and the advantage of lamination is largely nullified, since eddy-currents can flow in sheets from end to end of the core. The amount of turning should therefore be only just sufficient to secure a sensibly smooth and true surface, any parts which are not quite touched by the tool serving to break up the paths of the eddy-currents.

When the armature core is completed, and all rough edges have been filed smooth, the surface is varnished with shellac, and carefully *insulated* all over with alternate layers of stout paper and tape or calico varnished and ironed on to the core. Thus for pressures up to 200 volts, the surface of the core may be insulated as follows:—one layer of 2-ply insulating paper (such as rope or bond paper treated with linseed oil, or that known as "P. and B.") . . . . =0·010″
one layer of overlapping tape or calico or oiled linen . =0·020
second layer of paper . . . . . . =0·010

radial depth = 0·040″

For 500 volts, press-spahn or micanite sheeting should be employed; *e.g.*,

one layer of micanite sheet $\frac{1}{64}$″ thick = . . 0·016″
two layers of 2-ply Willesden paper, . 0·020 × 2 = 0·040

radial depth = 0·056″

The insulation thus rises in thickness up to 0·08″ or 0·1″ for 2000 volts with several layers of alternate micanite and paper.

The artificial forms of mica known as micanite and megomit are made of small and thin laminæ of mica reassembled into a sheet and cemented together with an insulating gum under great pressure and high temperature. Owing to their greater homogeneity their insulation resistance is more uniform than that of pure mica, and the voltage required to pierce a given sheet may be taken as 1000 for every thousandth of an inch in thickness. The specific resistance of organic materials such as paper, linen, or vulcanised fibre, falls considerably with a rise of temperature, but that of composite mica products is but little affected, and their heating under a rapidly alternating potential is but small. In order to apply the micanite or megomit to the armature core, it is heated to about 200° F. when it becomes soft and pliable, and can be easily moulded on to curved surfaces. Flexible micanite is also made, and lastly micanite cloth which has a backing of fine muslin or linen, or micanite paper in which the backing is of paper; the piercing voltage of the two latter is somewhat less than in the case of the sheet, and depends to a certain extent on their absolute thickness.

At the ends of the core, rings of paper, fibre, press-spahn or in high-tension machines of micanite are employed, and the insulation can be made thicker than on the cylindrical surface since space is not here so valuable. All joints or seams in the insulating covering require particular attention so that there may be no likelihood of the winding making contact with the metal core. The whole is then finally varnished and dried in a stove in order to rid it of all moisture preparatory to winding. The interior of ring armatures is similarly insulated, especial care being taken to cover with press-spahn or micanite protecting-pieces the driving spokes or arms against which the internal wires are tightly pressed.

§ 17. **Driving of the armature inductors against the magnetic pull.**—Of the total power absorbed by a dynamo when in full work, fully nine-tenths are expended in overcoming the drag which the inductors experience as they move through the magnetic field. Since it is in the inductors that the actual conversion of the mechanical energy delivered to the shaft into electrical energy (whether useful or not) takes place, it is evidently of the utmost importance that the inductors should be so firmly supported on the armature that they are positively and effectively driven through the magnetic field, without any possibility of their slipping round the surface of the core.

The driving-power is in the first instance transmitted from the shaft to the hub and discs of which the armature is composed, and it is therefore evident that the armature-core must be securely keyed to the shaft. The transmission of the power to the inductors from the core when the surface of the latter is smooth is, however, a more difficult matter. To a very large extent this is effected by simple friction; by means of the binding-wires the inductors are pressed down so tightly on to the surface of the armature that any slip of them relatively to the core would be opposed by a very large amount of friction. Again, in small armatures wound tightly with wire, the grip of the wires themselves materially assists in withstanding the pull on them, especially if they are ring-wound, since then the turns, threaded through the interior of the ring, are held in place by the arms of the hub.

But with larger drum-wound armatures, it becomes necessary to reinforce the influence of friction by some more positive means of driving, and various ways have been devised for effecting this. In many cases comparatively thick discs (say ⅛" thick) are inserted at intervals among the thinner discs of the core, and on the periphery of these are driving teeth which project up through the winding: several of these teeth are arranged in parallel line with insulation on either side of the row, or if they are few and dispersed over the surface of the core they are insulated separately with vulcanised fibre; in either case the inductors bed against them, and are thus positively driven through the field. (One such disc is shown at the centre of the ring armature of fig. 144.) In other cases small *drivers* of delta metal or other metal of high electrical resistance are let into the surface of the core; or shallow grooves are milled longitudinally along the core, and into these are driven hard-wood strips (fig. 154) which stand up level with the inductors and serve as drivers. Horn-beam cut across the grain and driven into the grooves with its grain radial to the core is especially suitable for this purpose owing to its great strength; it is further more durable than fibre. The drivers should be equally spaced round the core, the inductors being divisible by their number, and they should be a multiple of the number of arms in the hubs of ring armatures. Their

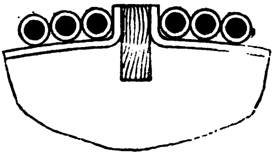


FIG. 154.—Hard-wood driving-strip.

width must be small, so as not to interfere with the symmetry of the spacing of the coils more than is necessary; otherwise they may cause sparking at the commutator, owing to the displacement of the coils relatively to the commutator sectors.

The magnetic pull to which each inductor is subjected, being at any moment $57\ B_g\ L'' c \times 10^{-8}$ pounds (eq. 8), is evidently dependent on the shape of the curve of B_g. As will be shown in Chapter XVII, the distribution of the flux in the continuous-current dynamo when at work is not uniform even under the pole-face, and the instantaneous pull on any inductor may at full load be as much as 20 per cent. greater than the average value. From equation (10) the total tangential pull on all the inductors is $\frac{p\tau \cdot cZ_a}{4{\cdot}26 \times 10^8 \times r}$ pounds, where r is the radius of the armature in feet. The average value of the air-gap induction on the approximate assumption that the lines are entirely confined to the pole-face area is $B_g = \frac{Z_a}{rL\phi}$; hence, if as usual ϕ is a constant fraction of $\frac{\pi}{p}$, the total pull on the inductors is $\propto \tau . c . B_g L.''$ The product of the total number of inductors into the current carried by each one is conveniently expressed as the "ampère-wires" (A_w) of the armature. Hence, if n be the number of driving strips extending along the length of the core, the average force acting per inch length on any one is $\propto \frac{A_w \cdot B_g}{n}$. For horn-beam strips $\frac{1}{8}''$ thick, the permissible value of the above expression may be taken as 7×10^6, and for strips $\frac{3}{16}''$ thick as 10×10^6.

In high-tension machines, the insulation must be arranged in troughs between the driving-strips to prevent any possibility of the potential causing a short circuit between the wires and the core at the edges of the slots into which the driving-strips are sunk.

§ 18. **Toothed armature-cores.**—A perfect system of driving is obtained by the use of *toothed* or *notched armatures*, somewhat similar to the form originally used by Pacinotti. All the core-discs of such an armature have a number of projecting teeth stamped out round the periphery (fig. 155; cp. fig. 140, where a few such teeth are shown in the end view, and fig. 162), so that when put together the surface of the armature is divided into a series of ridges and deep grooves running from one end to the other. The grooves are carefully lined with press-spahn (say 0·020″ thick), or flexible micanite, or moulded micanite troughs may be used for high pressures. Micanite or megomit plate is readily moulded in place into open slots; a strip of the required dimension is heated on a plate by a gas jet, laid in the slot and pressed down tightly into its corners by a wood or iron bar having the exact internal shape of the slot when insulated: a thickness of 0·020″ a side suffices for pressures up to 500 volts (fig. 156). Not only are the inductors well protected within the slots and securely driven, but further the force to which they are subjected is very largely reduced. The flux of

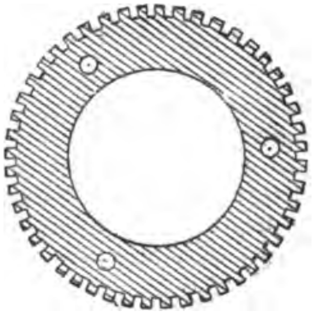


FIG. 155.—Toothed core-disc.

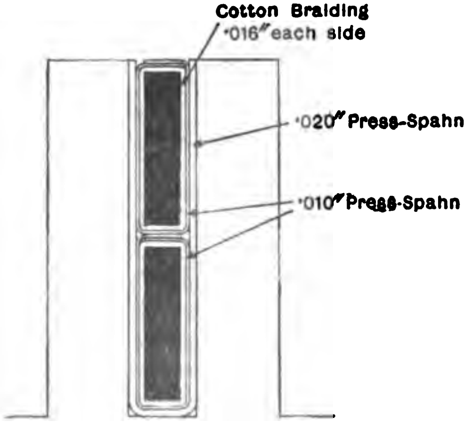


FIG. 156.—Insulated slot of toothed armature for 250 volts.

the field after passing through the air-gap proper divides between the alternative paths offered by the teeth and the slots in proportion to their permeances since the two paths are in parallel; the relative densities within tooth and slot are therefore $\frac{B_t}{B_s} = \mu$, where μ is the permeability of the iron teeth for the induction B_t within them. Since the permeability of good iron stampings even at as high a flux-density as 20,000 lines per square centimetre is to that of air as 66 : 1, it is evident that if the width of the teeth is comparable with that of the slots, almost all the lines will follow the iron path. The inductors in the slots are therefore situated at any moment in a very weak field, and the force acting upon them for a given armature current is only proportional to this weak field. Yet the total drag on the armature as a whole remains the same as if the inductors were wound on a smooth core, the magnetic pull being in fact transferred to the iron of the armature teeth. The explanation of this is to be found in the distribution of the lines over the surface of the armature core when current flows through the inductors. Instead of being uniformly distributed over the face of each tooth, they become denser on the trailing side than on the leading side (fig. 157; by the 'leading side' is indicated that side which during rotation first enters or leaves the gap between two poles, the 'trailing' side being that which is the last to enter or leave the interpolar gap) and the difference between the densities of the two edges

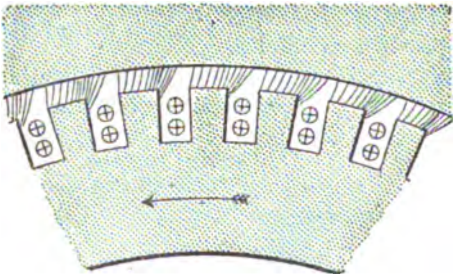


FIG. 157 —Distribution of flux in toothed armature carrying current.

increases as the current is increased. The lines tend to shorten and straighten themselves, so that the result of the unsymmetrical distribution is to produce on the armature core a magnetic drag dependent upon the amount of current that is flowing in the inductors. Thus the inductors themselves are relieved of the driving stress to an extent dependent upon the degree to which the teeth are saturated. The exact relation which the reduced stress on the inductors bears to that which they would experience on a smooth core with the same total flux depends on the average density in the air-gap and the relative proportions of the widths of tooth and slot. If $B_g = 6900$, and the width of the slot is about twice that of the tooth, the above-mentioned relative densities would be obtained, viz., $\frac{B_t}{B_s} = \frac{20,000}{300} = 66$; the stress on the inductors is then $\frac{B_s}{B_g} = \frac{300}{6900} = \frac{1}{23}$rd. of that to which they would be subjected on a smooth core with an equal average density, and $\frac{22}{23}$rds. of the total drag is transferred to the iron. On this account the toothed armature core is now very frequently employed, especially in cases where the driving stresses are large and rapidly varying as in dynamos for traction or motor work.

The shape of the slot permits of many variations, but if it is open at the top, the width of opening usually does not exceed twice the length of the single air-gap.[1] Otherwise the unequal distribution of the flux caused by the alternating slots and teeth becomes extended in a marked degree to the bored face of the pole-piece (cp. fig. 157); the passage of the lines as they sweep over the pole-faces will then set up eddy-currents in the solid mass of the pole, and the loss of energy and heating due thereto may be so great as to necessitate the lamination of the pole-shoes.

The liability to eddy-currents in solid poles is largely reduced by employing half-closed slots (fig. 158) and is entirely obviated by *tunnel* armatures, in which a number of holes are stamped in the discs close to the periphery and the inductors are threaded through these holes after they have been lined with tubes of micanite. Such half-closed slots or tunnels are, however, attended with the disadvantage that the inductance of the bars or wires is much increased, and the difficulty of commutation and of sparkless collection of the current is correspondingly greater; hence in practice their possible use is very limited.

---

[1] See Chap. XIX, § 27.

§ 19. **Insulation of armature wires.**—Armatures may be divided into the two classes of *wire wound* and *bar wound.* In either case soft copper is employed as the material of the inductors and connectors, and this is now commercially obtained with a conductivity equal to that of the pure copper of Matthiessen's standard. So long as the current to be carried by each inductor does not exceed about 50 ampères, solid wires of circular section can be used; their diameter will not exceed 0·150", and they can be readily bent or shaped. *Double cotton covering* adds 12 mils to the diameter of a wire measuring 0·060" or less, and 15 mils to the diameter for wires above that size, the cotton thread employed being slightly thicker on the larger wires. It is, however, only suitable for ring armatures and low voltages and on small wires, since it is apt to open out when the wire is bent. A *braided cotton covering* is therefore more usual, which with fine cotton may be taken as adding 13 mils on a diameter of 0·060" or less, 15 mils to a wire 0·100" in diameter, and 17 mils if the bare diameter be 0·160". For voltages above

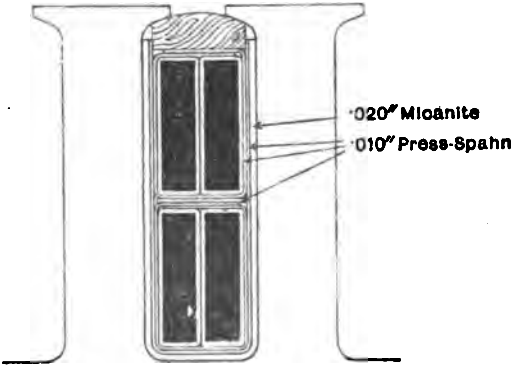


FIG. 158.—Half-closed slot insulated for 500 volts.

200, a combination of single or double cotton covering with an external fine braiding may be employed, the whole measuring 0·025" on the diameter. For currents above 50 ampères, the required sectional area of copper becomes so great that it is difficult to bend it, and the loops in general require to be divided into two or more portions which are afterwards soldered together, and must in any case be shaped prior to their being assembled on the armature core. Moreover economy of the valuable space round the external circumference of the armature dictates the use of a rectangular or trapezoidal section, so shaped that the inductors lying close by the side of each other leave little or no space wasted by interstices between them. We thus pass to 'bar-winding,' in which the inductors instead of being round wires are bars of rectangular cross-section. Double cotton covering must now be taken as adding 0·020" to their dimensions, braiding as adding 0·025," and a combination covering as adding 0·032". A combination of braiding with an internal spiral of thin paper next to the copper is suitable for 500 volts and measures 0·035" on the thickness of the bar, or 0·045" after shellacing which in every case causes the insulation to become somewhat thicker. If their shape permits it, the bars may be braided subsequently to bending; or in default, they must be wrapped round by hand or in a taping machine with a covering of tape or paper or micanite paper. In all cases the insulated wires or bars are afterwards varnished or soaked with shellac, and dried in the vacuum chamber or stove.

§ 20. **Lamination of armature inductors to avoid eddy-currents.**—Owing to the fact that any inductor must necessarily possess a certain width in the direction of rotation, its leading edge enters or leaves the field at the polar tips sooner than its rear edge, and in general, when moving through a field of varying density, one edge is cutting more lines of induction than the other. In consequence, when an inductor on a smooth-surface armature is either approaching or receding from the approximately uniform field under the centre of a pole-piece, a greater E.M.F. is set up along one edge than along the other, and if the inductor be solid, the unbalanced difference of these E.M.F.'s will cause a local eddy-current to pass up and down along the two edges and round at the ends, as shown in fig. 18. This production of eddy-currents in a solid inductor is naturally not very marked in the case of round wires, but if the inductors of any armature, whether ring or drum, continuous or alternating, are solid and of rectangular section, and have considerable width, the difference in the E.M.F.'s of the two edges of a bar will be appreciable and may lead to disastrous heating. Thus in a bipolar machine on open external circuit, but with excited field, twice in each revolution, an eddy-current will circulate round each inductor-bar as it passes through the fringe of lines within an interpolar gap where the density varies considerably, and although the eddy-current E.M.F. may be comparatively small, yet since it is acting round a circuit of very low resistance a large current may for a short time be generated: not only is mechanical energy thereby absorbed, but the temperature to which the bar is raised may become so high as to damage the soldered joints at its ends. If considerable current be taken out of the armature, no actual eddy-current will be produced, but the distribution of the current over the sectional area of the copper is no longer uniform; the proportion of the current carried by each of the thin laminæ of which the bar may be regarded as composed will be such that the terminal voltage of all is the same. The same distribution of the current may equally well be derived by supposing a local eddy-current to be superposed on the useful armature current, this latter being itself regarded as uniformly distributed. The additional expenditure of watts over the resistance of the bar which must result from any divergence from a uniform current-density is therefore called the *eddy-current loss*. The value of the eddy-current E.M.F. for a given width of bar will vary directly with the peripheral speed and with the rate at which the density of field changes in the neighbourhood of the polar tips, while the strength of the eddy-current will itself vary directly as the E.M.F. Hence the eddy-current loss will be proportional to the square of the speed. With high speeds and strong fields abruptly entered, it will readily be understood that the eddy-currents generated in wide bars may seriously impair the efficiency of the machine, even if the whole armature-winding does not become overheated. Something may be done to neutralise this source of loss by so shaping the pole-pieces that the edge of the field is not parallel to the length of the inductor, but is inclined to it: the whole of the leading edge of the inductor does not then become immersed in the field before the rear edge. Or the entry into and departure from the field may be rendered more gradual if the bore of the pole-pieces be slightly elliptical, so that the length of air-gap is increased slightly at the pole-tips and the field is gradually shaded off. In spite of such devices with an average peripheral speed of no more than 1500 feet per minute, and a fairly strong field, solid copper bars become inadmissible on a smooth armature core if their width much exceeds 0·150″. A satisfactory solution of the problem is, however, obtained if each bar be divided into a number of thin laminations, lightly insulated from each other, and be then twisted at its centre through an angle of 180° (fig. 159); although the laminations are at either end joined together, their position relatively to the direction of rotation is thus transposed, the lamination which forms the leading edge along one half of the bar's length becoming the trailing edge along the other half of its length. The E.M.F. that is set up along the entire length of one lamination is then equal to that of any other, since if

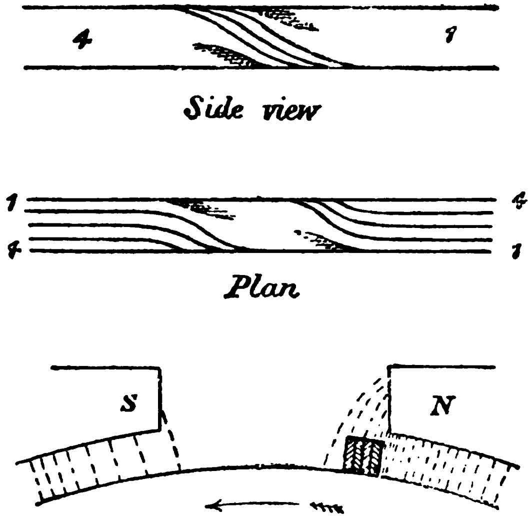


FIG. 159.—Laminated bar twisted at its centre to eliminate eddy currents.

one half of it is moving through a denser field, the other half is moving through a weaker field.[1]

In order to carry out this equalisation of the E.M.F. still more thoroughly, it is now customary to use bars of *stranded copper wire*; the separate strands of about ·036″ diam. are varnished or oxidised, or otherwise lightly insulated from each other, so that little or no contact is made along their length, and the cable is then moderately compressed into a bar of rectangular or trapezoidal section. By reason of the stranding, each separate wire passes from the one edge to the other, at least once midway along the length of the bar; the E.M.F. of the two sides is averaged, and the formation of eddy-currents is largely prevented.

The area of copper in the stranded bar is about 85 per cent. of the gross cross-section; further, owing to the lay of the strands, their length is slightly increased as compared with the length of the bar, so that in calculating the resistance of the bar its equivalent area may be taken as 84 per cent. of the area of a solid bar of the same over-all length.

§ 21. **Solid bars permissible with toothed armatures.**—A chief advantage of the slotted armature is now found to lie in the fact that it allows of the use of solid bars of much greater width than are permissible on the smooth-surface core. Since by far the greater proportion of the flux passes through the teeth, the density of the lines within the slots is but a small fraction of the average density in the air-gap of a similar armature with smooth surface; the possible range of density within any one slot is therefore very greatly reduced. Closely connected is the additional fact that the lines snap across the inductors in the slots with a velocity far exceeding that of the rotating armature. If B_g = the average flux-density in the air-gap, and B_s = the density within the slots, the product of the lesser density in the slots and the increased velocity of the relative movement v_s must be equal to the product of B_g and the speed of the armature v, in order that the E.M.F. may be the same as in the equivalent smooth-core armature; or $B_s \, . \, v_s = B_g \, . \, v$ and $v_s = \frac{B_g}{B_s} \, . \, v$. Thus in the case before taken $\frac{B_g}{B_s} = \frac{6900}{300}$ and the relative velocity of cutting is about twenty three times the speed of the armature, corresponding with the fact that the force on the bars is only $\frac{1}{23}$ of that which would exist if with the same flux the surface were smooth. The lines therefore move across the slot with such rapidity and cut the whole section of the inductor so nearly instantaneously,[1] that although the density in the slot must gradually decrease from B_s to zero as it approaches the neutral line between the poles, and increase as it approaches a pole, yet at any one moment the field within the slot is practically uniform and the change is made by a gradual decline or growth of the uniform density over the slot as a whole.

---

[1] Several methods of carrying out this same principle are illustrated in the original patent of Crompton, No. 12,880. 1886.

§ 22. **Ring armature winding.**—Passing to the operation of winding, let us first consider the wire-wound ring armature. Each section of its winding is wound on separately, the ends being left temporarily open and subsequently connected up when the commutator is put on. The double-cotton-covered or braided wire is cut off into lengths sufficient for an entire section of, say, two or three turns, and after being lightly varnished with shellac is allowed to dry. The winding of a section consists in passing one end of the wire through the interior of the ring, bending it round and over the core and drawing it tight from either end, so as to form a complete loop, the process being again repeated to form a second or third loop. In moderate-potential machines, giving 150 volts or less, there are seldom more than two, and usually there is but one, layer of wire on the outside; owing, however, to the smaller diameter of the central opening, and the loss of space by reason of the radial spokes, it is necessary to dispose the winding in at least two, and sometimes three or more, layers on the inside of the ring. The later-wound loops of a section are then made to ride over the first-wound, so that any section can be unwound and replaced without disturbing the rest of the winding. In order to secure an even and symmetrical winding, the ends of the core should at the outset be divided off by radial marks into the required number of compartments, so that no section may exceed its allotted space.

---

[1] See a paper on "Magnetism in its relation to Induced E.M.F. and Current" by Elihu Thomson, reprinted in the *Electrical Engineer*, June 21, 1889.

In former days, bi-polar bar-wound rings were commonly made for large outputs at moderate pressures; each U-shaped loop was formed separately on a mould, and was then sprung over the core. In some cases each loop was divided into an inductive and a connecting portion, and these two were of different section, the bars on the surface being rectangular and the connectors trapezoidal. The wedge-shaped section of the latter enabled them to be conveniently packed into the interior of the ring in a single layer, but the multiplicity of riveted and soldered joints was a serious drawback to the method.

At the present time, however, in continuous-current machines, ring armatures are seldom employed except for high pressures of 1000 volts or more. In such cases the absence of any high difference of potential between adjacent coils rendering them easy of insulation may outweigh other disadvantages. Chief among these are the greater inductance of the ring loop as contrasted with the drum loop, and the presence of the gun-metal arms of the hub in close proximity to certain of the coils which interferes with their perfect symmetry and affects their inductance. For both reasons the ring machine is more liable to sparking at the commutator; not only is the ring coil in itself more difficult to commute, but there is necessarily more dyssymmetry between the several coils than in the equivalent drum.

§ 23. **Hand-wound drum armatures.**—The wire-wound and bar-wound classes of drum armatures may each be divided into two sub-classes. In the former class, the coils may be either 'hand-wound' or 'former-wound.' If the output be small, the drum may be *hand-wound* in sections of, say, three or four complete loops, each section being an unbroken length of round wire wound along the surface and over the ends of the core without any joint. Where the turns cross one another at the ends they require to be neatly and systematically disposed, so as to form a compact 'head' round the shaft, and in order to prevent any slipping they should be interlaced with tape. If the armature core be smooth and the winding is to be disposed in one layer only, intervening spaces must be left between each successive coil of the first half of the winding, so that the remaining coils forming the second half of the winding may be wound into the spaces so left vacant. Further, in order that there may be no position in which one circuit through the armature contains coils whose end-connections lie closer to the core than those of the coils forming the other circuit—in other words, in order to equalise the length and resistance of the two circuits which are in parallel as much as possible,—the core of, *e.g.*, a bipolar machine should be turned through an angle of 180° after the first coil has been wound, and the coil belonging to a diametrically opposite commutator sector should be wound side by side with the first coil, but with its end-connections passing round the shaft on the opposite side. The core is then again turned round and another coil wound, the beginning of which would be afterwards joined to the same commutator sector as the end of the first. Thus in fig. 106 coils 1, 1′ and 5, 5′ would be wound first, and then coil 2, 2′, and so on until the whole armature surface is covered with one uniform layer of winding. Or after winding coils 1, 1′ and 5, 5′ instead of winding 2, 2′ a pair of coils approximately at right angles may be wound, viz. 3, 3′ and 7, 7′. Thus the equality of the two circuits at any moment is even better obtained, but the crossings of coils at different potentials is somewhat increased. The total number of loops to be wound may, however, be so large that with wire of a suitable diameter they will require to be disposed in two or more layers. When this is the case the simplest plan would be to entirely cover the surface of the core with half of the total number of coils, and then to wind the second half in a second layer on the top of the first. This method is, however, open to the great objections that the coils of the second half are longer and have more resistance than those of the first half, and further, that in one position one circuit through the armature consists of all the coils of the upper layer and the other of all the coils of the lower layer. If half the number of coils is an uneven number and in relation to the order in which they will afterwards be connected to the commutator every alternate coil is wound in the lower layer, and the intermediate ones wound in the upper layer, the two armature circuits will always have the same length and resistance, but the coils will themselves still be of unequal length. Two courses are open by which this evil may be minimised or obviated; either each coil must be built up in two or more layers, so as to exactly reproduce the arrangement above described for a single layer as is shown in fig. 160; or each complete coil must be split into two equal portions, and the second half wound on the top of the first half of the diametrically opposite coil, the two halves of one coil being taken round the shaft on opposite sides. Fig. 161 shows such a *split-coil* armature, one half of each coil being wound first, and the second half completed when the first half of the neighbouring coil has been wound; the

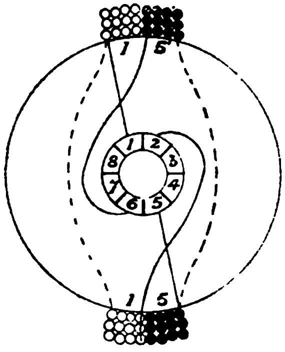


FIG. 160.

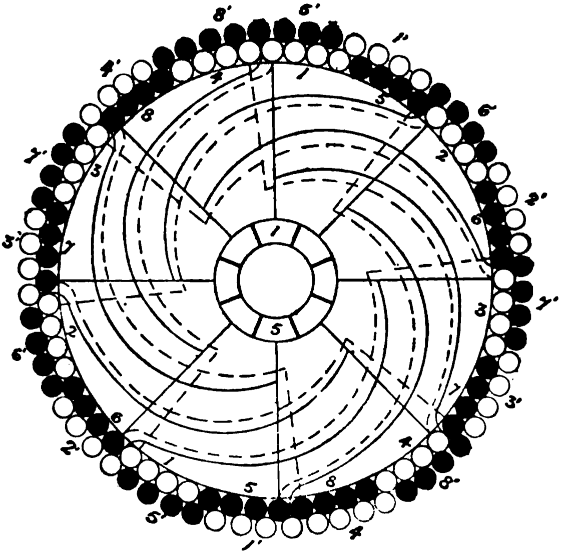


FIG. 161.—Split-coil drum armature.

beginning and end connections of each half are alone shown, the intermediate connections which are easily supplied being omitted for the sake of clearness.

While the above methods are equally suitable to slotted armatures, and are of easier application owing to the wires being confined in their proper places between the teeth, the slotted armature permits of still another kind of hand-winding which gives great regularity and equality between the coils. The wires are laid in the slots and thence brought down to a wooden ring on the shaft, being at the same time twisted through one half of the pitch; they are there tied down by a string band. If the number of wires in the bottom layer is too great for space to be found for them towards the centre, a certain number of equally spaced wires are first treated, and are then covered with a circular ring of insulating paper or press-spahn; a second batch is next taken, and so on until all the lower wires have been taken through half the pitch and lie extended along the shaft. An insulating ring of paper is again passed over them and the last layer of wires is brought back again to the armature circumference, at the same time being twisted through the remaining half of their pitch. The other layers of wires are similarly treated in reverse order, a layer of insulation being inserted between every batch. The end-connections at either end of the core are thus formed, and the several coils are now practically of equal length and no great difference of potential will occur between any neighbouring end-connectors in one layer, so that the likelihood of a breakdown in the insulation is much reduced.

§. 24. **Former-wound drum armatures.**—In the first-described hand-wound armatures where the loops overlap each other as they pass round the shaft at either end, the difference of potential at the crossing-points, since it amounts to the full E.M.F. of the machine, is apt to destroy the intervening insulation and lead to short-circuits; further, the repair of any one loop almost invariably necessitates the complete unwinding and rewinding of the armature. Owing to these objections hand-winding has largely been replaced by *former-wound* coils. These are shaped on 'formers' prior to being assembled on the armature core; they are therefore perfectly symmetrical and interchangeable, and have the further advantages that they can be well insulated by wrappings of paper or tape, can be tested before they are placed in position on the core, and are themselves inexpensive to wind in the first instance. Although former-wound coils may be used on both smooth and slotted armatures, they are especially suited to the latter with four or more poles, and chiefly on this account the multipolar form of machine now finds favour even for comparatively small outputs. Fig. 162 shows a small toothed drum armature, its slots insulated with press-spahn troughs, and ready for winding either by hand, or with coils formed on one of the systems to be now described. In the toothed armature, one inducing side of the coil forms the lower layer in the slot, and the second inducing side forms the upper layer in another slot separated by the correct pitch. The guiding principles on which the formed coils are then shaped are as follows; either the two halves of a complete coil are of different width, or the coil is lozenge-shaped with its centre wider than the two parallel and straight inducing sides; in either case one side of a coil being of smaller width can be passed through the wider sides or centres of other coils. If N_2 be the total number of coils, the first $\frac{N_2}{2p}$ coils are placed in succession on the core, one side of each being pressed down into position at the bottom of the slots and the other side being temporarily and loosely held in place. The winding now proceeds, both sides of each coil being fixed in their final position, one in the lower layer and the other in the upper layer, until there only remain $\frac{N_2}{2p}$ coils to be wound.

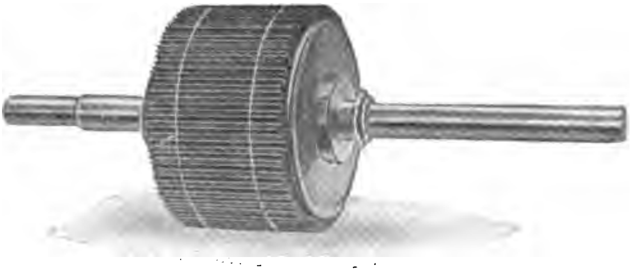


FIG 162.—Insulated core of slotted armature.

The short side of these has to be placed at the bottom of the slots beneath the sides of the first $\frac{N_2}{2p}$ coils, so that these latter are now lifted up a little to allow the sides of the remaining coils to be passed through the greater width of the upper layer. As each slot is filled with the lower layer, the sides of the upper layer can be successively closed down until the whole armature is finished. The shapers on which the coils are formed are of various kinds; the simplest form of coil is shown in fig. 163 where it will be seen that one side is of greater width than the other. Such a form is, however, only practicable in very small machines, and in cases where the depth of the slot can be given rather lavish proportions in order to bring the one side to a level well below that of the other side. A more developed form is shown in fig. 164; the wire is first wound round pegs (fig. 164 i), or on a special stepped shaper[1]; the coil is next forcibly opened out approximately into a rectangle of which the sides are not throughout of equal width, and is finally bent to the required curvature (fig. 164 ii). Or the coil may be wound immediately to the required shape, the wire being held in clips as it is formed upon a wooden or cast-iron mould. The latter system has been worked out in detail by Eickemeyer,[2] and it is well adapted to a great variety of cases. With a number of wires per side, these may be arranged one above the other along the periphery of the core, and side by side at the ends so as to obtain enough room

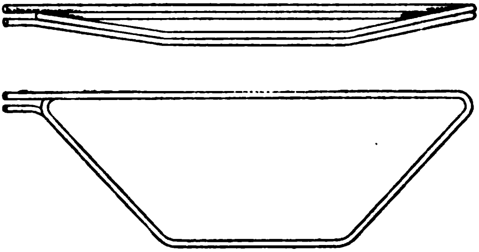


FIG. 163.

---

[1] See *Street Railway Journal*, vol. xviii. p. 3.

[2] Brit. Patent 2246, 1888.

towards the shaft, or any combination may be used giving a different cross-section to the coil at different parts of its contour. Fig. 164 iii shows several such coils in place on a four-pole armature; a certain number have their larger sides lifted up to allow of the introduction beneath and through them of the smaller sides of the last coils as indicated by a single dotted coil at the top of the diagram. It will be seen that the ends of the coils lie flat against the insulated ends of the core. An intermediate form through which we pass to barrel-winding makes the ends of the coils pass from slot to slot across a chord. The centre of the loops is now the widest portion of the coil and at the extreme points the wire is given a twist by which its level is altered so that one half lies at the bottom and the other half at the top of a slot (fig. 165). Fig. 166 shows the 4-pole armature of fig. 162 when finished, from which it will be seen that the ends of the coils

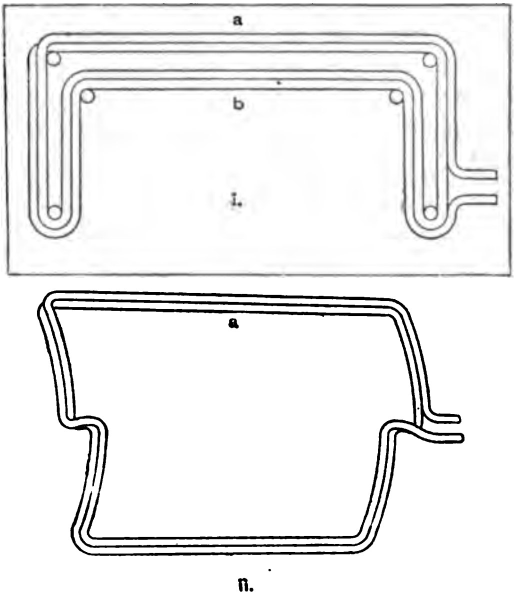


FIG. 164 i. and ii.

taper inwards towards the shaft. In *barrel-winding* proper the whole of the coil lies on the circumference of a cylinder practically of equal diameter with the core; it is by far the most widely used method, especially for multipolar machines with slotted armatures. The coil may be wound on a grooved shaper of the exact dimensions required; the wire which is at the top of the upper layer leads on to the wire which is at the bottom of the lower layer, so that the

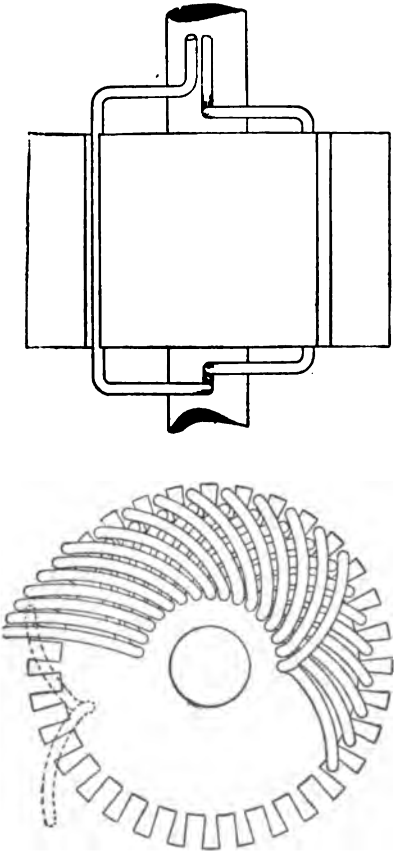


FIG. 164. iii.—Four-pole armature with former-shaped coils.

free ends come out respectively at the tops of the upper and lower layers (fig. 167 i). If the armature is lap-wound, both free ends are brought to the centre at the widest part of the coil, while if wave-wound, the ends are led away in opposite directions to be united to the appropriate commutator sectors. If the number of commutator sectors is some multiple of the number of slots, so that there are two or more sections per slot, the pair or more of coils may be wound together (as in fig. 167 ii, where three coils are shown; see also Rothert, *Electrical Engineer*, vol. xxviii. p. 266). The compound coil is then taped[1] or wrapped round with a common insulation of micanite paper, and varnished or

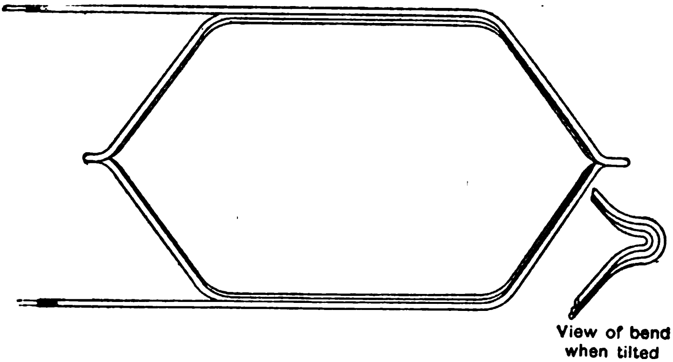


FIG. 165.

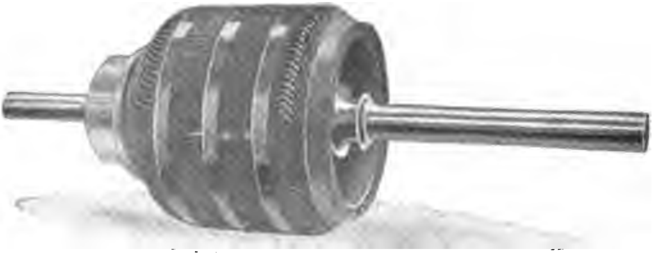


FIG. 166.

---

[1] See *Street Railway Journal*, vol. xviii. p. 4.

soaked in shellac; the impregnation is conveniently made under a vacuum in a small chamber communicating with a bath of the medium employed. The coil is next dried in a stove or drying chamber, and then forms a compact and easily handled element, the sides of which may be pressed and passed through a gauge to ensure an exact fit within the sides of slots. If the insulation be of sufficient electrical and mechanical strength, the slots themselves need not then be insulated.

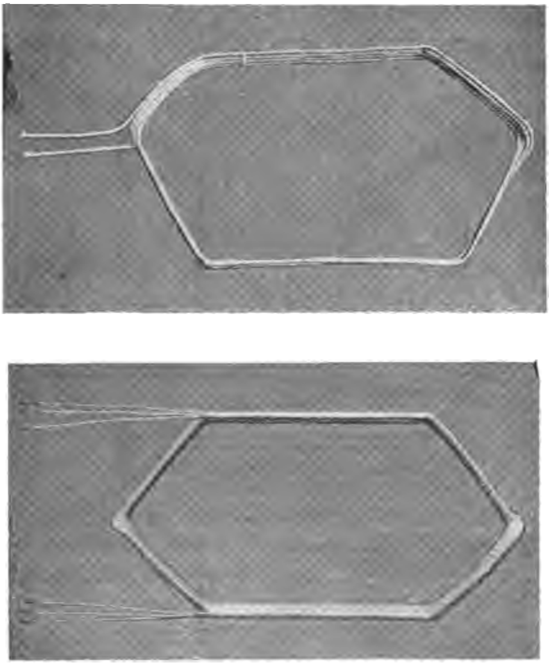


FIG. 167.—i. and ii.

§ 25. **Bar-wound barrel drum armatures.**—From the above it is easy to pass to the *bar-wound barrel armature*, the transitional form being a complete loop consisting of two inductors and end-connectors made of copper strip of rectangular section. The required length of copper is cut off and bent round at its centre, so that it falls into two levels with a little space between them; or if the copper strip is thin and deep, it may be folded over on itself so as to form the junction of the upper and lower layers (fig. 168). Each end is then bent through the correct pitch, and by means of a shaper is given the proper curvature so that it may lie on a cylindrical surface in two levels. Fig. 169 illustrates a lap-wound loop and fig. 170 the shaper on which it is formed. The loops after being taped are placed in succession on the armature, the lower half being pressed down into the bottom of the slot and insulation being inserted between the two layers wherever they are in close contiguity or contact. In finishing the armature the upper halves of the first-wound loops have to be lifted up to allow of the lower sides of the last loops being introduced beneath them. Fig. 171 shows the armature of a lap-wound 300 kilowatt dynamo manufactured by the International Electrical Engineering Co., in process of being wound. The connections between the loops are made at their

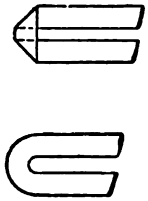


FIG. 168.

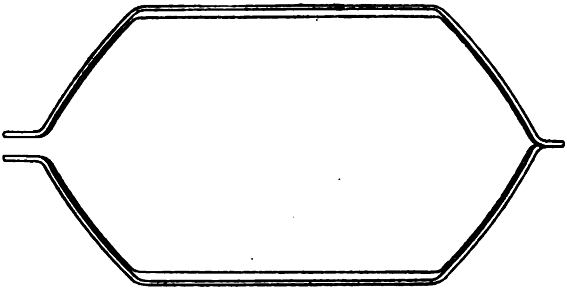


FIG. 169.—Lap-wound loop.

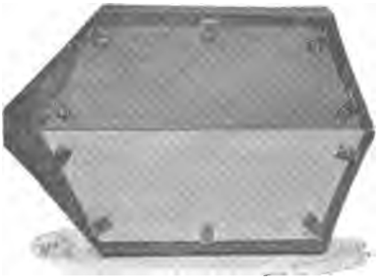


FIG. 170.—Shaper for lap-wound loop.

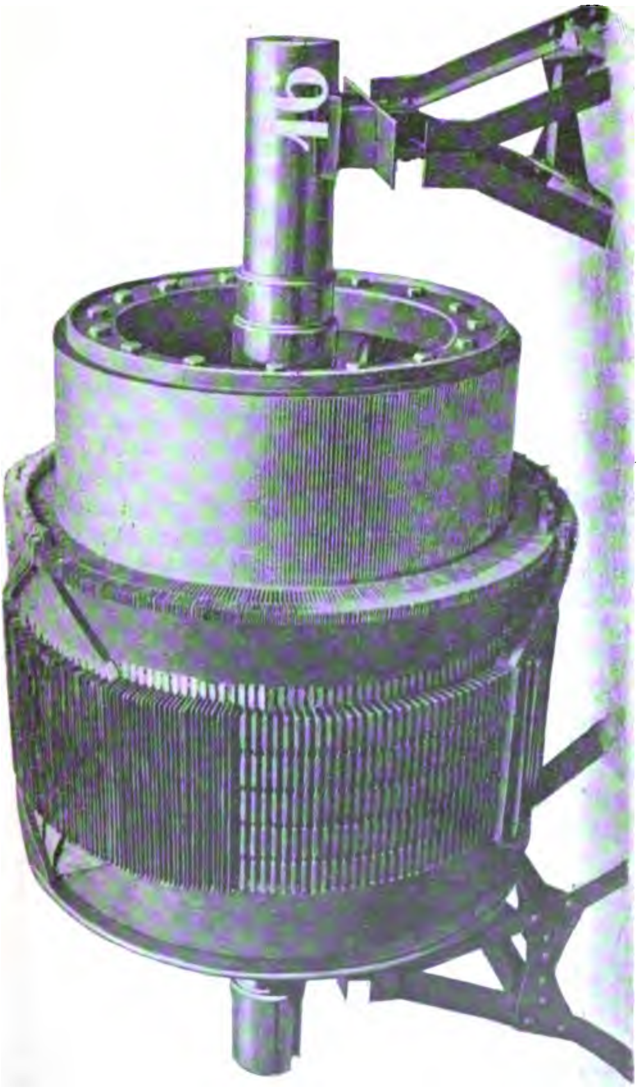


FIG. 171.—Winding of lap-wound barrel armature. 300 K. W. (International Electrical Eng. Co.)

junctions in the forks of the commutator connectors. Fig. 172

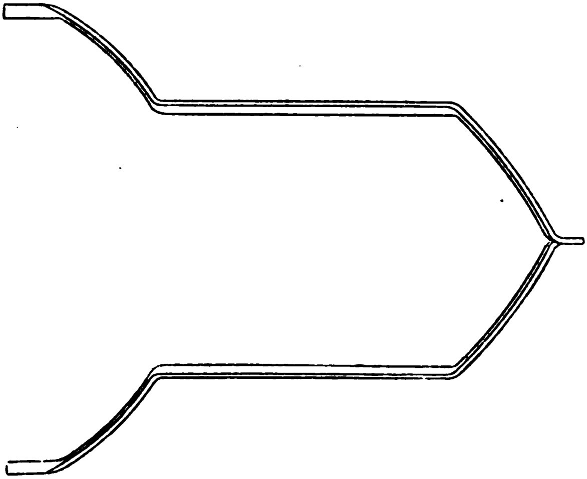


Fig. 172.—Wave-wound loop.

shows the loop of a wave-wound armature with its ends bent in opposite directions to join the commutator, while fig. 173 gives

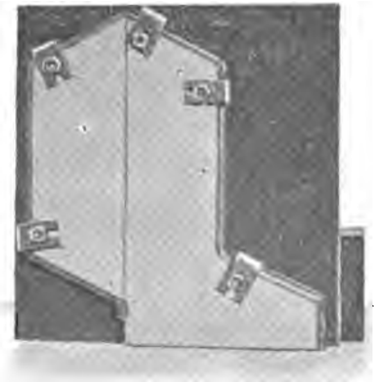


Fig. 173.—Shaper for wave-wound loop with two loops per section.

the shaper required for a wave-wound armature in which there are two loops per section divided between different slots, the method of connection with every other loop reversed being indicated in fig. 174.

The last form of the barrel armature is at once the simplest and commonest; by it each loop is composed of two separate bars which are first placed in position and subsequently soldered together in their due sequence when both layers have been arranged in the slots. Each end of the bar is bent through half the pitch, and is then curved to suit the armature circumference; fig. 175 shows the shaper used for this second operation, together with one bar in place for shaping and a second bar in front of the shaper. The shaped bars of the lower layer are then laid in position, one or more at the bottom of each slot, with their ends projecting so as to form a complete cylinder. After the insertion of insulation above the lower layer, the remaining bars are placed at the tops of the slots, their ends having previously been similarly bent so as to pass through the remaining half of the pitch, and forming a complete cylindrical envelope on the outside of the armature. At the ends the two layers are soldered together, small copper clips being passed over the bare ends of a pair of bars, one in the upper layer and the other in the bottom layer. Or at the commutator end the two layers of bars may be united by soldering within the lug which leads to the commutator sector. Fig. 176 shows the finished armature of a six-pole machine; as the slope of the end-connec-

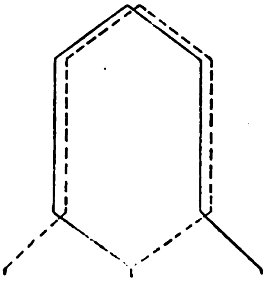


FIG. 174.—Connection of wave-wound-loops, two per section.

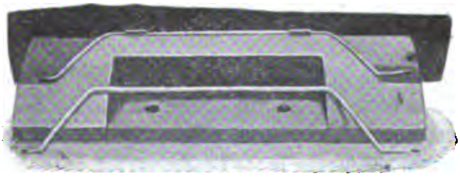


FIG. 175.—Lap-wound shaper and bars.

tions at either end is in opposite directions relatively to the armature core, it is lap-wound. If wave-wound, the direction of the slope at the two ends would have been approximately in parallel lines aslant across the armature core.

The simple bars above described involve a greater number of soldered joints than the method of completely formed loops, but on the other hand have the countervailing advantage that the amount of slack which must be allowed in the dimensions of the slot is reduced to a minimum; in either system the difference of potential between any two adjacent connectors in the same layer is never more than the E.M.F. due to two inductors, so that if the two or more layers are well insulated from each other by press-spahn or micanite, there is little likelihood of the insulation breaking down. They are further easily repaired, especially the simple bar-wound form; if a bar of the lower layer be damaged, it is only necessary to unsolder and take off a comparatively few number of bars in order to withdraw it. The sole disadvantage of the barrel-wound armature is in fact the axial length taken up by the end-connections, more especially when the number of poles is few. The minimum axial length L of the bent portions at each end of the armature may be calculated as follows; if (fig. 177) m = the distance measured on the circumference, through which the bar is bent, n = the number of bars in the distance in one layer, and t =

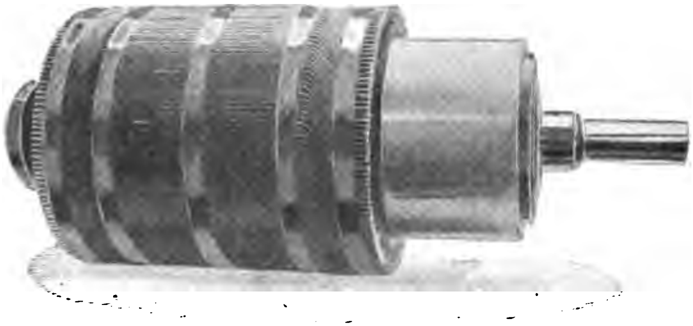


FIG. 176.—Lap-wound six-pole barrel armature.

the insulated thickness of one bar, $\frac{t}{t'}=\cos\theta$, or the width of a bar when cut through at right angles to the axis of the armature is $t'=\frac{t}{\cos\theta}$, and the width of n bars is $m=\frac{nt}{\cos\theta}$.

But $\cos\theta=\frac{\mathrm{L}}{\sqrt{\mathrm{L}^2+m^2}}$ whence $m=\frac{nt\sqrt{\mathrm{L}^2+m^2}}{\mathrm{L}}$

and $$\mathrm{L}=\frac{(nt)m}{\sqrt{m^2-(nt)^2}}$$

If l = the number of layers, and the bars are bent through half the pitch, $n=\frac{\tau}{4pl}$. Some addition must also be made to L to allow of the bars projecting well out of the slots before they become bent, and also to straighten before the point at which they are soldered together. In some cases in order to minimise this axial length the end-connections at the further end away from the commutator are cranked downwards towards the shaft, or the loops are thus shaped in the first instance; we thus approximate to the last division of drum armatures in which the axial length is a minimum owing to the employment of separate end-connectors arranged in planes perpendicular to the shaft.

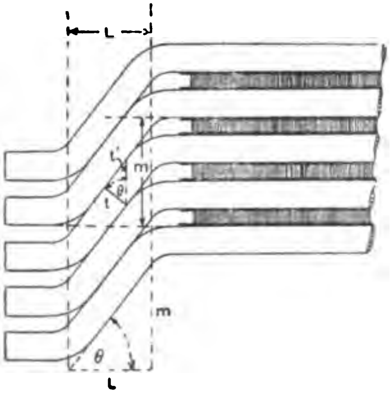


FIG 177.

§ 26. **Bar-wound drum armatures with separate end-connectors.**—While the term 'armature-winding,' however appropriate to small ring and drum armatures wound with round wire, is somewhat of a misnomer as applied to barrel armatures, it becomes still more so in bar armatures in which the end-connectors are entirely distinct from the inductors and are thin but wide strips of copper, built up separately and apart from the rest of the winding.

The development of the armatures of figs. 114 and 117 will serve to illustrate the general principle of all such end-connections for drums. When the connectors for either end of the armature are arranged in place, the lugs at their ends stand out in two circular rows, the one near to the armature core and the other farther away, each connector being so twisted or so formed that within its length it passes across from the one row to the other. At each end of the armature the ends of the inductors are alternately long and short, and, by means of the spirally-bent connector, the long end of one inductor is connected to the short end of another, situated at the proper distance apart round the circumference of the core.

The various forms which have been devised for drum end-connectors are too numerous to permit of detailed consideration,[1] but the best-known type may be here described. In general, they are thin strips of copper about 2¼″ to 4½″ wide; these have a slot punched down their centre, partially dividing them into two widths of about 1″ to 2″, and are then forced apart on a cast-iron shaper, so as to form a double evolute curve similar to the cross-connectors of fig. 108. When assembled, the gap in the centre between the outer and inner whorl has a width of about ¼″ in low-potential machines to ⅜″ for potentials up to 500 volts. The insulation between adjacent connectors is formed by strips of calico pasted on with shellac and wrapped over at the centre. If there are a great number of bars, a careful estimate must be made of the space which they will occupy in relation to the periphery of the innermost circle down to which they are to be drawn, and a margin must be allowed for slight differences in the thickness of the insulation. Thus if the calico be 0·007″ thick, and there is ample room on the inner circle, four thicknesses of calico may be permitted at the centre, which when shellaced may be taken to require an allowance of 0·040″ on the thickness of the copper. If space is limited, three thicknesses of calico at the centre will with care suffice, and will measure 0·025″. In calculating the radial depth of the end-connectors when built up into a whorl and drawn down to the correct diameter, allowance must be made for the fact that with thick copper there must be some space lost; if p be the number of pairs of poles, the number of connectors cut through in any section will be on the assumption of a double evolute shape $\frac{\tau}{8p}$, and if the thickness of copper in each be 0·040″ an addition of 0·030″ must be made, or if the copper be 0·070″, of ·040″; while for large currents if two connectors in parallel are employed each ·040″ thick, an allowance of ·060″ must be made for the insulation, since they cannot be made to lie absolutely close together, or of 0·075″ if the thickness of each separate strip be as much as 0·080″.

---

[1] See Weymouth, "Drum Armatures and Commutators," *Electrician* series.

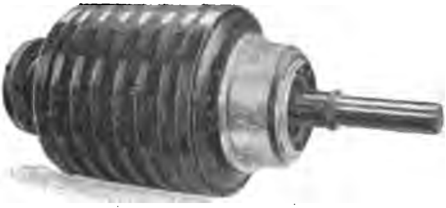


FIG. 178.—Four-pole bar armature with separate end connectors.

When assembled, the connectors for each end form a compact and symmetrical mass, which is subsequently passed over the shaft into its place. When the connections for one complete loop have been marked so as to start the winding, an inductor is laid on the surface of the core, and its ends soldered to the lugs of the appropriate pair of connectors; bar after bar in continuous succession is thus connected up at both ends by soldered joints until the whole surface of the armature is covered, and the winding results in a closed helix of many loops.

Fig. 178 shows a four-pole drum armature with smooth-surface and separate end-connectors; their evolute shape can be seen at the commutator end, and from their centre is taken the connection to each of the commutator sectors. In large multipolar armatures there is little difficulty in securing ample room at the ends of the core, so that the connectors can be kept further apart from each other; air can thus reach them, and their fanning action assists greatly in cooling the armature. The *bar-wound drums with separate end-connectors* share many of the advantages of the barrel-wound type; the difference of potential between the connectors is small, *e.g.*, in a two-pole dynamo with 100 inductors and giving 100 volts it will not be more than about 4 volts (cp. fig. 114), and any bar can be removed after the bands are cut without interfering with the rest of the winding. On the other hand the end-connectors themselves are not so readily accessible. In high-voltage machines with a large number of bars, the latter may require to be disposed in two layers with two sets of connectors at each end; a sheet of micanite should then be interposed between the layers, so as to secure thorough insulation between the bars which are at widely different potentials.

§ 27. **Binding-wire.**—When the winding of the armature sections is completed, the next step is to wind circumferentially round the armature several bands of *binding-wire*; by these the inductors are held securely in place, and the tendency for them to be thrown off the core by centrifugal force is counteracted. The materials used for binding-wire are phosphor bronze, hard brass, or in some cases non-magnetic steel, the requirements for it being great tensile strength with but little expansion under the heat of the soldering iron. The diameter of the wire varies from ·020″ (No. 25 S.W.G.) for small armatures, to ·056″ (No. 17 S.W.G.) in large armatures. A band of thin tape covered with strips of thin mica is fastened round the armature, and on this the wire is wound under considerable tension to form a belt about ¾″ wide. If wider, eddy-currents are liable to cause heating of the bands. At intervals of about 8–10 inches along the length of the band are placed small strips of sheet copper about ¾″ wide; after the band has been wound on over these, the ends of the strips are turned over to form clips round the wire, and the whole is soldered together (fig. 181). Such bands are placed at intervals of about 3″ along the entire length of the armature (figs. 179 and 180).

If we take No. 19 S.W.G. (= 0·042″) and allow 0·026″ for the mica on its tape backing, and 2 × 0·006″ = 0·012″ for the folded copper strip, a total addition of 0·08″ is made to the radius of the armature by the binding-wire, and the mechanical clearance must be reckoned from this over-all radius. In some cases when the core of a toothed armature is being built up, batches of discs of slightly smaller diameter are interposed at intervals along the length; by this means shallow grooves are formed round the periphery just deep enough to receive the binding-wire, so that the finished surface becomes practically flush

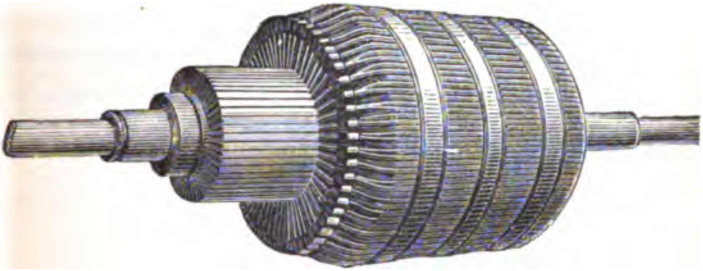


FIG. 179.—Completed ring armature.

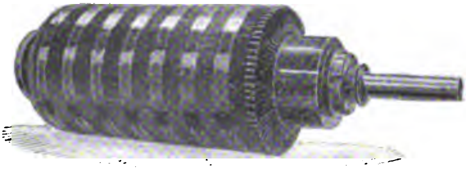


FIG. 180.—Completed drum two-pole armature.

If s = the cross-section of an inductor in square inches in a plane perpendicular to the axis of rotation, the total area of copper in the bars if the armature be cut through radially at any point is τs. Let d = the mean diameter in inches of the circle round which the inductors are disposed (fig. 182); then the thickness of the equivalent copper cylinder taken as uniformly enveloping the armature is $\frac{\tau s}{\pi d}$ inches. Taking any distance along the armature of l inches over which the disposition of the inductors is similar, let us consider a small sector of λ inches measured on the circumference corresponding to d (fig. 182). The cubic inches of copper in this small sector $= \lambda l \times \frac{\tau s}{\pi d}$, and its weight in lbs., allowing a little for the insulation surrounding the net cross-section of copper, is $0{\cdot}33\ \lambda l \,.\, \frac{\tau s}{\pi d}$. The weight per foot of length of the circumference corresponding to the radius of gyration r in feet will for the same width of inches along the core be $\frac{12}{\lambda}$ times as great, or $w = 3{\cdot}96\ l \,.\, \frac{\tau s}{\pi d}$. The centrifugal force of this mass will be $\frac{w}{g} \,.\, \frac{v^2}{r}$ lbs. and the internal pressure due to it per unit of width along the core will be $p = \frac{w \,.\, v^2}{grl}$ lbs. Taking any diametral plane A D through the cylinder, the resultant force acting across the sections A B, C D is due to the sum of the normal components when the individual pressures of all the similar sectors round the semi-circle are resolved at right angles to the plane. Thus just as in a flywheel, or as in the case

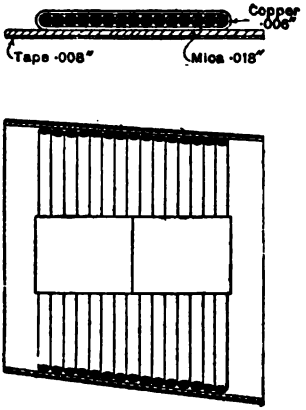


FIG. 181.—Binding-wire band.

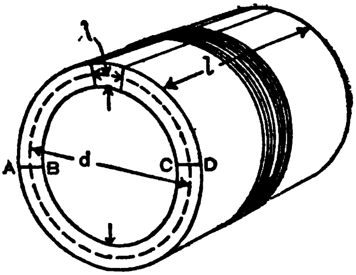


FIG. 182.

of a thin cylinder subjected to an internal bursting pressure, the resultant force on the two sides is equal to the product of the centrifugal force per foot transferred to the projection of the semicircle on the diametral plane; in other words, it is equal to the product of the intensity of the pressure p lbs. per foot and per unit width into the diameter in feet multiplied by the total width l, *i.e.* $=p \times \mathrm{D} l$. The force on the single section A B is then

$$=p\,r\,l=\frac{w \cdot v^2}{g}, \text{ or}$$

$$\mathrm{P}=\frac{3 \cdot 96\, l \cdot \tau \cdot s \cdot v^2}{\pi d \times 32}$$

and since $v=\dfrac{\pi d \mathrm{N}}{60 \times 12}$ feet per second,

$$\mathrm{P}=7 \cdot 5\, l \cdot \tau \cdot s \cdot d \cdot \mathrm{N}^2 \times 10^{-7} \text{ lbs.} \qquad (23).$$

This force acting on either side of the plane A B is balanced by the molecular tensions in the binding wires across the same section, so that if f_t is the safe permissible limit of tensile stress with the particular material employed, and a = the total area of cross-section of the wires in the width l, we have

$$\mathrm{P} \leqq f_t \cdot a.$$

The ultimate breaking strengths of brass and phosphor bronze wires are about 70,000 and 90,000 lbs. per square inch respectively, while that of steel is 200,000 and may even reach to 300,000 lbs. The stretching however must be inappreciable if the wires are not to work loose, and a large factor of safety of, say, 8 is necessary. A barrel-wound armature should be divided into two portions, corresponding to the length of the core, and to the length of the two sets of end-connections, as there is a greater mass per unit length at the ends; the necessary cross-section of all the wires should then be calculated for each of these two portions and the wires be afterwards distributed among a suitable number of bands approximately equally spaced. Still more in drum armatures with separate end-connectors unless these are specially held down must the additional mass concentrated at the ends be considered in relation to the strength of the bands towards the ends of the armature. In general so far as safety permits, the bands should be wound outside the limits of the pole-pieces, so as to reduce the eddy-currents in them; especially in barrel-wound armatures, for here the bars are tightly wedged in the slots and so are partly held by friction, while at the ends iron supporting rings can be cast on the hub upon which the end bands may firmly compress the bars (cp. figs. 140 and 141). On large armatures the calculation of the binding-wires becomes of great importance, since it is difficult to secure sufficient strength. Steel wire must then be resorted to, and this should be nickel-plated to render it easier to solder. In still larger armatures 6 feet or more in diameter, it becomes impossible to employ binding-wire at all, and the bars must then be held in place by half-closing the slots, or by driving beechwood keys into grooves stamped in the tops of the slots (figs. 158 and 183). Outside the pole-faces, thin bands of solid steel about 1½ inches wide may then be employed; these are fastened by a junction drawn together by a right- and left-handed screw, so that they can be easily removed and replaced in a few moments in case of repairs being necessary to the winding. The junction pieces riveted to the band must be arranged so as to balance one another on the armature.

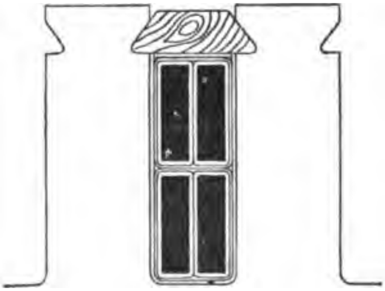


FIG. 183.

§ 28. **Commutators.**—To complete the armature winding it remains to put the commutator in position, and connect up each of its sectors to the loops or coils, junction being made between the end of one section and the beginning of the next in succession. The hand of the system on which the connections are made must follow a definite rule, so that it may agree with the designed direction of the field-winding, and so that a spare armature may be wound to the same hand and be self-exciting without reversal of the connections between the brush leads and the field-winding terminals. In the construction of *the commutator* various modifications are possible, but, broadly speaking, two types may be distinguished. In the first (fig. 184) which is suitable for small machines with gauze brushes, the wedge-shaped strips of copper and the intervening plates of mica are held in place by a sleeve or bush with a coned mushroom-shaped head and a coned collar (*c*) forced tightly up against the sectors by a nut (*n*) screwing on to the bush. The latter may be of gun-metal or cast iron, the collar and nut being of wrought iron or gun-metal. The angle at which the coned surfaces are inclined to the horizontal axis should not exceed 50°. Complete insulation of the sectors from the supporting structure between which may exist the full stress of the voltage of the machine is obtained by the conical rings (*ff*); these are turned out of hard red vulcanised fibre, and are backed by thin slips of mica cut to the required curve and cemented together, or, better, are formed entirely of mica or micanite moulded exactly to the required taper. The nut (*n*) is slotted, as shown at (*h*), so as to permit of its being

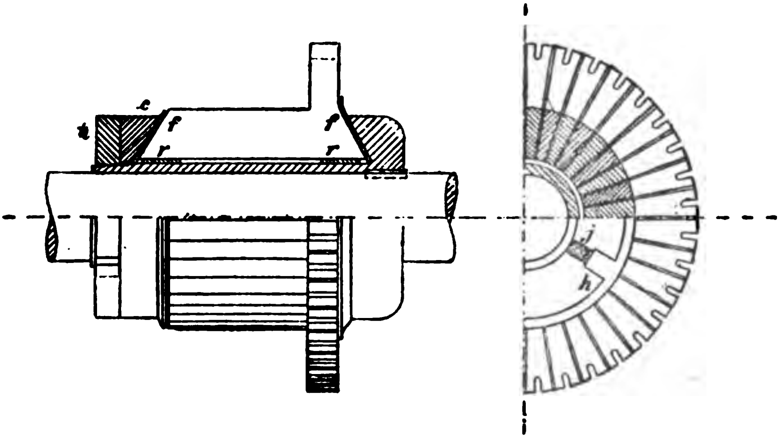


FIG. 184.—Commutator for small dynamo.

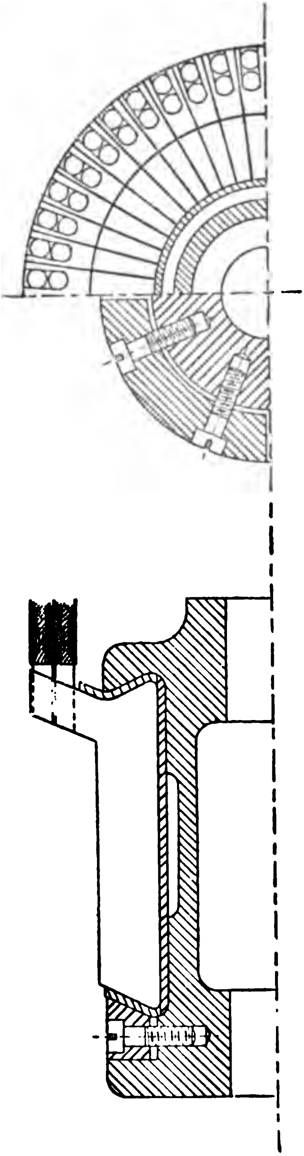


FIG. 185.—Commutator for small dynamo.

tightened up by a horned spanner. During this process the sectors are forced inwards under the sloping faces at either end, and thus bind on one another like the stones of an arch. It is still, however, possible, if their taper be but slight, that they may be driven inwards and put out of truth by an accidental blow. It is advisable, therefore, to support them on two rings of micanite (*r r*) moulded over the body of the sleeve, care being taken that they do not bed down on to the rings before they are thoroughly tightened up sideways. In order to prevent the collar from twisting the sectors round, a screw or pin in the collar is in some cases made to engage in a slot in the sleeve. Finally, to prevent the nut (*n*) from slacking back, one or two set screws (*j*) are put radially through the nut so that their points bite into the sleeve. In place of the nut and collar, a coned ring divided into four or more segments and screwed down to the sleeve may also be employed (fig. 185).

In the second type (fig. 186) which is more usual in larger machines, especially with carbon brushes, the coned ends of both sleeve and collar are let into recesses in the sectors, and for greater rigidity the sleeve is usually of cast-iron with collar of cast or wrought-iron. At each end of the commutator a V-shaped groove is turned, into which a taper ring of built-up mica or micanite is fitted.

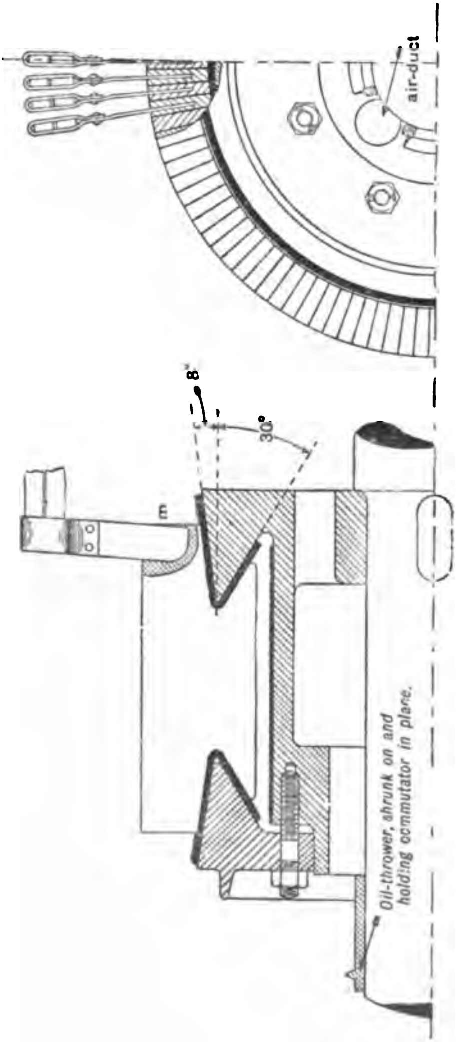


FIG. 186 i.—Commutator of second type with double taper and set screws or studs.

When the dynamo is at work, the temperature of the commutator is raised by the passage of the current over the contact-resistance of the brushes (especially if of carbon) and by their friction with its surface; the difference in the expansion of the copper and the cast-iron shell or sleeve then sets up very considerable stresses in the structure. It is consequently difficult to devise any mechanical construction that shall never fail to keep the surface perfectly cylindrical after repeated heating and cooling, since between metal and metal must intervene the layer of insulation, and upon this comparatively compressible medium falls all the strain. Any relative movement of neighbouring sectors by which one is raised above or lowered below the other even to a minute degree suffices with carbon brushes to cause them to jump as they pass over the displaced sector owing to the carbon having no elasticity such as is given by copper gauze brushes. Sparking is thereby set up, and the surface of the sector which is at fault is rapidly eaten away until finally the commutator is rendered unworkable. Smoothness of surface is in fact more essential than perfect concentricity with the axis of the shaft; the brushes have time to follow any eccentricity of the commutator as a whole, but owing to the inertia of the brushes and brush-boxes their tension-springs cannot take into account with sufficient rapidity any unevenness in adjacent sectors; the brush-box or brush is alternately jerked off and drawn on again to the surface—an operation accompanied by a chattering noise and excessive sparking. In order to prevent any shifting of the sectors relatively to one another, various constructions and various angles of taper have been tried, and experience seems to show that a double taper such as is shown in figs. 186 and 187 is better than a

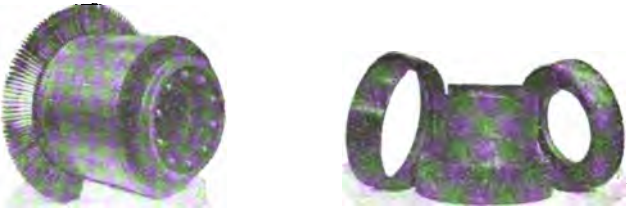


FIG. 186 ii.

single taper and a flat band. The two angles are, however, usually unequal, a general proportion being a total angle of 38° divided into 30° below the horizontal and 8° above. By means of the double taper the sectors are held endways and centred round a circle corresponding to the diameter of the apex of the cone, and as the retaining rings expand and contract during heating and cooling all the sectors are maintained concentrically on this circle. Owing, however, to the greater angle below the horizontal, more force is exerted inwards than outwards as the metal cones are driven home in the initial process of construction, so that the sectors are at that time not only pinched together axially but jammed tight against each other sideways. The apex of the cones is rounded off so as not to press on the internal angle of the insulation. Between the inner periphery of the copper and the sleeve is a clear air-space, or if the voltage is above 100, and this space is small, the sleeve must be covered with a moulding of micanite or paper to prevent sparking taking place to the iron. But to prevent any sector being knocked in by an accidental blow, reliance is still to be placed entirely on the support of the circular rings. Fig. 186 ii. shows a finished commutator, and by its side the cast-iron body with the two tapered micanite rings. For greater strength in large commutators, instead of the nut (*n*) the pressure on the coned rings is applied and maintained by several set screws (fig. 186) or in still larger machines by bolts, as in fig. 187.

When fastened directly to the shaft, if there be any flexure of the shaft between the armature hub and the commutator sleeve, the connecting lugs are alternately extended and compressed every revolution. This stress in time hardens the copper, and causes it to break either at the junction with the armature bar or at the root where it enters the commutator. This evil may be avoided by cranking the commutator lugs in the middle of their length so as to enable them to give slightly at this point or by the employment of flexible stranded ribbon or cable for the connections. It is, however, very usual in large machines subjected to heavy strains to fasten the commutator directly to the armature cast-iron hub either on a projecting sleeve or by bolting it up to its end (fig. 188) so that it is entirely free from the shaft. No relative displacement of the armature and commutator can then take place even if the shaft does bend in a minute degree. With large commutators

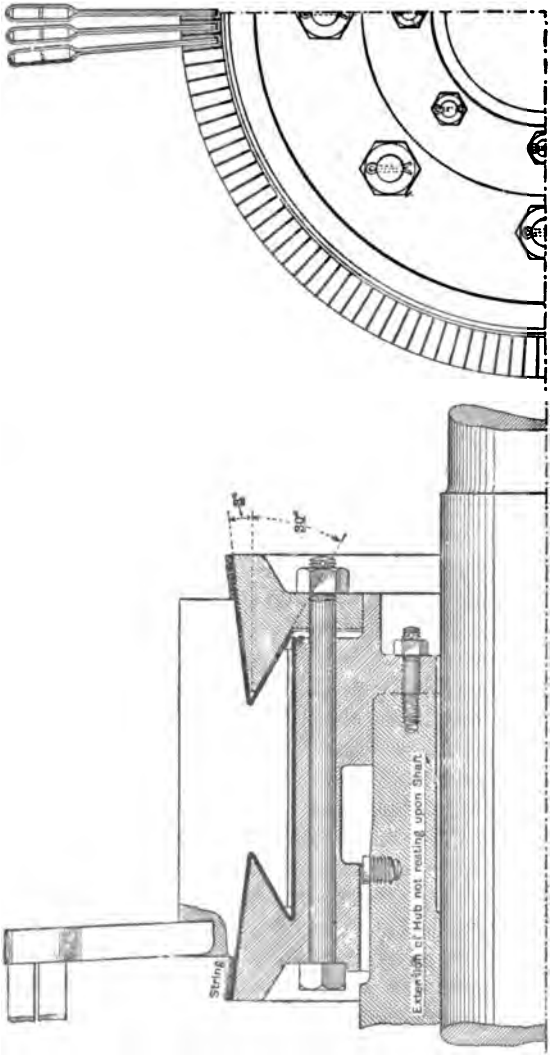


FIG. 187.—Commutator of second type with double taper and bolts.

running at high peripheral speeds, great care must be exercised in the design to ensure amply sufficient mechanical strength in the rings, bolts or screws and in the copper sectors, so that there may be no fear of the commutator bursting under the stress of centrifugal force.

It will be seen that for a given length of brush working surface commutators of the first type are slightly longer than those of the second type; on the other hand, the latter cannot be worn down below the level of the end rings, and therefore for a given diameter their radial depth of wear is less than in the first type. They present, however, a greater cooling surface and if of considerable diameter airways can be arranged so as to allow air to circulate through the inside.

In both types of commutator, in order to prevent the sleeve twisting round out of its proper position it is secured either by a small sunk key under its head, as in fig. 186, or by a pin driven into the shaft which engages in a slot in the sleeve. The small screw in fig. 187 serves as a register when the commutator is being fitted in place.

The connections from the individual sectors to the sections of the armature winding are most commonly made by thin strips of copper. A saw-cut is made in each sector, as shown at *m* (fig. 186), and into this one end of the strip is soldered; at the other, it is carried up to the level of the ends of the sections, and there embraces and is soldered to the armature wire. The strips must be bent over the top of the armature bars, and may each be composed of a pair of thinner strips in parallel, as in fig. 187. Since the current only flows through the commutator lugs for very short intervals of time, the current-density within them may be high, but should not exceed 5000 ampères per square inch of section. In some cases, cast sectors of phosphor bronze alloyed with copper or gun-metal are used, and a projecting lug is directly cast on each sector, as in figs. 184 and 185; the wires are then soldered into a groove at the top of each lug. In such cases the mica is extended up to the full height of the lug, and an additional advantage claimed for the construction is that copper dust worn off the commutator by the brushes cannot be blown through the solid wall formed by the close-fitting lugs. On the other hand, a considerable amount of ventilating effect is lost, which on the first method is secured by the moving blades of copper. In many Continental machines, round wires are used to unite the armature winding with the sectors and are fastened into holes in the latter by two or more grub screws. In America, drop-forged sectors have been largely employed, and in these the lug can be forged in the mould in one piece with the sector. Nothing, however, is so suitable for commutators as bars of hard-drawn copper sawn up to the required lengths; homogeneity and perfect uniformity of the sectors in hardness is essential to good working, and in these qualities hard-drawn copper notably surpasses either drop-forgings or castings.

After it has been built up, the surface of the commutator is turned true either on a mandril or preferably after it is in place on the armature shaft. With carbon brushes in order to secure the perfect smoothness so essential to their successful use, the commutator should be ground while slowly revolving by a small rapidly driven emery wheel.

§ 29. **Insulation of commutator sectors, etc.**—For the insulation between the separate sectors of the commutator up to 8 inches long *mica* is now almost universally used in plates about $\frac{1}{40}$th to $\frac{1}{25}$th of an inch thick (0·025" – 0·040"; 1 cubic inch weighs $\frac{1}{10}$th of a pound) and these plates should project through at the inner end and between the lugs by some $\frac{3}{32}$". The difference of potential between neighbouring sectors may be small, inasmuch as it is only the potential generated within the limits of one section of the winding. When, however, the sectors pass under the brushes, sparking is liable to occur, and a small arc is formed which bridges the insulation. Under this action almost every insulating substance, except mica, is apt to char and become conductive, and even with mica insulation, if the thickness be less than $\frac{1}{40}$th of an inch, small particles of copper may bridge over two adjoining sectors. In machines giving over 1000 volts, the thickness may be increased to $\frac{1}{20}$th of an inch (·050"). Air insulation is impracticable unless the insulating spaces be large, as in the open-coil machines of Chapter X, since otherwise they quickly become choked with copper dust. Besides being incombustible, mica possesses the advantages of being non-hygroscopic (so that it does not deteriorate if the dynamo be situated in a damp place), and of being mechanically strong to withstand great pressure; further, it can be easily split into flat plates of very uniform thickness.

Its specific insulation resistance varies considerably in different qualities, and for electrical purposes it must be free from iron oxide in patches of any thickness. It need not, however, be entirely transparent, and in commutators green or black-spotted mica is employed. It is extremely important especially with carbon brushes that the rate of wear of the mica should be the same as that of the copper sectors; it must not therefore be too hard, and the best degree of hardness is obtained with Indian mica. Plates more than 8 to 10 inches long occur in nature with such comparative rarity that their price is prohibitive, and it becomes an economical necessity to employ either built-up plates or the artificial forms, micanite and megomit. When the latter are used for commutators, the component mica must be specially chosen so as to have an equal rate of wear with the copper sectors, and there must be no tendency for the cement to ooze out when the material becomes heated; hence powerful pressure is employed in order to remove all excess of cement during the process of manufacture. The latter condition applies with equal force to end-rings and sleeves made of micanite or megomit, and when they are used, it is advisable after the commutator has been assembled to heat it to a temperature of about 300° – 400° F. in a stove or by means of gas-jets, and when at this high temperature to compress the whole structure by tightening up the fastening screws or bolts, at the same time carefully hammering the sectors so as to bed them well into the micanite until a compact and solid mass results. The taper end-rings are perferably not formed of two cones or of a coned ring and flat band cemented together, but are made in one piece by bending small strips of mica round at a sharp angle or by squeezing a sheet when hot in a mould. In machines for 250 volts and upwards, the end-rings should project outwards beyond the edges of the sectors to prevent sparking to the metal case, and on the ledge of some ½″ to ¾″ width may be wound a layer of string to prevent flaking away of the mica (figs. 186 and 188). The thickness of the insulation on the body of the sleeve and of the end-rings should rise from 0·06″ for 100 volts and 0·1″ for 250 volts up to 0·125″ for 500 and 0·15″ for 1000 volts. In closed-coil machines the sectors should be as numerous as possible within reasonable limits, and, roughly speaking, the volts per commutator sector or $\frac{2p \cdot E_b}{N_2}$ should not exceed 20 to 25; otherwise an arc may be established between two adjacent sectors and short-circuit the winding between them. Except, however, in high-tension machines for small currents, the maximum named above is seldom reached, since for other reasons it is necessary to limit the number of inductors embraced between two neighbouring sectors. A large number of sectors implies a large diameter of commutator unless their width be very small, and mechanical considerations require that their width should not be less than $\frac{1}{10}''$ at the top tapering downwards say to $\frac{1}{32}''$ at the bottom. Even then the top of the sector cannot be sawn to receive the lug, and it becomes necessary to increase the depth of the sector until it measures 0·190″ at the top, and perhaps to turn it down along the brush working surface. In any case the top of the commutator lugs should fall slightly below the level of the armature surface.

It is thus evident that the commutator is, speaking generally, of a somewhat delicate nature, and on this account it becomes practically impossible (except by the use of special devices) to build closed-coil machines to give more than 3000 or at the outside 4000 volts. A natural limitation is thereby set to the employment of continuous currents for the transmission of energy over very great distances, and for this purpose especially the continuous-current dynamo must yield place to the alternator.

In conclusion, to the above diagrams is added (fig. 188) a complete sectional drawing of a four-pole barrel-wound slotted armature, 21 in. in diameter by 11 in. long with commutator in place. The armature is lap-wound to give 60 kilowatts at 500 revs. per min. and 250 volts; and there are two complete turns per commutator section, the bars being arranged in four layers within the slots.

§ 30. **Insulation resistance of armature.**—After the completion and drying of the winding, the armature is finally subjected to a test for *insulation resistance* between the winding and commutator as a whole and the iron core. Under a difference of potential proportioned to its working pressure, the insulation must not give way, even when warm. Thus if the armature is to

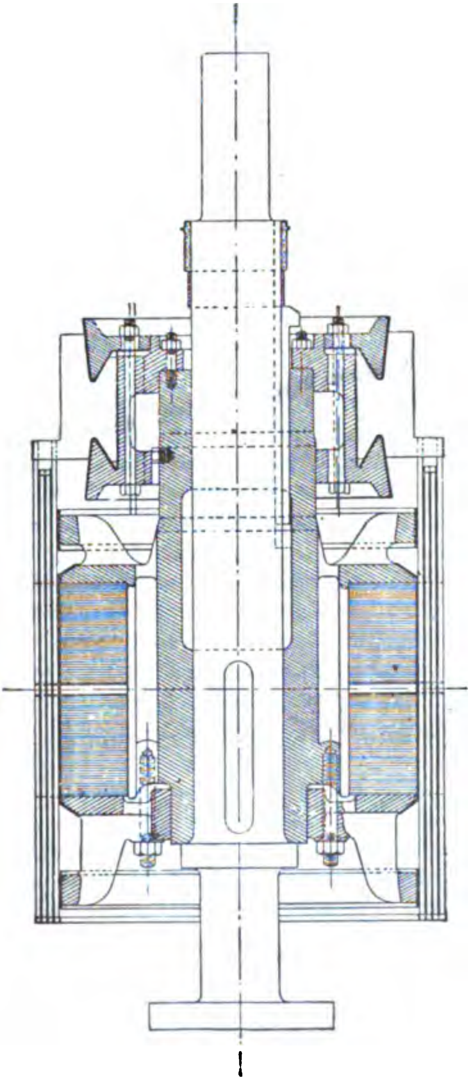


FIG. 188.—Barrel-wound armature and commutator in section.

give 100 volts on lighting circuits, it should be tested when warm with 500 to 1000 volts between winding and shaft, or if its working pressure is 500 volts, by the application of an alternating difference of potential of 2000 volts for some little time, say, not less than five minutes. It must be borne in mind that not only does the alternating difference of potential strain the dielectric resistance of the insulation more than an equivalent continuous potential, but from the shape of the alternating E.M.F. curve, its maximum value is considerably greater than its effective value; hence while such a test must not be pushed to such an extreme as to permanently weaken the insulation, it is far more valuable than any measurement of the actual insulation resistance, since the measured megohms will vary greatly with the state of the armature and the amount of the surface leakage, especially from the exposed ends of the commutator.

§ 31. **Electrical resistance of armature.**—The calculation of the *electrical resistance of an armature* from brush to brush is made as follows: From the dimensions of the core, the length of one inductor and of one connector by which it is joined to the next inductor in series can be estimated. Let l be this length in some unit, and let ω = the resistance of unit length of copper wire of the given sectional area; then the entire length of conductor with which the armature is wound is $\tau \times l$, where τ is the total number of inductors, and its resistance if extended out in series is $\tau \times l \times \omega$. Since the armature is divided into q parallel circuits, the resistance of each circuit is $\frac{\tau \,.\, l \,.\, \omega}{q}$, and the resistance of q such circuits in parallel is

$$R_a = \frac{\tau \times l \times \omega}{q^2} \quad . \quad . \quad . \quad . \quad (24).$$

Thus in the simple bipolar machine the resistance of the armature is one-fourth of the resistance of the total length of wire if in series; and, in the multipolar parallel-connected machine, unless multiplex wound, $q = P$, the number of poles, and $R_a = \frac{\tau \times l \times \omega}{P^2}$. While in a ring-wound armature l is the length of one complete loop, in drum machines it is the length of a half loop. If the sectional area of inductor and connector be different, as is often the case in drums with separate end-connectors, it is simplest in calculating l to substitute for the actual length of the connector the equivalent length which would give its actual resistance supposing it to have the same area as the inductor itself: thus if the ratio $\frac{\text{area of connector}}{\text{area of inductor}} = 1{\cdot}25$, and l_r and l_c be the actual lengths of the inductor and connector respectively, their combined resistance is $\left(l_r + \frac{l_c}{1{\cdot}25}\right) \times \omega$, where ω is the resistance per unit length of the inductor.

§ 32. **Peripheral speed of armature.**—A high peripheral speed is to some extent desirable, as reducing the size and weight of a machine for a given output; but it must be limited by considerations of mechanical strength and durability, and therefore its permissible value depends largely on the method on which the armature is built up. But apart from the question of the mechanical strength of the rotating portion, its perfect balancing is of almost equal importance: with high speeds, even a comparatively light armature, unless accurately balanced, will set up such excessive vibration throughout the whole machine as will in the end considerably shorten its life. High-speed armatures are usually balanced on knife edges before leaving the workshop and a little material is added as by solder or by a lead plug to whichever side is lighter. But even this only secures a statical balance when at rest, and does not necessarily imply perfect balance under rapid rotation.

With the ordinary ring or drum armature driven by belt, a peripheral speed of about 2500 feet per minute is commonly found, and a well-constructed armature of this type should mechanically allow of a speed of at least 3000 feet. With slotted armatures in which binding-wires can be replaced by wood keys, the peripheral speed in large machines reaches 5250 feet per minute. A limit is, however, set to the advantage of high speeds by the great increase in the eddy-current loss which is thereby produced, and by the increased difficulty of securing sparkless commutation.

§ 33. **Ratio of length and diameter of armature.**—The relative proportion of the length L to the diameter D of armature cores depends on the type of magnet and on the number of poles, and in each type considerable variations are permissible without transgressing the limits fixed by mechanical or economical considerations. In bipolar rings a general proportion is $L = D$, or $L = 1{\cdot}25\ D$; but in the case of double-horseshoe fields, where a large diameter of ring is frequent, the length is often not more than half the diameter. The diameter of the internal hole in the core-discs is usually so chosen that after allowing for the internal insulation, and deducting the width of the supporting arms, the number of layers on the inside is twice that on the outside. This results in a radial depth of core, $a = {\cdot}19\ D$ approximately, the proportion being somewhat larger for large ring armatures.

In bipolar drums the length is usually somewhat more than in a ring of the same diameter, and the ratio rises to $\frac{L}{D} = 1{\cdot}75$, or even higher. This, however, in armatures of large diameter leads to an inconveniently long span of shaft between the bearings, and therefore L is reduced to 1·25 or 1·5 D. The radial depth of core is limited only by the diameter of the shaft, unless internal spaces be desired for purposes of ventilation.

The proportions of multipolar machines will be further considered in Chapter XIV § 17.

Since the total flux which enters the armature from any one pole, or Z_a, divides within the core, half of the lines passing in either direction onwards to an adjacent pole of opposite sign, the area of the iron through which the Z_a lines flow is twice the cross-sectional area at any one part of the core if it be cut through from the outside to the shaft. Hence, in a ring or drum, if $a =$ the radial depth in cm. of the discs of which the armature is built up, and $b =$ the net length in cm. of iron parallel to the shaft (after allowance has been made for the insulating varnish or paper between the discs and for any ventilating air-spaces that there may be in the core), the maximum flux-density in the core is $B_a = \frac{Z_a}{2ab}$. As stated in § 2, b will vary from 0·85 to 0·90 of the gross length of the core L. In the case of toothed armatures, the value of a must be reckoned from the bottom of the slots only; in other words, as equal to the difference between the internal radius and the radius to the bottom of the teeth or slots. In the discoidal ring the allowance for the insulation must be made in the radial depth a instead of in the length b.

§ 34. **Radial depth of armature core.**—The iron of which the core is composed being soft and permeable, a high induction is permissible in the armature without impairing the efficiency or economy of excitation. This is especially the case with ring armatures, since in them a high induction implies a small section of core for a given total number of lines, and correspondingly greater room in the interior for the disposal of the inner connecting wires. In rings, therefore, B_a is seldom less than 16,000, and may be taken as high as from 17,000 to 19,000 C.G.S. lines per sq. cm. of actual iron. If the induction be carried further, leakage occurs across the internal opening of the ring, and the loss by hysteresis becomes more important. In bipolar drums, as there are no inside connecting wires, the radial depth of the core is greater than in rings of the same diameter and the induction is seldom carried above $B_a = 16{,}000$. In multipolar drums, the average values of B_a decrease with the number of poles and do not exceed 15,000 in a 4-pole or 13,000 in a 6-pole machine. Since $B_a = \frac{Z_a}{2ab}$, and V_c the volume of iron in the armature $= ab.\, 2\pi r$, where r is the mean radius of the core, it follows from eq. (18) that with a given Z_a the hysteresis loss is $\propto \left(\frac{r}{V_c}\right)^{1\cdot 6} \times V_c \propto \frac{r^{1\cdot 6}}{V_c^{0\cdot 6}}$. Taking the external diameter as fixed by other considerations, it will then be found that an increase in the radial depth reduces the hysteresis loss more than in proportion to the increased volume of iron. But when B_a is confined within the upper limits named above, and is made to decrease with the number of poles, the absolute value of the hysteresis loss is so small that the slight gain in efficiency with a much lower induction does not warrant the increased cost of the iron: hence it is not economical to reduce the induction much below 14,000 in a 2-pole drum, 12,000 in a 4-pole or 10,000 in a 6-pole machine.

CHAPTER XIV

FIELD-MAGNETS

§ 1. **Economical limits to induction in magnets.**—That the number of ampère-turns required per unit length of iron in order to produce a certain induction therein increases very rapidly as the induction is itself increased, is sufficiently shown by the induction-curves of fig. 125. Thus in the case of iron forgings or steel castings, while an induction $B = 16,000$ may be obtained with 20–30 ampère-turns for each centimetre length of the circuit in the iron, it requires nearly treble that number of ampère-turns per centimetre length to obtain the increased density of 18,000. The iron magnet of a dynamo necessarily has some length, and the number of ampère-turns required to propel the flux through that length forms a considerable item in the total number required for the whole magnetic circuit. But the excitation of the field by magnetising coils implies, not only a certain first cost for the copper wire used therein, but also a continuous outlay during the working of the machine; for electrical energy is absorbed by the passage of the ampères through the turns, so long as the machine is at work. Evidently, therefore, there must be some approximate limits within which it is economical to keep the value of the induction in the field-magnet. If the induction has less than the lower of the two limiting values, the iron magnet becomes too heavy and expensive; in a self-exciting dynamo there is the further disadvantage that the magnetism may be unstable (as will be more particularly described in Chapter XVI § 10), and the machine may become difficult to excite at all. On the other hand, if the induction in the magnet be pushed up to a very high figure, the ampère-turns required thereby will bear an excessive proportion to the whole number, and will involve too large an expenditure either in the first cost of the copper or in watt-hours during the working of the machine; the result is that for the same total number of lines to be carried it would be more economical to increase the area of the iron so as to lessen the flux-density, and consequently the ampère-turns required to produce it. Since, when designing a dynamo, the number of lines, z_m, to be carried by the iron of the magnet is approximately known, we are enabled from the above considerations to determine approximately the area which the field-magnet must present for the flow of lines. For magnets of forged ingot iron or cast steel the limits within which it is advisable to keep the induction may be set at $B_m = 14{,}000$, and $B_m = 16{,}500$, a good intermediate value being $B_m = 16{,}000$; while for cast iron the limits are 5500 and 9000, an average value being $B_m = 7000$. If pressed beyond these values, the horizontal divergence of the curves, even with materials of the same class, becomes so marked that any slight inferiority of the metal may lead to difficulties owing to the impossibility of increasing the ampère-turns sufficiently to obtain the desired voltage and speed. A small error in the estimate of the densities will in fact produce a disproportionately great error in the result.

§ 2. **Comparison of cast iron with wrought iron and cast steel.**—From the above average values for the induction, it follows that it is found advisable to magnetise cast iron less than half as strongly as forged iron or steel castings; and consequently the weight of a cast-iron magnet as compared with that of a forged-iron or cast-steel magnet to carry the same number of lines must be roughly as 2¼ to 1. Where considerations of weight and bulk are paramount, it is therefore essential to build up the magnet out of ingot-iron forgings or to cast it in steel. The forgings in their rough state as they come from the steam-hammer may be cheaper than castings of good soft iron, but when machined to the required dimensions their cost is so far increased that they become more expensive per hundredweight than cast iron; yet this increased price is more than counterbalanced by the lesser total weight that is required. In the same way steel castings are considerably more expensive than iron, but not more than twice as costly, so that for the same magnetic work when their lesser weight in handling, lesser freight, etc., are taken into account, the balance of advantage is again in favour of their use rather than of cast iron. On the other hand, castings of complex shape can be produced easily and cheaply in iron, and of such accuracy of dimension that they require but little further machining. Further, even if the form of magnet be simple, it is frequently desired to adapt the supporting framework or bedplate of the dynamo, so as to form a portion or the whole of the magnetic circuit, and such framework is most conveniently made in one single casting. Hence, for small belt-driven dynamos (up to an output of about 5 kilowatts) in which the cost of machining bears the largest ratio to the total cost of the machine, it is very usual to make the magnets of cast iron, and to cast them wholly or partially in one with the bedplate. In large machines an additional set-off which to a great extent counterbalances the higher cost of cast iron is found in the shape of its flux-density curve as compared with that of forged-iron or cast-steel. A reference to fig. 125 shows that the curve for cast-iron over the working range rises in a gentle sweep, while that for forgings or cast steel has a marked point of flexure and then rises very slowly. Hence in the case of a dynamo which has to work over a fairly long range of voltage, if forged iron or steel are employed for the material of the field-magnet and these are worked on the steep part of the curve for the lower limit of voltage, the machine may prove magnetically unstable (Chap. XVI § 10), or if the working point be carried further up, an undue and unexpected increase in the exciting turns may be required to give the higher voltage, should the material prove slightly inferior to the curve. But with cast-iron both the lower and upper limits of voltage can be reached with more certainty, and the designer is less at the mercy of the quality of the material. Again if the dynamo is to be compound-wound, as will be further explained in Chapter XVI, the compounding may be made more accurate if the magnet be wholly or chiefly of cast-iron owing to a similar reason connected with the shape of its B, H curve.

A cast-iron magnet, then, may be simpler and cheaper to manufacture, but must necessarily be heavier and bulkier than one of forged iron or cast steel; furthermore, if the poles are likewise of cast iron it is more expensive in copper, or exciting energy, or both. It will be seen from fig. 125 that inductions of 16,000 and 7000 in forged and cast iron respectively are produced by nearly the same number of ampère-turns per unit length; hence, if the exciting bobbins on two equivalent round magnet-cores have the same length, and are wound with the same number of turns of the same wire, the length of each turn encircling the cast-iron magnet as compared with that encircling the forged-iron magnet is as $\sqrt{2 \cdot 25}$ to 1, that is, as 1·5 to 1: the weight of copper used will therefore be 50 per cent. more on the cast-iron than on the forged-iron magnet, and, further, the energy absorbed in the excitation will be correspondingly increased. If the latter is to be kept the same in the two cases, a further increase of the copper wire on the cast-iron magnet is necessary; while if the weight of copper is to remain the same as on the forged-iron magnet, the rate of loss of energy in the coils must be increased, and to meet this, as will be seen later, the axial length of the bobbins, and consequently the length of the cast-iron magnetic circuit, must be increased. As compared with cast steel, the same holds good, since in permeability there is but little difference between ingot-iron forgings and steel castings. Evidently, therefore, cast-iron should never be employed in any part of the magnetic circuit on which exciting coils are to be wound. As between ingot-iron and cast-steel, the relative advantage of the one or the other resolves itself chiefly into the question of shape. While the former is very suitable for simple forms which can be easily machined out of plain slabs, the latter lends itself to designs of more complex shape with curved surfaces, and thus combines many of the advantages of both forged and cast iron. Steel is commonly employed for large multipolar dynamos; by casting the magnetic system in two or more large pieces, the necessity of joining together a number of forgings is avoided. If an exceptionally high induction such as $B_m = 18{,}000$ should be required in any special case, it is perhaps more likely to be obtained economically in a steel casting than in an ingot-iron forging.

§ 3. **Composite magnets.**—Lastly, it is very usual to find a composite magnet, built up partly of cast iron and partly of forged iron or cast steel, the latter forming the 'cores' on which the magnetising coils are wound. The most common instance of this is to be found in the 2-pole dynamo of fig. 2, where a part of the bedplate forms the yoke, *y*, to which the magnet limbs of iron or steel are bolted. Again, the upper and lower pole pieces of the 'Manchester' field of fig. 192 are usually castings, the magnet cores of iron or steel being let into them at either side. In both cases the portion of the magnetic circuits on which the exciting coils are placed consists of forged iron or mild steel. Large multipolar machines again are frequently made with steel poles and cast-iron yoke-ring, the advantages of economy in excitation and stability of the voltage over a long range with accurate compounding being thus more or less combined by the joint use of the two materials.

Since the permeability of any casting is much reduced if it be hard, it is important in the design of the shape of any casting to avoid introducing into the magnetic circuit any thin flange or narrow, outstanding edge: any such piece of small area is likely to chill when it is being cast, and so to become mechanically hard. Hence all corners and projections should be well rounded, or, if need be, cast massive, and subsequently machined to the required dimensions.

The design in short should be such that there is no disproportionate difference in area between different portions of the same casting, whether of iron or steel; otherwise as it cools unequal contraction takes place, and hollow cavities or sponginess in the casting is difficult to avoid. For this reason in large multipolar machines, it is on the whole better for the massive poles to be cast separately from the yoke-ring.

§ 4. **Comparison of sectional shapes for magnet cores.**—The same economical considerations which determine approximately the sectional area of iron required to carry a given flux, viz., the first cost of the copper coils and the allowable expenditure of energy in magnetising them, will also bear upon the geometrical shape or figure by which the required area is obtained—only, however, in that portion of the magnetic circuit whereon the magnetising coils are actually wound. Beyond the limits of the coils it is immaterial what shape the section of the iron takes.

Since for a given area the circle has the least periphery, the theoretically best section which can be given to the magnet cores where the field-winding is placed is the circle; the length and resistance of the wire for a given number of turns embracing a given sectional area have then their minima values. For this reason, in the 'Manchester' dynamo (fig. 192) or in multipolar alternators similar to figs. 76–78, the magnet cores, *m*, *m*, are usually of circular section. If, however, the magnet cores are of forged iron or cast steel, it is often necessary to joint them into cast-iron pole-pieces or yokes, as in the above cases, or possibly also into the cast-iron bedplate or framework of the machine, which itself forms part of the magnetic circuit. Especially is it necessary to present a proper area of polar surface where the lines pass through the air-gap in order to minimise the reluctance of the interferric space between the iron of the pole and the iron of the armature core; and the cast-iron pole-pieces employed for this purpose, if not of large sectional area, may themselves perhaps increase the total number of ampère-turns required owing to their being magnetically inferior to forged iron. Instead, therefore, of a circular section it may be more economical to give the magnet forgings a rectangular shape, which may be continued up to the armature, and so admit of being bored out to embrace the core, as in the case of the single horseshoe magnets of figs. 2 and 190. Or, again, an oval or trapezoidal section of magnet core may be arranged so as to approximate to the required polar area at the air-gap, as in the alternators of figs. 76 and 79.

Next to the circle, the square must be ranked as the most economical form of section, its periphery being 1·13 times that of the circle containing the same area. Finally, the larger the ratio between the length and breadth of any rectangular figure, the more uneconomical becomes the shape as regards length of wire in each turn encircling it. If the two sides of the rectangle be as 2 to 1, the periphery becomes 1·2 times that of the equivalent circle, while for a ratio of 3 to 1 it increases to 1·3. When wound, therefore, with the same number of turns and the same size of wire, the weight of copper employed will be increased respectively 20 and 30 per cent. above that of the circular magnet, and the resistance being similarly increased, the rate of expenditure of electrical energy to produce the same excitation will likewise be increased by 20 and 30 per cent. If the same efficiency is to be retained in all cases, the area of the increased length of copper wire must also be increased, making in all, for a ratio of 3 to 1, an increase in the amount of copper of nearly 70 per cent. A larger ratio, therefore, is very uneconomical, and is seldom required in ordinary designs.

§ 5. **Length of the magnet-core.**—The sectional area and shape of the magnet-cores having now been considered, the question of the length of the magnet remains to be dealt with. Since the field-magnets of commercial dynamos are now invariably electro-magnets, this length must necessarily be such as to admit of the requisite magnetising coils. It has been said that the maintenance of the excited field during the working of the dynamo requires the continuous expenditure of energy at a certain rate in watts, such expenditure being simply that due to the passage of the magnetising current through the electrical resistance of the wire, and appearing as heat in the coils. This heat must be carried off by radiation, convection, and conduction in order to avoid such a rise of temperature in the coils as will endanger the insulation of the field-winding; and this is secured by so disposing the coils that they present a considerable cooling surface to the air. As soon as the dynamo is set to work, the temperature of the coils begins to rise above that of the surrounding air: this gradual rise continues until, finally, the rate at which the heat is generated in the coils is balanced by the rate at which it is dissipated under the combined action of radiation, convection, and conduction. After reaching this limit the temperature of the coils remains stationary so long as the conditions are unchanged. The effect of radiation, &c., being dependent on the entire cooling surface of the coils, it is evident that the rise of the temperature of the coils above that of the surrounding air may be kept within any required limit by allowing a due proportion of cooling surface for each watt expended in the coils. The effective cooling surface of a given coil wound round an iron magnet will depend on a large number of conditions, difficult to calculate, since the mass of iron itself helps to dissipate the heat by conduction, and, again, the depth of a winding of many layers will largely affect the temperature of the turns forming the middle layers. Roughly speaking, however, the comparative cooling power of a coil may be taken as proportional to its external surface, reckoned as the product of its external periphery multiplied by its length, the cooling influence of the end-flanges or depth of winding in the coil being neglected. Thus, if the over-all dimensions of one such coil, as is shown in fig. 2, are a width of 14″, a depth of 8½″, and a vertical height or length of winding of 9½″, the external surface, $45'' \times 9\frac{1}{2}'' = 428$ square inches, will be a measure of its cooling power, and this may in our present connection be reckoned as its cooling surface. Having thus decided on the method upon which the cooling surface is to be estimated, experience will guide us to a certain ratio which the cooling surface must bear to the watts so that the rise of temperature of the coil may not exceed a certain assigned limit. Of the values for this ratio more will be said in Chapter XIX. If the rate in watts at which energy may be lost in the field winding is approximately determined by the efficiency required in the machine, the length of the magnet coils must be such as to provide a suitable amount of cooling surface per watt dissipated in them. More generally, however, the settlement of the number of watts which may be regarded as an allowable rate of dissipation in the field-coils will be a matter of compromise between the efficiency of the machine and the first cost of the copper wire; since the smaller the number of watts expended in the field-coils, the greater the weight of copper wire required. Further, the total number of ampère-turns required for the field excitation is not exactly known until the length of the magnetic circuit has been determined; experience, therefore, alone can furnish guidance for a first approximation to the required length of coil, and a method of trial and error must be resorted to in order to exactly determine the ampère-turns on the field, the loss of energy in magnetising the field, the dimensions of the coils and their weight of copper. When, however, the designer has assigned an adequate length to those portions of the magnetic circuit whereon the coils are to be placed, he will complete the magnetic circuit in as direct a line as possible; since any length beyond the minimum thus required unnecessarily adds to the reluctance, and is therefore uneconomical in both iron and copper.

§ 6. Types of field-magnets. A (1) Bipolar single horseshoe.—The most compact form which the single horseshoe-magnet can have is that in which it is bent round into a more or less circular shape, as in fig. 43, the magnet-coil being wound uniformly all over its curved length. The length of the magnetic circuit in the iron is then a minimum, and the form is both economical in iron and light in weight. It is, however, expensive to wind, and it is therefore customary to concentrate the winding into one or more straight coils: these may be wound directly on to the iron magnet-cores, the wire being of course entirely insulated from the iron; but more often, for convenience of manufacture and to facilitate their removal, they are wound separately and are subsequently slipped on over the magnet. We thus arrive at the simple *C-shaped magnet* of fig. 189, formed of two curved castings screwed together at one faced joint under the single coil. The same form may also be used with a forged-iron core let into two curved cast-iron pole-pieces with a consequent saving in weight of both iron and copper; or it may be arranged with the coil vertically above the armature. For bipolar machines, however, *the single horseshoe with straight limbs* and two magnet-bobbins is by far the most widely adopted form. For belt-driven machines up to moderate sizes, a composite magnet with forged-iron or steel limbs and a cast-iron yoke forming part of the bedplate has been much used (fig. 2); since the armature and pole-pieces are above the yoke, it may be called the 'overtype' single horseshoe, in distinction to the 'undertype' magnets of figs. 190 and 191. The advantage in these latter of placing the armature under the yoke is that the axis of rotation is kept low, and the necessity of high pedestals or standards to carry the two bearings is obviated; hence this form lends itself admirably to direct coupling to

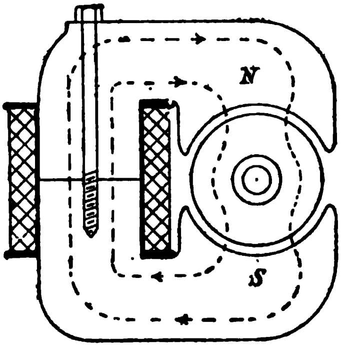


FIG. 189.—C-shaped single horseshoe.

the engine crank-shaft. Since the whole machine is usually supported on a cast-iron bedplate, it is necessary that the magnet-poles should be carried on feet or brackets of non-magnetic material, such as gun-metal or zinc (*n n*); otherwise the magnetic circuit would be directly closed by the iron bedplate, and many of the lines of induction would be shunted across from pole to pole through the bedplate, instead of passing through the armature as required. In fig. 190 the whole of the magnet is formed of forged iron or steel slabs of rectangular cross-section bolted together at the top. The vertical depth of the non-magnetic base must be such as to avoid excessive leakage of the lines into the bedplate, and should therefore be roughly proportional to the length of the interferric air-gap. A common proportion is for the depth of the base to be about eight times the length of the single air-gap, which with an armature of 12″ diameter leads to a depth of from 4–5 ins.

In fig. 191 the upper part of the magnet is a single casting of steel which can be lifted vertically away from two lower C.I. pole-pieces, these being attached by side-brackets of gun-metal to the extended baseplate of the steam-engine.[1] The length of each of the two coils on the single horseshoe-magnets above described varies usually from 1 to 1·5 times the diameter of the armature.

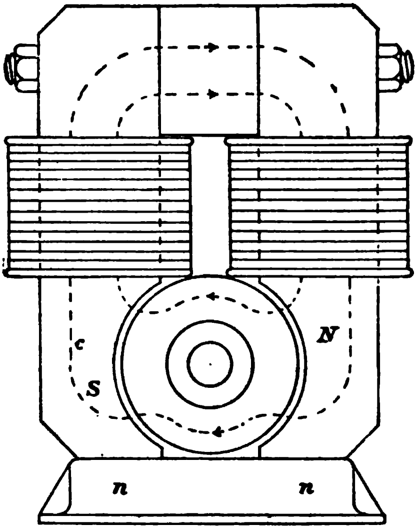


FIG. 190.—Undertype single horseshoe with rectangular magnet-cores.

§ 7. **Types of field-magnets.** A (2) **Bipolar double horseshoe.** A further modification consists in dividing the single magnet into two portions, so arranged that their like poles abut on each other, and have a common polar face (figs. 192 and 193). Each

---

[1] Parker, *Brit. Patent*, 21,821, 1894.

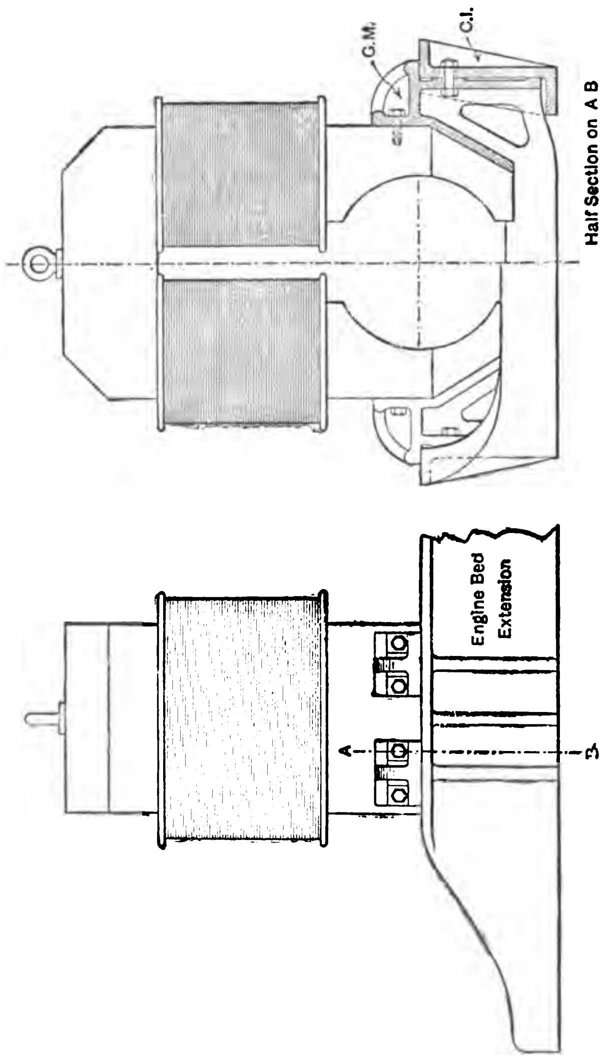


FIG. 191.—Undertype single-horseshoe magnet with G.M. cradle.

half of the double magnet carries half the total number of lines that pass through the armature, the right-hand magnet carrying the lines that pass through the right-hand half of the armature core, and the left-hand magnet those that pass through the left-hand half. Both halves of the field require, therefore, to be exactly similarly magnetised. The '*Manchester*' *field* of fig. 192 is in reality a pair of C-shaped magnets (like that of fig. 189) placed together, each magnetic circuit requiring its own magnetising coil, which is placed on the yoke, and these coils being wound so as to produce 'consequent' poles at N and S. In the same way, the *double horseshoe magnet* of fig. 193 consists of two magnets, each similar to that of fig. 190 with two magnetising coils on its limbs, but carrying half the total number of lines which pass through the armature.

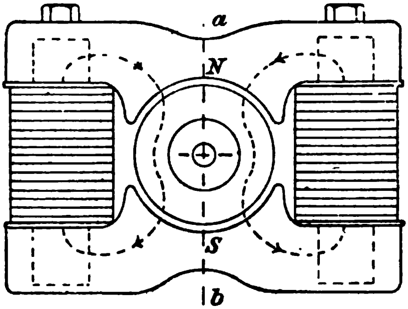


FIG. 192.—Double horseshoe two-pole field, 'Manchester' type.

The advantages gained by such 2-pole double-magnets are complete symmetry of the field, and in certain cases avoidance of the necessity for massive and heavy pole-pieces; the leakage of lines by paths other than that through the armature is, however, considerably greater than in the single magnet, so that for the same size of armature and the same total flow of lines through it, the cross-sectional area of each double-magnet limb must be more than half that of the single magnet. On calculating the respective weights of iron in two equivalent fields, it will be found that the single magnet is about 1·3 times heavier than the double magnet, chiefly owing to the thin section which is allowable in the poles of the latter on the line *a b*. Against this advantage, however, must be set the fact that the exciting power on *each* of the two circuits of the divided magnet must be the same as that on the single magnet, and that therefore the double magnet is expensive in copper. If the magnet cores of the two dynamos to be compared are both circular (as in figs. 192 and 189), and if taking into account the increased number of leakage lines the area of each core in the double magnet is 0·55 of that of the single magnet, the increase in the length of wire required by the double magnet over that of the single magnet will depend upon the ratio of the depth of the winding to the diameter of the magnet-core, but under the most favourable condition of a very thin coil it cannot be less than 46 per cent., and the weight of copper is also increased in the same proportion. If the diameter of the single magnet-core be 5 inches and the depth of winding in either case be 2 inches, the increase in the length and weight of wire amounts to more than 60 per cent. If the magnet limbs in both cases are formed of rectangular slabs (as in figs. 193 and 190), and the two dimensions of the cross-section in the single magnet are as 2 : 1, the ratio in the case of the double magnet would be so very uneconomical that it would become advisable to increase the diameter of the armature and to shorten its length in order that the increased weight of wire might fall even within the percentages named above.

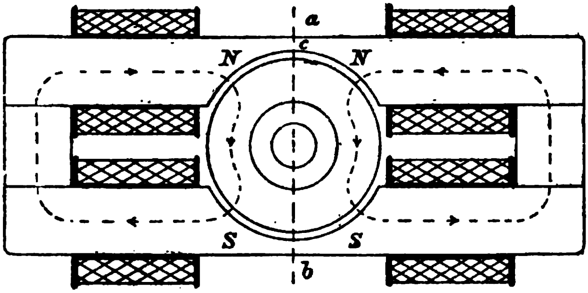


FIG. 193.—Double horseshoe two-pole field.

The rate in watts at which energy is dissipated in the coils of the double magnet will also be correspondingly increased by the same amount as the length and weight, *e.g.*, by 40 to 60 per cent. in the case of the circular cores; the square inches of cooling surface of the bobbins are, however, increased in a slightly higher degree, so that there will be no detrimental effect so far as their heating is concerned. But if the same efficiency is to be attained in the two machines, the weight of wire must be increased still further: this may, however, again be followed by a proportionate shortening of the length of the coils, and a consequent shortening of the magnetic circuits, the net result being that the double magnet may be light in weight, but is very expensive in copper, and must therefore rely for its advantage chiefly on its complete symmetry of field distribution, the importance of which will be again alluded to in § 15.

Another form of divided field is shown in fig. 194, and is adapted to flat-ring machines; the like poles, which face each other on either side of the armature, may or may not be connected together by pole pieces arching over the core, and in any case may be reckoned as one polar surface. The lines which emanate from one such joint polar surface and pass into the armature core are divided into two groups, one on either side of the armature: the circuit of each group requires to be similarly magnetised by its own coils. If an overarching pole-piece unites the side poles, the lines enter into or issue from the core in two planes, and it becomes difficult to laminate the core so as effectually to eliminate eddy-currents; hence such extensions of the polar area have in practice been discarded, and the magnetic circuits are entirely divided.

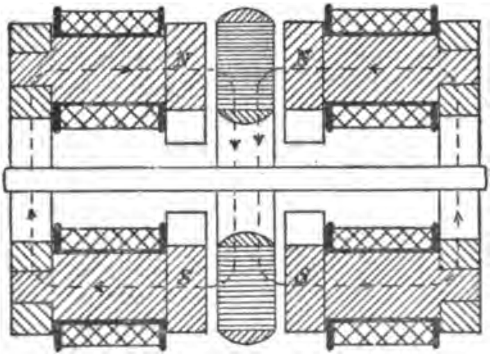


FIG. 194 —Two-pole divided magnet for discoidal rings.

§ 8. **Types of field-magnets.** A (3) **Bipolar iron-clad.**— In fig. 195 we revert to a single-magnet dynamo. In this type the yoke is divided, and passes above and below the armature if the poles are horizontal, or on either side if the machine is turned so that the poles are vertically above and below the armature. It has been called an *iron-clad* dynamo, since the field-magnet incloses the magnetising coil. With a single exciting coil as in fig. 195(i) the leakage across the air from the polar edges and out of the armature flanks becomes excessive unless the yoke be kept well away; except, therefore, in very small machines it is better to divide the excitation between two coils as in fig. 195. The distribution of the field then becomes symmetrical, and the magnetomotive force acting across the air path between the pole edge and yoke is halved; precisely on a line midway between the poles, there is no leakage between armature and yoke since both are there at the same magnetic potential.

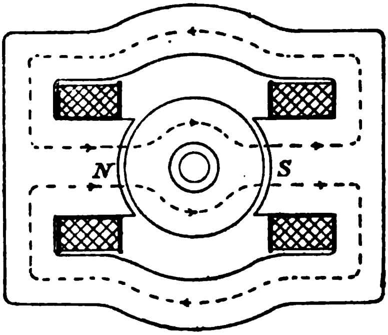


FIG. 195.—Two-pole 'iron-clad' dynamo.

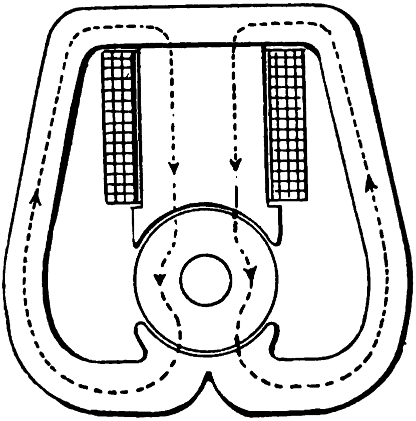


FIG. 195(i).—Two-pole iron-clad magnet with single exciting coil.

§ 9. **Types of field-magnets.** (B) **Multipolar.**—We now turn to multipolar magnets and will derive them from their bipolar forms, taking these in the reverse order. From fig. 195 is easily derived the four-pole magnet of fig. 197, and this type is by far the most common for any number of poles from four upwards (cp. fig. 76). The yoke ring is usually divided

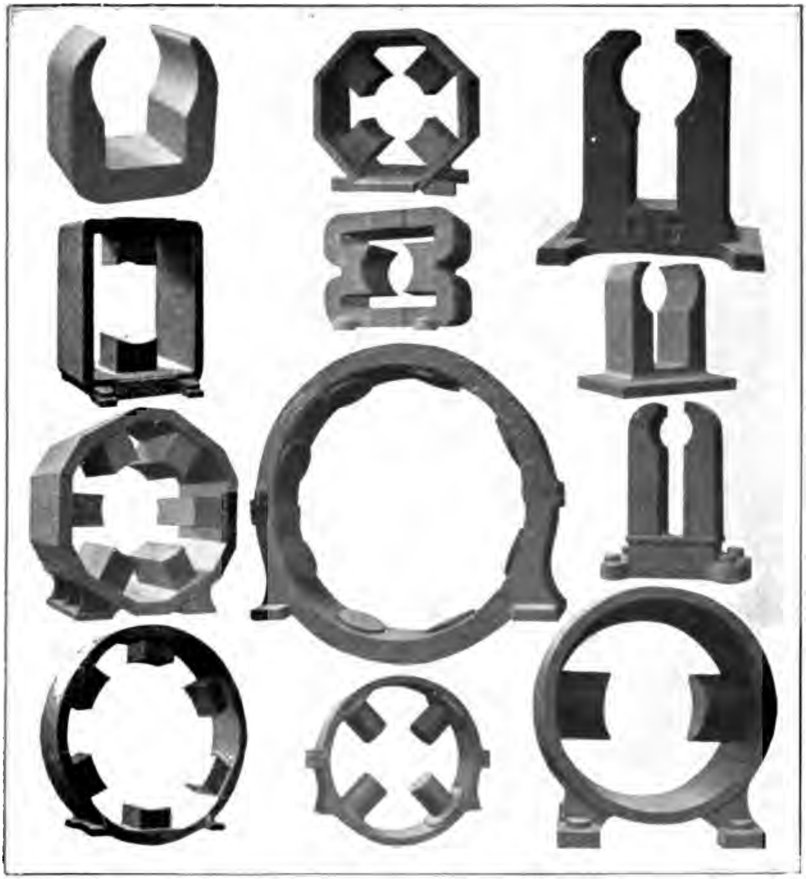


FIG. 196.—Types of field-magnets, cast in steel by Otto Gruson & Co. of Magdeburg-Buckau.

[*To face p.* 398.

on the horizontal diameter to enable the upper half to be removed vertically for examination or removal of the armature. For small machines the yoke may be of cast iron, since the section of metal required in cast steel makes the depth of the ring somewhat thin in external appearance even if it be strong enough mechanically. The separate pole-pieces of cast steel, laminated or solid as the case may be, are either fastened to the ring by screws or are cast into the yoke. The proportion of leakage in this type is but small, and it is economical in both iron and copper, especially if the section of the poles on which the bobbins are placed is circular. The shape of the yoke ring may be a polygon with a number of sides depending on the number of poles, or it may be made circular even with four poles. From fig. 197 it is but a step to the converse case, in which the yoke is internal and the magnet cores project radially outwards (fig. 77).

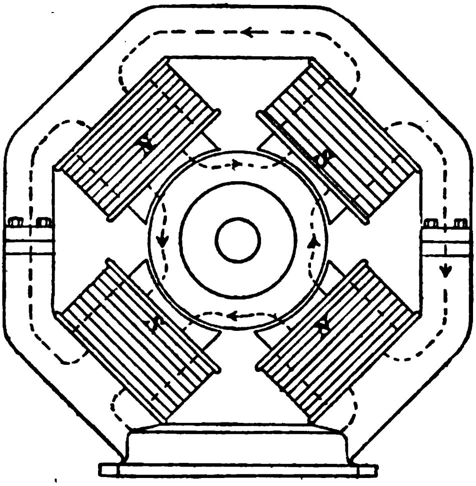


FIG. 197.—Four-pole dynamo.

Another form of four-pole field may also be derived from fig. 195 if the two sides are brought in so as to give two more polar areas, and the direction of the winding of one coil is altered so as to produce two consequent poles on the vertical diameter (fig. 198). The type is, however, not advantageous for continuous-current dynamos since the leakage is considerable.

The principle of the divided magnetic circuit is often introduced in multipolar fields. Thus figs. 78 and 199 show the multipolar form suitable for the discoidal ring, while from fig. 193 may be evolved the 6-pole form of fig. 200. The latter type is identified with the name of Thury, and is composed of wrought-iron slabs bolted to wrought-iron pole-pieces and is therefore light in weight but expensive to manufacture. In all such cases

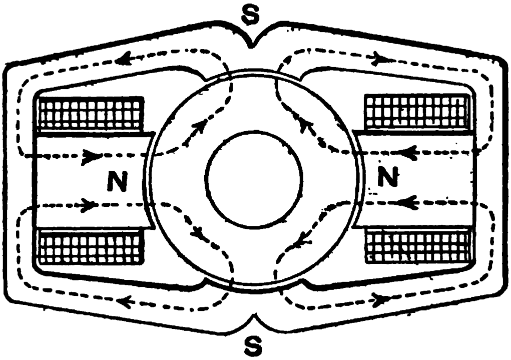


FIG. 198.—Four-pole iron-clad magnet with consequent and salient poles.

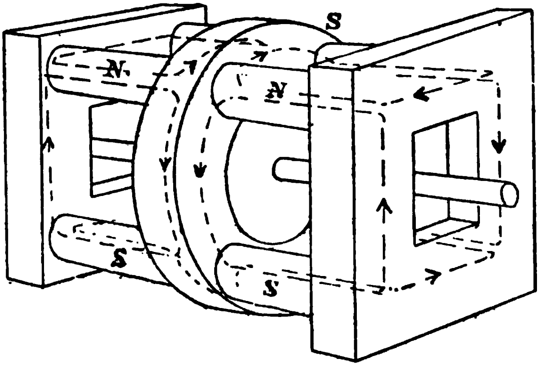


FIG. 199.—Four-pole dynamo with divided magnetic circuit.

of divided magnetic circuits, where the flux of each pole-face is divided before passing through the magnetising coils, the weight of copper on the field must necessarily be considerably more

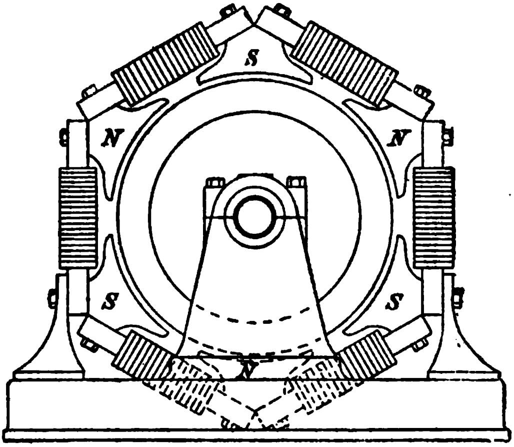


FIG. 200.—Thury six-pole dynamo.

than in machines where the coils are wound on the poles themselves.

An entirely different type of 4-pole field with only two horseshoes is shown in fig. 201 and may be derived from fig. 193 if

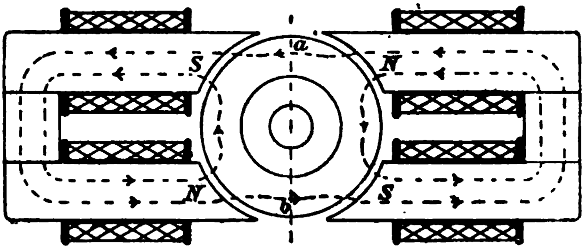
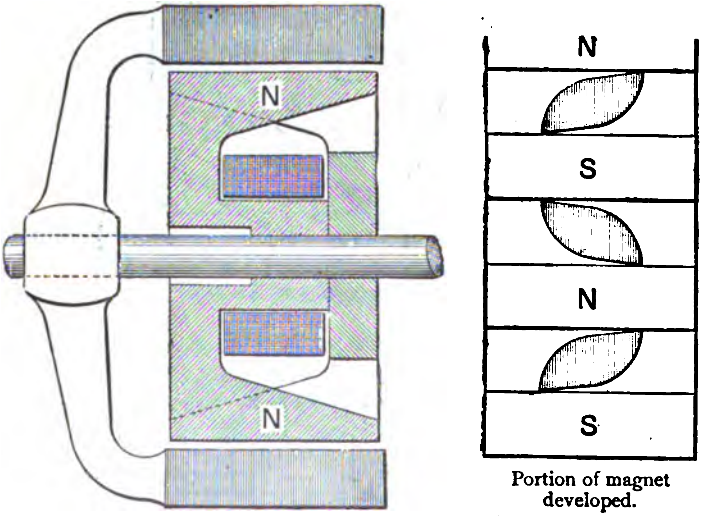


FIG. 201.—Four-pole field with two single horseshoes.

the pole-pieces are imagined to be divided and the direction of the winding on one side to be reversed. It is usually constructed of forged slabs, the magnets being laid horizontally on their sides, and supported by gun-metal brackets at a suitable height above a common bedplate. The paths of the lines are shown dotted and it will be seen that the group of lines issuing from one pole divides, part returning at once through the armature core to the neighbouring pole of the same magnet, and part passing through the armature onwards to a pole of the second magnet.

Lastly, from fig. 189 may be derived a multipolar form in which there is only one magnetising coil although there are several fields. Such a type may be combined with a multipolar continuous-current winding if the overarching claws be made to interlace (fig. 202), but its use is confined to very small machines. The single exciting coil is in fact best adapted to homopolar or inductor alternators, as shown in figs. 82 and 88, where the two end-castings are branched out to give the several fields, or a number of separate pole-pieces are fastened to a common central core, all the N poles being on one side, and all the S poles on the other side.



Portion of magnet developed.

FIG. 202.—Multipolar field with single exciting coil.

§ 10. **The design of field-magnets.**—That the field-magnet and armature core of any dynamo should form approximately a closed magnetic circuit, only broken by the two interferric air-gaps, has already been stated at the end of Chap. III. and only needs to be again enforced in our present context. The aim of the dynamo-designer being to produce a good magnetic circuit, economical in its manufacture and in its working, the requirements which he has to consider may be summed up under the heads of compactness and mechanical strength, suitable proportion of the areas of cross-section at different parts of the circuit, small proportion of the leakage relatively to the useful lines, and lastly symmetrical distribution of the field; while, in certain special cases, lightness of weight may become of first importance.

How far the requirement of compactness is met in the usual designs of field-magnets will be evident from the preceding figures; in older designs, originated when the principles of the magnetic circuit were not so fully understood as now, long columnar magnets were frequently adopted, and great use was made of divided magnetic circuits with consequent poles. Experience, however, has shown that the simple U-shaped magnet with comparatively short limbs of massive size stands pre-eminent in point of simplicity, economy, and compactness, while the double-magnet fields of figs. 192–4 occupy more room, and require more winding and more copper. For large outputs or for alternators it becomes necessary to adopt a multipolar field; but again on the score of economy the simple types of figs. 197, 76, and 77, must be given the preference if the single-exciting-coil form be excepted.

§ 11. **Mechanical strength of field-magnet frames.**—As regards mechanical strength, it must be remembered that between the armature core and the polar surfaces of the magnet a very considerable attractive force acts; thus apart from the torque exerted on the stationary part of the dynamo by the rotating part, when current flows through the armature, the magnet requires to be rigidly and firmly supported, so as to preserve the requisite clearance and avoid any liability of the poles to collapse upon the armature. The attractive force acting between each square centimetre of a pair of divided surfaces is proportional to the square of the number of lines that pass through from one surface to the other, *i.e.*, to the square of the induction in the gap, B_g; and for a field of lines uniformly distributed over an area of A square centimetres the total pull[1] in pounds between the two surfaces is

$$\text{P} = \frac{\text{B}_g{}^2\text{A}}{8\pi \times 981 \times 453{\cdot}6} = \frac{\text{B}_g{}^2\text{A}}{11{,}183{,}000}$$

acting along the direction of the lines, and tending to shorten their length. If A be in square inches

$$\text{P} = \frac{\text{B}_g{}^2\text{A}^{\square''}}{1{\cdot}735 \times 10^6} \text{ pounds} \quad . \quad . \quad . \quad . \quad (25)$$

while if B_g is itself reckoned in lines per square inch

$$\text{P} = \frac{\text{B}_g{}^2\text{A}^{\square''}}{72{\cdot}134 \times 10^6} \text{ pounds} \quad . \quad . \quad . \quad . \quad (25a)$$

A pull of this nature exists, for example, between each half of the armature and the opposite pole-face in the dynamo of fig. 2, and has to be withstood by the mechanical rigidity of the magnet as a whole. A similar pull must in fact exist in every case between the armature, whatever may be its type, and the several poles which surround it. A distinction is, however, to be drawn between such types of field-magnets as have a geometrical shape which is symmetrical about the armature, *e.g.*, figs. 192 and 195 and 197, and those which are not geometrically symmetrical, such as fig. 189 or 190. In the former the effects of the several poles still remain and must be withstood alike by armature core and magnet, but their combination when the armature is exactly central within the bore leaves no unbalanced pull in any one direction. Such a condition is to be regarded as the normal state, but in many cases it is not easily secured; the consequences of departure from it will be further considered under the head of a closely-connected subject, viz., the symmetrical distribution of the field in §§ 15 and 16, but the mechanical effects that may be present even with a symmetrical type of magnet need here to be further detailed. So long as the armature is central, and the flow of lines on the one side of the core is similar to the flow on the opposite side, the pulls will be in exact balance, and there will be no tendency for the magnet frame to move as a whole. But should it shift owing to mechanical vibration while the machine is running, the air-gap or gaps will be increased on the one side and decreased on the other side; the fields will be correspondingly reduced and increased, and the result is that for a very small displacement an unbalanced pull of considerable magnitude is set up tending still further to displace the magnet system. The latter must therefore be so bolted down as to retain the due clearance on either side of the armature and any difference in the strengths of the fields must be avoided, since, even if the magnet be constrained from moving, the unbalanced pull acting on the armature may easily reach such an amount as to bend its shaft. Especially is this likely to be the case in a short-air-space dynamo, wherein any displacement will bear a large proportion to the total air-gap. In a bipolar field displacement of the armature along the centre line or axis of the poles will produce but little effect, as compared with displacement along the neutral line dividing the interpolar gap between the two poles. But in a multipolar field, although displacement along the neutral line has the greatest effect, yet any displacement along the centre line of a pole will produce considerable resultant tension, the two effects becoming more nearly equal the greater the number of poles. In any case, as the bearings of the armature shaft of such a machine as fig. 197 wear and allow the core to sink slightly, the air-gaps at the lower poles will be shortened, and those at the upper poles lengthened, thus producing on the whole stronger fields in the lower half than in the upper half. The mechanical result will be an increase in the bending moment on the shaft, and in the pressure on the bearings, even if the magnet itself be sufficiently strong to be practically unaffected.

---

[1] This equation is strictly applicable only when the gap is at right angles to the direction of the lines of induction, and when the opposing surfaces are large as compared with the distance between them, so that the induction in the gap is appreciably uniform and there is no side leakage (cp. Du Bois, *The Magnetic Circuit*, chap. vi. § 102); it is, however, sufficiently accurate for the ordinary purposes of dynamo calculations.

Apart, however, from any magnetic reasons, since the field-magnets often form part of the supporting framework, the whole structure must be sufficiently rigid to withstand any mechanical stresses or vibration due to the working of the machine, and all jointed surfaces must be securely bolted or screwed together. Absence or fewness of joints in the magnetic circuit has also been frequently regarded as desirable from magnetic reasons, and it is therefore necessary to consider shortly what is the exact effect of a joint on the magnetic reluctance of an entire circuit.

§ 12. **Influence of joints in the magnetic circuit.**—If a bar of iron be divided transversely in two, and the two pieces be then placed in contact and magnetised, it is found that the total reluctance of the iron is slightly increased owing to the influence of the joint, and this increase in the reluctance may be expressed as that of an air-gap equal in area to the cross-section of the iron, and having a certain width depending on the exact conditions of the experiment. If the two surfaces of contact are such as are produced by ordinary planing or other machine-tools, and the joint may be regarded therefore as, comparatively speaking, rough, the width of the equivalent air-gap is equal to about 0·005 cm., which is only reduced to 0·004 cm. when the two pieces are squeezed together under a very considerable pressure. If, however, the two surfaces are carefully scraped up, so that they are true planes, the equivalent width may be taken as 0·0035 cm. when the joint is not compressed, although under the influence of considerable compression the effect of the joint may be made almost entirely to disappear. It would seem that the increased reluctance due to a joint is in either case due partly to the interposition of a thin film of air between the surfaces, and partly to an alteration in the molecular structure of the iron at the joint, which decreases its permeability.[1] It will be seen, however, that the effect even of a comparatively rough joint, although appreciable, is not of great magnitude, and is especially unimportant in the case of the magnetic circuits of dynamos, which necessarily have air-gaps beside which the equivalent air-gaps of joints may be regarded as negligible. The conclusion to be drawn is that it is not worth while to use forgings of complicated shape, and therefore expensive to manufacture, in order to avoid joints in forged-iron magnets, nor is it worth while to spend time and labour in carefully facing or scraping up any joints which it is decided to introduce into the design of a field-magnet. Absence or fewness of *joints in a magnetic circuit* must therefore be regarded as a minor consideration from an electrical point of view, and is only of importance as bearing on the mechanical strength of the design, and as lessening the first cost of the machine to the manufacturer.

---

[1] Vide *Magnetic Induction in Iron and other Metals* (Ewing), § 165, and Du Bois, *The Magnetic Circuit*, chap. ix. §§ 151–2.

§ 13. **Proportioning of areas at different parts of magnetic circuit, and in different materials.**—The question of joints where different portions of a magnet, possibly one of steel and the other of cast iron, are united, leads us naturally to the third question—of the suitable proportioning of the sectional areas at different parts of the magnetic circuit. It is highly important that the lines of induction should never be 'throttled,' as it is termed, by having to traverse in the course of their path a portion which is of insufficient area; any such 'throttling' implies a high induction, and therefore a large number of ampère turns for each inch in length of the contracted portion. If the disproportion between the areas at different parts of the magnetic path be carried to excess, the ampère-turns required to pass the lines through the area of the narrowest part may form so large a proportion of the whole number as almost to nullify the advantage of the larger section at other parts. In magnets composed throughout of iron of the same quality the area of section should be approximately identical at all parts of the circuit which carry nearly the same total number of lines; and, generally, in any portion of a circuit of the same magnetic quality, the minimum number of ampère-turns is obtained if it be worked at a uniform induction. Hence in designing, the distribution of the lines must be carefully studied. Thus, for example, in the forged-iron magnet of fig. 190, a reduction of section is only allowable at the pole-pieces after a certain proportion of the lines have already passed into the armature; even then the sectional area of the pole-piece at *c* must be quite ample enough to carry half the total number of lines, since otherwise their path to the lower half of the armature would be throttled. When, however, we compare the single magnet with the double magnets of figs. 192 and 193, it will be seen that the area of the pole-pieces at *c* may be very much thinned down; in fact, if consistent with mechanical strength, the two horseshoes may even be separated along the line *a b*, and made to merely abut on each other over an area of very small depth; no flux need pass through the joint, since each magnet will separately feed its own half of the armature with lines.

Next, when the lines pass from one material into another, as from forged iron or steel into cast iron, or *vice versâ*, as in figs. 2, 192, 78, the areas of the two parts should be so proportioned that both are worked at a suitable induction: hence it follows from § 1 that the area of the cast-iron should be more than double that of the forged-iron or steel portion. Further, the nature of the joint should be such as to give ample area of contact, and afford an easy passage from the one metal into the other: this is secured in the dynamo of fig. 192 by sinking the magnet-cores into accurately turned holes in the cast-iron yokes, and drawing the two firmly together by screws. So also, in the dynamo of fig. 197, the cast-steel cores may be drawn tightly up against machined facings on the cast-iron yoke by means of screws, or they may be cast in, suitable precautions being taken that the cast metal is not chilled as it flows round the cores and that intimate adhesion of the two is secured.

In applying the above rules it must be borne in mind that 'throttling' is a relative term, and under certain circumstances a high induction and somewhat contracted area of certain parts may be legitimate; especially is this the case with those portions of the magnetic circuit which are actually encircled by the magnetising coils. It has been stated that the induction in an armature of the ring type may be given a high value, since by so doing the length of each turn of the copper wire, and consequently its resistance and weight, are lessened, and precisely the same consideration applies to the magnet-cores on which the field ampère-turns are wound. On the other hand, portions of the magnetic circuit which are not overwound with copper, especially if of cast iron, which is comparatively inexpensive, may often be advantageously given somewhat lavish proportions and worked with a low induction. Thus, in the dynamos of figs. 194 and 197, it is economical to keep the induction in the magnet-cores at a fairly high figure; the single cross-area of a cast-iron yoke-ring should then be more than equal to that of the magnet core; the induction in the former will thus be less than half that of the latter, since the lines divide after passing through each core, half only passing in either direction round to the next pole. As, however, the material of the yoke is cast iron, and in this part gives strength and solidity to the whole structure, it may be economical to even enlarge its area to considerably more than equality with the area of the wrought iron, so as to further lessen the induction and with it the ampère-turns on the field. In many other cases the yokes and pole-pieces, not being surrounded by wire, may advantageously be of comparatively large area.

§ 14. **Magnetic leakage.**—In all calculations for the purpose of determining the proper area for any portion of the magnet, the question of *leakage lines* must not be left out of sight when estimating the induction. As was pointed out in Chap. III. § 14, lines of induction will flow between any two points or surfaces between which there exists a difference of magnetic potential, and consequently, in all dynamos, there is a considerable number of lines that leak across from one pole to another without passing through the armature core, where alone their presence would be of use. Thus, in the 2 pole dynamos of figs. 2 and 190, lines will leak across from one pole-piece to the other, from one magnet limb to the other, and even from one part to another part of the same limb: a few of these paths are traced out in figs. 203 and

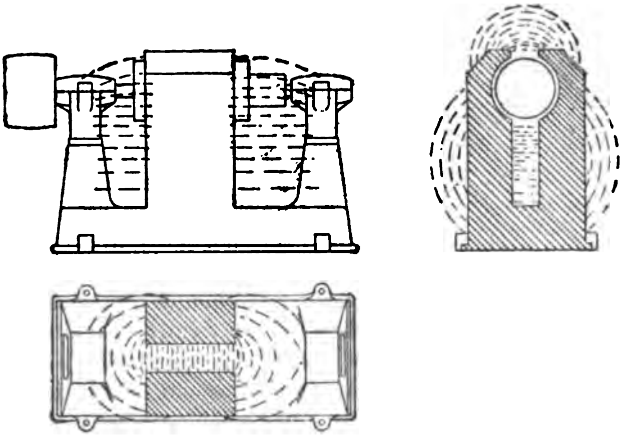


FIG. 203.—Magnetic leakage in overtype bipolar dynamo.

204. The presence of such leakage lines in the air is shown in a disagreeable manner by the magnetic effects which they produce in watches, electro-magnetic measuring instruments, or any piece of iron held in the vicinity of an excited dynamo.

If the magnet is to be worked with uniform induction throughout its length, its area of cross-section should, strictly speaking, be continually varied as lines leak into or out of it. Since, however, the chief fall of magnetic potential occurs over the two air-gaps and the armature between them, it will be approximately sufficient (cp. Chap. III. § 14 and Chap. XV. § 8) to regard all the leakage lines, ζ, as flowing between the one pole-piece and the other, and then to add them to the lines through the armature in order to estimate the total number of lines which pass through the magnet. The induction, therefore, in the magnet will be reckoned as equal to the sum of ζ and z_a divided by the area of the magnet; and, apart from any difference in the permeability of the iron of the magnet and that of the armature core, the area of the former should be larger than that of the armature.

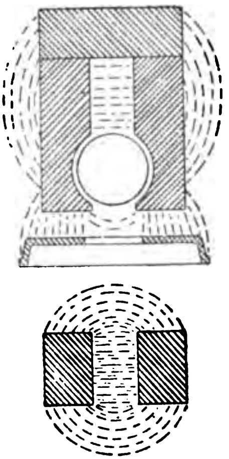


FIG. 204.—Magnetic leakage in undertype bipolar dynamo.

Since it is the aim of the designer to reduce ζ to a small proportion of the total number of lines z_m passing through the magnet, he will avoid bringing any large mass of iron into close proximity to the pole-pieces. In especial the magnetic leakage will be largely affected by the nearness or otherwise of the bed-plate, bearings, or the flywheel of the machine, since these by their comparatively high permeability lessen the total reluctance presented by any leakage path. The effect of the bedplate has been already pointed out, in the case of the undertype single horseshoe, as rendering its leakage greater than in the corresponding overtype machine. In the Manchester field it is very usual to support the bearings of the armature shaft on pedestals formed by extensions from the lower cast-iron pole-piece, and due to this there is considerable leakage from the bearings into the upper pole-piece. Even in the overtype single horseshoe, where the bearings are supported from the yoke of the magnet, lines leak from the sides of the pole-pieces into the pedestals and bearings on either side of the armature. For the same reason, in the single magnet of fig. 195 the over-arching halves of the yoke must be kept at some distance away from the armature and poles in order to avoid considerable leakage. When this type is adopted for multipolar machines, as in fig. 197, the leakage from the armature into the yoke becomes less important, but the close neighbourhood of the poles and yoke leads to considerable leakage across the air from one to the other (fig. 205); and this is almost inevitable in any case where there are numerous poles of alternate sign presented to the armature.

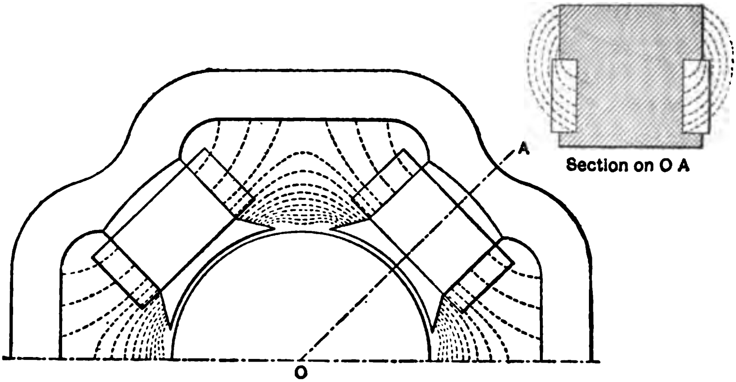


FIG. 205.—Leakage of multipolar magnet.

§ 15. **Symmetrical distribution of field in air-gaps of bi-polar machines.**—Lastly we come to the symmetrical distribution of the field: by this requirement we are to understand that if a line be drawn through the centres of a pair of fields existing between the armature and two separate polar surfaces, and each of the fields be thus divided into two halves, the distribution of the lines of induction in the two halves on the one side of the dividing line should be as nearly as possible similar to their distribution in the two halves on the opposite side. Thus in fig.

48 it is requisite that the two halves, *a* and *c*, of the fields above the line N S should be symmetrical with the two halves, *b*, *d*, below that line as regards their distribution of flux. It is not implied that the two halves, *a*, *b*, of one and the same field must be identical; and, in fact, in the working of the dynamo (as will be explained in Chap. XVII) this cannot be secured, for at the corner of *b* the lines will be more densely crowded together, and the field will be stronger than at *a*. But granting this to be so, what is implied is that the field at the corner *c* should be stronger than that at the corner *d*, to the same extent and in the same way as *b* is stronger than *a*, so that the entire distribution above and below the dividing line is symmetrical. The importance of this is due to two reasons, one mechanical and the other electrical. Taking first the case of the overtype single-magnet dynamo (fig. 2), it is evident that there is a tendency for the lines of induction to crowd into the lower half of the armature, since the path which they then follow is shorter and of less reluctance than that of the lines which pass on into the upper half of the armature. Especially will this be the case under two conditions: first, if the sectional area of the pole-piece across the neck at S or N be too small, and the passage of the lines to and from the upper half of the armature be thereby throttled; or, secondly, if the armature be of large diameter, and the length of iron traversed by the lines flowing to and from the upper half of the armature be therefore considerably greater than that traversed by the lines of the lower half. The general direction of the lines through the air-gap being more or less radial (*vide* fig. 48), the attractive force of each half-field upon a quarter of the armature may be regarded as concentrated along a line inclined at an angle of about 30° to the line N S; it has therefore a vertical component in an upward direction in the case of the two upper quarters of the armature, and downwards in the case of the two lower quarters. Now, owing to the density of the field at *b* being greater than at *a*, as described above, the downward component of the pull, due to the field at *b*, may be greater than the upward component of the pull due to *a*; yet if the distribution of the field is symmetrical on either side of the line N S, according to our previous definition, this difference will be balanced by the greater upward pull of the field at *c* as

compared with the downward pull of *d*. The result is that the armature as a whole experiences no unbalanced pull, either upwards or downwards. But if the field be unsymmetrical, and the quarters *b*, *d* are more crowded with lines than *a* and *c*, the downward pull of *b*, *d* will more than counterbalance the upward pull of *a* and *c*, and there will thus be an unbalanced pull downwards. Thus even when placed symmetrically within the bore, the armature of fig. 2 as soon as it is excited must exert a pressure on its bearings greater than that due to its mere weight; and if the want of symmetry of the upper and lower halves of the fields be considerable, the downward pressure may reach such a value as to bend the shaft and perhaps cause serious heating of the journals. If, for example, we assume a uniform density in the upper halves of the air-gaps (B'_g) 5 per cent. less than the average (B_g), and a uniform density in the lower halves (B''_g) 5 per cent. more than the average, and further assume the pull on each quarter of the armature to result from the tension of these uniform densities[1] acting along lines 30° above and below the horizontal (fig. 206), then if A be the area of one-half of a pole-face in sq. cm., the upward pull due to one upper quadrant is $\frac{(B'_g)^2 A}{11,183,000} \cdot \sin 30°$, and the downward pull on one quarter is

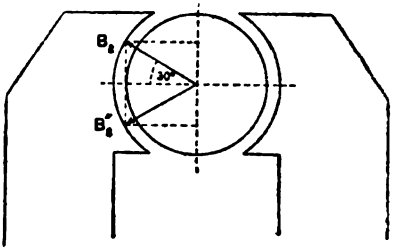


Fig. 206.

---

[1] Strictly speaking, in estimating the magnetic pull, the mean square of all the values of the induction under the pole-face should be calculated, but the effect of any want of symmetry is so closely bound up with the saturation or otherwise of the iron magnet that the problem becomes too complicated to be expressed even approximately by any simple formula. Distortion of the polar induction by armature reaction under load in a magnet which is geometrically symmetrical and with an armature centrally situated will have no further effect, but if the magnet is unsymmetrical to start with, so that one pole-face is affected differently from another, as in the case above considered, the result may again be entirely altered.

$\frac{(B''_g)^2 A}{11,183,000} . \sin 30°$. The difference between the downward and upward pulls from both halves is therefore

$$\frac{(1{\cdot}05B_g)^2 - (0{\cdot}95B_g)^2}{11,183,000} . A = \frac{0{\cdot}2B_g{}^2 . A}{11,183,000}.$$

The weight of the armature slightly bending the shaft downwards and any wear in the brasses of the bearings intensify this effect, since they combine to shorten the length of the air-gaps in the lower quarters and to increase that of the upper quarters.

Mutatis mutandis, the whole of the above applies equally well to the undertype single horseshoe, or again to the iron-clad field of fig. 195(i) with a single coil above the armature, in which, owing to lateral leakage, the density of the upper field is greater than that of the lower. But in these cases the unbalanced pull will be upwards, and will tend to lift the armature, so taking its weight off the bearings; in fact, in some cases of large drum armatures placed underneath a single 2-pole magnet the distribution of the fields has been so arranged that the upward pull almost balances the weight of the armature. We may, indeed, conceive of an undertype armature revolving in mid-air without any bearings, its only support being the unbalanced pull exerted by the upper polar surfaces. Generally speaking, however, it is desirable that the fields in which both ring- and drum-wound armatures are to be placed should be symmetrically distributed, and the extreme importance of this from the electrical point of view in all cases of ring-wound armatures has already been pointed out in Chap. XI. (§ 19). Now the chief merit of the double-magnet type of 2-pole dynamo is that complete symmetry of field-distribution is much more easily attained; from an inspection of figs. 192 and 193, it is evident that, provided the armature be placed concentrically within the bore of the pole-pieces, there will be complete similarity between the two halves of the fields on the right of the dividing line, *a b*, and the two halves on the left of that line; the double-magnet field is therefore specially suited to ring armatures of large diameter.

In the case, however, of the simple horseshoe magnet especial care must be taken in the design that the area of the pole-piece be not so much reduced at its middle or thinnest part as to throttle the lines and cause the majority of them to take the shortest path through the armature by passing through that half which is nearest to the yoke. The area of the pole-pieces must be well maintained up to their furthest extremities, so that, as the lines gradually filter out into the armature, the reduced density of those that remain lessens the reluctance and compensates for the greater length of the path which they follow. In fact, by careful shaping of the pole-pieces, much may be done to secure a uniform distribution of lines in each air-gap between the polar face and the armature core. Special devices have also been adopted in order to equalise the reluctances of the two paths—the upper and the lower. The simplest method is to place the armature eccentrically in the bore, its centre in the overtype dynamo being farther away from the yoke than the centre from which the pole-pieces are bored out. Another method is to make the width of the gap between the pole-pieces farther from the yoke less than that of the gap nearer the yoke; or, again, the rectangular magnet slabs may have polar extensions forged on them at their further ends ($c_1 c_2$, fig. 208), the place of these extensions at the ends of the polar surface nearer the yoke being taken by cast-iron 'horns,' $c_3 c_4$, screwed on to the wrought iron; the inferior permeability of the cast iron then makes the path through the half of the armature nearer the yoke present equal reluctance to that of the path farther from the yoke.

§ 16. **Symmetrical distribution of field in multipolar machines.**—The same principles may also be extended to multipolar machines. The mechanical effect of any eccentricity of the armature in the bore or any inequality in the field strengths has been already touched upon in § 11. The best approximate solution of the former problem is that given by Mr. Knowlton (*Electrical World and Engineer*, vol. xxxvii. p. 969, to which the reader is referred for a full proof of the following formula). If d be the displacement of the armature from a central position, in a vertical direction due to wear of the bearings, and a be the angle which the centre line of any pole makes with the vertical, the addition to or subtraction from the length of its air-gap is practically $= d \cos a$. The total reluctance of one of the $2p$

magnetic circuits may be expressed as a multiple of that of the double normal air-gap, or as $c \cdot \frac{2l_g}{A}$, where A = the half area of an air-gap and $c = \frac{X}{X_g}$. Taking then one magnetic circuit half of which passes through a pole making an angle α_1 and the other half through a pole making an angle α_2 with the vertical line, and these poles being in the lower half so that their air-gap is shortened, the new flux after displacement is

$$\frac{1 \cdot 257\, X}{c \cdot \frac{2l_g}{A} - \frac{d \cos \alpha_1}{A} - \frac{d \cos \alpha_2}{A}},$$

while the flux before displacement was $\frac{1 \cdot 257\, X}{c \cdot \frac{2l_g}{A}}$. The air-gap densities are in the same proportion, so that if B_g = the normal density before displacement and B_1 = the new density in the one half of the pole-face,

$$B_1 = B_g \frac{1}{1 - \frac{d}{c \cdot 2l_g}\left(\cos \alpha_1 + \cos \alpha_2\right)}.$$

Taking the next magnetic circuit, the density in the other half of the same pole-face is $B_2 = B_g \frac{1}{1 - \frac{d}{c \cdot 2l_g}\left(\cos \alpha_2 + \cos \alpha_3\right)}$. If we now assume the actual pole-face density to be the mean of the two densities in its two halves, or $B = \frac{B_1 + B_2}{2}$, and the attraction of this is balanced against that of the pole situated diametrically opposite, their difference multiplied by $\cos \alpha_1$ gives the vertical downward component from the pair of poles, and multiplied by $2 \cos \alpha_1$ gives the vertical downward component due to two pairs of poles symmetrically situated. The total downward pull on the whole armature when thus calculated may be expressed as a certain multiple of the normal pull on one pole-face or $= \frac{B_g^2 A}{11{,}183{,}000} \times C$, in which c varies with the value of $\frac{d}{c\,.\,l_g}$. It is, however, found that with usual values of d and l_g such as may be met with in practice, $\frac{C}{d/c\,.\,l_g}$ is constant for a given number of poles, and has the following values, viz. :—

4 poles, 2
6 „ 4·7
8 „ 7
10 „ 9·6

and for 12 poles or over is practically equal to the number of poles. The total pull is then

$$\text{for 4 poles } \frac{B_g^2A}{11{,}183{,}000} \times \frac{2d}{c\,.\,l_g};$$

$$\text{„ 6 „ } \frac{B_g^2A}{11{,}183{,}000} \times \frac{4{\cdot}7d}{c\,.\,l_g};$$

$$\text{„ 8 „ } \frac{B_g^2A}{11{,}183{,}000} \times \frac{7d}{c\,.\,l_g};$$

$$\text{„ 10 „ } \frac{B_g^2A}{11{,}183{,}000} \times \frac{9{\cdot}6d}{c\,.\,l_g};$$

and for 12 poles or over $\frac{B_g^2A\,.\,P}{11{,}183{,}000} \times \frac{d}{c\,.\,l_g}$ where P = the number of poles.

The electrical effect of any displacement of the armature, as described in Chap. XI. §§ 17 and 24, may again shortly be summarised. In the case of all parallel-connected lap-wound armatures, if, *e.g.*, the armature sinks in the bearings out of the centre, the inductors which at any time form the lower parallels of the winding will always produce a slightly higher E.M.F. than those which form the upper parallels; on open circuit the unbalanced E.M.F. will produce large local currents and consequent waste of energy, while on closed circuit the current is unequally distributed among the different parallels, and the efficiency of the machine is again impaired even if sparking at the brushes be not produced. Any considerable wear of the bearings must therefore be corrected by either raising the armature or lowering the field-magnet. For this purpose in large multipolar machines centring screws are often provided by means of which the magnet frame may be adjusted horizontally and sideways. These may be used at the outset to erect the magnet so that its parallel circuits give very approximately equal voltages on open circuit as checked by a volt-meter applied to the several pairs of brushes; if the iron paths vary slightly in permeance, it may even be found advisable to set the magnet so that it is not quite concentric with the armature, yet so that the magnetic pulls of the poles and the electrical E.M.F.'s are closely balanced. At any later time the screws may again be called into use to follow up the effect of wear of the brasses. The electrical objection from want of symmetry in large multipolar fields is entirely removed if the armature be wave-wound with only two sets of brushes, since then the zigzag winding passes continuously round in series from the weaker to the stronger poles. If, however, more than two sets of brushes are employed, other difficulties may arise (Chap. XI. § 24), whether the armature be series-connected or parallel-connected. Instead of the wave-wound parallel-connection, it is therefore better to employ lap-winding and equalising connections between different points of the winding or commutator, which should be at the same potential; the initial evil is not thereby prevented but its injurious effect on the commutation is minimised

When, however, we pass to the multipolar form of fig. 199 or to the homopolar form of fig. 61 with flat-ring armature, no objection on the score of dissymmetry can be raised, even though the armature is ring-wound and in several parallels; the armature may sink relatively to the fields as wear takes place in the bearings, but the electrical effect of such decentralisation on the equality of the flux of the several fields, and on the voltage of the parallels, will be practically negligible.

§ 17. **Proportioning of multipolar magnet-frames.**—Although the weight of copper on the multipolar field of fig. 197 is, *ceteris paribus*, more than that on the single horseshoe bipolar field, the total weight of the former is less, and this advantage becomes more and more marked as the specific output in watts per rev. per min. is increased. Apart, therefore, from any other reasons the multipolar field will be adopted when the specific output exceeds a certain limit, although the exact point at which the transition is made will vary greatly according to circumstances.

An important consideration in deciding this question is the relative economy in copper given by different sections of magnet-core. For a two-pole dynamo of large output, an armature core of considerable length as compared with its diameter would be chosen, but such proportions give a section to the magnet limb which is inefficient in exciting copper. On the other hand, in the multipolar field the diameter of armature can be larger and its proportion to the length can be so chosen that the section of the magnet-core approximates to a square or to a circle and is therefore economical in exciting wire. Thus, although the subdivision of the total flux between several magnetic circuits is in itself expensive in wire and exciting energy, the proportions of the magnet-cores in the multipolar case can be made so economical that it may actually take less weight of exciting wire than the two-pole massive horseshoe; it then has the advantage not only in weight of iron in the magnet but also in weight of copper. In order to obtain such advantageous proportions, the two dimensions M O and O P (fig. 207)

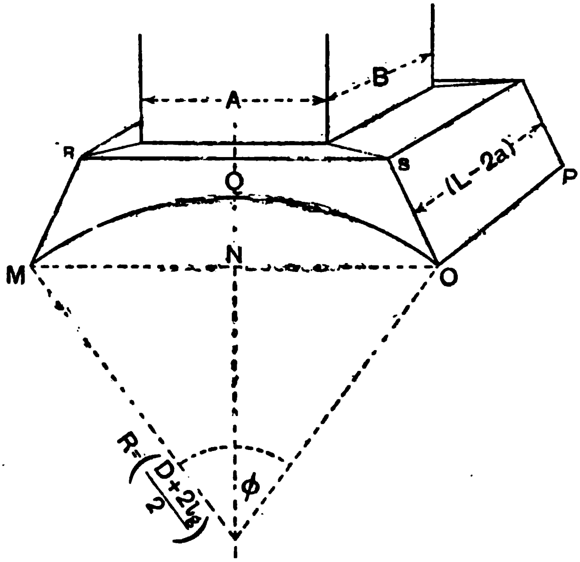


FIG. 207.

must not be far different from each other. Since $\sin\frac{\phi}{2}=\frac{\mathrm{MN}}{\mathrm{R}}$, $\mathrm{MO}=2\mathrm{R}\sin\frac{\phi}{2}=(\mathrm{D}+2l_g)\sin\frac{\phi}{2}$, where D is the diameter of the armature core, and $(\mathrm{D}+2l_g)$ may be taken as approximately $=1{\cdot}04\,\mathrm{D}$. If a be the amount by which the length of the armature core exceeds the length of the pole-face at each end, $\mathrm{OP}=\mathrm{L}-2a$, and this may be taken as equal to 0·93 L. If therefore MO is to be equal to OP, $1{\cdot}04\,\mathrm{D}\sin\frac{\phi}{2}=0{\cdot}93\,\mathrm{L}$, or

$$\frac{\mathrm{L}}{\mathrm{D}}=1{\cdot}12\sin\frac{\phi}{2}.$$

The polar angle bears an approximately constant proportion to the pitch of the poles, or $\phi=\frac{360^\circ\times\beta}{2p}$, where β is a constant and is usually about 0·75. The favourable ratio of length to diameter of core for different numbers of poles on these assumptions is

$$\frac{\mathrm{L}}{\mathrm{D}}=1{\cdot}12\sin\frac{135^\circ}{2p},$$

whence

No. of Poles.	$\frac{\mathrm{L}}{\mathrm{D}}$
2	1·03
4	0·62
6	0·43
8	0·324
10	0·233

It is, however, advisable to keep the diameter as small as possible and consequently to make the ratio $\frac{\mathrm{L}}{\mathrm{D}}$ greater, provided there is no great departure from the most economical section of magnet-core. Hence the above may be regarded rather as the minima values which the ratio should take, and a closer analysis may be made by taking into account the necessary reduction which there must be in the area A,B carrying lines at a density of 16·000 per square centimetre, as compared with the area of the bored pole-face carrying lines at a density of, say, $B_g = 6000$. Expressing the latter in terms of the chord M N O, we have

$$AB = MNO \times OP \times \frac{\pi}{\sin \frac{135^\circ}{2p}} \times \frac{\beta}{2p} \times \frac{6000}{16{,}000}$$

or $$AB = MNO \times OP \times xy,$$

and a little calculation shows that the product xy decreases from 0·476 for two poles to 0·396 for four, 0·384 for six, and then becomes nearly constant for eight or more poles at 0·378. This product has now to be divided into the two factors x and y, being respectively the ratio of A to the chord $\left(= 1{\cdot}04 \text{ D} \sin \frac{\phi}{2}\right)$, and of B to the length O P ($= 0{\cdot}93$ L). Equating A and B, we thus obtain

$$\frac{L}{D} = \frac{x}{y} \,.\, 1{\cdot}12 \,.\, \sin\frac{\phi}{2} \,.$$

There must be no undue thinning out of the lines towards any edge of the pole-face, so that roughly speaking A,B must stand in the middle of M O, O P, but there is no necessity for the amount by which the width A is less than the chord to be precisely the same as the amount by which B is less than the pole-length. It is therefore advantageous to make x somewhat greater than y, the two values ranging from 0·8 and 0·6 in the two-pole down to 0·71 and 0·53 in the ten-pole machine. The ratio $\frac{x}{y}$ is then a constant, $= 1{\cdot}34$, and we finally obtain

$$\frac{L}{D} = 1{\cdot}5 \sin\frac{\phi}{2},$$

whence

No. of Poles.	$\frac{L}{D}$
2	1·38
4	0·83
6	0·57
8	0·43
10	0·35

While the above proportions admit of considerable variations according to different conditions of the design, the method by which they have been arrived at serves to show how they depend on the relative densities in the air-gap and magnet-core, and upon the amount by which it is considered advisable for either edge of the pole-piece to project beyond the magnet-core.

CHAPTER XV

THE AMPÈRE-TURNS OF THE FIELD

§ 1. **The three divisions of the magnetic circuit.**—In our previous equations for the internal E.M.F. induced in an armature, one of the three determining factors has been z_a, or the number of lines of induction which enter into the armature-core from one pole or leave the core to pass into another pole. We have now to determine the ampère-turns of exciting power which must be placed on the magnet in order to produce z_a lines in that part of the magnetic circuit where they are required, viz., in the armature, so that they may be cut by the inductors. The actual process has been already illustrated in Chap. III, § 14, and consists in the subdivision of the entire magnetic circuit into such lengths as may be considered to be of the same sectional area and permeability and to carry the same number of lines, the calculation of the magnetic differences of potential required to drive the lines through each of these lengths, and their subsequent summation as one magnetomotive force. In nearly all cases, any magnetic circuit in a dynamo may be divided into the three principal parts: (1) the *air-gaps*, (2) the *iron of the armature*, (3) the *iron of the field-magnet*; the latter may further require to be subdivided into two parts, viz., (a) the *magnet-cores* or *limbs*, and (b) the *yoke*, and in some cases, where the pole-pieces differ much in area and quality from the rest of the magnet, a third subdivision becomes necessary, viz., (c) the *pole-pieces*. The subscript letters g, a, m, y, and p, will in future be used to denote these several parts. Owing to leakage the total flux of lines will vary in different parts of the circuit, but, as explained in Chap. III, on the supposition of the leakage paths being all in parallel with the armature and air-gaps, it will suffice to consider z_a lines as flowing through the armature, and a larger number, Z_m, as flowing through the magnet.

§ 2. **The equation of the magnetic circuit.**—If Z lines flow through a portion of a magnetic circuit, having a total area of cross-section of a square centimetres normal to their path, the value which H must have is solely determined by the density per square centimetre, *i.e.* by the induction $B = \frac{Z}{a}$; in other words, whatever be the total area normal to the flow, if a difference of magnetic potential, H, be maintained between two surfaces one centimetre apart, B lines will flow through each square centimetre of the cross-section. Each centimetre length of the substance, therefore, requires a specific difference of magnetic potential, H (dependent on B), to be maintained between its ends; and if the same induction, B, be continued over a portion whose length is l centimetres, the total fall of magnetic potential between opposite faces of the portion will be $H \times l$, and that difference of potential maintained between its ends will cause a total flow of $Z = Ba$ lines through it. Reckoning, therefore, the lengths of the three portions of the magnetic circuit in centimetres, and l_g being the length of a single air-gap, we may express the fundamental equation for the ordinary bipolar dynamo with its two air-gaps, one on either side of the iron armature, in the following form :—

$$\text{Total M.M.F.} = H_a l_a + H_g 2 l_g + H_m l_m,$$

where H_a, H_g, and H_m are the magnetising intensities required to produce B_a, B_g, B_m in the armature, air-gaps, and field-magnet respectively. Now H has been shown to be a function of the induction, so that the equation may equally well be written—

$$\text{Total M.M.F.} = f(B_a) \,.\, l_a + f(B_g) \,.\, 2l_g + f(B_m) \,.\, l_m.$$

And finally, since $B = \frac{Z}{a}$,

$$\text{M.M.F.} = f\left(\frac{Z_a}{a_a}\right) . \, l_a + f\left(\frac{Z_g}{a_g}\right) . \, 2l_g + f\left(\frac{Z_m}{a_m}\right) . \, l_m.$$

In curves of flux-density, such as figs. 126–8, are connected together corresponding values of B and H for various substances

Having calculated, therefore, the different values of B_a, B_g, B_m for a given value of z_a, all that is necessary is to look out in the flux-density-curve of the substance in question the particular value which B now has in it, and then to read off on the scale of abscissæ the corresponding particular value which H must have for that induction: this value can then be substituted for the specific H in the above equation.

So far we have dealt with C.G.S. units of magneto-motive force, their number being equal to 1·257 times the ampère turns; but for our immediate purpose we require an equation expressed directly in ampère-turns of exciting power (X), or

$$X = X_a + X_g + X_m,$$

where, as before, the three right-hand terms are the exciting powers required respectively by the armature, air-gaps, and field-magnet. The magnetomotive-force equation will require to be divided throughout by 1·257 in order to express it in ampère-turns; but since the values of H in the curves of induction were originally derived from a measured number of ampère-turns per centimetre length, it is still simpler to show on the horizontal axis of abscissæ a scale of ampère-turns per centimetre length (as well as a scale of H in C.G.S. units), and thence read off the exciting power required per centimetre length to produce a flow of B lines per square centimetre. This has been done in the lower of the two horizontal scales of figs. 125–8, whence, therefore, we can read off the new value which the function of the induction takes when expressed in ampère-turns, or

$$f'(B) = \frac{f(B)}{1{\cdot}257} = 0{\cdot}8 f(B).$$

We thus obtain for one magnetic circuit

$$\text{Total ampère-turns} = X = f'(B_a) . l_a + f'(B_g) . 2l_g + f'(B_m) . l_m.$$

It may here be objected that no curve of induction appropriate to the air-gaps of the dynamo has been given, and that therefore $f'(B_g)$ cannot be at present determined. When, however, it is remembered that the permeability of air is strictly constant, or, in other words, that the magnetising intensity required per cm. length of path in the air is directly proportional to the induction, it is evident that the curve for air would be simply a straight line inclined at a certain angle to the axes, and that it is therefore unnecessary to plot it. As stated in Chap. III, the permeability of air on the C.G.S. system = 1, so that to produce an induction of one line per square cm. in air requires $\frac{1}{1{\cdot}257} = 0{\cdot}8$ of an ampère-turn, and to produce any other induction B_g requires $0{\cdot}8 \times B_g$ ampère-turns per cm. length. The air-gap of a dynamo is made up partly of air and partly of cotton-covered copper wire, but since the permeability of the latter two materials is sensibly the same as that of air, the whole interferric space between the iron of pole-piece and iron of armature core may be treated as a gap of air alone. Hence for $f'(B_g)$ in the above equation we may at once substitute the value of $0{\cdot}8B_g$, and

$$X_g = 0{\cdot}8B_g \, . \, 2l_g, \text{ or } 0{\cdot}8\frac{Z_g}{a_g} \, . \, 2l_g.$$

As hinted in Chap. III, § 15, in calculating a_g a certain additional area must be reckoned over and above the actual area of the polar face in order to allow for the spreading out of the lines in a fringe, which increases the effective area of the air-gap. The quantitative value for this increase will be discussed later.

The expression now obtained for the total ampère-turns of excitation for one complete magnetic circuit is therefore

$$X = f'\left(\frac{Z_a}{a_a}\right) . \, l_a + 0{\cdot}8 \, \frac{Z_g}{a_g} \, . \, 2l_g + f'\left(\frac{Z_m}{a_m}\right) . \, l_m, \quad . \quad . \quad . \quad (26)$$

or $X = f'(B_a) \, . \, l_a + 0{\cdot}8 \, B_g \, . \, 2l_g + f'(B_m)l_m$.

The actual process in designing will be to determine the inductions, B_a and B_m, and then to read off from a flux-density curve the corresponding number of ampère-turns required per cm. length; while even in the case of the air-gap it is useful to calculate B_g as affording something of a clue to the probable amount of losses by eddy currents which may be expected in the armature winding and surface of the core.

§ 3. **Use of flux-density or B,H curves.**—The flux-density curves from which the values of the specific exciting powers are read must be suited to the exact nature of the materials of which the dynamo's magnetic circuit is composed. On referring to fig. 125, it will be seen that if the magnet be worked with a low induction, say, *e.g.*, 10,000 in wrought iron or 6000 in cast iron, the value of H for a given induction is indeterminate, and may have any value lying between the ascending and descending curves; in the cases supposed, the limits will be 1·5 or 4 ampère-turns per cm. length for wrought iron, and 16 or 25 for cast iron. *Vice versâ*, with a given H, the induction may vary between certain limits according as it has been reached from a higher or lower value. Hence a certain number of ampère-turns on the field may induce a slightly larger number of lines at one time than at another if the excitation be raised to a higher value and then decreased; and the student may therefore feel a doubt as to what value of exciting power he is to select. In practice, however, any such effect of hysteresis resulting in a variation of the voltage given by a dynamo when running at a certain speed is hardly perceptible; not only is a dynamo seldom worked with a very low induction in the iron of the field-magnets (and with higher values for the induction the vertical difference between the ascending and descending curves becomes negligible), but also the exciting power expended over the reluctance of the field-magnet or X_m forms only a certain proportion, usually less than half, of the total number of ampère-turns, $X = X_a + X_g + X_m$, and therefore an increased number of ampère-turns is required for the armature and air-gaps, if an increased number of lines pass through the armature. Further, in actual working the excitation is never varied through a very wide range; in fig. 125, the magnetising intensity per cm. length of iron has been carried up to a high figure, and then gradually reduced, so producing the greatest possible difference between the ascending and descending curves. Such a condition would seldom occur, except in the conduct of workshop or laboratory experiments on dynamos; in the ordinary course, the field-magnets become gradually excited as the machine is set to work, and then the excitation is maintained fairly constant until on the stoppage of the machine the magnetism dies away. Hence in the design of dynamos it is sufficient to invariably use the ascending curve of induction; any difference, due to the exciting power having been differently reached, may be neglected, since its only effect will be to slightly reduce the speed required to generate a given voltage.

§ 4. **The back ampère-turns of the armature.**—There still remains one point on which our previous equation requires to be supplemented. When a considerable current is passing through the armature of any continuous-current bi- or multipolar dynamo, it gives rise to a magnetomotive force and the armature *reacts* on the field, as will be explained in greater detail in Chap. XVII. Further, it usually becomes necessary to move the brushes forward through a small angle in the direction of rotation in order to prevent sparking at the commutator; the reason for this *shifting of the brushes* will also be more fully explained in Chap. XVIII. Suffice it here to say that when the brushes are given the requisite forward *lead* in order to obviate sparking, the ampère-turns due to the current flowing in the belt of armature inductors inclosed within twice the angle of lead are directly opposed to the ampère-turns of the field. In fig. 246 those wires in which the current, whether of the field or of the armature, is directed towards the observer are marked with a dot, and those in which the current is away from the observer are shown crossed; and from this figure it will be apparent by application of the rule of the hand that the current-turns between $k\,l$, $m\,n$, i.e. within the angle 2λ, are opposed to the current- turns of the field, and tend to magnetise the circuit in exactly the reverse direction: they are, in fact, *back ampère-turns*, and the result is that a certain number of extra ampère-turns have to be added to the field ampère-turns in order to neutralise the demagnetising effect of the armature current. Let this additional number $= \mathrm{x}_b$; then the complete equation for the ampère-turns required from the field coils must contain this additional term, and thus becomes

$$\mathrm{x} = \mathrm{x}_a + \mathrm{x}_g + \mathrm{x}_b + \mathrm{x}_m, \quad . \quad . \quad . \quad (27)$$

When the armature current is very small, x_b may be regarded as negligible, but for any except very small loads the appropriate number of compensating ampère-turns must be added, their number increasing as the armature current and its demagnetising effect are increased.

§ 5. **The magnetic leakage due to the difference of potential between the poles.**—The reason for the insertion of x_b in the third place needs further explanation. It has been said that the flow of leakage lines takes place in the main under the magnetic difference of potential existing between the poles of the dynamo, and even if this be not true of all the leakage, allowance can be made for the inaccuracy involved in the assumption. Before z_m or B_m can be determined, the amount of leakage ζ requires to be estimated. In order to do this, the joint permeance of the leakage paths is calculated; it is then only necessary to multiply the magnetic difference of potential at the poles by this joint permeance to determine ζ. The magnetic difference of potential between the poles is equal to 1·257 times the ampère-turns required to produce the flow of lines between the poles; let these be expressed as X_P; then $\zeta = 1{\cdot}257\ X_P \times P$, where P is the joint permeance of the leakage paths. Thence $z_m = z_g + \zeta$, from which the magnet induction B_m and the ampère-turns required to produce it, X_m, can be determined.[1]

Now X_b, or the ampère-turns required to counterbalance the back ampère-turns of the armature current, take effect between the poles, and consequently must be included when estimating the difference of magnetic potential between the poles: thus

$$X_P = X_a + X_g + X_b$$

and

$$\zeta = 1{\cdot}257\ (X_a + X_g + X_b) \times P \qquad . \quad . \quad . \quad (28)$$

Our final and complete *equation for the ampère-turns or exciting power of a magnetic circuit* will thus take the form

$$X = f'(B_a) \,.\, l_a + 0{\cdot}8 B_g \,.\, 2l_g + X_b + f'(B_m) \,.\, l_m \qquad . \quad . \quad (29)$$

Simple though this equation is in its form, its application presents certain difficulties which preclude us from calculating the ampère-turns required with absolute and complete accuracy. Apart from the somewhat indefinite nature of the leakage paths, the lengths of the paths in the iron portions of the magnetic circuit can only be calculated approximately, a mean having to be struck between the longest and the shortest; or, again, the area of cross-section normal to the lines of induction may be continuously varying (as, for example, in the pole-piece), and this in practical work necessitates a mean being taken which it requires considerable judgment to estimate accurately. A second source of error is that our calculations are based on certain flux-density curves, yet the particular class of iron used may not be exactly similar to that for which a curve has been obtained; cast iron in particular varies considerably in its magnetic properties. Still, when tested experimentally, a dynamo should certainly not require to be run at a speed differing by more than 5 per cent. from the designed speed in order to give the designed voltage.

---

[1] Cp. Chap. III, § 14.

§ 6. **Calculation of ampère-turns on a bipolar horseshoe field-magnet.**—In order to render the above methods of calculation clearer by an actual example, they will now be applied to the design of a 14-kilowatt shunt-wound machine with drum armature, giving an output of 140 ampères at a pressure of 100 volts or to allow for the drop of volts over the leads, etc., to the switchboard and lamps, say 103 volts at the dynamo terminals. The chief dimensions of the iron carcase which forms the magnetic circuit are given in fig. 208, from which it will be seen that it is a bipolar overtype horseshoe with forged ingot-iron limbs bolted up to a cast-iron yoke in the centre of the bedplate. The armature core is 9″ in diameter × 12″ long, and a preliminary estimate will lead us to allow a loss of about $3\frac{1}{2}$ volts over the resistance of the armature and brushes, and to determine upon 132 inductors. The speed is not to exceed 950 revolutions per minute, and therefore, by the equation (15)

$$E_a = 106{\cdot}5 \text{ volts} = z_a \times 132 \times \frac{950}{60} \times 10^{-8},$$

whence $z_a = 5{,}100{,}000$.

Taking the separate elements of the magnetic circuit in the order of our equation (29), we have

(1) The armature, requiring X_a ampère-turns.

The internal diameter of the discs is $4\frac{1}{4}$,″ so that their radial depth is $a = 2\frac{3}{8}$.″ Allowing 10 per cent. of the gross length of the armature core as occupied by the insulation between the discs, the net sectional area of iron on the two sides of the drum is $2ab \times 0{\cdot}9 = 2\,(2\frac{3}{8} \times 12) \times 0{\cdot}9$ square inches, or $51 \times 6{\cdot}45 = 330$ sq. cm. The maximum induction in the armature core is thus

$$B_a = \frac{5{,}100{,}000}{330} = 15{,}450.$$

The mean length of path in the armature, as shown by the dotted line, l_a, in fig. 208, is about 8½", or 22 cm.

By reference to the curve of fig. 128, for stamped armature discs, it will be seen that to produce an induction of 15,450 lines per sq. cm. requires an exciting power of about 24 ampère-turns per cm. length of path. It is, however, only over about 4¼" or 10·8 cm., between the polar tips, that the maximum density obtains: along the horizontal diameter, B_a is practically 0, and midway between these extremes, where B_a = about 7,800, the necessary exciting power is only about 4 ampère-turns per cm. length of

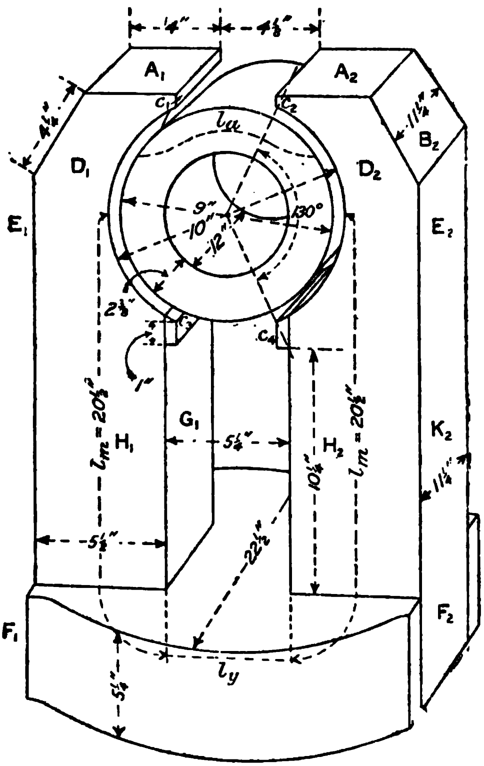


FIG. 208.—Typical two-pole field.

path. We may therefore with fair accuracy regard the whole of the exciting power required by the armature as expended in producing an induction of about $B_a = 15{,}200$ over a path of 12·5 cm., or a length somewhat more than the distance between the polar tips $c_1\,c_2$, or $c_3\,c_4$. For such an induction $f'(B_a)$ is 22, and therefore

$$X_a = f'(B_a).\,l_a = 22 \times 12{\cdot}5 = 275 \text{ ampère-turns.}$$

Even if this estimate be not absolutely correct, an error here will produce but little effect on the total result.

(2) The air-gap, requiring X_g ampère-turns.

Owing to the spreading of the lines of induction as they issue out of the polar face and pass into the armature core, the area of the air-gap is greater than the area of the bored face of the pole-piece. Not only do lines pass in a sloping direction into the core from the vertical edges of the 'horns,' or extended tips of the poles (cp. fig. 159), but some also curve round into the armature from the outer flanks of the pole-pieces at the edge of the bore, and this is especially the case when, as here, the length of the armature core is slightly greater than the width of the poles parallel to it.

The normal value which the induction has under the greater portion of the pole-face is not maintained right up to the edges, the point where decrease begins being at a distance from the pole-edge practically equal to the air-gap, l_g;[1] at the extreme edge the value is 0·84 of the normal, and thence it rapidly falls. The additional number of useful lines entering the armature core as a *fringe* along any edge may be taken into account by assuming the normal induction B_g to hold over the entire pole-face, and by then adding along the edge in question the permeance of an additional strip of air. This strip must be of such width that with a length of path through it equal to the normal l_g, and with the normal induction B_g assumed to be caused in it by $\frac{X_g}{2}$, the required addition to the flux is obtained. The width of the equivalent strip of air along the flanks of the pole will in general be different from its width along the edge parallel to the axis of rotation. Thus if a be the amount by which at each end the armature length exceeds that of the pole-piece, the effective width of the strip producing the equivalent of the end-fringe will depend upon the ratio $\frac{a}{l_g}$. On the other hand, if c be the distance measured on the surface of the armature core between a radius drawn to the pole-tip and the interpolar line of symmetry, the width of the strip producing the equivalent of the interpolar fringe will depend upon the ratio $\frac{c}{l_g}$. In both cases the width of the strip may be expressed as a certain fraction of l_g. In Table I.[1] are

---

[1] If the pole-edge is at right angles to the core. See F. W. Carter, "Note on Air-gap Induction," *Journal Inst. Electr. Eng.*, vol. xxix. p. 929.

TABLE I.

$\frac{a}{l_g}$	Values of K_1.
0·5	0·736
0·75	1·0
1·00	1·2
1·25	1·38
1·5	1·54
1·75	1·68
2·00	1·81
4	2·7
8	3·6

given the approximate values of the constant K_1 by which l_g must be multiplied to obtain the joint effective width of the two strips, one on each side-flank of the pole-piece, for various values of $\frac{a}{l_g}$. For small values of this ratio below 1, the table may appear to give too high a value for K_1, but it must be remembered that in such cases when the pole-piece is nearly as long as the armature core, some lines do not enter on the circumference of the core, but curve round into the ends, and for these lines allowance must be made.

---

[1] Hawkins and Wightman, "The Air-gap Induction in Continuous-current Dynamos," *Journal Inst. Electr. Eng.*, vol. xxix. pp. 436 ff.

In the above table it is assumed that the side-flank of the pole-piece is at right angles to the cylindrical surface of the armature core, but in the case of the interpolar strips the angle which the edges of the pole-tips make with the armature must also be taken into account by the following approximate construction. The length of path of the lines entering the armature at any point distant x cm. (measured on the armature core) from the pole-tip is assumed to be made up of a curved portion, the length of which is a constant multiple of x, and a straight portion $(=l_g)$; *i.e.* the total length of the air-path is $\xi x + l_g$, where ξ is the constant multiplier whose value must be estimated from the drawing in relation to the shape of the pole-tip. In ordinary cases if the radial distance of the pole-face from the core is not increased at the pole-tip, ξ is about 1·05 times the angle which the pole-tip makes with the armature core expressed in circular measure. Thence can be obtained the following table II. of values for the constant K_2 by which l_g must be multiplied in order to give the joint width of the two strips one along each interpolar edge.

TABLE II.—Values of K_2.

$\frac{c}{l_g}$	$\xi = 1·6$ Angle 90°.	1·8 100°	2 110°	2·2 120°	2·4 130°	2·6 140°
4	1·96	1·85	1·76	1·67	1·59	1·52
5	2·19	2·06	1·95	1·84	1·75	1·67
6	2·38	2·23	2·10	1·99	1·89	1·8
7	2·56	2·39	2·25	2·12	2·01	1·91
8	2·7	2·52	2·37	2·24	2·12	2·01

If D be the diameter of the armature core, and l_g the length of one air-gap, the length of arc subtending the polar angle at the mean radius of the air-gap is $A' = \pi(D + l_g)\,\frac{\text{polar angle}}{360^\circ.}$

The *effective area of the air-gap* is then

$$a_g = (L + K_1 l_g)(A' + K_2 l_g) \quad . \quad . \quad . \quad . \quad (30)$$

and $B_g \,.\, a_g = Z_g$.

In the present case the bore of the pole-pieces is 10″, whence, the diameter of the armature being 9″, $l_g = \frac{1}{2}''$. The angle subtended by the polar face is 130°: hence

$$A' = \pi(9\tfrac{1}{2})'' \frac{130^\circ}{360^\circ} = 10{\cdot}76.''$$

The ratio $\frac{a}{l_g} = 0{\cdot}75$, and $\frac{c}{l_g} = 4$, while allowing for the somewhat different shape of the two pole-tips c_1 and c_2, ξ may be averaged as $= 1{\cdot}85$.

The effective area of the air-gap is therefore

$$(11{\cdot}25 + 0{\cdot}5)\ (10{\cdot}76 + 0{\cdot}92) \times 6{\cdot}45 = 885 \text{ sq. cm.},$$

and since $Z_g = Z_a$, the induction when averaged over the whole of it is

$$B_g = \frac{5{,}100{,}000}{885} = 5{,}750,$$

and $$X_g = 0{\cdot}8 B_g .\ 2 l_g = 0{\cdot}8 \times 5{,}750 \times 2{\cdot}54 = 11{,}700.$$

(3) The back ampère-turns, X_b.

The method by which these are calculated will be discussed in Chap. XVII, but for the machine now under consideration they may be reckoned at full load as = 515.

The result so far is
$$\begin{aligned} X_a &= \;275 \\ X_g &= 11{,}700 \\ X_b &= \underline{515} \\ \text{Sum} &= 12{,}490 = X_p \end{aligned}$$

or the exciting power acting at the poles of machine.

The next step necessitates a knowledge of P, the leakage permeance: for the present this may be taken as = 75, the method by which this numerical value is obtained being explained in §§ 7 and 8. Thence the leakage lines

$$\zeta = 1{\cdot}257 \times 12{,}490 \times 75 = 1{,}175{,}000$$

and $$Z_m = Z_a + \zeta = 6{,}275{,}000.$$

(4) The magnet limbs, requiring X_m ampère turns.

The area of the wrought-iron limb is

$$5\tfrac{1}{2} \times 11\tfrac{1}{4} \times 6{\cdot}45 = 400 \text{ sq. cm.},$$

and the average induction in them is

$$B_m = \frac{6{,}275{,}000}{400} = 15{,}700.$$

By reference to the forged-iron curve (3) of fig. 126, we see that when $B_m = 15{,}700$, the ampère-turns required are 15 per cm. length, or slightly less on the descending curve. The length of path in each magnet-core under the exciting coils is $10\tfrac{1}{4}''$, and at the yoke, where the lines bend round into the bed-plate, their curved path to the edge of the forged iron may be taken as $5{\cdot}3''$ long. Within the pole-piece the induction decreases slightly, for the passage of the lines outwards is hardly counterbalanced by any corresponding decrease in the cross-sectional area of the iron. Hence the equivalent length of the mean path in the pole-piece may be taken as $5''$. The average length of path within each limb may therefore be reckoned as

$$10{\cdot}25 + 5{\cdot}3 + 5 = 20{\cdot}5'';$$

and the total length within the two as

$$20{\cdot}5 \times 2 \times 2{\cdot}54 = 104 \text{ cm.}$$

Hence

$$X_m = f'(B_m) \,.\, l_m = 15 \times 104 = 1{,}560 \text{ ampère-turns.}$$

(5) The yoke, requiring X_y ampère-turns.

The area of the cast iron $= 22\tfrac{1}{2}'' \times 5\tfrac{1}{4}'' \times 6{\cdot}45 = 760$ sq. cm.;

and

$$B_y = \frac{6{,}275{,}000}{760} = 8260,$$

whence from the cast-iron curve of fig. 125, $f'(B_y) = 84$. The lines spread somewhat in the yoke, and their mean length of path is about 16 cm., whence

$$X_y = 84 \times 16 = 1{,}350.$$

The total excitation required is therefore

$$X = 12{,}490 + 1{,}560 + 1{,}350 = 15{,}400 \text{ ampère-turns.}$$

§ 7. **Formulæ of the leakage permeances.**—We have now to return to the question of the magnetic leakage, ζ, and in the first place to the calculation of the *leakage permeance*, P, of any dynamo. In the following pages the method of Prof. Forbes[1] is described with slight modification. Since we have to do with a large number of paths, which are assumed to be all in parallel with one another, it is simplest to deal with their permeances, for these can be immediately added together to discover the joint permeance of the leakage paths. The lines are assumed to follow certain directions in the air according to the situations and distances of the two surfaces between which they flow, and to meet these different cases three general propositions are, as a rule, sufficient.

(I) In the case of two parallel surfaces facing each other, the areas of which are approximately equal, the lines of induction may all be assumed to pass straight across from the one surface to the other. The permeance of the air-gap between the two surfaces is then, on the C.G.S. system, equal to the mean of their areas divided by their perpendicular distance apart (fig. 209); or

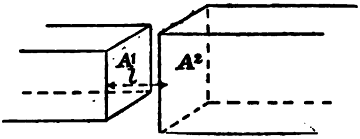


FIG. 209.

$$p = \frac{\frac{1}{2}(A_1 + A_2)}{l}, \quad . \quad . \quad . \quad . \quad (31)$$

(II) In the case of two equal rectangular surfaces situated near each other in the same plane, with their neighbouring edges parallel, the lines of induction may be assumed to be semicircles described about a central line drawn between the two surfaces; the permeance of the air-path from one to the other is then, on the C.G.S. system—

$$p = \frac{a}{\pi} \log_e \frac{r_2}{r_1}, \quad . \quad . \quad . \quad (32)$$

where r_1 and r_2 are respectively the distances from the central line to the nearest and farthest edges of either rectangle, and a is the depth of each rectangle at right angles to r, *i.e.* along the parallel edge (fig. 210).

---

[1] *Journal Soc. Tel. Eng.*, vol. xv. part 64 p. 555.

(III) In the case of two equal rectangular surfaces situated similarly in one plane, but at some distance apart, the lines of induction may be assumed to be quadrants connected by straight lines, the quadrants being described from the neighbouring edges of the rectangles as lines of centres: the permeance of the air between the two surfaces is then, on the C.G.S. system—

$$p = \frac{a}{\pi} \log_e \frac{\pi w + b}{b}, \quad . \quad . \quad . \quad . \quad (33)$$

where a is again the depth of each rectangle, w its width, and b is their distance apart (fig. 211). If the two surfaces are not in the same plane but are inclined to one another at some angle other than π, then the value of this angle in circular measure is to be inserted in place of π in equations (32) and (33).

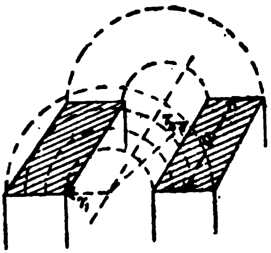


FIG. 210.

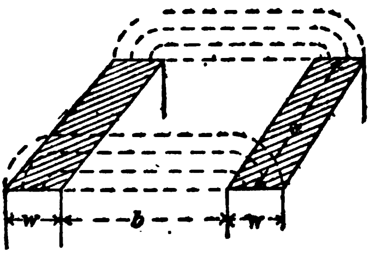


FIG. 211.

To these may be added other extensions of the same principles, but in general the three main propositions above considered will meet the most common cases, or in default of any other guidance it must suffice to map out a probable course for the leakage on the above lines, and thence by scaling the mean areas of the two surfaces and the mean length of path between them to deduce the permeance $= \frac{\text{area}}{\text{length}}$.

It must be remembered that in all the foregoing equations the dimensions are in centimetres and the logarithms are to the Napierian base ϵ: hence, for English dynamo-designers, it may be useful to give the equivalent equations when the dimensions are in inches, and the areas in square inches; they are:—

$$(\text{I})\ p = 2{\cdot}54 \cdot \frac{\frac{1}{2}(A_1'' + A_2'')}{l''} = \frac{1{\cdot}27(A_1'' + A_2'')}{l''} \quad . \quad . \quad . \quad (31)$$

$$(\text{II})\ p = \frac{2{\cdot}54\, a''}{\pi} \log_{10} \frac{r_2''}{r_1''} \times 2{\cdot}3 = 1{\cdot}86 \times a'' \times \log_{10} \frac{r_2''}{r_1''}. \quad . \quad (32)$$

$$(\text{III})\ p = 1{\cdot}86 \times a'' \times \log_{10} \frac{\pi w'' + b''}{b''}. \quad . \quad . \quad . \quad . \quad (33)$$

§ 8. **Calculation of the joint leakage permeance.**—We are now in a position to calculate the leakage permeance of our dynamo (fig. 208), taking the permeance of the several paths in succession, and assuming them to be approximately as shown in fig. 203.

(1) Between the flat tops of the pole-pieces, A_1 and A_2, taking the paths by proposition (III) as partly quadrants and partly straight lines,

$$p_1 = 1{\cdot}86 \times 11{\cdot}25 \times \log_{10} \frac{\pi \times 4 + 4{\cdot}125}{4{\cdot}125},$$
$$= 12{\cdot}7$$

(2) Between the sloping surfaces, B_1 B_2, at the top of the pole-pieces, the lines curve round in a zone outside the lines of region (1); hence by a combination of the principles of propositions (III) and (I), if β is in circular measure (fig. 212)—

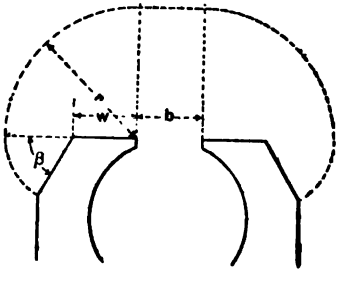


FIG. 212.

$$p_2 = \frac{2{\cdot}3 \times 2{\cdot}54}{(\pi + 2\beta)} \times a'' \times \log_{10} \frac{(\pi + 2\beta)r'' + b'' - 2\beta w''}{\pi w'' + b''}$$

and in our case since $\beta = \frac{\pi}{3}$

$$p_2 = \frac{1\cdot 86}{1\cdot 666} \times 11\cdot 25 \times \log \frac{1\frac{2}{3}\pi \times 8\cdot 25 + 4\cdot 125 - 8\cdot 37}{16\cdot 685},$$
$$= 4\cdot 6.$$

(3) Between the neighbouring edges of the opposing polar tips c_1 and c_2, and the corresponding surfaces, c_3 and c_4, below the armature.

Many of the lines which issue from the extreme edge of the polar projections converge upon the armature core, and are therefore not to be reckoned as lost by leakage. The lower horns are, however, of greater depth to allow of their receiving the fixing screws; it may therefore be regarded as sufficiently accurate to take the permeance of the lower pair only, or by rule 1—

$$p_3 = 2\cdot 54 \times \frac{1 \times 11\cdot 25}{4\cdot 125} = 7.$$

(4) Between the sides of the pole-pieces, D_1 and D_2, and the corresponding surfaces at the further end of the magnet. Each surface may be treated approximately as a rectangle 3⅜″ wide × 10½″ deep, the two being separated by a mean direct distance of 9″. Therefore, by proposition (III) the permeance between the one pair D_1 and D_2 is

$$1\cdot 86 \times 10\tfrac{1}{2} \times \log \frac{\pi \times 3\cdot 375 + 9}{9} = 6\cdot 6,$$

and the joint permeance on both sides of the magnet is $p_4 = 13\cdot 2$.

(5) From the back of the pole-pieces into the yoke, *i.e.* from E_1 to F_1, and E_2 to F_2. The surfaces E_1 and F_1 may be averaged as of equal area, 13″ wide × 6″ deep, and in order not to intersect the lines of (6. iii.), the paths may be taken as semicircles; hence, by proposition (II), the permeance between E_1 and F_1 or

$$p_5 = 1\cdot 86 \times 13 \times \log \frac{5\cdot 125 + 6}{5\cdot 125} = 8\cdot 2.$$

Now the magnetic difference of potential between the two surfaces E_1 and F_1 is approximately half that existing between the pole-pieces, for the yoke may be regarded as at zero potential, and the limbs as gradually increasing in potential to a positive or negative maximum respectively, as we proceed from the yoke to the poles. The lines, therefore, which flow through each limb, due to the leakage now under consideration, are

$$\zeta_5 = 1{\cdot}257 \frac{X_P}{2} \times p_5.$$

If p_5 be calculated as above, and we wish to express the leakage on the assumption that it is all due to the difference of magnetic potential at the poles, the value p_5 must be halved before it can be regarded as in parallel with the preceding permeances; when so halved it may be directly added to them, and, therefore, for our purpose it will be necessary to take the value $\frac{p_5}{2} = 4{\cdot}1$.

(6) From limb to limb. This leakage is divisible into three portions: (i) straight across from the inside of the one limb to the inside of the other, *i.e.* from G_1 to G_2; (ii), in curves from the neighbouring sides of the magnets, *i.e.* from H_1 to H_2, and between the corresponding surfaces at the further end of the magnet; and (iii) in semicircles, within the limits of the bobbins themselves over the outer surfaces K_1 and K_2, the lines not passing through the yoke.

The permeance of path (i) is

$$2{\cdot}54 \times \frac{10{\cdot}25 \times 11{\cdot}25}{5{\cdot}25} = 56,$$

and the joint permeances of path (ii) is

$$1{\cdot}86 \times 10{\cdot}25 \times \log \frac{\pi \times 5{\cdot}5 + 5{\cdot}25}{5{\cdot}25} \times 2 = 24.$$

Now the leakage along these paths from limb to limb takes place under varying differences of potential, as we pass from the yoke to the pole-pieces; the magnetomotive forces of the ampère-turns are uniformly distributed along the two limbs, but the difference of magnetic potential between the two limbs being the gradually increasing sum of these forces, less the fall of potential over the iron, rapidly rises in value as we proceed from the zero potential of the yoke towards either pole-piece.

The loss of potential over the iron may approximately be assumed to be confined to the limits of the bobbins and to be uniform in value over each cm. of their length; the average difference of potential acting between the two limbs is then half the maximum difference of potential, *i.e.* $= 1{\cdot}257 \frac{X_P}{2}$.

As regards path (iii.), the permeance of a small strip distant x cm. from the centre of a bobbin and of width dx is $\frac{dx \times a}{\pi x}$; this is acted on by a M.M.F. $= 1{\cdot}257 \frac{X_P}{2} \cdot \frac{x}{\frac{l}{2}}$, where l is the length of the bobbin, so that its flux is $1{\cdot}257 \,.\, X_P \,.\, \frac{x}{l} \times \frac{dx \times a}{\pi x}$. Since $\int dx = \frac{l}{2}$, the total flux issuing from one half of the bobbin and entering into the other half is $1{\cdot}257 \frac{X_P}{2} \cdot \frac{a}{\pi}$. As the area is increased by increasing the length of the bobbin, the length of the semicircular path is increased in the same proportion, so that for the same excitation the total flux is independent of the bobbin length. In order to rank the flux of paths (iii) with those of (i) and (ii), also acted on by $1{\cdot}257 \frac{X_P}{2}$, the permeance of each may be reckoned as $\frac{2{\cdot}54 \times a''}{\pi}$, or in our case $\frac{2{\cdot}54 \times 11{\cdot}25''}{3{\cdot}14} = 9{\cdot}1$, and the total of the three[1] is thus $p_6 = 89{\cdot}1$. Let $\zeta_6 =$ the actual number of lines that leak across under the average difference of magnetic potential; then $\zeta_6 = 1{\cdot}257 \frac{X_P}{2} \cdot p_6$.

These lines are distributed over the space about the limbs, and do not all pass through the entire length of limbs and yoke; the total number of lines carried by the limbs is, in fact, continually varying along their length, as lines leak out of or into them. We may, however, assume without much error that within the limits

---

[1] For round magnet-cores placed side by side as in fig. 194 or 78, see Prof. S. P. Thompson's Cantor Lectures, *Electr. Eng.*, Nov. 21, 1890.

$z_m = z_a + \zeta_1 + \ldots + \zeta_5$ and $z_m = z_a + \zeta_1 + \ldots + \zeta_5 + \zeta_6$ the number of ampère-turns required by the magnet will closely correspond to rather more than the arithmetical mean number of lines, or say $z_a + \zeta_1 + \ldots + \zeta_5 + \frac{\zeta_6}{1\cdot 5}$. The question, therefore, is, what permeance will give $\frac{\zeta_6}{1\cdot 5}$ lines when a magnetic difference of potential of $1\cdot 257\ X_P$ acts over it, and this is evidently $\frac{p_6}{3} = 29\cdot 7$.

All the permeances have now been reduced to such values as may be regarded as in parallel with the armature and air-gaps, and may therefore be immediately added together; and our final result is that the joint permeance

$$P = p_1 + p_2 + p_3 + p_4 + \frac{p_5}{2} + \frac{p_6}{3}, \quad . \quad . \quad . \quad . \quad (34)$$

$$= 12\cdot 7 + 4\cdot 6 + 7 + 13\cdot 2 + 4\cdot 1 + 29\cdot 7 = 71\cdot 3,$$

or in round numbers say 75, and the average number of lines passing through the entire length of the magnet is $z_a + \zeta$, where

$$\zeta = 1\cdot 257\ X_P \times P.$$

In addition to the several paths already considered, there are others which would have to be taken into account if greater exactitude were required, such, for instance, as from the pole-pieces to the bearings and their supporting pedestals on either side of the magnet. Again, the surfaces already taken into account may be paired differently; for example, we might calculate the permeance between A_1 and B_2 or between B_2 and F_2; but if this be done, it would be incorrect to add the several permeances together, since they are not all in parallel. In their actual distribution the lines of induction never cross one another or intersect; further, if two sets of lines are regarded as traversing the same air-space for a part of their course, *e.g.* leakage from A_1 to A_2, and leakage from A_1 to B_2, the fall of magnetic potential over that part of their course which is common to both must correspond to their joint density, and not to that of either set taken separately. Hence the addition of fresh paths would involve a distribution of the lines different from that assumed in our first approximation and a re-calculation of their amount, and such rearrangement might result in a slightly higher value for the leakage permeance. Still, if the leakage field be regarded as distributed mainly after the fashion indicated above, a close approximation may be made to the ampère-turns actually found to be required by the field-magnet when the machine is tested. A certain amount of judgment is required in the first instance in selecting the direction of the several paths, none of which intersect: but to guide us in this, actual experiment may be called to our aid, if a machine with the required type of field-magnet is at hand.[1] If a needle be fastened to a thread passing up the centre of a hollow tube of wood or cardboard, and the end of the thread be held so as to prevent the needle from being drawn to a pole, it can be used as an exploring magnet, and on plotting the directions in which it sets itself a good idea can be obtained of the distribution of the leakage flux.

A partial compensation for the difficulty of accurately determining the somewhat vague and indefinite factor, P, lies in the fact that a considerable percentage error in its determination produces, under ordinary conditions, but a small percentage of error in the total number of ampère-turns required to produce the useful field through the armature; for this total number of ampère-turns is the sum of those required for the armature and air-gaps and those required for the magnet, and an error in our estimate of P only affects the latter. Evidently, the smaller the proportion which the ampère-turns required for the magnet form of the total number, the less will an error in their determination affect the whole. Hence the percentage of error thus introduced will depend on the relative amount of X_m as compared with the total X, and will be greater in a toothed than in a smooth armature, and greater in the case of the magnet which has a long length of circuit. But, further, it will in especial depend upon the degree of saturation of the field-magnet, since the higher the induction the greater is the difference between the ampère-turns required to pass the supposed and the actual number of lines Z_m through the magnet.

§ 9. **Empirical formulæ for leakage permeance in bipolar machines.**—If the value of P for a particular machine of a given type has once been determined by calculation and checked by experiment, certain further conclusions may be drawn applicable to other machines of the same type, but differing in their dimensions. If the linear dimensions of one machine were simply magnified n times in the design of another, the new value for P would be n times its former value; the lengths of the leakage paths would be increased n times, but their cross-sections would be increased n^2 times. More often, however, the ratio of the two chief dimensions of the dynamo, viz., the diameter and length of the armature core, is altered, and this involves a more detailed consideration of the way in which the leakage permeances are affected by any change in these dimensions. The most important principle must be the fact that an increased or reduced length of armature core involves no further alteration than an equal increase or reduction in the width of the magnet parallel to the armature; while, on the other hand, a change in the diameter of the armature usually involves a more or less proportionate change in the depth and length of the magnet limbs, for the alteration in the area of iron in the armature necessitates an alteration of the area of iron in the magnet, and also of the length of the exciting coils. Now, considering the various leakage paths of the dynamo above discussed, it will be found that the permeances between A_1 and A_2, B_1 and B_2, E_1 and F_1, G_1 and G_2, and over K_1 and K_2 are almost solely dependent on the length of the armature, and vary directly with it; for if its diameter alone be altered, the length between them is increased, but so also to the same extent is one of the two dimensions which go to make up their area. On the other hand, the permeances between D_1 and D_2, or H_1 and H_2, are almost solely dependent on the diameter, being unaffected by any change in the length of the armature; and, further, they vary directly as the diameter of the armature, for although an increase in the diameter lengthens their paths, it increases both dimensions of their cross-section. Lastly, the permeance between the edges c_3 and c_4 of the pole-pieces varies directly as the length of armature and inversely as the diameter, for, with an increased diameter, the width of the gap between the pole-pieces would be increased, but the vertical depth of the edges would be scarcely altered. Hence an approximate formula for the total leakage permeance would take some such form as $P = k_1 L + k_2 D + k_3 \frac{L}{D}$, where k_1, k_2, k_3 are suitable constants for the particular type of machine. By this differentiation of the leakage permeances it is possible to judge of the relative effect of any alteration in the diameter or length of armature, and by the substitution of the new values of L and D to approximate to the resulting permeance of the new dynamo. In the case of the ring overtype dynamo, proportioned somewhat after the fashion of fig. 208, calculations similar to the above will lead us to the approximate formula

---

[1] For such actual experiments, vide *Journal Inst. Elect. Eng.*, vol. xix. part 86, pp. 244–246 (Trotter).

$$P = 3.25L'' + 2D'' + 7\frac{L''}{D''}, \quad . \quad . \quad . \quad . \quad (35)$$

where L'' and D'' are in inches; and from this it will be seen that an increase in the length of the armature produces a considerably greater increase in the leakage permeance than the same increase in the diameter. If the ratio $\frac{L}{D} = 1{\cdot}33$, as before, and the diameter is not very small, over one-half of the leakage takes place from surfaces the permeances of which are solely affected by the length of the armature, over one-quarter will be affected by a change in the diameter, and the remainder is affected by a change in either dimension.

In the case of a similar overtype dynamo with drum-armature, for a given diameter of core, the depth of iron will be greater than in the ring; hence the depth of the magnet will be greater, the second term which has to do with leakage between the outside faces, D_1 and D_2, H_1 and H_2, will acquire greater prominence, and k_2 must be increased. The equation will now take some such form as

$$P = 3{\cdot}5L'' + 2{\cdot}5D + 7\frac{L''}{D''}, \quad . \quad . \quad . \quad . \quad (36)$$

and in general the relative importance of the two dimensions depends on the thickness and length of magnet which suits a given design of armature.

The leakage of undertype bipolar drum dynamos is so largely dependent on the exact method by which they are supported on the bedplate, and the shortest distance between the iron of the bedplate and the iron of the magnet, that it is more difficult to give even approximate values for the constants k_1, k_2, and k_3 Roughly, it may be said that the leakage permeance varies between $\text{P} = 4\text{L}'' + 2{\cdot}5\text{D}'' + 8\,\frac{\text{L}''}{\text{D}''}$ and $\text{P} = 5\text{L}''' + 3\text{D}'' + 8\,\frac{\text{L}''}{\text{D}''}$; in both cases the normal distance between the bedplate and magnet has been taken at about 8–9 times the air-gap on one side between armature and pole-piece, but in the former case no sheet of metal runs under the pole-pieces; in fact, a continuous stretch of iron lying under the pole-pieces should, as far as possible, be avoided, or broken up by a large central air-gap, so as to force the lines to pass round through a considerable length of cast iron.

The above equations will, however, serve to show the varying importance which must be attached to the two dimensions L and D in different types, and from the form in which they are cast they are better adapted to prevent us from being misled when any radical alteration of dimensions is made than other and simpler equations which have been from time to time proposed.

§ 10. **Empirical formulæ for leakage permeance in multipolar machines.**—Similar empirical formulæ for multipolar dynamos with from 4 to 10 radial poles external to the armature are more difficult of establishment, owing to the greater diversity of their shapes and proportions. A rough approximation is, however, of service as indicating the general effect of passing to a greater number of poles. Taking first the sloping edges of the pole-shoes M R, O S (fig. 207), the opposing surfaces are usually inclined to each other at an angle which may be expressed by the term $75^\circ + \frac{240^\circ}{p^2}$, or in circular measure, $\pi\left(0{\cdot}415 + \frac{1{\cdot}33}{p^2}\right)$, where p is the number of pairs of poles. Between these surfaces the lines pass in curves under a M.M.F. $= 1{\cdot}257\ \text{X}_\text{P}$. The same distribution also extends up the bobbin for a distance which increases with the number of poles (*cp.* fig. 205), and with this extension the ratio of the two radii from the point of intersection of lines drawn from the neighbouring pole-edges is of the order 2·5 : 1. The width across the magnet may be taken as equal to that of the armature, seeing that the lines curve slightly outwards and that the winding usually projects beyond the core. If the paths of the lines were portions of circles, the permeance between one pair of edges would be

$$\frac{1{\cdot}86 \times \mathrm{L}''}{\left(0{\cdot}415 + \dfrac{1{\cdot}33}{p^2}\right)} \times \log 2{\cdot}5 = \frac{0{\cdot}74\,\mathrm{L}''}{\left(0{\cdot}415 + \dfrac{1{\cdot}33}{p^2}\right)},$$

and the permeance between the two pairs of edges which correspond to a magnetic circuit would be

for 4 poles, 2 L″
„ 6 „ 2·62 L″
„ 8 „ 2·96 L″

with a gradual approach to 3·56 L″ as the number of poles is increased. Or if the paths are reckoned after the analogy of proposition (III), the two surfaces at their nearest point may be regarded as separated by a distance $b = \dfrac{\pi \mathrm{D}(1-\beta)}{2p}$ where β is the ratio of the polar arc to the pole-pitch or in general $= 0{\cdot}75$, and the depth of each surface is $w = k.\dfrac{\mathrm{D}}{2p}$, where $k = 0{\cdot}6$ with 4 poles, 0·72 with 6, and 0·78 with 8 poles, whence, *e.g.*, with 4 poles,

$$p = \frac{1{\cdot}86\,\mathrm{L}''}{\left(0{\cdot}415 + \dfrac{1{\cdot}33}{4}\right)} \times \log \frac{0{\cdot}6\,\dfrac{\mathrm{D}}{4} \times \left(0{\cdot}415 + \dfrac{1{\cdot}33}{4}\right)\pi + \dfrac{\pi\mathrm{D}(1-\beta)}{4}}{\dfrac{\pi\mathrm{D}(1+\beta)}{4}}$$

$$= \frac{1{\cdot}86\,\mathrm{L}''}{\left(0{\cdot}415 + \dfrac{1{\cdot}33}{4}\right)} \times \log \frac{0{\cdot}6\left(0{\cdot}415 + \dfrac{1{\cdot}33}{4}\right) + (1-\beta)}{1-\beta} = 1{\cdot}1\,\mathrm{L}''$$

The permeance of both edges is then

with 4 poles, 2·2 L″
„ 6 „ 2·78 L″
„ 8 „ 3·04 L″.

Neither assumption is exact, but the latter values, as the higher, more closely represent the curved paths of the leakage

Nearly as large an amount of leakage passes from the inside surfaces of the poles across to the inner surfaces of the yoke (cp. fig. 205); the distance along the bobbin at which the leakage passes into the yoke rather than across into the adjacent pole is determined by the point at which the former path is half that of the latter, and its decrease with an increasing number of poles has been roughly allowed for in the previous calculation. Thus the yoke leakage may be taken as extending from the root of the pole downwards to a distance of say four-fifths of the bobbin length in the 4-pole machine to three-quarters with 6, and two-thirds with 8 poles. The length of the air-path at any point varies in proportion to its distance x from the root of the pole, and is approximately equal to x in the 4-pole octagonal machine of fig. 197, and in other cases with a circular yoke-ring equal to $1{\cdot}2\,x$. The flux on one side of a pole in a small strip of depth dx is then, *e.g.*, in a 4-pole machine with circular yoke-ring

$$\frac{dx \times a}{1{\cdot}2x} \times \frac{1{\cdot}257\,X_P \times 0{\cdot}8}{2} \times \frac{x}{0{\cdot}8l};$$

but $\int dx = 0{\cdot}8\,l$, so that the total flux on both sides is

$$1{\cdot}257\,X_P \times \frac{a'' \times 2{\cdot}54 \times 0{\cdot}8}{1{\cdot}2}.$$

The permeance in relation to X_P is thus

in the 4-pole machine with octagonal yoke-ring	$2{\cdot}04\,L''$
„ „ „ „ „ „ circular „	$1{\cdot}7\,L''$
„ „ 6-pole „ „ „ „	$1{\cdot}58\,L''$
„ „ 8-pole „ „ „ „	$1{\cdot}4\,L''$.

The leakage from the flanks of the poles into the yoke (fig. 205, side view) is proportional to their transverse width, and so depends upon the diameter of the armature, the number of poles, and the polar arc that is used. The area of the yoke surface is considerably greater than that of the poles; if a be the width of the pole-flank, and l be the length of the bobbin, the flux may be assumed to be concentrated between two surfaces of average width $2{\cdot}5\,a$, and of depth $\frac{l}{4}$, the two surfaces being separated by a distance

$\frac{3l}{4}$; then by proposition (III), the permeance of one flank

$$= 1\cdot86 \times 2\cdot5\, a'' \times \log \frac{\pi\frac{l}{4}+\frac{3l}{4}}{\frac{3l}{4}}, \text{ acted on by } \frac{X_P}{2},$$

and of both flanks in relation to X_P is $p = 1\cdot45a''$. As a general rule, a'' is approximately $= D'' \times \sin\frac{90^\circ}{2p}$, whence the flank permeance

with 4 poles $= 1\cdot45\ D'' \sin 22\cdot5^\circ = 0\cdot55\ D''$
" 6 " $= 1\cdot45\ D'' \sin 15^\circ = 0\cdot375\ D''$
" 8 " $= 1\cdot45\ D'' \sin 11\cdot2^\circ = 0\cdot28\ D''$.

In the above no difference has been made between poles of rectangular and of circular section, but with the proportions that are customary in multipolar machines the variations due to the two shapes largely balance one another.

Lastly, the leakage between the flanks of the pole-shoes is independent of the length, and is also to a great extent independent of the diameter; since, as their area increases with increase of diameter, so also does the length of path between them. Treated as rectangles of some $1\frac{1}{2}''$ depth, and of width $= 0\cdot75$ of the pole-pitch, their mean distance apart is about $0\cdot4$ of the pole-pitch, so that their total permeance is

$$\frac{1\cdot5'' \times 2\cdot54 \times 0\cdot75}{0\cdot4} \times 2, = \text{say } 14.$$

If the maximum flux occurring at the root of a pole be experimentally measured and expressed in terms of the useful flux, the leakage given by the above errs on the small side, but since all the leakage lines do not flow through the entire length of the magnet-core and yoke, the use of the calculated values for P in determining the mean flux without deduction leads to results which are justified in practice when the magnet is not highly saturated. The calculations are, however, at best only approximations, and other surfaces and edges may also add their quota of leakage; further, when the magnet is strongly saturated, the difference of magnetic potential acting across the air may along certain paths be somewhat more than proportional to the excitation between the poles. For the purpose of designing, maxima values are required rather than minima, and the error will be on the safe side, if the assumed permeance be taken as $1\frac{1}{2}$ times the above.

The addition of the several items then gives for the leakage permeance in multipolar machines with circular yoke-ring external to the armature the following values, as the maxima likely to be found under ordinary conditions :—

with 4 poles, $P = 5{\cdot}85L'' + 0{\cdot}82D'' + 21$ . . . (37*a*)
„ 6 „ $= 6{\cdot}55L'' + 0{\cdot}56D'' + 21$ . . . (37*b*)
„ 8 „ $= 6{\cdot}66L'' + 0{\cdot}42D'' + 21$ . . . (37*c*)

It will be found that as compared with the two-pole machine the diameter of the multipolar plays a less important part, and this might be expected from the fact that to a great extent the air-gap and length of bobbin remain the same, however large the diameter and the number of poles.

§ 11. **The leakage coefficient, ν, and its values.**—Since the leakage permeance of any given dynamo is due to the air-spaces between certain surfaces, its value is practically a constant quantity for that dynamo, and independent of the actual amount of leakage. But the difference of magnetic potential between the poles of the dynamo increases as the number of useful lines through its armature is increased, so that the leakage ζ will also continuously increase as Z_a is increased. Further, it will increase faster than Z_a; for as the armature core becomes more and more highly saturated, an increase in the ampère-turns expended over the armature will produce less and less increase in the lines through it, while the leakage still continues to increase almost directly as the difference of magnetic potential between the poles.[1] Hence the proportion which the leakage lines bear either to the useful or to the total number of lines through the magnet will continuously increase, and for each degree of magnetisation of the dynamo $\frac{Z_a+\zeta}{Z_a}$ or $\frac{Z_m}{Z_a}$ will have a different value. In the case of the drum overtype dynamo above considered, $Z_m = 1{\cdot}23 Z_a$ when it is magnetised as it would be under ordinary working conditions, so as to obtain an induction of about 5750 in the air-gap; in other words, the leakage is about 23 per cent. of the useful lines, or 18·7 per cent. of the total flux. Strictly speaking, the factor by which the useful lines through the armature must be multiplied in order to obtain the greater number of lines flowing through any other part of the circuit will vary all along the length of the magnet, as lines leak into or out of it. With sufficient accuracy, however, we may assume the flux of lines to be constant over each of our main subdivisions of the magnetic circuit, so that $Z_m = \nu_m Z_a$, $Z_y = \nu_y Z_a$, $Z_p = \nu_p Z_a$; the different parts of the magnet are denoted by the subscript letters as before, and each part has its appropriate factor, ν, greater than unity. The fundamental equation for the ampère-turns of a simple magnetic circuit may now be expressed as

---

[1] Although generally correct in practice, the above statement is not universally true. Indeed both experiment and theory have shown that in certain special cases the leakage may actually decrease as the magnetising intensity is increased, this somewhat surprising result being due to the tangent law of refraction of the lines of induction as they leave the bounding surface of the iron (Du Bois, *The Magnetic Circuit*, §§ 88 and 123). The true law depends upon the distribution of the magnetising coils with reference to the geometrical form of the iron magnet, and the case of a decreasing leakage with increasing magnetomotive force is one in which the analogy of the magnetic and electric

$$\mathrm{X} = f'\left(\frac{Z_a}{a_a}\right) . l_a + 0{\cdot}8\,\frac{\nu_g Z_1}{a_g} . 2l_g + \mathrm{X}_b + f'\left(\frac{\nu_p Z_a}{a_p}\right) . l_p$$
$$+ f'\left(\frac{\nu_y Z_a}{a_y}\right) . l_y + f'\left(\frac{\nu_m Z_a}{a_m}\right) . l_m,$$

and such was, in principle, the original form of the equation as first published by Drs. J. and E. Hopkinson in their classical paper on *Dynamo-electric Machinery*.[1] Having established the

---

circuits entirely breaks down. For the same reason it is impossible to apply the method of Kirchhoff's laws to branch magnetic circuits with any approach to scientific accuracy. This objection does not, however, prevent the analogy from being of great service to the dynamo designer, since the deviating cases are seldom met with.

[1] *Phil Trans.*, 1886, part i. p. 331, reprinted in *Original Papers on Dynamo Machinery*, pp. 79–133. This important paper may be said to have first laid down a sure groundwork of theory for the scientific design of dynamos, and raised it from the level of empirical 'rules-of-thumb.' A study of it is essential to every student.

theoretical equation, they further measured experimentally[1] the number of stray lines of induction that leak through different portions of the air-space about the field-magnets of two dynamos of different types, each of which was excited with its normal magnetising current. Thence they were enabled to deduce the values which the factors v_g, v_p, etc., have for the given dynamos when magnetised to their working degree of saturation.

The first factor v_g requires further explanation. It has been said that, owing to the difference of magnetic potential which exists between opposite sides of an armature under the two pole-pieces, a certain number of lines pass diametrically across the interior of the armature through the shaft without passing round through the core between the two pole-pieces; hence, if z_a be reckoned as the number of lines actually flowing round through the two halves of the core, and passing through sections of the core taken midway between the pole-pieces, the number of lines flowing through the air-gaps must be greater than z_a; for the additional lines passing diametrically across the shaft must have passed through the air-gaps, and must, therefore, be taken into account. This is effected when z_a is multiplied by the factor v_g. In the case of drum armatures it is unnecessary to introduce this factor, owing to the lower induction which is usual in them and the absence of internal wires; but in rings strongly magnetised, the lines across their interior may become of sufficient importance to warrant their being taken into account. These lines are cut by the internal wires and give a back E.M.F., so that their net effect on the E.M.F. is zero, and in the E.M.F. equation z_a is used as contrasted with z_g. Thus, in the ring armature tested by the Drs. Hopkinson, which was magnetised to a very high induction at the part of maximum density, the internal leakage was then found to amount to as much as 5 per cent. of the lines through the armature, or $v_g = 1{\cdot}05$. The dynamo, upon the ring armature of which the above experiment was carried out, was of the 'Manchester' double-magnet type (fig. 192), and it was found that the number of lines flowing through a section of the upper pole-piece near the magnet core, when expressed in terms of z_a, was $= 1{\cdot}13\ \frac{z_a}{2}$, and through the lower pole-piece $1{\cdot}24\ \frac{z_a}{2}$, the difference between the two being due to leakage from the magnet-cores into the bearings, which were supported on extensions from the lower cast-iron pole-piece. The greatest number of lines flows through a section at the centre of each magnet-core, the value gradually decreasing from this point up to their ends; the *mean* number of lines flowing through either magnet was $1{\cdot}49\ \frac{z_a}{2}$, or $\nu_m = 1{\cdot}49$. In designing the field-magnet, it is this latter factor which is in reality the most useful to us, since we wish to obtain some mean value of z_m whence we may calculate the ampère-turns required to overcome the reluctance of the magnet, assuming that that number passes through the entire length of the magnet core. Thus, in the above case of the 'Manchester' double-magnet field, if we require z_a lines through the armature, 49 per cent. more lines must be allowed for in calculating the area and ampère-turns required for the magnet-cores, since out of the total number of lines through them only about two-thirds pass through the armature core as well. Again, in the case of the undertype Edison-Hopkinson machine, with drum armature, which was also examined by Drs. J. and E. Hopkinson, it was found that out of 100 lines passing through the cross-section of each magnet limb at the middle of its length, only about 75 reach the armature core, or $\nu_m = 1{\cdot}32$. Out of the observed loss of 25 per cent., nearly half, or 11 per cent., was due to leakage through the zinc base into the cast-iron bedplate which stretched directly across below the pole-pieces; the leakage between the limbs and between the upper edges of the pole-pieces was 8·3 per cent., the remainder being accounted for by leakage between the lower edges of the pole-pieces, from their sides, and from their backs into the yoke.

---

[1] For certain points relating to leakage testing with a ballistic galvanometer, see (Giles) *Elec. Eng.*, vol. viii. p. 343.

In most cases we may without much error assume the different factors for the different parts of the magnet to be identical, and reckon the number of lines through any part of it to be $z_m = \nu z_a$ where ν may be called the *leakage coefficient* of the dynamo. The values of ν and the percentage distribution of the leakage for several types of dynamos have been experimentally determined and recorded by different observers;[1] but, unfortunately, little or no information has at the same time been given of the degree of saturation of the armature and magnet, beyond the fact that they were magnetised to their normal working extent.

Such recorded values of ν may be used for the purpose of approximately calculating the area and ampère-turns required by the field-magnet. Thus the usually adopted value of ν for the 2-pole overtype dynamo of fig. 2, with cast-iron pedestals on either side of the magnet, is about 1·25, which agrees closely with our previous calculation as to the amount of leakage when normally excited with 15,400 ampère-turns; and for the similar undertype of fig. 190, $\nu =$ from 1·3 to 1·35. For the Manchester field of fig. 192, with the armature supported on cast-iron extensions from the lower pole-piece, ν was, as stated above, $= 1·49$. If the air-gap is long, even this is too low, and ν approaches the value of 2. In multipolar dynamos of the type of fig. 197 with external poles, ν ranges from 1·1 at no load to 1·25 at full load. In the case of an alternator of the type shown in fig. 79, without iron in its armature, ν has been found to be as high as 2; in other words, only half the total flux passed through the armature, the remaining half leaking directly across from one pole to the neighbouring pole on either side without being cut by the rotating inductors. Implicit reliance cannot, however, be placed on any such values of ν. As already mentioned, it varies decisively if the degree of saturation be considerably varied, and since this is so, it varies with different values of the 'back ampère-turns,' *i.e.* with different values of the armature current, even though the total number of ampère-turns on the field remains unchanged; for the armature current directly affects the degree of saturation of the core. It further varies decisively if the transition is made from a smooth-surface armature with long air-gap to one with a short air-gap and toothed core. On this account the first method of obtaining the ampère-turns, which involves a term $\zeta = \frac{\text{X}_{(a+g+b)}}{0{\cdot}8} \times \text{P}$, is decidedly preferred by the authors to the second form, which contains the factor ν. In default of such experimental determination of the values which it takes (as was, in fact, made by the Drs. Hopkinson before they put into use their original equation), we are more liable to be misled by false values of ν than by errors in determining P. The somewhat laborious task of determining P, even if it be, at best, only an approximation, is thus very strongly to be recommended, especially in the case of the adoption of a new type of field-magnet.

---

[1] *Vide* Esson, "Some Points in Dynamo and Motor Design," *Journal Inst. Elec. Eng.*, vol. xix. part 85; "Magnetic Leakage in Dynamos and Motors" (Ives), *Electr. Review*, Jan. 22 and 29, 1892; "Magnetic Data of Sprague Motor" (Parshall), *Elec. Eng.*, June 13, 1890; (Puffer) *Electr. Review*, April 15, 1892; (Giles) *Elec. Eng.*, Oct. 9, 1891; and especially (Thornton) *Electr. Eng.*, vol. xxix. pp. 523 ff.

Finally, it may be remarked that in all heteropolar machines, and to a much less degree in all homopolar machines, a certain amount of leakage is an inevitable necessity, and, so long as it be kept within due limits, is but a small evil. If the energy required to magnetise a dynamo is from 3 to 5 per cent. of its output, as it is in most modern dynamos, magnetic leakage cannot very greatly affect the efficiency and cost of working, for a supposed complete absence of leakage would but slightly decrease this percentage; and the practical endeavour to reduce it appreciably may reduce the weight of iron and wire, but may also lead to a considerably increased first cost of the machine, owing to a too lavish use of non-magnetic materials, such as gun-metal.

§ 12. **Flux-curves of dynamo.**—It has been shown in § 6 that the number of ampère-turns required to give 5,100,000 lines through the armature of a particular dynamo is 15,400 when the armature current has its full value (*i.e.* the sum of 140 ampères in the external circuit and about 4 ampères in the shunt circuit of its field-magnet, or 144 ampères in all). If a number of different values be assigned to z_a, and the ampère-turns required in each case be determined (always on the assumption that the armature current has its full value), a curve may be plotted, connecting together the corresponding values of z_a and X, and this curve may be called the *flux-curve* of the particular dynamo for its full current, or more strictly for the full value of its armature ampère-turns. Curve 3 in fig. 213 shows such a full-current curve of flux for our 9″ × 12″ dynamo, C_a being maintained throughout at its full value of 144 ampères. The corresponding values of z_a and X may either be worked out by the same methods of calculation as have been previously applied, or, if the machine be already built, they may

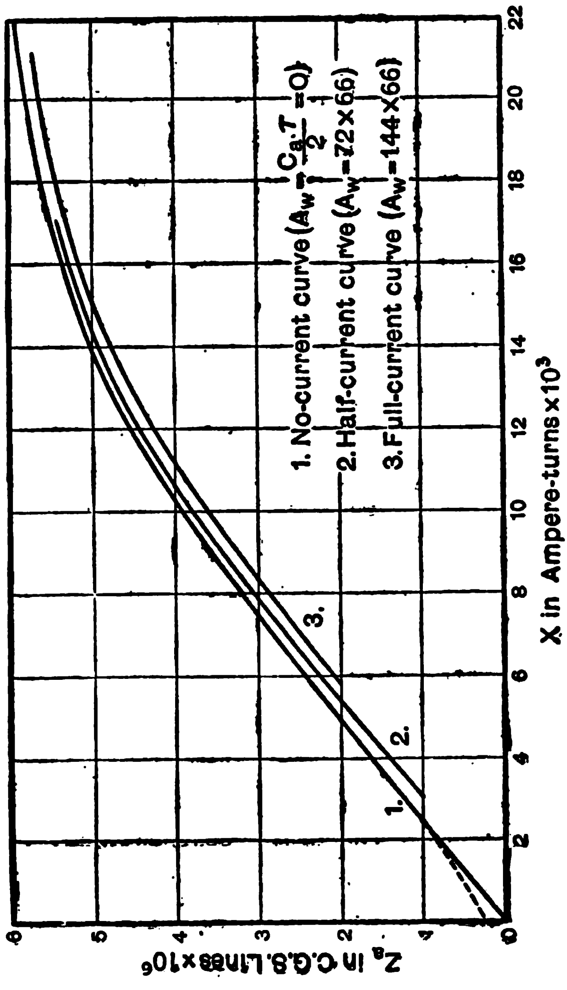


FIG. 213.—Curves of useful flux of 9" x 12" dynamo.

be determined experimentally, as will be shown in Chap. XVI: in either case the brushes are throughout assumed to be adjusted to the point of minimum sparking, and the abrupt termination of the curve marks the point below which it is impossible to reduce the magnetisation without causing excessive sparking at the brushes. If we further carry out the same process for a different and smaller value of c_a, say half its maximum value, or 72 ampères, a second curve of flux will be obtained for half-current: this will fall higher than curve 3, inasmuch as the back ampère-turns are less, and therefore fewer ampère-turns are required on the field to produce a given number of lines through the armature. Finally, if c_a be taken as $=0$, there is no reaction of the armature-current on the field, the back ampère-turns, x_b, are zero, and the highest or 'no-current' curve of flux (1, in fig. 213) is obtained. It should be observed that the horizontal distance between the curves for no-current and full-current at, say, 5,100,000 lines, is more than the direct value of the back ampère-turns, viz. 515, inasmuch as these latter increase the leakage, and therefore the ampère-turns required over the iron of the magnet. The use and importance of these curves will be more apparent in the next chapter. A tangent to the initial part of the curve 1 may be called the 'air-line,' since it gives the ampère-turns required by the air-gap for any value of the flux, and it is of service to determine this experimentally in order to check the calculated values, especially in toothed armatures. This leads us to a consideration of the air-gap permeance in the case of toothed armatures.

§ 13. **Calculation of air-gap permeance of toothed armature.**—Owing to the presence of open slots in a toothed armature, the direction of the lines in the interferric gap is no longer strictly radial from pole to armature core, nor is their density within the bore of the pole-piece uniform. Along the edges of the slots the lines curve round and enter the iron through the sides or flanks of the teeth, and as their path is then longer than that of the lines which enter directly through the face of the teeth, their density is less. Thus the total flux is divided into bands of dense and weak induction corresponding to the teeth and slots, as may be made visible by iron filings introduced into the gap after the method described by Dettmar (*E.T.Z.*, vol. xxi. p. 944). The permeance of the air-gap of the smooth armature (apart from the fringe) being equal to the area under the pole-face divided by the normal length of the air-gap, the permeance in the case of the toothed armature is evidently less by some amount depending mainly upon the relative proportions of the width of tooth and of the opening of the slot to the air-gap. The question is, however, still more complicated, since with any given values for the ratios of the width of the top of the tooth to the opening of the slot or of the latter to the air-gap, *i.e.* with given values of $\frac{k_{t1}}{k_s}$ and $\frac{k_s}{l_g}$ (fig. 219), the permeance of the slotted armature does not necessarily always bear the same ratio to that of the smooth armature of the same dimensions. The air-gap permeance is in fact also dependent, although to a less extent, upon the depth of the slot and upon the degree to which the density of the lines rises towards the root of the tooth, owing to the taper of its sides. As a general rule the sides of each slot are parallel to one another, so that the width of the tooth tapers towards the centre by an amount depending upon the diameter of the armature and the depth of the slot. Roughly speaking, it may be said that the flux passes in straight lines across from the iron of the pole-face to the face of each tooth, and in straight lines joined by arcs of circles to the sides of the teeth within the slots. The actual curved paths of the fringe at the slot-edges may be shown by the beautiful figures obtained from photographs of the stream-lines in a viscous liquid forced between two sheets of glass. This method of investigation can be made to imitate with exactitude the analogous passage of lines through media of different permeability, and from the original paper on "Lines of Induction in a Magnetic Field"[1] by Prof. Hele-Shaw and Alfred Hay, who first employed the method, we are enabled, by the kind permission of the authors and of the Royal Society, to reproduce figs. 214 and 215. The lines forming the fringes within the slots are as a fact more gradually curved than has been assumed above, but in the main bear out its correctness. Since the average density of the lines over the air-gap proper does not in ordinary cases exceed 7000 to 8000 C.G.S. lines per sq. cm., and the area of the iron teeth at a distance $\frac{k_s}{2}$ cm. from their top is seldom less than h the area of the air-gap reckoned as in a smooth-core machin it follows that the density across the section distant by half t width of a slot from the top can never exceed 16,000, even if the lines are here confined to the iron. Under these circum stances the difference of magnetic potential required to pa the flux through the $\frac{k_s}{2}$ cm. of the teeth is inappreciable, an the tips of the teeth to this depth may be treated as a

---

[1] *Phil. Trans*, series A, vol. cxcv. pp. 303–327.

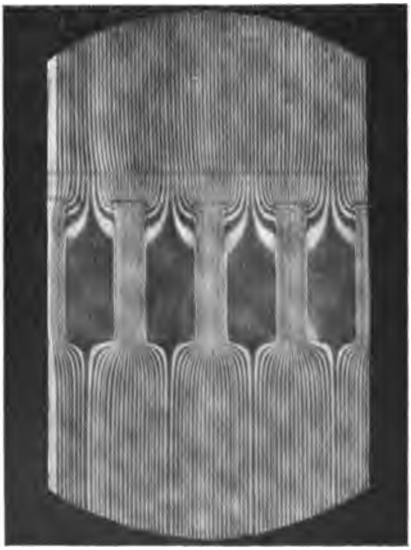
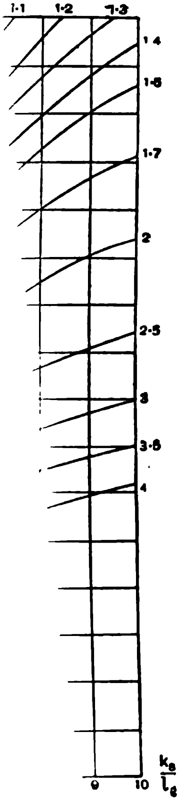


FIG. 214.—Induction in air-gap of toothed armature. Permeability of teeth = 100.

equipotential surface between which and the surface of the pole there exists a difference of magnetic potential due to $\frac{X_g}{2}$ ampère-turns. We thus obtain indirectly a definition of the air-gap permeance of the toothed armature which may be compared against that of the smooth core with the same normal air-gap of L_g cm. A more accurate mathematical solution of the

'arter, and to his pape
indebted for the curve
coefficient *m* by whicl
must be multiplied i



d armatures.

re. It remain
nt do not trul

problem has been given by Mr F. W. Carter, and to his paper on "Air-gap Induction"[1] the authors are indebted for the curves of fig. 216, which give the values of the coefficient *m* by which the $\mathbf{x}_g$ as calculated for a smooth core must be multiplied in

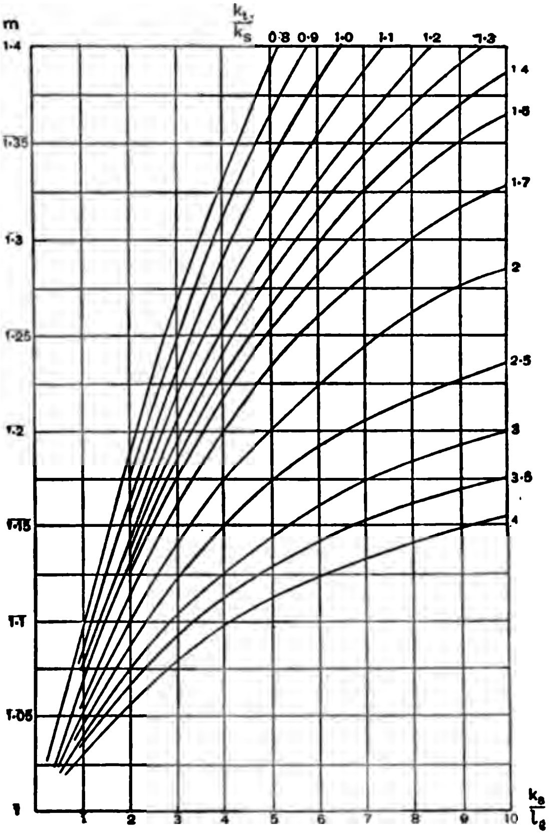


FIG. 216.—Coefficient for equivalent air-gap in toothed armatures.

order to obtain the real $\mathbf{x}_g$ of the toothed armature. It remains to point out in what way the results to a slight extent do not truly

[1] *Elec. World and Engineer*, Nov. 30th, 1901

represent the facts. In the reproductions of figs. 214 and 215, the sides of the teeth are parallel and their permeability is constant and high, so that little potential is absorbed over the length of the tooth even though the depth of the slot is considerable. The gradual broadening of the lines within the slots in figs. 214 and 215 indicates, however, the weakening of their density, and in reality there must be down the centre of the slot a very weak field passing into the iron at the bottom. Such a field is assisted onwards by the ampère-turns expended over the teeth, and has the effect of slightly straightening the lines within the slot. If the slots were very shallow, the tendency for the lines towards the centre of the slot to pass straight across from the pole-face to the iron at the bottom would be strengthened; all the lines would then be still further straightened (fig. 217), and the proportion of the fringe to the flux entering the faces of the teeth would be higher, so that the permeance would be slightly higher than calculated. The same effect is produced in another way, if the depth of the slot and the taper be great, and at the same time the density at the root rises to such a high value that lines begin again in appreciable number

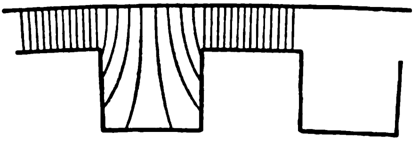


FIG 217.

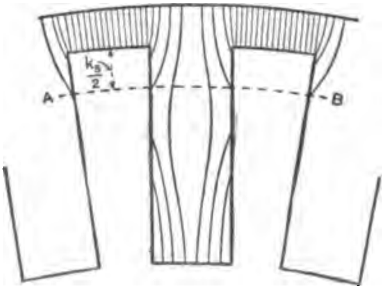


FIG. 218.

to issue forth into the slot as shown in fig. 218. A large number of ampère-turns are then being expended over the teeth; the line A B practically becomes again an equipotential surface, and any flux which passes straight through the air-gap is carried onwards through the slot by the ampère-turns which are subsequently to be credited to the iron teeth, so that as before the fringe of lines without the tooth-face is higher than was assumed, or conversely, less excitation is required over the air-gap proper. It will now be evident on what grounds our statement is made that the true X_g of the toothed armature cannot be absolutely determined from the mere ratios of k_{t1} and k_s to l_g, but is also dependent upon the condition of the teeth and the depth of the slots.

In conclusion, it may be added that the useful fringe that extends from the edges and flanks of the poles into the armature core may without much error be taken as the same as in the smooth armature, since the longer the path of the lines from the pole to the armature, the less becomes the influence of the broken surface of the core. If, therefore, it be required to calculate the excitation necessary for the air-gap of the toothed armature, it is convenient to be able to divide the total permeance or multiply the total reluctance by m; hence the two coefficients K_1 and K_2 of § 6 must first be multiplied by m. The total air-gap reluctance corresponding to one field or pole and one air-gap is then

$$\frac{ml_g}{(L + mK_1l_g)(A' + mK_2l_g)}$$

and

$$X_g = 0{\cdot}8 \, . \, Z_g \, . \, \frac{2ml_g}{(L + mK_1l_g)(A' + mK_2l_g)} \qquad . \quad . \quad . \quad (38)$$

It will be noticed that the quotient

$$\frac{Z_g}{(L + mK_1l_g)(A' + mK_2l_g)}$$

is of the nature of a flux-density, so calculated as to permit of its being multiplied by the effective length of the air-gap in order to find X_g. The real average B_g, or the value of the induction averaged over the whole of the effective area of the air-gap, remains as before in the smooth-core armature, viz.:

$$B_g = \frac{Z_g}{(L + K_1 l_g)(A' + K_2 l_g)}$$

and should still be reckoned out in order to judge of the performance of the machine.

§ 14. **Calculation of reluctance of teeth in toothed armature.**—Next must be calculated the ampère-turns necessary to carry the lines through the teeth and slots from the assumed equipotential circle A B to the unbroken circle of the core at the bottom of the teeth (fig. 219). As has been already described, the flux is divided between the two parallel paths offered by the iron teeth and slots, together with any non-magnetic spaces existing in the line of the teeth. In order to prevent the reaction of the armature reaching an undue amount, it is advisable to work the teeth at a high density; it therefore becomes very necessary to calculate closely the ampère-turns required over the teeth,[1] and in the first place to determine the proportion in which the flux divides between the teeth and slots, as fixed by their relative permeances.

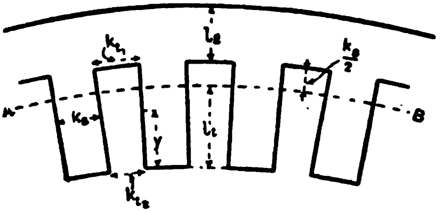


FIG. 219.

Between the laminations of the teeth there exist the small non-magnetic spaces corresponding to the insulating material between the discs which form the armature core: there may also be ventilating gaps arranged at intervals between the divisions of the core. If L be the gross length of the core, let η L be the net length of iron, η being the ratio which the added thicknesses of the iron laminations bear to the gross length. Then the cross-section of iron in a tooth is $k_t \cdot$ L $\cdot \eta$, of air in a slot is $k_s \cdot$ L, and

---

[1] Especially when the magnitude of the field left for reversing is being estimated.

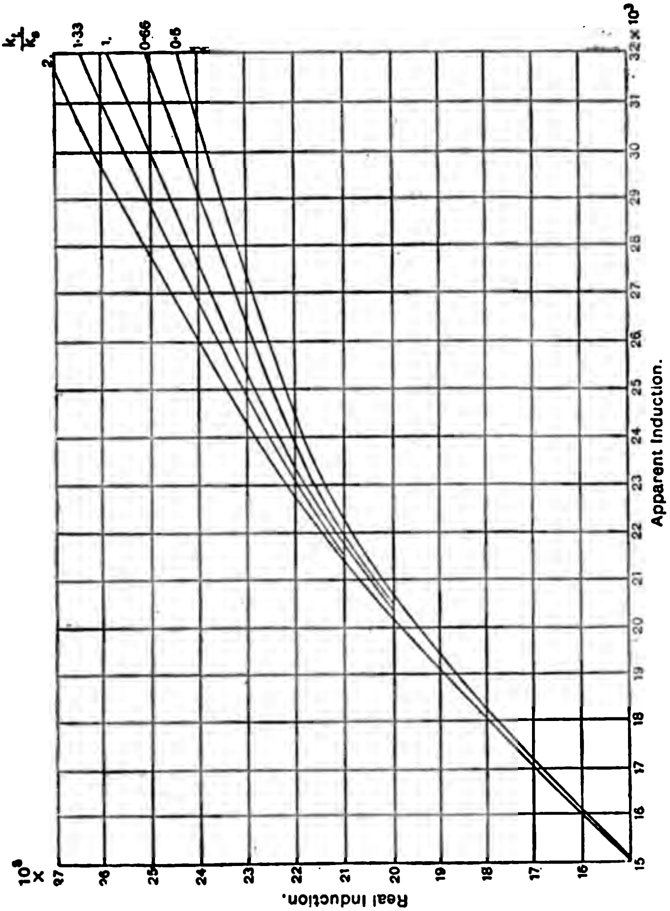


FIG. 220.—Curve connecting real and apparent inductions for different values of $\frac{k_t}{k_s}$.

of air or non-magnetic material between the discs composing the teeth k_t L $(1 - \eta)$. Taking the number of teeth under a pole, together with one or two extra to allow for the spreading outwards of the fringe along the pole-edges, let z = the total number of lines to be carried by one tooth and one slot per cm. length of the core. Then at any section through a tooth, $z = k_t \, . \, \eta \text{B} + \text{H}\{k_s + k_t(1 - \eta)\}$. The induction increases in the tapering tooth as we approach its root, and the leakage of the flux outwards into the

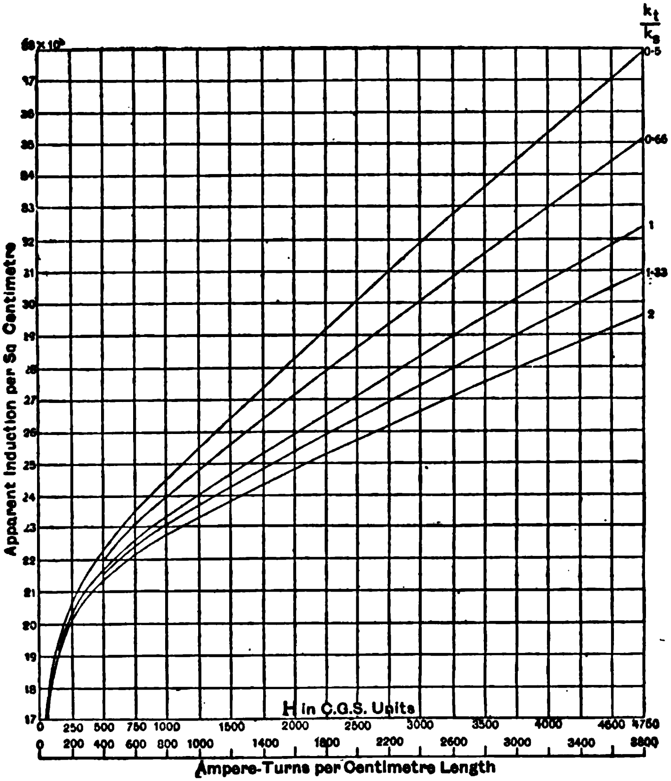


FIG. 221.—Curve connecting ampère-turns with apparent induction in teeth.

slot becomes more and more important. The apparent induction in the teeth on the assumption that all the z lines pass solely through the iron is $\text{B}' = \frac{z}{\eta k_t}$, and the divergence of this value from the real induction becomes greater and greater as the tooth becomes more saturated. Thus at any point at which the width of the tooth is k_t, the apparent induction is

$$B' = B + \frac{H\{k_s + (1 - \eta)k_t\}}{\eta \,.\, k_t} \quad . \quad . \quad . \quad . \quad (39)$$

For any given values of η and of the ratio of width of tooth and slot, a curve can be drawn connecting the true iron induction with the apparent induction, and in fig. 220 this has been done for several values of the ratio $\frac{k_t}{k_s}$. In order, then, to allow for the continually varying value of the ratio $\frac{k_t}{k_s}$, the tooth must be divided into, say, five sections, the average value of B′ calculated for each section and the corresponding value of B read off from the curves of fig. 220, the ratio of k_t to k_s being taken into account for each section. The mean of the values of H corresponding to the five values of B multiplied by the length of the tooth l_t in cm. reckoned from the circle A B, will then give with close accuracy the magnetic potential or ampère-turns required over the tooth. Or the curves of fig. 220 may be immediately converted into the corresponding curves of fig. 221 with ampère-turns per cm. length as abscissæ.

Another method of solving the problem has been given by W. B. Hird in a note on the reluctance of the teeth in a slotted armature,[1] and this method in a slightly modified form is described in the following words. The line integral of H over the path within the teeth of length l_t is

$$\int_0^{l_t} H \,.\, dy = l_t \,.\, \frac{1}{k_{t1} - k_{t2}} \int_{k_{t1}}^{k_{t2}} H \,.\, dk_t.$$

But $$k_t \,.\, \eta \,.\, B + H\{k_s + (1 - \eta)k_t\} = z.$$

The last term on the left-hand side, which gives the amount by which the width of the slot must in effect be increased in order to take into account the lamination spaces or ventilating gaps between the discs, really varies, but, as the whole of the second term only becomes of importance at the root of the teeth, the value k_{t2}

---

[1] *Journal Inst. Electr. Eng.*, vol. xxix. p. 936.

at the root may without error be substituted for k_t, and the expression within the bracket be treated as a constant. It can then be shown that

$$\int_{0}^{l_t} \mathrm{H} \,.\, dy = l_t \left\{ \frac{z}{\eta(k_{t1} - k_{t2})} \int_{\mathrm{B}_2}^{\mathrm{B}_1} \mathrm{H} \,.\, d\left(\frac{1}{\mathrm{B}}\right) + \frac{k_s + (1 - \eta)k_{t2}}{\eta(k_{t1} - k_{t2})} \int_{\mathrm{B}_1}^{\mathrm{B}_2} \mathrm{H} \,.\, d\left(\frac{\mathrm{H}}{\mathrm{B}}\right) \right\} \qquad (40)$$

where B_1 is the real density at the top, and B_2 is the real density at the root of the tooth corresponding respectively to the widths k_{t1} and k_{t2}. Although complicated in form, this expression is easier to work with than would appear at first sight. From the known (H, B) curve of the iron used for the armature discs—*e.g.*, figs. 128 and 129—we can plot once for all two curves having B as abscissæ and $\int \mathrm{H}d\left(\frac{1}{\mathrm{B}}\right)$ and $\int \mathrm{H}d\left(\frac{\mathrm{H}}{\mathrm{B}}\right)$ respectively as ordinates. This has been done in figs. 222–3, and from these curves the values corresponding to the B_1 and B_2 of a given tooth can be immediately read off and substituted in the expression for the line integral of H. The values of B_1 and B_2 can be easily found from the known values of the apparent induction B' for a given ratio of $k_t : k_s$ at any part of the tooth, and thence by interpolation from the three curves of fig. 220. It should be noticed that the upper and lower limits are reversed in the two integrals, and that $\int \mathrm{H}d\left(\frac{1}{\mathrm{B}}\right)$ has the negative sign, so that the difference between the sum for the upper limit with the lower induction and the lower limit with the higher induction leaves a positive surplus. Further, as the lower induction at the top of the tooth seldom will exceed some 16,000 or 17,000, it is sufficient to simply read off the value of $\int_{0}^{\mathrm{B}_2} \mathrm{H}d\left(\frac{\mathrm{H}}{\mathrm{B}}\right)$ from fig. 223 and add it immediately to the first term.

§ 15. **Magnetic equation for various types of field-magnets.**—For the purpose of comparing different types of machines, or of recording the magnetic data of a dynamo in the form of a flux-curve, the ampère-turns corresponding to one complete magnetic circuit, or, in other words, X, should be plotted in relation to one field of flux Z_a. When, however, the field-winding is being determined in the first instance during the process of design, and there are several magnet coils on the complete machine, it is simpler to calculate the ampère-turns which must be given by one coil, or AT_c (where T_c is the number of turns per coil), and then to multiply by N_c the number of coils, in order to obtain the total number of ampère-turns which must be provided on the whole machine, or $AT = AT_c \times N_c$. An inspection of the type of machine quickly decides for what proportion of one magnetic circuit each coil must be answerable. Thus, in the simple overtype or undertype horseshoe with two coils, if a vertical line be drawn symmetrically through the centre so as to divide both armature and yoke,

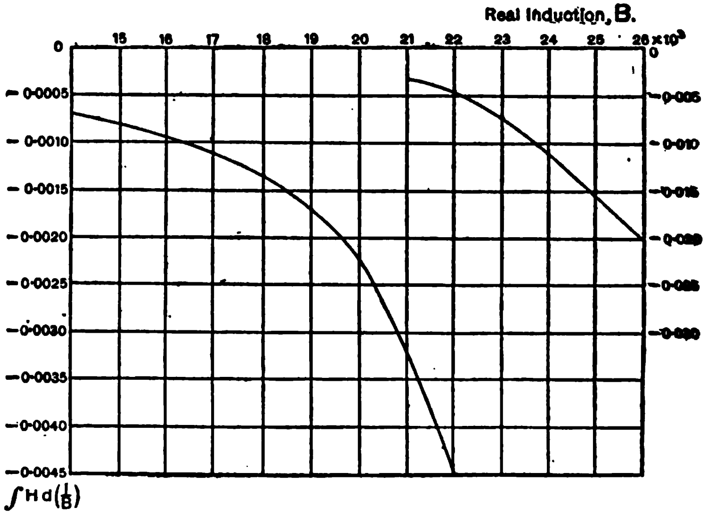


FIG. 222.—Curve of $\int H d\left(\frac{1}{B}\right)$.

$$AT_c = \frac{X}{2} = f'(B_a) \cdot \frac{l_a}{2} + 0{\cdot}8 B_g \cdot l_g + \frac{X_b}{2} + f'(B_m)\,\frac{l_m}{2} + f'(B_y)\,\frac{l_y}{2} \quad . \qquad (41)$$

In other words, it is only necessary to estimate from the drawing the lengths of the paths on one side of the dividing line, the other half being precisely similar and $AT = AT_c \times 2$. In the case of the divided magnetic circuits of double horseshoe fields such as figs. 192 and 200, Z_a being as before the number of lines that leave or enter one polar surface, each magnetising coil must furnish sufficient exciting power to cause half that number to pass into and out of one side of the armature through two air-gaps, each of which has an area half that of the entire air-gap. We arrive, therefore, at the same inductions, B_a, B_g, whether we consider the entire number of lines, Z_a, passing through the areas of the air-gaps and armature as a whole, or half that number passing through half these areas. Let ζ = the number of leakage lines for each magnet-core, and a_m = its area, then

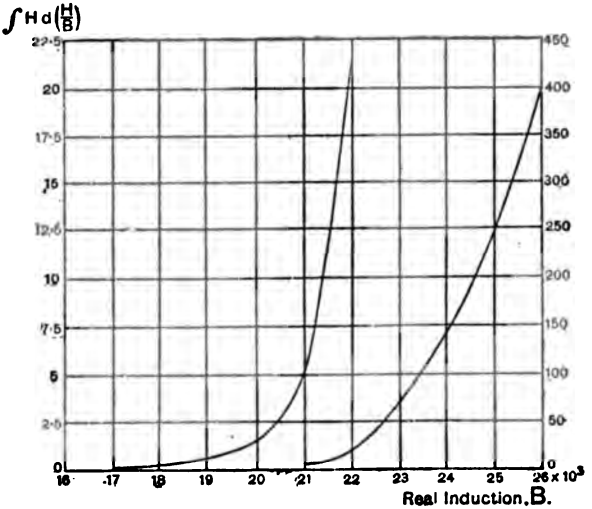


FIG. 223.—Curve of $\int Hd\left(\frac{H}{B}\right)$.

$$AT_c = X = f'(B_a)\, l_a + 0{\cdot}8\, B_g \,.\, 2\, l_g + X_b + f'\left(\frac{\frac{Z_a}{2} + \zeta}{a_m}\right) .\, l_m \quad . \quad . \quad (42)$$

and the total number of ampère-turns on the whole machine is $AT = AT_c \times P$, the number of coils being equal to the number of poles. In the case of the field of fig. 193, the right-hand side of equation (42) is divided by 2 to obtain AT_c and $\text{AT} = \text{AT}_c \times 2\text{P}$.

Similarly, in figs. 194 and 199 two opposite poles are reckoned as one polar surface, out of which or into which flow z_a lines, but in these cases not only is the group of z_a lines split up into two before they pass through the magnetising coil, but further, each sub-group of $\frac{z_a}{2}$ lines again bifurcates when passing through the two separate paths of the yoke frame. The induction in the yoke is therefore $\dfrac{\frac{z_a}{2}+\zeta}{2a_y}$, where a_y is the area of either half of the yoke frame. Hence

$$\text{AT}_c = \frac{\text{X}}{2} = f'(\text{B}_a) \, . \, \frac{l_a}{2} + 0{\cdot}8 \, \text{B}_g \, . \, l_g + \frac{\text{X}_b}{2} + f'\left(\frac{\frac{z_a}{2}+\zeta}{a_m}\right) . \frac{l_m}{2} + f'\left(\frac{\frac{z_a}{2}+\zeta}{2a_y}\right) . \frac{l_y}{2} \quad (43)$$

and the total number of ampère-turns on the machine is $\text{AT} = \text{AT}_c \times 2\text{P}$, where P is the number of poles counted on one side only of the ring.

In the single magnet with divided yoke of fig. 195 with two exciting coils, or in its multipolar form of fig. 197,

$$\text{AT}_c = \frac{\text{X}}{2} = f'(\text{B}_a) \, \frac{l_a}{2} + 0{\cdot}8 \, \text{B}_g \, . \, l_g + \frac{\text{X}_b}{2} + f'\left(\frac{z_m}{a_m}\right)\frac{l_m}{2} + f'\left(\frac{z_m}{2a_y}\right)\frac{l_y}{2} \quad (44)$$

and $\text{AT} = \text{AT}_c \times \text{P}$.

In the 4-pole field of fig. 201 each coil must be wound with sufficient ampère-turns to pass z_a lines through one air-gap and through each armature path up to a horizontal and a vertical dividing line. The equation (41), therefore, for a single circuit applies exactly to each coil. In the case of the disc multipolar dynamo (fig. 79), which usually has no iron in its armature, if l_g be the length of each air-gap between opposite poles, each coil must be wound with

$$\text{AT}_c = 0{\cdot}8 \, \text{B}_g \, . \, \frac{l_g}{2} \, . + \frac{\text{X}_b}{2} + f'(\text{B}_m) \frac{l_m}{2} + f'\left(\frac{z_m}{2a_y}\right)\frac{l_y}{2} \quad . \quad . \quad . \quad (45)$$

and $\text{AT} = \text{AT}_c \times 2\text{P}$.

§ 16. **Determination of size of field-wire.**—The method by which the field-magnets of dynamos are excited admits of several variations, according to the sources whence the magnetising current is derived; and these will be explained in the following chapter. Apart, however, from such differences of source, it remains to determine the necessary gauge and weight of copper wire required for the winding of magnetising coils which are to give a certain number of ampère-turns, AT: these may form either the whole or a part of the total excitation required by the machine, the following being a general solution of the problem applicable to all cases. The data which we have at our disposal may vary in different cases, but usually it will be found that our starting-points will be a knowledge, direct or indirect, of (1) the voltage that will be applied to the ends of the wire, and (2) the mean length of one turn of the coils. Knowing the thickness and width of a rectangular magnet limb, or the diameter of a round magnet-core, it is easy to estimate fairly closely the mean length of one turn of a coil encircling either the one or the other: a certain allowance must be made for the depth of the winding, by reason of which the mean length of a turn in the central of several layers of winding will be greater than the actual perimeter of the magnet itself; and the correctness of this allowance must be subsequently checked when the winding has been determined on. Experience, however, forms a ready guide on this point, and we may therefore assume that the mean length of one exciting turn is known when the dimensions of the iron encircled by the coils are known. Let this mean length $= l_x$, and let $E_x =$ the voltage which will be applied to the ends of the exciting coils. Let $\omega =$ the resistance in ohms of unit length of the required wire at a certain standard temperature, say 60° F.; and let A and T_c be the two factors, the magnetising current in ampères, and the number of turns per magnet coil, so that if N_c be the number of coils, $T_c N_c = T$, the total number of turns on all the bobbins of the machine, and $AT =$ the required multiple of X or of AT_c according to the number of magnetic circuits into which the field-system is divisible. The resistance of the magnetising turns (assumed to be all in series) at the standard temperature is $= T \times l_x \times \omega$; but when a current passes through them, then, as explained in Chap. XIV, § 5, their temperature will gradually rise, and, in consequence, the resistance of the magnetising coils after a run of some hours will be higher than at starting. It will thus be necessary to multiply their resistance at 60° F. by some coefficient, *k*, dependent upon their rise of temperature in working, and on the temperature of the surrounding air from which that rise is reckoned; or $R_x = T \times l_s \times \omega \times k$. The final temperature of the coils will be attained when the rate at which the heat is carried away by radiation, convection, and conduction is equal to the rate at which it is generated. The maximum rise of the temperature of the outside of the coils above that of the surrounding air will depend upon the ratio which their cooling surface bears to the rate of generation of heat in them. Hence from a knowledge of this ratio, and also of the temperature of the surrounding air, the maximum temperature which will be attained by the outside of the coils in continuous work can be predicted; or, in the case of a finished machine, it can be measured by a thermometer. But with a large number of layers wound closely over one another, as in a field-magnet coil, an appreciable difference of temperature is required to produce the flow of heat from the central layers to the outer surfaces, by conduction partly through the length of the wire itself, and partly from layer to layer across the intervening cotton covering, this latter being a bad thermal conductor. The temperature of the central layers of the winding is, therefore, considerably higher than that of the outer layer as measured by a thermometer applied to the external surface. The value of *k* is fixed by the mean temperature of the whole mass of the coil, and is thus dependent not only on the maximum temperature of the external surface, but also upon the depth of the winding. For the field-magnet coils of ordinary machines, the ratio of the mean rise of temperature to the surface rise as measured by a thermometer varies from about 1·25 for a depth of winding of ¾ inches to about 1·5 for a depth of 2½ inches, or 1·6 for 3 inches.[1] For each degree Fahrenheit that the temperature of a piece of copper rises between the limits of 50° and 150° F., its electrical resistance rises about 0·21 per cent. Hence, to give an idea of the practical value of *k*, if the field-coils are 1½ inches deep, and the maximum temperature attained by their outside be 55° F. above that of the surrounding air, the mean rise of the temperature of the whole mass will be, say, $1{\cdot}39 \times 55 = 76{\cdot}5°$ F.; the value of k will then be $1 + (76{\cdot}5 \times 0{\cdot}0021) = 1{\cdot}16$, if the temperature of the surrounding air be assumed to be 60° F., or $1 + (86{\cdot}5 \times 0{\cdot}0021) = 1{\cdot}18$ for the same rise of temperature, but with 70° F. as the initial temperature of the engine-room.

---

[1] Cp. "The Heating of Dynamos" (Wilson), *Electrician*, Oct. 11, 1895. Even higher ratios were obtained in the case upon which Mr. Brown experimented (*Journal Inst. Electr. Eng.*, vol. xxx. p. 1159, qu. v.).

Now $\mathrm{A} = \frac{\mathrm{E}_x}{\mathrm{R}_x}$; therefore $\mathrm{AT} = \frac{\mathrm{E}_x}{\mathrm{R}_x} \times \mathrm{T} = \frac{\mathrm{E}_x \times \mathrm{T}}{\mathrm{T} \times l_x \times \omega \times k}$,

whence
$$\omega = \frac{\mathrm{E}_x}{\mathrm{AT} \times l_x \times k};$$

or if l_x be reckoned in feet, and ω' be the resistance of 1,000 ft. of the required wire,

$$\omega' = \frac{\mathrm{E}_x \times 1000}{\mathrm{AT} \times l_x \times k} \quad . \quad . \quad . \quad . \quad (46)$$

or
$$= \frac{\mathrm{E}_x \times 1000}{(\mathrm{AT}_c \times \mathrm{N}_c) \times l_x \times k}.$$

The area and diameter of the required wire having a resistance of ω' ohms per 1,000 feet is easily obtained by reference to any table of the resistance of copper wire,[1] or by direct calculation. The resistance at 60° F. of 1,000 feet of a No. 1 (Birmingham or Standard Wire Gauge) wire of 100 per cent. conductivity is 0·115 ohm, its diameter being 0·3 inch, and its area 0·0707 square inch. Hence the area of the required wire $= 0{\cdot}0707 \times \frac{0{\cdot}115}{\omega'} = \frac{0{\cdot}00813}{\omega'}$ square inch; or combining this with the above equation,

$$\text{area of wire} = \frac{0{\cdot}00813 \times \mathrm{AT} \times l_x \times k}{\mathrm{E}_x \times 1000} \quad . \quad . \quad . \quad (47)$$

If the wire is to be rectangular, the two dimensions which go to make up this area may be chosen to suit our own convenience in winding; but if it be round, the necessary diameter is

---

[1] Such as may be found, *e.g.*, in Munro and Jamieson's *Pocket-book of Electrical Rules and Tables*, p. 293, 13th ed. If such a table be not at hand, a very convenient *memoria technica* is afforded by the fact that at 60° F. the resistance of 1,000 feet of wire, of 100 per cent. conductivity, and having an area of ·09 sq. inch, is ·09 ohm: or the resistance of a wire of area ·05 square inch is very nearly ·05 ohm per 100 yards.

$$\sqrt{\frac{4 \times \text{area}}{\pi}} = \sqrt{\frac{4 \times \left(\frac{0{\cdot}00813}{\omega'}\right)}{\pi}} ;$$

whence $$d = \frac{0{\cdot}1015}{\sqrt{\omega'}} \quad . \quad . \quad . \quad . \quad (48)$$

From the above formulæ it is evident that if E_s and l_s be fixed, there is but one area or diameter of wire which will satisfy the equation and give the required number of ampère-turns, and, further, that this area or diameter is entirely independent of the actual number of turns in the coils, since neither T nor A appears separately in the final equations. This result may at first sight seem surprising, but is easily followed when it is remembered that if the number of turns be, for instance, doubled, the resistance of the coils is also doubled, which, with a given E_s, halves the current through them, and therefore leaves the total number of ampère-turns unaltered. If the number of turns be doubled by winding twice as many layers on the same length of bobbin, it is true that l_s is increased, since the depth of the winding is doubled; but the effect of this upon the necessary diameter of wire is more or less counterbalanced by the reduction which must be made in the value of k. When the number of turns is doubled, the current and the rate at which it generates heat are halved; more than this, the cooling surface is itself increased, owing to the perimeter of the coil being greater; and therefore, for both reasons, the temperature attained by the coils will be less than before. The lesser value of k thus almost compensates for the increased value of l_s.

§ 17. **Determination of weight of field-wire.**—In order, therefore, to determine the actual number of turns which must be used, or the weight of wire, the necessary area or diameter of which has been determined, it is essential that some other factor of the problem be known to us. This may be either the number of watts, W, to be lost in the field-winding under consideration, or the current A; the latter case is, in reality, identical with the former, since $A = \frac{W}{E_s}$, and in many cases W and A form two of the data given at the outset, whence $E_s = \frac{W}{A}$ is at once derived. From our knowledge of W or of A, the total number of turns is at once fixed as

$$T = \frac{AT}{A} = \frac{AT \times E_s}{W},$$

and the total length of wire required is T × l_s feet. The weight of one foot of a No. 1 B. or S.W.G. wire is 0·273 lb., therefore the weight of wire is

$$= T \times l_s \times 0{\cdot}273 \times \frac{0{\cdot}115}{\omega};$$

whence by simple substitution

$$\text{weight in lbs.} = T \times l_s \times 0{\cdot}273 \times \frac{0{\cdot}115 \times AT \times l_s \times k}{E_s \times 1000}$$

$$= \frac{AT}{A} \times l_s \times 0{\cdot}0314 \times \frac{AT \times l_s \times k}{E_s \times 1000}$$

$$= \frac{(AT)^2 \times l^2_s \times k \times 3{\cdot}14}{W \times 100{,}000} \quad . \quad . \quad . \quad (49)$$

It should be observed that if our object is simply to form an estimate of the weight of wire required from the above data, it is unnecessary to first determine ω, or the actual number of turns and layers of the wire. The final settlement of the winding will, however, require the latter to be determined, and when the number of layers has been decided, it will be well to check the correctness of the assumed depth of winding underlying the first estimate of l_m or the mean length of a turn. It may be worth while to mention that if a continuous length of round wire is wound on to a bobbin in several layers, the layers are of alternate hand, and at one point each turn of an upper layer has to cross over the turn of the layer below it; hence, if $d_1 = d + \delta$ be the diameter of the wire with its insulating covering, and n be the number of layers, the depth of winding at this point is nd_1. Except, however, at the crossing point, the turns of the upper layers bed into the hollows between the turns of the layers underneath. The depth of winding is then

$$d_1 + (n-1)\,d_1 \sin 60^\circ = d_1\,\{1 + (n-1)\,0{\cdot}866\}.$$

While allowance must be made in the space allotted for the bobbins for the larger value, the mean depth of winding from which l_s is to be calculated may be taken as lying between the two values, or

$$t = 0{\cdot}9nd_1.$$

Round or rectangular wire, double-cotton-covered, is usually employed for field-magnet bobbins, the total thickness of the insulation being 15 and 20 mils in the two cases respectively.

§ 18. **Winding of field-magnet bobbins.**—Large and heavy coils are usually wound on sheet-iron spools, with iron or brass end-flanges, or on zinc spools; these are insulated with several layers of shellaced paper or micanite cloth according to the voltage of the machine, and after winding are slipped over the magnet-cores and fixed in place. The end-flanges in the case of two-pole machines are sometimes provided with insulating cheeks of wood from ¾" to 1¼" thick, and the wire leading to the bottom layer is then conveniently brought out to the exterior in a groove cut in the wood. With metal flanges, in order to obviate the danger of a break of the leading-in wire rendering the lower layer inaccessible, a thin insulating partition may be placed against the end-flange, and within this, as each layer is wound, a turn of the leading-in wire is taken in the reverse direction until it is finally brought up to the top, flush with the bulk of the winding and in a readily accessible state. In many cases the spools are specially moulded from insulating material, such as vulcasbeston or isolit, grooves being formed in the flanges to receive the wire leading to the bottom layer. In small machines, and also in large multipolar continuous-current dynamos in which the several coils are not very large, they may be wound on a removable former; the numerous layers of wires are built up in the process of winding with interlaced strips of tape, so that the former may be subsequently withdrawn without the coil losing its shape. It is then wrapped with a double layer of webbing or tape, and to use an Americanism, is 'mummified' by soaking it in bitumen or an insulating varnish, and thoroughly dried, the whole being finally bound over with cord or strong tape.

CHAPTER XVI

SERIES, SHUNT, AND COMPOUND WINDING.

§ 1. Methods of excitation of the field-magnet. Separate excitation.—By the principles stated in the previous chapter the designer is enabled to calculate the number of ampère-turns required to excite the magnetic system of a dynamo. There are, however, several different methods of supplying the exciting current, and since these have different effects on the working of the machine, the four principal modes require to be considered in detail. The excitation of the electro-magnet of a dynamo is, in general, effected either (1) by coils forming a shunt to the external circuit, the dynamo being then known as a 'shunt' machine; or (2) by coils in series with the external circuit, when the dynamo is called a 'series' machine; or (3) by both shunt and series coils in combination, a method known as 'compound winding'; or lastly (4) by coils connected to a separate source of current, the machine being then said to be 'separately excited.'

The excitation of a machine from a separate and entirely external source of electrical energy is the most obvious method, and was the one first adopted in practice, a small dynamo with permanent magnets of steel being used to furnish current for exciting the field-magnet of a larger dynamo. Fig. 224 shows a *separately-excited machine*, the electric circuit of the magnetising coils being entirely distinct from the circuit of the main dynamo: in the diagram the main external circuit, R_e, is indicated by incandescent lamps strung in parallel across from the positive to the negative lead. The source of the magnetising current is represented by a battery of cells, but may, of course, equally well be a separate continuous-current dynamo.

§ 2. **Shunt excitation.**—The second step in the order of development was made when it was suggested that a part of the electrical energy generated in the main dynamo might be used to maintain or increase the magnetism of its own field-magnet. This was, however, only possible if the machine were furnished with a commutator by which the current was commuted into a steady flow in one direction, and was thus rendered suitable for magnetising purposes. Two distinct methods were then invented, by which this suggestion was realised.

By the first, magnetising coils were arranged as a *shunt* to the external circuit proper; thus, in fig. 225, from the brushes of the dynamo two paths, R_e and R_s, are presented, and the armature current divides into two portions, the relative magnitudes of which will vary inversely as the resistances of R_e and R_s; while the one portion of the total armature current flows through the external circuit, R_e, wherein the useful electrical energy is manifested, the other is shunted through the magnetising coils, and both reunite to flow through the armature. The voltage on the shunt is, of course, the same as that on the external circuit, since the same terminals, A and D, serve for both. If the resistance of the shunt, R_s, be relatively high as compared with R_e, only a small proportion

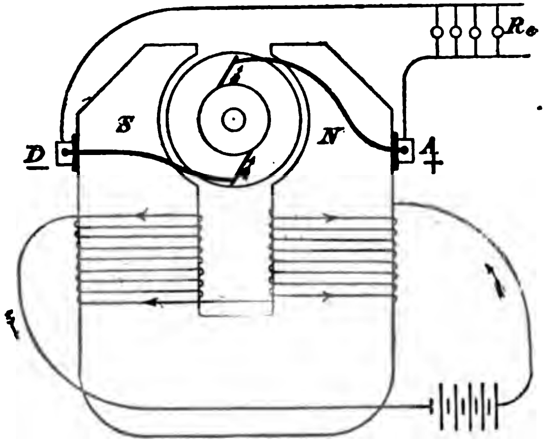


FIG. 224.—Separately-excited dynamo.

of the total energy developed will be absorbed in exciting the field-magnet; hence the shunt coils are usually composed of a large number of turns of small wire, and are represented by fine lines in contrast to the thicker lines, which indicate the lower resistance of the external circuit.

§ 3. **Series excitation.**—By the second distinct method, magnetising coils are arranged in *series* with the external circuit, and the whole of the armature current passes alike through the turns of the field-magnet coils, R_m, and the external circuit, R_e (fig. 226). A portion of the total pressure developed at the

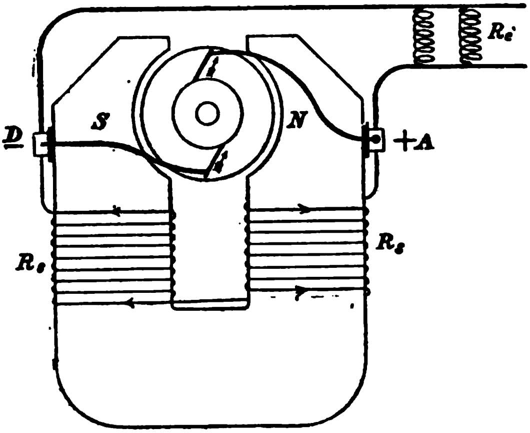


FIG. 225.—Shunt-wound dynamo.

brushes is expended in the magnetising coils, and the remainder is available for useful work at the terminals, A, D, to which the external circuit is applied. If the resistance of the series coils, R_m, be low as compared with the resistance of the external circuit, the percentage of energy absorbed in the field will be small as compared with the useful output; on this account the series coils are usually a few turns of thick wire, and are so represented in our diagram. The number of ampère-turns on the magnet of the dynamo may be the same whether it be shunt- or series-wound, since in the one case a small current flows through a large number of turns, and in the other case a large current flows through a few turns; and in both cases the amount of energy absorbed in securing any given number of ampère-turns is simply a question of the amount of copper in the field-coils.

§ 4. **Self-excitation.**—A further and most important step was now found to be practicable, viz., the *self-excitation* of the machine, whether shunt- or series-wound. The field-magnet of a dynamo is slightly magnetised, even when the machine has ceased running: it may then appear to be perfectly demagnetised by its failing to attract, say, a bunch of keys held near the poles, but a more delicate test will show that both forged and cast iron or steel retain a certain amount of flux (Chap. XII, § 4), and if the field-magnet be approached with a small compass needle, one pole of the machine will attract and the other will repel the N. pole of the delicately poised needle. Now the presence of this feeble residual flux is sufficient to start the process of self-excitation, since it implies that a few lines of induction are passing round the magnetic circuit of the dynamo through the armature, even when at rest. Hence, when the armature of either fig. 225 or fig. 226 is rotated, the inductors cut the residual lines, and a small E.M.F. is thereby set up within the winding of the armature. If the brushes are 'down' on the commutator, and a closed circuit is thereby made, this small E.M.F. sends a feeble current through

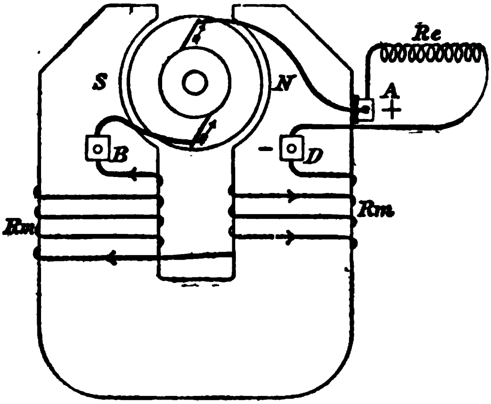


FIG. 226.—Series-wound dynamo.

the magnetising coils ; the ampère-turns of the latter, although as yet they may be small, still serve to increase the number of lines passing through the armature, and these, as they are cut by the inductors, produce an increased E.M.F. ; the exciting current is thereby in turn increased, and its increase is again followed by a further increase in the flux and E.M.F. Thus the magnetism gradually grows, the increments becoming less and less as the iron of the field-magnet becomes more and more saturated The time taken by the 'building up' process will vary from a few seconds in a small machine to a minute or more in a large magnet, but after a short time a small increase in the exciting current produces very little increase in the induction; finally, for a given speed of rotation, the voltage and exciting current reach a settled state, and the machine will run for any length of time, maintaining its own constant flux. Thus the presence of residual flux renders it unnecessary to impart to the field-magnet any initial excitation, and the continuous-current machine becomes self-exciting by the mere rotation of the armature. One difference, however, between the shunt- and the series-machine will be apparent from their respective diagrams. In the latter case, the external circuit must be closed before the process of excitation will begin, since the circuit of the field-coils is only completed through the external circuit. In the former case, if an external circuit of very low resistance be closed on the brushes, so small a portion of the feeble initial current will be shunted through the magnet-coils that the machine may fail to excite. Hence, in the shunt machine, the excitation is most quickly and surely obtained if the external circuit be left open until the magnet is thoroughly excited.

It only remains to remark that even when a machine has just come from the workshops, and is run for the first time, there is usually sufficient residual flux in the magnet, due to the effect of the earth's magnetic field or of other magnetic bodies upon it during the process of its manufacture, to enable it to excite itself. The first excitation may, however, require a higher speed than will be subsequently necessary, and in cases of large shunt machines, running at slow speeds, separate excitation may have to be resorted to in the first instance.

§ 5. **Energy stored in the excited field.**—When once the machine is normally excited, and is running under settled conditions, the energy spent in the magnetising coils is entirely converted into heat; but during the process of creating the magnetic field, a certain amount of energy is absorbed which is given back as the field demagnetises. If the circuit of a series-wound dynamo be opened while it is running, the stored-up energy reappears in the form of a spark at the switch; the rapid collapse of the lines of induction which circle through the turns of the field-coils generates a high E.M.F., which tends to keep up the strength of the current, and if the field be powerful and the turns encircling it be numerous, the intensity of the spark becomes very considerable. In the case of a shunt-wound dynamo the opening of the external circuit does not interrupt the flow of the shunt current round the coils; in fact, these latter themselves afford a bye-pass for the discharge of electrical energy. Owing, however, to the large number of turns in the shunt, the effect of suddenly breaking the magnetising circuit is even more marked than in a series machine. Thus, if a person accidentally lifts the brushes of a large shunt machine running on open circuit, the self-induced E.M.F. may rise to thousands of volts, may damage the insulation of the machine, and, further, may give perhaps a fatal shock. If the machine be running on a closed circuit, say, of incandescent lamps, the person would be less liable to receive such a severe shock, since the so-called 'extra-current' is discharged through the external circuit, and causes the lamps to momentarily flash up. In order, therefore, to open the shunt circuit of a machine when running, a shunt-breaking switch is necessary, in which at the moment of opening a non-inductive resistance is closed upon the terminals of the shunt.

§ 6. **Determination of shunt-winding.**—The application of the formulæ of Chapter XV to the design of the field-winding of the shunt machine is easy. Let E_e be the terminal or external voltage which the machine is required to give when supplying its full external current; then the difference of potential on the ends of the shunt is likewise E_e, and it is only necessary to substitute its value for E_s in the equations 46–49 in order to determine the necessary area or diameter of wire. In order to further determine the number of turns and weight of wire, it is necessary to know, at least approximately, the rate in watts at which energy is to be expended in the field-coils.

While, however, the necessary size of wire is rigidly determined, there is, in fact, no hard-and-fast rule for deciding the weight that is to be used, and this must be left to the designer's judgment. If a large quantity be used, the weight and cost of the machine are increased (double-cotton-covered round wire costs approximately from £4, 10s. to £5 per cwt.), while if a smaller quantity be used, the heating of the field-coils is greater, and the efficiency of the machine is decreased. A further disadvantage which arises when the weight of copper is reduced is, that there is a greater difference between the resistances of the shunt when cold and when hot, owing to its rise of temperature being greater: this difference causes a difference in the shunt current, and therefore in the exciting power and total induction produced, so that when the dynamo runs at a constant speed, the E.M.F. induced is greater at starting than it is after it has run for several hours continuously, and has attained its final temperature. If the dynamo be supplying incandescent lamps direct, and this difference be great, it will necessitate an alteration in the speed of rotation in order to maintain the correct voltage on the lamps, or it must be corrected by means of a variable rheostat in series with the shunt. The settlement of the rate in watts at which energy may be expended in the shunt coils depends, therefore, upon their allowable heating and upon the efficiency which the dynamo is to have: from both considerations combined, experience enables us to fix upon a preliminary estimate from which the completed design need differ but little. In practice, the loss of energy in the field of a shunt machine varies from about 1·5 per cent. of the output in a 100-kilowatt machine to 7 or 8 per cent. in a 4-kilowatt machine. The heating question will be more fully discussed in Chapter XIX. If it be settled that w watts may be lost in the field-winding, then $w = E_e \times C_s$ watts, and $C_s = \frac{E_e}{R_s}$, whence the resistance of the shunt and its composition can be at once determined. The preliminary result thus arrived at should only require such slight revision as will lead to the winding forming a complete number of layers.

§ 7. **Example of shunt-winding calculation.**—Thus in the case of the dynamo of Chapter XV, § 6, $E_s = 103$. The perimeter of the insulated spool before winding will be a little more than that of the magnet limb, an allowance of say 2·5″ on the whole being required, or of $\frac{5}{8}''$ on each of its component dimensions. Thus the length of a turn in the bottom layer is $2\{(A + \frac{5}{8}'') + (B + \frac{5}{8}'')\} = 2\{(5·5 + 0·625) + (11·25 + 0·625)\} = 36''$. The length of a turn in any other layer is made up of this length plus a quadrant of a circle at each corner, the radius of this circle being equal to the depth of the winding at the layer in question. Hence if t be the total depth of winding $= 0·9nd_1$, the mean length of a turn is $36'' + \pi t$; or in our case, assuming an average depth of $1\frac{1}{2}$ inches, $l_s = 36'' + 4·7'' = 40·7'' = 3·39$ ft. If the temperature attained by the surface of the coils be about 58° F. above the normal temperature of the surrounding air, the mean rise of temperature of the wire will be 58° F. × 1·39 = 80·5° F.; and since the temperature of the engine room may be taken as 70° F. while the standard temperature of the wire table is 60° F., $k = 1 + (0·21 \times 90·5'') = 1·19$.

Thence, by equation (46),

$$\omega' = \frac{103 \times 1000}{15{,}400 \times 1·19 \times 3·39} = 1·66 \text{ ohms},$$

and the necessary diameter of the shunt wire is 0·079″.

In the present case the axial length of each bobbin is to be 10″ over-all, and allowing $\frac{1}{2}''$ at either end for the end-flanges after insulation, the winding space is 9″ long. The perimeter of the outer layer will be $36'' + 2\pi t = 45·4''$. The cooling surface of the two coils together (measured as in Chapter XIV, § 5) may then be reckoned as 45·4″ × 9″ × 2 = 816 sq. inches, and (as will be explained in Chapter XIX) an allowance of about 460 watts will be within the mark as regards the permissible heating of the coils. After a preliminary trial, a winding which forms a complete number of layers is thus arrived at; the diameter of the double-cotton-covered wire is $d_1 = ·079 + ·015 = ·094''$, and there will be 96 turns per layer. With eighteen complete layers on each coil the total number of turns on the two will be 3458; 11,710 ft. of wire will be required, and the resistance of the shunt when cold will be

$11\cdot71 \times 1\cdot66 = 19\cdot5$ ohms, or when hot $19\cdot5 \times 1\cdot19 = 23\cdot2$ ohms. $C_s = \frac{103}{23\cdot2} = 4\cdot45$ ampères, and $AT = 4\cdot45 \times 3{,}458 = 15{,}400$ as required. The rate of expenditure of energy in the field-coils will be $103 \times 4\cdot45 = 460$ watts, or nearly $3\cdot2$ per cent. of the output. The depth of the winding at its thickest part will be $18 \times \cdot094 = 1\cdot69''$, agreeing sufficiently with our preliminary assumption of $1\frac{1}{2}'' = t = 0\cdot9 \times 1\cdot67$, and the weight of copper is approximately 2 cwt.

§ 8. **External characteristic curve of shunt machine.** —If a dynamo be run at a constant speed, and the resistance of its external circuit be varied so as to alter the value of the external current, the curve connecting simultaneous values of the terminal E.M.F., E_e and the external current, C_e for a given speed of rotation is known as the *external characteristic* of the machine for that speed; since from it the behaviour of the machine under varying conditions of load can be graphically studied. The ampères of such a curve are usually plotted horizontally as abscissæ with vertical ordinates for the corresponding volts. The two may either be obtained by direct measurement, voltmeter and ammeter being read off simultaneously, or the curve may be derived from the flux curves of the machine. In the case of a shunt-wound machine the curve of external E.M.F. and current will be found to take the characteristic form shown in fig. 227, O L K D. The manner in which it may be derived from the flux-curves has now to be explained, and first it will be necessary to consider the process by which the three curves of fig. 213 may be determined by direct experiment on a shunt-wound dynamo.

Considering, for simplicity, a bipolar machine, the total number of lines passing through the armature is $Z_a = \frac{E_a \times 10^8 \times 60}{N \times \tau}$ (eq. 15), and the ampère-turns on the field-magnet are $X_s = C_s \times T_s$. $C_s = \frac{E_b}{R_s}$ may be obtained by measuring the E.M.F. at the brushes and dividing it by the resistance of the shunt; but for greater accuracy it is preferable to measure it directly by an ammeter, since the resistance of the shunt is continually altering if the machine is in process of warming up during the tests. In a shunt machine

$$C_a = C_s + C_e; \text{ and } E_a = E_b + C_a R_a;$$

whence $$z_a = \frac{(E_b + C_a R_a) \times 10^8 \times 60}{N \times \tau} \quad . \quad . \quad . \quad (50)$$

When the external circuit is open, the only current flowing is that through the shunt. As a rule, this is so small that $C_a R_a$ becomes negligible, and E_b and E_a are practically identical. If, therefore, the armature be run at different speeds, and a series of simultaneous readings of the speed, the E.M.F. at the brushes, and the shunt current be taken, a curve can be plotted from the readings which will connect z_a and X_s; the armature ampère-turns due to the shunt current alone being very small, their influence on the field may practically be neglected, and the curve thus obtained is the highest or 'no-current' curve of flux of fig. 213. To obtain the lower curves of flux for either 'half-current' or 'full-current,' the same readings must be taken; but between the readings at each different speed the resistance of the external circuit must be altered, so that when the measurements are made for either curve, the current through the armature ($C_a = C_s + C_e$) has the constant value required. The loss of volts over the resistance of the armature must now be added to the terminal E.M.F. to obtain E_a.

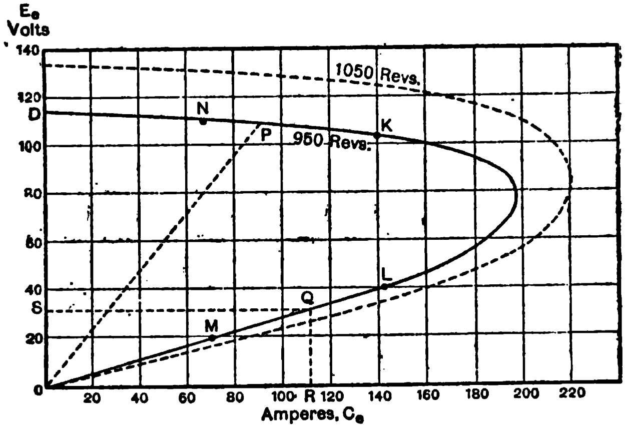


FIG. 227.—Characteristic curve of shunt-wound dynamo.

Since $E_a = \frac{z_a \times N \times \tau}{60 \times 10^8}$, it follows that $E_a \propto z_a$, if the speed be kept constant; further, $X_s = \frac{E_s}{R_s} \times T_s$, whence E_s or the potential difference applied to the terminals of the exciting coils, is $\propto X_s$ when R_s is constant, as it is when a machine has been running for some time, and has attained a steady temperature. If, therefore, we possess the flux-curves of fig. 213, these can be converted

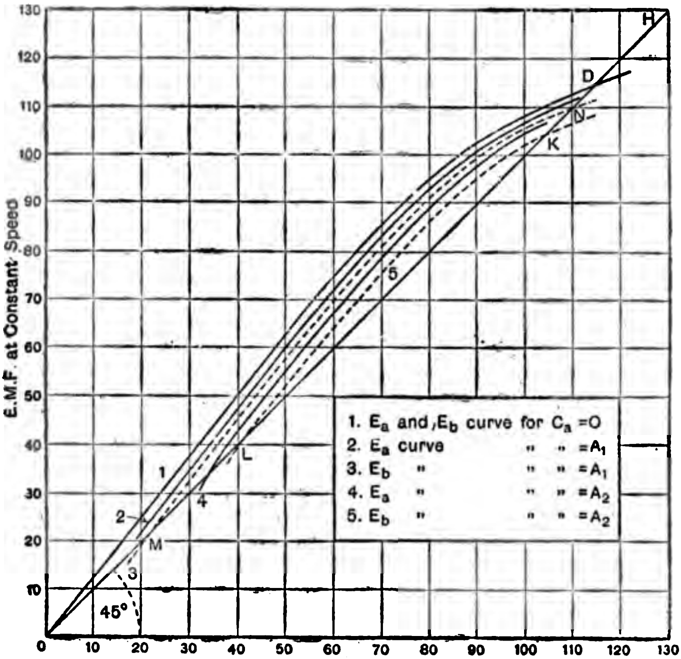


FIG. 228.

into curves connecting armature E.M.F. and exciting voltage when some fixed speed is assumed. The full-line curves of fig. 228 show the curves of fig. 213 thus converted on the assumption that in our 9″ × 12″ dynamo the number of armature inductors is 132, the number of shunt turns 3458, and their resistance 23·2 ohms and that the constant speed of rotation is to be 950 revolutions per minute. Curve 1 connects E_a or E_b and E_s when $C_a = 0$.

Curve 2 connects E_a and E_s when C_a has the value A_1 (= 72 ampères), and curve 4 connects E_a and E_s when $C_a = A_2$ (= 144 ampères). From curve 2 is obtained the dotted curve 3, connecting E_b and E_s when $C_a = A_1$ ampères, by deducting from the ordinates of curve 2 the loss of volts over the armature resistance when warm; in fact, the vertical distance between curves 2 and 3 is throughout equal to $A_1 \times R_a = 1{\cdot}75$ volts. Similarly, from curve 4 is derived curve 5, connecting E_b and E_s when $C_a = A_2$, or 144 ampères. Now, since in a shunt-wound machine E_b is the same as E_s, those points on curve 5 (such as L or K) for which the ordinate E_b = the abscissa E_s are the only values of voltage which the machine would give when run at the fixed speed with A_2 ampères flowing through the armature; and in general those points on any curve connecting E_b and E_s, for which ordinate and abscissa are equal, give the only possible values of the terminal E.M.F. for the particular armature current. Deducting the shunt current from the total armature current, we obtain the external current; thus at point K, $C_s = \frac{103 \text{ volts}}{23{\cdot}2 \text{ ohms}} = 4{\cdot}45$, C_a = nearly $144\frac{1}{2}$ ampères, and therefore $C_e = 140$ and $E_e = 103$ are simultaneous values of external current and terminal voltage. All such points are necessarily passed through by the straight line O H, which is drawn at an angle of 45° from the axis of abscissæ, as, *e.g.*, K and L, N and M, D and O; and after deduction for the shunt current, these may be plotted as part of the external characteristic, as shown by the corresponding letters in the full-line curve of fig. 227.

Given the no-current curve connecting E_a with the ampère-turns of excitation on open external circuit, *i.e.*, such a curve as the upper one of fig. 228, the corresponding curve of terminal E.M.F. for any particular value of the current may be deduced by the following construction. The ampère-turns required to balance the armature reaction may be expressed as a certain proportion of the armature ampère-turns, or as $= k.\ AT_a$, where k is some constant. It should be noted that this number of turns takes into account all the effects of the armature reaction in increasing the required excitation, including the increase in the magnet-turns necessitated by the increased leakage; it is not therefore identical with X_b. In fig. 229 let $k.\ AT_a = ab = cd$; if then the no-current curve be shifted horizontally along to the right by the uniform distance *cd*, we obtain the curve connecting E_a with the excitation for some given value of the current C_a, and for the same constant speed. Let *de* the constant loss of volts over the armature resistance for the particular current in question, *i.e.*, $= C_a R_a$; deducting this value from the second E_a curve, a third curve is obtained connecting E_b with the excitation. If, therefore, the right-angled triangle *cde* be applied to any part of the E_a curve for no-current, the position of the point *e* marks one point in the required curve; or a line may be drawn at any point parallel and equal in length to the hypothenuse *ce*. Although the above construction is a useful approximation, it cannot be regarded as completely accurate, since the ampère-turns required to balance the

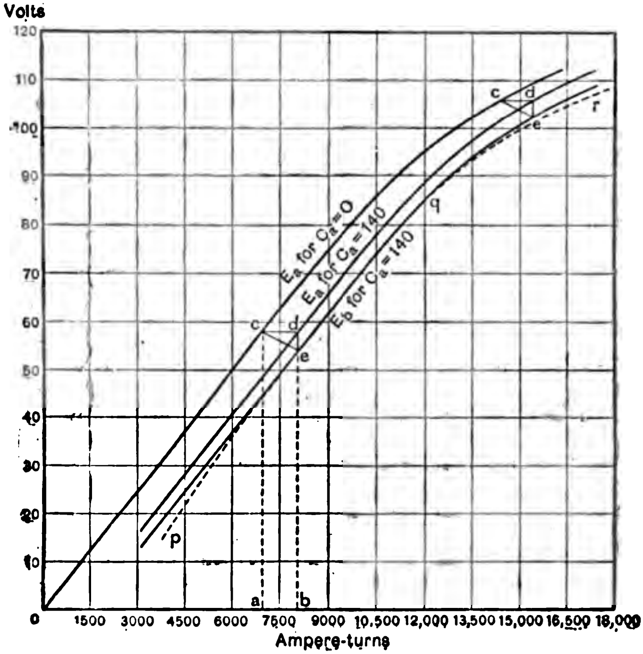


FIG. 229.—Method of deriving voltage curve for shunt dynamo.

armature reaction do not always bear a constant ratio to the armature ampère-turns; *e.g.*, with weak field, the brushes may require to be shifted forwards, and with strong field the saturation of the magnet by leakage may require a disproportionately large number of turns, so that in both cases k is not constant, but increases somewhat, and the curve should then be as shown dotted at pqr (fig. 229).

§ 9. **Deductions from characteristic of shunt dynamo.**—It will be seen from fig. 227 that for the shunt-wound machine running at any one fixed speed there is a maximum value for the external current, beyond which it is impossible to go. Owing to the heat which would be generated in the armature winding by a long-continued passage of this maximum current, it may be, and usually is, impossible to work the machine at the point of maximum current: apart, however, from the question of the heat damaging the winding, it is for another reason inadvisable to work close to this point. After rounding the point of maximum current the characteristic curve descends very rapidly and almost in a straight line to the origin. Thus in the diagram (fig. 227), the part O Q is practically a straight line, and this implies that for an external resistance, $= \frac{QR}{OR}$, the terminal E.M.F. or the external current may have any value between nought and OS volts or between nought and OR ampères; the machine may therefore give widely different voltages at different times, although running at exactly the same speed on the same external resistance. The flux of the field is, in fact, unstable owing to the voltage on the shunt not being sufficient to duly magnetise the iron; hence, if, when working near the point of maximum current, the resistance of the external current becomes lowered to a value considerably below that which corresponds to the maximum current, the machine loses its magnetism altogether, and the voltage runs down to zero. The practical importance of this is that if a shunt-wound machine be accidentally short-circuited when at work, or, in other words, if R_e is reduced almost to $= 0$, the armature winding is not burnt up by the continued passage of an abnormally large current; the machine, on the other hand, becomes demagnetised, and gives no current, although the driving engine may be running at a speed higher than the normal.[1] Neither will a shunt-wound machine excite if an external circuit of abnormally low resistance be closed, so that it is unharmed if an accidental misconnection of the leads has resulted in its terminals being short-circuited. For every value of the external current except the maximum there are two values of the external E.M.F., and which of the two E.M.F.'s is obtained depends entirely upon the resistance of the external circuit: this latter is equal to $\frac{E_e}{C_e} = \tan \alpha$, or the slope of the line to any point, P, on the curve when E_e and C_e are plotted to the same scale, as in fig. 227. The slope of the full-line curve between L and O marks the *critical resistance* of the external circuit for a speed of 950 revolutions per minute; at this value the magnetism is unstable, and at any lower value the machine will fail to excite or maintain its excitation.

For each constant speed the same shunt-wound dynamo gives a different external characteristic. Thus, in fig. 227 the dotted curve shows the external characteristic of the same dynamo at a constant speed of 1050 revolutions per minute, instead of 950, and it will be seen that at the new speed the maximum current is increased. The higher the speed, the less is the slope of the descending branch of the curve; or, in other words, the smaller is the critical resistance which the external circuit may have, without the dynamo losing its magnetism.

§ 10. **Instability of magnetism.**—Instability of the magnetism requires to be carefully guarded against in the design of self-exciting dynamos. It will be seen from figs. 227 and 228 that the unstable portion of the curve between L and O is obtained if the flux for which the machine is designed falls on the initial straight portions of the curves in fig. 213, where the ampère-turns over the air-gaps form almost the whole of the excitation; and, in general, if the working number of lines falls on the lower part of the flux-curve where it descends rapidly towards the origin, the magnetism of the machine as a self-exciting dynamo will be *unstable*. Even if the field hold its magnetism, a very

[1] Occasionally a complete short-circuit of a shunt dynamo will, through certain secondary reactions, end in magnetising its field slightly in the reverse direction, so that, when again self-excited, its polarity is reversed.

slight variation of the speed will cause a large variation in the E.M.F.—a result which is always undesirable, and especially so if the machine be feeding lamps directly. Thus, in the case of a shunt-wound machine, supppose that its speed is raised slightly; the increased E.M.F. causes an increase in the current through the shunt, which will again increase the induction and the E.M.F., and the lower down the curve of magnetisation that the machine is worked, the greater is the effect of any variation in the shunt current upon the E.M.F., owing to the magnet being less saturated.

A measure of the percentage variation in the voltage which follows from a certain percentage variation in the speed may be obtained by a construction due to Messrs. Poynder and Wimperis.[1] Let O Z S (fig. 230) be a portion of the flux-curve containing the working point corresponding to Z lines at which it is desired to investigate the effect of change of speed. The same curve with a different scale of ordinates will also represent the voltage, or $E = kNZ$, where k is a constant of the required value to convert lines of flux into volts at the speed of N revs. per min. At the point Z draw a tangent to the curve; let it cut the vertical axis at z_1, making an angle a with the horizontal. The equation to the tan-

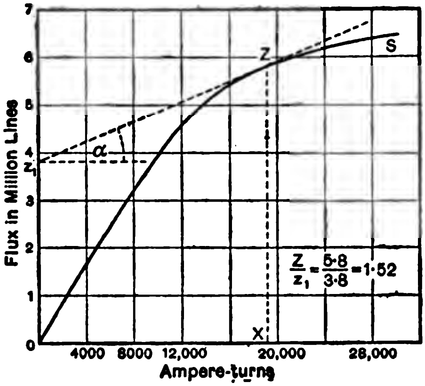


FIG. 230.

---

[1] *Engineering*, May 3, 1901.

gent line is $Z = z_1 + X \tan \alpha$; and in the shunt machine the excitation is practically proportional to the armature voltage, or $X = c\,E$, where c is some constant depending upon the field winding. For a very small alteration of the voltage the working point may be taken as moving up or down the tangent, and for a very small alteration of the speed k may be assumed constant. Hence $E = k\,N\,Z = k\,N\,(z_1 + c\,E \tan \alpha)$. Differentiating with respect to the speed, the rate of change of the voltage when the speed begins to alter is

$$\frac{dE}{dN} = k\left\{ z_1 + c \tan \alpha \left(N \frac{dE}{dN} + E \right) \right\},$$

whence

$$\frac{dE}{dN}\left(1 - k\,c\,N \tan \alpha\right) = k\left(z_1 + c\,E \tan \alpha\right) = k\,Z.$$

Since $Z = z_1 + k\,c\,N\,Z \tan \alpha$, we have $Z\,(1 - k\,c\,N \tan \alpha) = z_1$

or $$\left(1 - k\,c\,N \tan \alpha\right) = \frac{z_1}{Z};$$

therefore $$\frac{dE}{dN} = k \,.\, \frac{Z^2}{z_1}.$$

Multiplying both sides by $\frac{N}{E}$, and substituting $k\,N\,Z$ for E on the right-hand side,

$$\frac{dE}{dN} \times \frac{N}{E} = \frac{1}{Z} \,.\, \frac{Z^2}{z_1}$$

or $$\frac{\frac{dE}{E}}{\frac{dN}{N}} = \frac{\text{fractional change of voltage}}{\text{fractional change of speed}} = \frac{Z}{z_1} \quad . \quad . \quad . \quad (51)$$

The percentage change of the voltage corresponding to a given percentage change of the speed is thus directly given by the ratio $\frac{Z}{z_1}$, and this must not be allowed to exceed a certain limiting value if instability of the voltage is to be avoided. Fig. 203 illustrates the case of a dynamo working on a stable part of the flux-curve; the ratio $\frac{Z}{z_1} = 1{\cdot}52$, and a one per cent. variation of the speed implies a 1·52 per cent. variation of the voltage. As the working point approaches the origin, the ratio $\frac{z}{z_1}$ rises, and in practice in order to avoid instability should not exceed 3, so that the rate of change of the voltage becomes 3 per cent. for a 1 per cent. variation of speed. In the compound-wound dynamo of § 18 this value as being a safe limit is approached in regard to the open-circuit voltage given by the shunt winding. As a matter of fact the change of voltage for any change of speed is to a certain extent held in check by the hysteresis, which assists in maintaining the stability of the magnetism; but the determination of the ratio $\frac{z}{z_1}$ affords a very useful warning to the designer when in doubt as to the lowest point on the flux-curve at which it is safe to work.

A further disastrous consequence of designing a shunt-wound machine to work on the initial straight portion of the flux-curves is that, even when the external circuit is open and the machine is run very nearly at the normal speed for which it is designed, it may be difficult to reach the proper excitation. Just as excitation begins, the magnet reaches a state which satisfies all the conditions of speed, &c., and the process stops abruptly. The no-current flux-curve owing to the presence of residual magnetism really ends at some point on the vertical axis slightly above the origin as shown by the dotted line in fig. 213. If now a number of voltage curves similar to that of fig. 228 are plotted for several speeds, the points at which they cut the diagonal (fig. 231) give the voltages for the different speeds, and these when plotted as in fig. 232 show that there is a *critical speed* at which the voltage rapidly rises. Below this the effect is almost entirely due to the residual magnetism, and as in practice this is very small, it has been exaggerated in the diagram in order to render the matter clearer. If the dynamo is run up to its full speed before the circuit of the shunt is closed, then on closing the latter (and neglecting the secondary effects from self-induction) the rising voltage follows the upper curve of fig. 231; the E.M.F. at the brushes is initially higher than that corresponding to the required excitation, and up to a certain point the surplus ampère-turns continue to increase, after which they decrease until a condition of stable equilibrium is reached at P. If the speed is run up with the shunt closed, the voltage follows the diagonal line, and a rapid increase is obtained when the speed reaches the value of about 750 revs. per min. This critical speed is determined by the air-line, and is easily calculated from the flux-curve.

In the shunt machine the ampère-turns are themselves dependent upon the voltage at the brushes which is nearly proportional to the flux at any given speed. Hence X ∝ flux × speed, or speed $\propto \frac{X}{\text{flux}}$. In fig. 233 if Z be the working point at no load with the designed speed N_1, $N_1 \propto \frac{X}{Z}$. At the critical speed N_0 the ampère-turns are practically proportional to the air-gap reluctance, and the ratio of the two is given by any point on the air-line, and

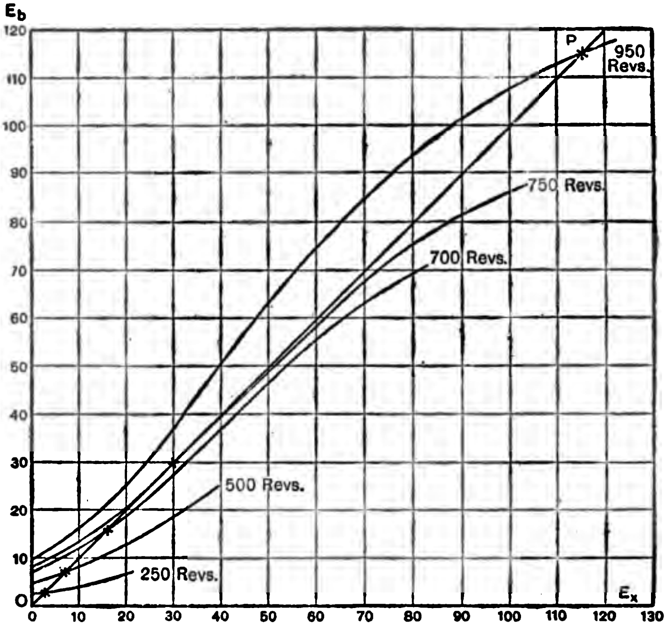


FIG. 231.

therefore by point z_2. Hence $N_0 \propto \frac{X}{z_2}$, and

$$\frac{N_0}{N_1} = \frac{Z}{z_2}, \text{ in fig. } 233 = \frac{5{\cdot}45}{7{\cdot}1} = 0{\cdot}77.$$

In other words the critical speed is $N_1 \times \frac{Z}{z_2}$, in our case $= 950 \times 0{\cdot}77$

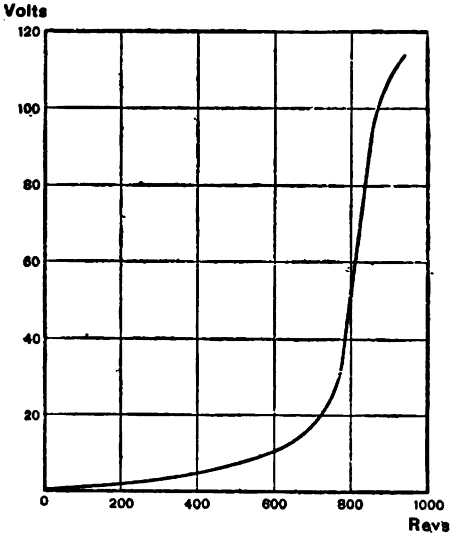


FIG. 232.

= 730 revs.; or it may be immediately calculated as

$$= \frac{0{\cdot}8 \times 2l_g}{\tau \,.\, A_g} \times \left(\frac{R_a + R_s}{T_s}\right) \times 10^{-8}.$$

It is therefore extremely important in designing a machine which is to become excited on its shunt to work fairly high up on the flux curves, and in fig. 213 the best portions of the curves whereon to work fall within the horizontal lines marking five and six million lines respectively (corresponding to a working range from D to K on the characteristic curve of fig. 227). These portions may be said to lie on the bend or knee of the curve, but it must be remembered that the apparent position of this bend depends largely on the relative scale to which lines and ampère-turns are plotted. Another advantage of working high up the curve is that the difference of the ampère-turns of the shunt when hot and when cold then produces but small effect upon the induction and E.M.F. of the machine, since the magnet is well saturated. Also, the vertical distance between the no-current and full-current curves decreases as the total induction is increased, so that the variation in E.M.F. of the machine between full load and no load is small when the magnet is strongly excited. If, however, the working induction be taken too high, a large amount of copper will be required on the field, and if for any reason the actual curve comes below the predetermined curve, it may be practically impossible to rewind the magnet so as to obtain a greater induction, so that care must also be exercised not to exceed the due limit in this direction.

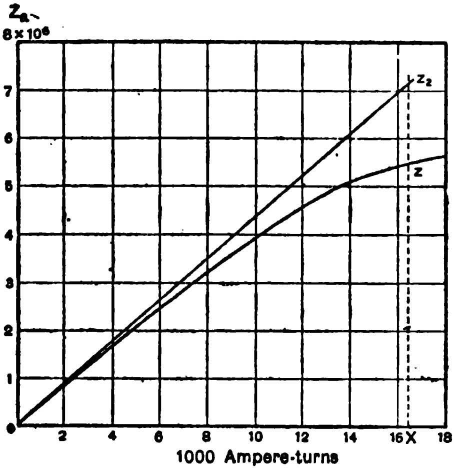


FIG. 233.

§ 11. **Fall of volts in shunt-wound dynamo, and its regulation.**—It will already be apparent that much information can be obtained from the shape of the characteristic curve of the dynamo; in the case of a shunt-wound dynamo run at a constant speed, we see that its E.M.F. is highest when the external circuit is open, and the only current flowing is that through the shunt. When the external circuit is closed, and the armature current is swelled by the addition of an external current, not only does the loss of volts over the resistance of the armature progressively increase as the armature current increases, but the demagnetising effect of the back armature ampère-turns increases. Hence, not only does the exciting voltage at the brushes of the dynamo fall, but the flux due to a given number of shunt ampère-turns is decreased. Owing, therefore, to the combined effect of the two causes, viz., the increase of the back ampère-turns, X_b, and the loss of volts over R_a, the characteristic gradually falls throughout the entire working range of the dynamo. If the armature current much exceeds its working limit and passes the maximum, the machine is then rapidly demagnetised and loses its voltage entirely. If, however, the armature be of low resistance, the drop in volts for any current within the working range will be but small, and consequently the shunt-wound dynamo may in such cases be practically regarded as giving a nearly constant voltage when run at a constant speed.

In order to regulate the voltage of a shunt machine, it is usual to insert in series with the magnet winding a rheostat or resistance, the coils of which can be successively thrown into or out of the shunt circuit. Since the exciting current for a given voltage at the brushes is thereby reduced or increased, the voltage of the machine can be lowered or raised by small steps. Less often the speed of rotation is altered, or both methods are used in conjunction.

§ 12. **Uses of shunt-wound dynamos.**—Large continuous-current dynamos for central-station work, which supply current at 220 or 550 volts to a network of mains, or at higher pressures to continuous-current transformers are usually shunt-wound. The voltage at the station end of the feeders to the network is in such cases conveniently regulated by means of rheostats. For the direct lighting of incandescent lamps in smaller installations, shunt-wound dynamos are not so suitable, owing to the necessity for constant attendance in order to regulate their E.M.F. according to the load, if a constant voltage is to be maintained on the lamps. They are, however, frequently used in conjunction with accumulators, and in such cases, by means of a supplementary resistance in the shunt, they are arranged to charge accumulators during the day up to about 130 volts, and at night to work in parallel with the battery, lighting incandescent lamps at 100 volts, the speed under the two conditions being maintained at nearly the same value.

On the other hand, for the electro-deposition of metals, shunt-wound dynamos giving only some four or five volts are employed, this being the voltage required by each electrolytic bath with its leads. For charging accumulators, and for electrolytic or electro-metallurgical work, shunt-winding is alone suitable; if the E.M.F. of the dynamo becomes less than the E.M.F. of the cells, or the current through the electrolytic baths becomes reversed through polarisation at their electrodes, the direction of the current round the shunt remains unchanged, although the current through the armature will be reversed. The result is that the machine runs as a motor in the same direction as it did when acting as a generator, and therefore opposes a back E.M.F. to the discharge from the cells: this back E.M.F. prevents the flow of a large current which might burn up the armature and also deteriorate the cells.

At high voltages, the necessary diameter of wire for a shunt-winding on a small machine becomes very small, and fine wire is not only expensive in itself, but the time and trouble of winding the great length of it which is required become very considerable. Hence the maximum E.M.F. for a small machine which has all or most of its excitation provided by a shunt-circuit may be set at about 500 volts, but in large generators of say 400 kilowatts output, this may be raised to 2250 volts.

Arc lamps are in many cases run in parallel from shunt machines, but in each parallel a steadying resistance is usually required. It is therefore advantageous to make each parallel circuit consist of several arc lamps in series. This method may be extended until finally all the arc lamps are connected in series; they are then fed from a series-wound dynamo, giving, say, 10 ampères and any E.M.F. up to about 3,000 volts. By this arrangement, not only is any necessity for steadying resistances avoided, but the weight of copper in the line is reduced, since the higher the voltage, the less the weight of copper required for a given percentage loss in the line.

§ 13. **Determination of series winding.**—The determination of the field-winding for a series dynamo follows the same lines as in the case of a shunt machine, save that now the magnetising current and number of turns are fixed, and the necessary size of wire has to be determined from the permissible loss of watts in the field. Thus, if it be desired to convert our 9″ × 12″ dynamo into a series-wound machine without alteration of the armature, then, while the full armature current becomes available externally, the E.M.F. at the terminals will be less than the E.M.F. at the brushes by the loss of volts over the series-winding, *i.e.*, approximately by about $2\frac{3}{4}$ per cent.; the output therefore may be fixed at, say, 100 volts and 145 ampères, at a speed of 950 revolutions per minute. The ampère-turns required on the field in order to give 5,100,000 lines through the armature and an internal E.M.F. of close upon 106·5 volts will be 15,400 as before. The allowable loss of watts over the field-winding being assumed at about 400, the voltage between the ends of the series-winding is $\frac{400}{145} = 2{\cdot}75$, and the resistance of the series coils will be when warm $R_m = \frac{2{\cdot}75}{145} = 0{\cdot}019$. The necessary area of wire is by equation (47)

$$= \frac{{\cdot}0081 \times AT \times l_s \times k}{1000 \times E_s} = \frac{{\cdot}0081 \times 15400 \times 3{\cdot}39 \times 1{\cdot}17}{1000 \times 2{\cdot}75}$$
$$= 0{\cdot}18 \text{ sq. in.}$$

We further have $AT = 15{,}400 = C_s T_m$, whence $T_m = \frac{15{,}400}{145} = 106$ nearly.

As a round wire, the necessary diameter will be so large as to render it troublesome to wind; hence it will be better to adopt a copper strip of rectangular section, and we are at liberty to select such a width and thickness as will give a whole number of layers, and will render it easy to wind on flatwise. Such dimensions will be ·610″ × ·295″ double-cotton-covered to ·630″ × ·315″: four layers, each of $13\frac{1}{4}$ turns, will give 53 turns on each magnet bobbin, or 106 in all. In calculating the number of turns that can be wound within the winding length of 9″, one turn must be deducted from the quotient $\frac{\text{winding length}}{\text{width of wire}}$ since the space lost from the spiral way in which the strip must be wound amounts to the width of one turn; or *vice versâ*, one turn must be added to the required number to form the divisor in the fraction $\frac{\text{winding length}}{\text{number of turns}} = \text{width of insulated wire}$. If the strip be wide and the length of coil be short, this loss amounts to a large percentage of the total length. Hence, in such cases, it may become necessary or advisable to wind the strip on edge, although such a method requires special tools and manipulation. In the present case $13\frac{1}{4} + 1 = 14\frac{1}{4}$ and $\frac{9''}{14{\cdot}25} = 0{\cdot}63''$. Whether a quarter turn can be secured depends upon where the terminals are placed, or the wire is led on to or off the bobbin.

If the thickness of the strip is so great as to render it difficult to handle, two or more strips may be wound in parallel, or the bobbins may be placed in parallel; such parallel connection always involves a greater loss of space in insulation, but its other advantages in many cases outweigh this objection. With a rectangular section of wire, less space is wasted in interstices than if it had been round, and the depth of the series-winding will then not be so great as that of the shunt-winding of § 7; hence, if greater accuracy were required, the values assigned to l_s and k should be slightly modified. For ordinary work the above result will, however, hardly require further correction, since, having regard to the close packing of the rectangular wire, and the consequently reduced cooling surface, a somewhat smaller expenditure of watts in the field was in the first instance assumed for the series-winding, and also a slightly lower value for k to correspond with the lower temperature rise. The actual weight of copper on the field in the two cases is practically the same; it is, however, a little more in the series than in the shunt machine (250 lbs. as against 224 lbs.), and due to this the efficiency of the former is slightly the greater. The current-density in the field-winding, if, as usual in continuous-current dynamos, the magnet is stationary, should not exceed 1000 ampères per square inch; it usually works out to about 750-850 ampères per square inch.

§ 14. **External characteristic of series-wound dynamo.**—The external characteristic of the above dynamo when series-wound for a speed of 950 revolutions per minute is shown in fig. 234. It may be derived from the flux-curves of fig. 213 in the following manner. Taking any current, C_1, multiply it by T_m, the number of turns in the series-winding, and then find on the flux-curve for the particular current C_1 the Z_a corresponding to the exciting power, C_1T_m. Thence $E_a = \frac{Z_a \times \tau \times N}{60 \times 10^8}$ can be determined; the loss of volts over the resistance of the armature and series winding $= C_1 \times (R_a + R_m)$, so that E_e corresponding to a value of $C_e = C_1$ is $E_a - C_1\,(R_a + R_m)$, and the corresponding point on the external characteristic can be plotted. Next take another value for C_e, and in the same way find the corresponding value of E_e, and so on until sufficient points have been obtained to draw in the curve. Of course, if only three or four flux-curves are to hand, it will be necessary to interpolate or add other curves in order to determine the characteristic throughout a large range of current. Thus the

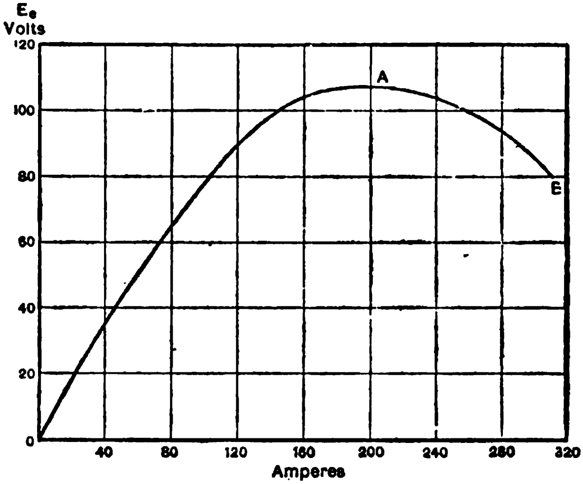


FIG. 234.—Characteristic curve of series-wound dynamo.

curve for ¾ths of full current will fall approximately midway between the curves for half and full current, but slightly nearer to the former; the back ampère-turns of the armature are less at three-quarters than at full load, owing to the decreased current, and the proportion of leakage is thereby decreased, but, more than this, the lead of the brushes may be slightly decreased, and therefore, on both grounds, the effect on the field is to cause the ¾-current curve to fall nearer to the half- than to the full-current curve.

The construction of the external characteristic may also be made graphically by a method similar to that employed for the shunt machine in § 8. On the assumption that the ampère-turns required to balance all the effects of the armature reaction bear a constant ratio to the armature ampère-turns, draw a line OK (fig. 235) at such an inclination to the vertical that the abscissa to it ($ab = cd$) at any point gives the value of $k \cdot \mathrm{AT}_a$. Inclined to the horizontal draw a line OL such that the ordinate to it measures the loss of volts over the resistance of the armature and series coils, or $de = C_a\ (R_a + R_m)$. Taking corresponding points on the two lines for any given value of the current, a right-angled triangle cde is obtained of which the sides are continually altering as the current is varied. The application of this triangle to any corresponding point on the no-current E_a curve marks by its apex e one point on the E_t curve. If the magnet becomes highly saturated, the effect of armature reaction increases more rapidly than the armature current, and the curve of E_t turns over more quickly as shown dotted by qr.

The shape of the external characteristic for a series-wound dynamo is widely different from that for a shunt machine. Since the external current is also the magnetising current, it resembles in the main a curve of flux. It will be seen, however, that after a certain point, A (fig. 234), is reached, the terminal E.M.F. decreases as the external current is increased. This fall of the external characteristic is due to the loss of volts over the resistance of the armature and series winding, and to the demagnetising effect of the back ampère-turns on the armature. The former loss is directly proportional to the current, while the effect of the back ampère-turns increases somewhat faster than the current. On the other hand, when the magnet is approaching saturation, if the external current is increased, the increase in the total flux is by no means proportional to the increase in the exciting power of the field ampère-turns. Hence the induced E.M.F. rises less and less rapidly as the current is increased, and after a certain point the increase in the induced E.M.F. does not so much as compensate for the increased loss of volts over the resistances of the machine; the external characteristic curve therefore attains a maximum height, and thence bends gradually downwards.

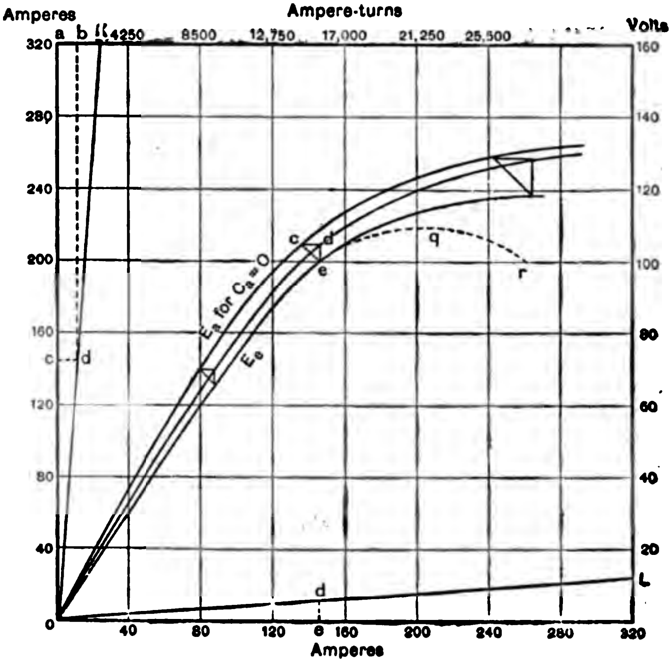


FIG. 235.—Method of deriving voltage curve for series dynamo.

The external characteristic at any other speed is obtained by simply altering the height of the ordinates; the slide rule is set up for the ratio of the speeds, and the calculated values of E_o are altered in proportion to the alteration of the constant speed.

§ 15. **Uses of series-wound dynamos.**—Series-wound dynamos are chiefly used for the electric transmission of large powers over considerable distances,[1] when a single generator is employed to supply current to a single series-wound motor of approximately the same power; and also, as already mentioned, for high-potential arc-lighting. For this latter purpose the drooping external characteristic is a distinct advantage, and the series-wound arc-lighter is frequently worked on the part of the curve marked A B in fig. 234. For the proper working of arc lamps a nearly constant current is required, which implies that the E.M.F. on the series of lamps must be varied in proportion to the resistance which at any moment they present. Now if the resistance of any one or more lamps be decreased owing to the carbons being fed together, or if a lamp be entirely cut out of circuit, the momentary increase in the external current is, in the case of a series-wound dynamo working on a falling characteristic, accompanied by a decrease in the terminal volts: this helps to bring the current back to its normal strength, and although it may not be sufficient to make the machine entirely self-regulating for constant current at varying potentials, it tends in the right direction, and leaves less for the automatic constant-current regulator to do. Thus in constant-current dynamos, the drooping characteristic is expressly exaggerated.[2]

Series dynamos are unsuitable for charging accumulators, since if by any chance the E.M.F. of the machine be allowed to fall below that of the cells, the latter send a reversed current through the dynamo: this reverses the field-magnetism, and consequently the direction of the machine's E.M.F. The result is that both dynamo and cells act in series round a circuit of almost negligible resistance, and generate a current which increases until either a safety fuse is melted or armature and cells are damaged.

§ 16. **Compound winding.**—While a series-wound dynamo may be made to yield an approximately constant current at varying pressure by working it on the drooping portion of its characteristic, it has been shown that a shunt-wound dynamo with armature of low resistance will give an approximately constant voltage over a considerable range of current. This latter requirement, which is the essential of direct incandescent lighting, is, however, much more nearly fulfilled by the *compound-wound* dynamo (fig. 236), to a consideration of which we now pass. If a dynamo be shunt-wound to give a certain voltage at no load, and the magnet be in addition wound with a certain number of turns connected in series with the external circuit, then, as the load is increased, the current flowing through the series or as they are also sometimes called 'main' turns will progressively increase the exciting power. Now this increase in the exciting power may be made, not only to balance the increase in the back ampère-turns, as more load is thrown on, but further to increase the total induction through the armature. As a consequence the induced E.M.F. is increased, and when properly proportioned this increase may be made to exceed the loss of voltage over the series turns, and in addition to compensate for the increased loss of volts over the armature resistance. Thus, while the the armature E.M.F. rises with the load, the terminal E.M.F. remains practically constant, however the current be varied over a very considerable range. The external characteristic is therefore practically a straight line throughout the entire working range of current. In the compound-wound

---

[1] Kapp's *Electric Transmission of Energy*, 4th ed., p. 179 ff.

[2] For hints on the problem of sparkless commutation in constant-current dynamos, see Carhart, *Elect. Eng.*, vol. vii. p. 580 ff. and vol. xiii. p. 140 ff.

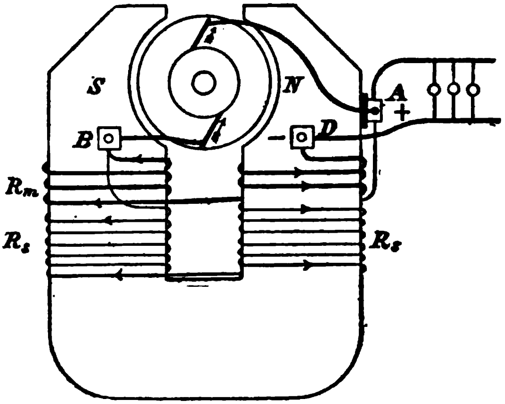
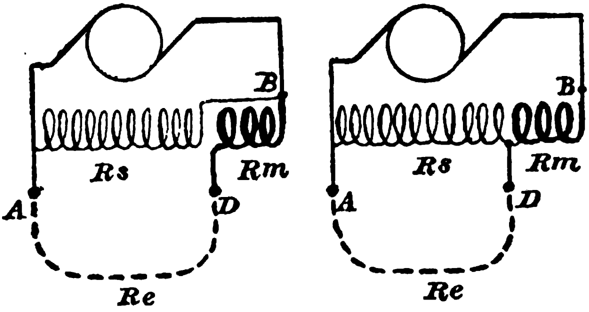


FIG. 236.—Compound-wound dynamo.

dynamo the shunt may, in fact, be regarded as providing a certain initial flux and voltage: these latter are maintained and reinforced against the combined causes which tend to reduce them by means of the series turns, the exciting power of which varies directly with the external current.

The connections of a compound-wound dynamo may be arranged in two slightly different ways, shown diagrammatically in fig. 237. By the first, which is also shown in fig. 236, and is known as the 'short-shunt' winding, the shunt is placed across the brushes of the machine, and the E.M.F. acting on it is $E_b = E_e + C_e R_m$, the current flowing through the main being simply the external current C_e. By the second, or 'long-shunt' winding, the shunt is placed across the terminals of the external circuit: the E.M.F. acting on it is then E_e, while the whole armature current $C_a = C_e + C_s$ flows through the main. Of the two methods, the first is the more common, but their practical difference is not very important. As regards the winding of the two sets of magnetising coils on the magnet bobbins, it is purely a matter of convenience whether the series be wound over the shunt or *vice-versâ*, or whether they form separate coils. In the latter case, however, care must be taken that the equality of the turns on each pole is such as to suit the necessities of the armature winding. Thus in the two-pole horseshoe,



Short Shunt. *Long Shunt.*

FIG. 237.

the series-winding is sometimes arranged entirely on one limb; in the multipolar machine, it may be confined to alternate poles if the armature be series-connected, but if it be parallel-connected, all the poles are preferably similarly wound, each with its proportion of shunt and series turns.

§ 17. **Over-compounding.**—A constant speed and a required constant voltage have been tacitly assumed above for the compound-wound dynamo; but such are not in strictness the ordinary conditions of working, and it must therefore be designed to meet the more usual working conditions. When a dynamo is driven by an engine, and forms a considerable portion of the load on that engine, the greater the output, the slower will be the speed of the engine, even if it be governed for approximately constant speed. The difference of speed between full and no load on an electric-light engine varies; but it should not exceed 4–3 per cent. of the no-load speed. Further, if the dynamo be belt-driven, there will be extra slip of the belt at full load, which will diminish the speed of the dynamo; an allowance of 5 per cent. variation may, however, be taken to cover this further reduction. Apart from this mechanical lowering of the speed, there is also the electrical loss of volts over the main leads between the dynamo and the lamps, which increases in direct proportion to the current passing. If, therefore, a constant potential is required at the far end of the line (as, for example, in a house with 100-volt incandescent lamps run direct from a dynamo placed at some distance) the dynamo must give a terminal E.M.F. of, say, 105 volts when supplying full current in order to allow for the loss of potential over the leads. When, however, only a few lamps are burning, the dynamo must give little more than 100 volts, and for any external current between zero and the maximum, the external E.M.F. must be between 100 and 105 volts. Hence in such a case the compound-wound dynamo must give a higher E.M.F. at full load than at light loads, and that at a lower speed. The desired external characteristic is thus a straight line, not horizontal, but slightly inclined upwards, or, as it is termed, the dynamo is to be *over-compounded*.

§ 18. **Example of calculation of compound-winding.**—Let us suppose that the 9″ × 12″ dynamo of Chap. XV, § 6, has to be compounded to fulfill the following conditions: at 950 revolutions per minute, $E_e = 103$ volts when $C_e = 140$ ampères, and at 980 revolutions $E_e = 100$ when $C_e = 0$, *i.e.*, the voltage is to rise by 3 volts while the speed drops about 3 per cent. as the load is increased from no current to full current.

To give the required voltage at no load, z must be 4,650,000, and from the no-current curve of fig. 213 we see that the shunt-winding must then give $X_{s1} = 12{,}115$ ampère-turns. At full load, allowing a loss of nearly $3\frac{1}{2}$ volts over the armature and brush resistance (as before), and of 0·8 volts over the series-winding, we find that the induced E.M.F. must be 107·3 volts; the total number of lines to give this E.M.F. must be 5,150,000, requiring a total excitation of $X = 15{,}865$ ampère-turns. At full load the exciting voltage on the shunt is 103, if the machine has a 'short-shunt winding,' and the ampère-turns due to the shunt are then

$$X_{s2} = X_{s1} \times \frac{E_{b2}}{E_{b1}} = 12{,}115 \times \frac{103}{100} = 12{,}500.$$

The ampère-turns which the series-winding must produce at full load are therefore $X - X_{s2} = 15{,}865 - 12{,}500 = 3365$, and the number of series-turns is $T_m = \frac{X_m}{C_e} = \frac{3365}{140} = 24$.

Assuming that a loss of about 460 watts in the field is decided upon, we have still to settle the question of how many watts are to be lost in the shunt and how many in the series coils in order that the total weight of wire used may be a minimum. That the total weight will vary with the distribution of the loss of energy between the shunt and series coils is evident if we consider the extreme case in which all the watts are lost in the one portion and none in the other; then, while the one has a definite weight, the copper in the other must be infinitely great, so that the total weight required will be infinitely great. It can be shown that the most economical distribution is to make $\frac{W_m}{W_s} = \frac{X_m}{X_s}$, or, which is the same, to make $\frac{W_s}{W} = \frac{X_s}{X}$ and $\frac{W_m}{W} = \frac{X_m}{X}$[1], where W_s and W_m represent the watts to be lost in the shunt and series respectively. We thus arrive at the result that the watts lost in the shunt should be roughly $= 460 \times \frac{12{,}500}{15{,}865}$, or about 360, and the watts lost over the series turns about 100.

---

[1] Neglecting any differences in the price of the two kinds of copper wire. To allow for differences in the lengths of a turn of the shunt and series, and for

The mean lengths of a turn of the shunt and series coils will be slightly different according as they form the inner or the outer layers of the coil; and also their temperature coefficients will vary slightly, the outer layers being less hot than the inner. Assuming both shunt and series to be equally divided between the two bobbins, and the series turns to be wound on the outside above the shunt, the mean length of a shunt turn will be 40·3″, and that of a series turn 45·4″ or $l_s = 3{\cdot}36$ ft., $l_m = 3{\cdot}78$ ft., while k_s and k_m may be estimated at 1·2 and 1·14 respectively. We are now in a position to definitely determine the winding. The resistance of 1000 ft. of the necessary shunt wire is (by equation 46),

$$\omega' = \frac{100 \times 1000}{12{,}115 \times 3{\cdot}36 \times 1{\cdot}2} = 2{\cdot}04 \text{ ohms},$$

and the necessary diameter is 0·071″. When double-cotton-covered to 0·086″, there will be 105 turns per layer, and if there are 18 layers on each bobbin, $\text{T}_s = 3780$, $\text{R}_s = 25{\cdot}9$ ohms at 60° F, or, when hot, 31 ohms; thence $\text{C}_{s1} = \frac{100}{31} = 3{\cdot}23$, and $\text{C}_{s2} = 3{\cdot}32$. Hence, under the full-load condition, $\text{W}_s = 342$ watts.

At this point it will be advisable to refer back to the data assumed above, so as to verify the correctness of our allowances for the loss of volts over the armature and series winding, and of our estimate of l_s and l_m as affected by the depth of winding. Since the shunt watts at full load are rather less than 360, the series watts may be increased to say 115, and our original allowance

---

differences of temperature, according as the one or the other is wound outside, the above formula may be modified to

$$\frac{\text{W}_m}{\text{W}_s} = \frac{\text{X}_m \sqrt{\dfrac{l_m^2 \times k_m}{l_s^2 \times k_s}}}{\text{X}_s}, \text{ whence } \frac{\text{W}_s}{\text{W}} = \frac{\text{X}_s}{\text{X}_s + \text{X}_m \sqrt{\dfrac{l_m^2 \times k_m}{l_s^2 \times k_s}}}.$$

of 0·8 volt loss over the series winding may be permitted to stand. Hence its necessary area (from eq. 47), is

$$\frac{0·00813 \times 3365 \times 3·78 \times 1·14}{1000 \times 0·8} = 0·147 \text{ sq. inch.}$$

If of rectangular section and wound in one layer of 12 turns on each bobbin, the width, which when double-cotton-covered with a thickness of 20 mils will form a complete layer, is $\frac{9}{12+1} - 0·020 = 0·670''$ and the required thickness of the copper is therefore 0·220″. The weight of shunt being about 200 lbs., and of series 55 lbs., the total is nearly the same as that of the simple shunt or series machine.

§ 19. **Necessary imperfections of compound winding.**—Although a dynamo may compound perfectly at full and no load, it will not do so with equal accuracy at intermediate points. This is due partly to the flux curves being rounded between the working points of no and full load, as, *e.g.*, between the limits of 4,650,000 and 5,150,000 lines in fig. 213, and partly to the fact that the back ampère-turns at half current are somewhat less than half the back ampère-turns at full current, and those at quarter current less than half of those at half current, and so on.[1] The effect of these two causes is seen in fig. 238, which shows a portion of the flux-curves of fig. 213 on an enlarged scale. At no load (or practically no load) the working point of our compound-wound dynamo is C, and at full load F; since the length G K gives the increase in the ampère-turns, when the current is increased up to full load, the increase in the ampère-turns at half load will be $GH = \frac{GK}{2}$, and the corresponding Z_a on the half-current curve is HL. But for perfect compounding only HM lines are then required, and the result is that the E.M.F. is rather higher than it should be: this, of course, increases the shunt-current, and the excess of E.M.F. is thereby rendered even greater. The terminal E.M.F. is higher than is required, not only at half load, but at all loads between zero and the maximum. The excess of E.M.F. is roughly proportional at any point to the vertical distance between the full line C L F and the dotted line C M F, from which it will be seen that it is greatest between quarter and half load. In order to minimise this imperfection, a compound machine should be worked as far as possible on the upper part of the flux-curves; the higher the working limits, the flatter and more horizontal do the curves become; and the effects of the unequal spacing of the intermediate curves, and of the increase of the shunt-excitation, are lessened. But in slow-speed machines, *e.g.*, in an 18-kilowatt dynamo, running at 250 revolutions per minute, the percentage loss of volts over its electrical resistance may amount to about 8 per cent. of the terminal voltage (6 per cent. over the armature and brushes, and 2 per cent. in the series coils); allowing a further loss of 3 per cent. in the leads between the dynamo and the lamps,

---

[1] Due to the shifting of the brushes necessary to avoid sparking, and to the effect of the armature reaction in increasing the leakage.

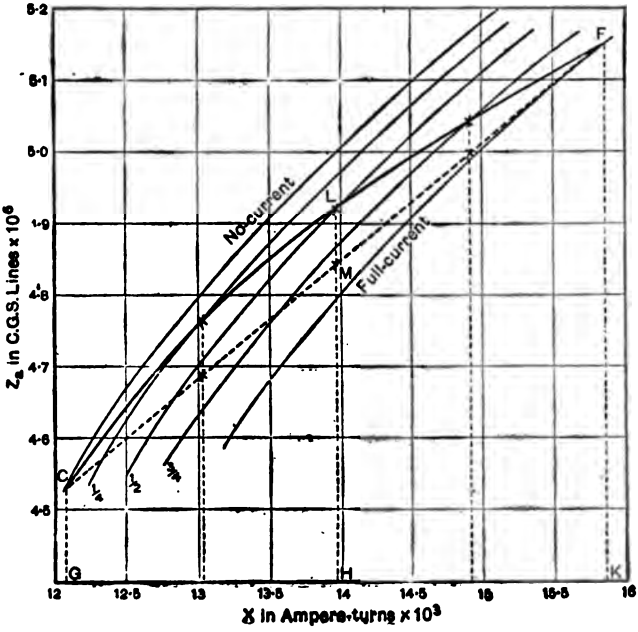


FIG. 238.

and also a decrease of 5 per cent. in the speed of the engine at full load, it will be found that z_a at full load is $1·11 \times 1·05 = 1·17$ times the no-load z_a. In such a case it is practically necessary to work the machine on the rounded knee of the magnetisation-curve, since to work higher up would require an excessive amount of series wire. It results that slow-speed machines and those which have to work over a large range of E.M.F. always compound more or less imperfectly, and the terminal E.M.F. for any current between ¼ and ¾ of full load may be from 3 to 4 per cent. too high. In machines running at comparatively high speeds the voltage should not exceed the required amount by more than 3 per cent., even at the point of greatest difference.

Small machines compound less perfectly than large machines, and multipolars being equivalent to several smaller two-pole machines in parallel show in general a greater percentage rise at intermediate loads than do large two-pole dynamos. Since the compound-wound machine is usually required to excite itself on open circuit, the same precautions must be taken against instability of the magnetism as in shunt machines.

Differences of temperature considerably affect the perfection of the compounding of a dynamo: if worked on the rounded portion of the curve at no load, and if the temperature of the field-coils rises during working 50° or 60° F., a machine designed to compound when hot will give too high a voltage at no load, and, in general, as the temperature of the dynamo changes, the relative value of the shunt- and series-turns is altered. Such effects can be compensated by inserting a small variable resistance in the shunt-circuit, which can be gradually short-circuited as the dynamo warms up during working.

Traction generators are very frequently wound to give 500 volts at no load and are then over-compounded to give 550 volts at full load, so as to balance the loss of volts in the transmission line. Compound winding is also largely employed in the dynamos of isolated installations, as for driving motors in a factory or for the lighting of steamships, in order to maintain an approximately constant voltage on the motors or lamps under all conditions of load.

The field-winding of separately-excited machines can be at once calculated from eq. 46–48. Such machines are never unstable, since the excitation is maintained from an external source. Separate excitation is most used in the case of alternators, or in machines which have to yield a high E.M.F. of over 500 volts, and a variable current.

CHAPTER XVII

ARMATURE REACTION IN CONTINUOUS-CURRENT DYNAMOS

§ 1. **Diameter of commutation and brush-position.**— In the continuous-current closed-coil dynamo the winding is divided by the brushes into as many current-sheets as there are poles. Thus in the case of an armature, whether ring- or drum-wound, in a two-pole field, two sheets of current as shown by the dotted and crossed circles of fig. 239 flow along the external surface from end to end, and the direction of the currents in the inductors changes at the diameter of commutation (DC) corresponding to the position of the brushes. It may here be recalled that the relative position of the line of the brushes on the commutator and the line passing through the coils undergoing commutation on the armature core depends upon the way in which they are connected to the commutator sectors (see Chap. XI, § 15). If the connecting lugs are straight, the two will coincide

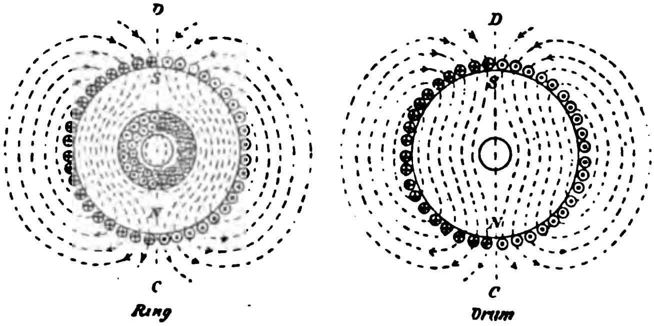


FIG. 239.—Magnetic field of armature.

in a ring machine, but such coincidence is not necessary since the lugs may be twisted so as to bring the brush line into any convenient position. In a drum machine the connection is often made at the centre of the end-connectors, so that the true line of commutation is shifted $\frac{90^\circ}{p}$ relatively to the brushes. In every case then the line or *diameter of commutation* (a term extended to multipolars by analogy from the two-pole dynamo) must be understood to refer to the actual position of the coils undergoing commutation rather than to the position of the brushes corresponding thereto.

§ 2. **Distortion of the resultant field.**—Considering first an armature carrying current but in which the brushes are so set that the diameter of commutation is coincident with the line of symmetry (VL) bisecting the interpolar gap, let a system of lines be drawn through the armature similar to the symmetrical distribution which holds when the armature is run on open circuit (fig. 240). When the wires are paired to form current-turns, each line is found to pass through a certain number of armature ampère-turns; and of these turns the one half acts in the same direction as the exciting M.M.F., and the other half against it. The relation of the armature-turns to the direction of the flux on the two sides of the line of symmetry is reversed in opposite halves of the armature, but the forward and back ampère-turns are exactly balanced, so that they neutralise one another. Such a distribution is, however, not the one that holds in reality; in nature the total flux is always the maximum possible with the given number of ampère-turns that are present, and is always linked with the maximum possible number of forward ampère-turns, or conversely passes through a minimum number of back ampère-turns; due regard being had to the reluctivity or permeability of the material of the circuit which may alter as the flux density at different points is varied

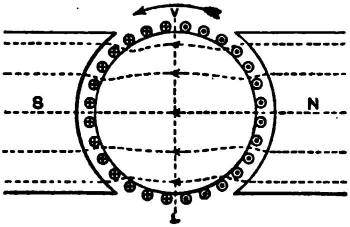


Fig. 240.

with different distributions. It follows that the lines tend to become bent so as to pass through as many forward armature turns as possible; in other words, the field is distorted, the general tendency being for the lines to enter, say, in the upper right quadrant and to emerge at the lower left quadrant. All the lines cannot flow along the path indicated (fig. 241), since the additional forward ampère-turns through which the lines when thus concentrated and crowded together pass would not suffice to meet the combined effect of the two restraining causes, viz., the longer length of the path and the greater fall of magnetic potential that must result from the increased density of the flux even if the permeability of the material remained constant. The lines must therefore be more or less spread out, the total flux being retained at its highest possible value and linked with as many ampère-turns as possible on the armature, so that at one and the same time the energy stored in relation to both the exciting coils and the armature turns is a maximum. When the lines are thus more or less distorted and concentrated, the question is how far the two checking causes that have been mentioned are counterbalanced by the increased number of forward armature turns through which they pass. The average length of the distorted lines could however only be approximated by guess-work, and the actual number of forward and back ampère-turns traversed by any group of lines would be uncertain. A more detailed analysis is therefore necessary for the dynamo designer, and in the first instance let it be assumed that the iron whether of the poles or of the armature core has a constant permeability and is therefore on the same footing as the air-gaps.

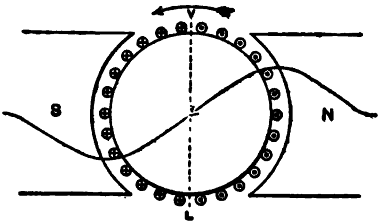


FIG. 241.

§ 3. **The magnetic field of the armature alone.**—On such an assumption the necessary distribution is obtained by superposing on the initial symmetrical field due to the field ampère-turns a second symmetrical field due to the cross ampère turns of the armature. In the first place, then, what is the nature of the field due to the armature ampère-turns considered by themselves? In the armatures of fig. 239, the general effect of the two current-sheets is to convert the iron armature into a cylindrical electro-magnet with two polar surfaces at opposite ends of the diameter of commutation, and stretching along the entire length of the core. If the armature be ring-wound, with the turns of wire passing round and inside the core, the analogy to a pair of magnets of semicircular shape, each wound with a magnetising coil and placed so that their like poles abut on each other to form a common N. and a common S. pole, is at once apparent. If the armature be drum-wound, this effect is not immediately so apparent, but we may for simplicity neglect the actual end-connections of the wires and imagine each inductor connected across, in a plane at right angles to the diameter of commutation, to a corresponding inductor on the opposite side of this diameter. A number of loops are thus obtained, each carrying a current equal to half the total armature current, and magnetising the core in a general vertical direction. Hence, whether the armature be ring- or drum-wound, lines of induction would flow through the two halves of the core on either side of the diameter of commutation, issue from the lower surface and circle round to enter again at the upper surface; or, as it is otherwise expressed, poles are formed on the armature at opposite ends of the diameter of commutation.[1]

Apart from the field-magnet these lines of induction will be comparatively few in number (as shown in fig. 239), but when the armature is surrounded by the iron pole-pieces, their length of path in the air is enormously reduced, and a strong flow will arise, passing through the two pole-pieces and traversing the short air-gaps of the dynamo. Fig. 242 serves to show the course of the increased number of lines due to the currents in the inductor-loops when the armature is placed within the embrace of the pole-pieces, the field being otherwise entirely unmagnetised. When the armature is rotated, the inductors pass successively from one half of the armature into the other, but the current-sheets remain fixed in position on either side of the diameter of commutation, and, therefore, the field of lines which they produce will likewise remain stationary and unchanging. The magneto-motive force of the armature ampère-turns rises as we pass from a central horizontal line towards either the upper or lower limits of the armature core. Taking any angle, $k\,o\,l$, the magneto-motive force between the limits k and l is proportional to the number of ampère-turns which they include: thus, if $\tau =$ the total number of inductors evenly spaced round the armature, and the current flowing in each be c, the magneto-motive forces of the ampère-turns summed up between k, l $= 1{\cdot}257\, c\tau \dfrac{\theta}{360^\circ}$ where $\theta =$ the angle $k\,o\,l$ expressed in degrees. The greatest magneto-motive force is thus obtained between the extreme top and bottom of the core, and is there equal to $1{\cdot}257 \dfrac{c\,.\,\tau}{2}$, or when expressed more generally so as to include multipolar machines $= 1{\cdot}257 \dfrac{c\tau}{2p}$ where p is the number of pairs of poles.

---

[1] In the case of the ring armature some lines will cross the interior of the ring, passing through the shaft and supporting hub (or spider) as indicated in fig. 239.

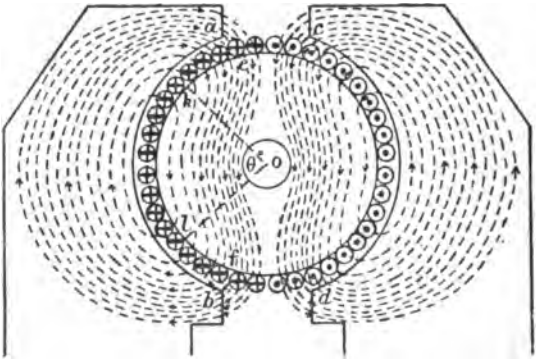


FIG. 242.—Magnetic field of armature alone when placed within pole-pieces.

Taking any closed path, as between $k\,l$, the flux which traverses it is equal to the magneto-motive force of the armature ampère-turns within those points, divided by the reluctance of the double air-gap and of the iron through which the lines pass. At the centre of the pole-face there is no M.M.F. and no flux; thence the induction rises to a maximum at the pole-tips. Outside the limits of the polar arc, the armature M.M.F. continues to increase up to its maximum on the line of symmetry, but for l_g must now be substituted the increased length of air-path, viz. $(l_g + \xi x)$ as explained in Chap. XV, § 6, so that the density decreases to a second minimum but not to zero on the line of symmetry. If the armature be imagined to be cut across and opened out, and the values of the induction in the air-gap at each point of its surface be plotted, the dotted line of fig. 243 corresponding to one pole or 180° is obtained. The direction of the lines changes at 90°, and each half of the curve is repeated in reverse order under the second pole.

§ 4. **Superposition of armature field on main field.**—Let us suppose that the sheets of armature current shown in fig. 242 are due to rotation of the armature as a dynamo, say, in a counter-clockwise direction; then in order to produce armature currents in the direction shown, there must be a N. pole on the right, and a S. pole on the left hand, *i.e.*, such a field as that shown in fig. 48. Let that edge or corner of the pole under which a coil first enters after passing through the interpolar gap be termed the 'leading' edge as opposed to the trailing edge from under which it emerges into the interpolar gap. If we now compare together the general direction of the two sets of lines, viz., those of fig. 48, that would be due to the field-magnet by itself, and those of fig. 242, due to the armature ampère-turns alone, it will be found that the direction of the lines due solely to the armature current is immediately opposed to the direction of the field-magnet lines in the air-gap under the leading half of each pole-face as, *e.g.*, under the two leading pole-tips (*a*, *d*), while they are in the same direction in the trailing half of each air-gap as, *e.g.*, under the two trailing pole-tips (*b*, *c*). But when the dynamo is at work and supplying current, the magneto-motive forces of both the field ampère-turns and the armature ampère-turns are simultaneously present, and in nature there is only one resultant distribution of field due to the combined effect of the two acting together. In the present case the resultant field which satisfies all the conditions may be arrived at by an imaginary superposition of the two sets of lines, the one on the other, followed by a process of compounding the two vectorially, with due regard to their directions, sense, and strength. In fig. 243 the thin-line curve shows the density in the

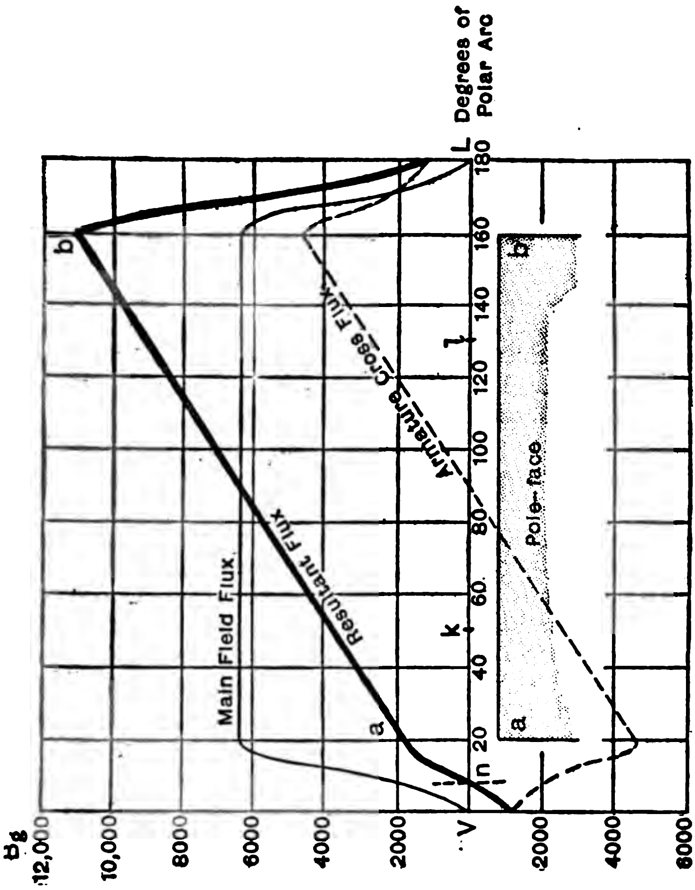


Fig. 243.—Flux distribution in smooth-core armature.

air-gap of the symmetrical flux due to the field-magnet turns acting alone. The composition of the two fluxes with due regard to their sense or algebraic sign yields the curve of resultant air-gap induction as shown by the thick line of fig. 243. At the centre of the pole-face the induction has its normal value $B_g = \frac{1{\cdot}257 X_g}{2 l_g}$, and on the one side it rises to a maximum B_g'' at the trailing pole-tip *b*, while on the other side it decreases to B_g', the slope within the limits of the pole being uniform as shown by the inclined straight line. In the other half of the armature the distribution of the field in the air-gap is altered in exactly the same way, but now the leading pole-corner where the induction is below the average is at *d* while the trailing half where the induction is above the average is at *c*.

Since the system of armature lines passes through the core and pole-piece in a direction across that of the main field, it is termed the *cross flux*, and the ampère-turns which produce it are known as *the cross ampère-turns of the armature*.

The general result of the reaction of the armature currents on the field is shown in fig. 244, from which it will be seen that the flux is, as it were, displaced and twisted round in the direction of rotation. The distribution of the field or the induction in the air-gap on either side of the armature, instead of being uniform over the greater part of the bored face of the pole-piece, varies continuously from a minimum value at *a e* to a maximum at *b f*; the densest part of the field is shifted up to and beyond the trailing pole-tip, while the field under the leading pole-corner is weakened. The distortion of the resultant field now corresponds to the inductance of the armature current-turns. If the armature current were reduced in strength, the lines would straighten themselves and cut the inductors, giving a forward E.M.F., and tending to keep up the previous value of the current. In so doing the stored energy of the armature current-turns, due to their reaction on the magnetic field, would reappear.

§ 5. **No diminution of total flux from cross turns with iron assumed of constant permeability.**—So far it has been assumed that the brushes are given no forward lead, so that the diameter of commutation coincides with the vertical line of symmetry, and the materials of the circuit have been supposed to have a constant permeability. Under these conditions it is now evident that the average induction in the air-gap, or, taking into account the length of the armature, the total number of lines entering into it on the one side and leaving on the other is entirely unaffected by the current-turns of the armature, since the induction is as much strengthened over the one half of each polar face as it is weakened over the other half. At any two points such as *k*, *l* (figs. 242 and 243) equidistant on either side of the line through the centre of a pole, the cross induction has the same numerical value, but in the one case must be deducted from and in the other case added to the induction of the main flux. The M.M.F. of the cross ampère-turns within any angle *k o l* is in fact divided equally between the two halves of the cross path, and the distribution of the cross-flux on either side of the pole centre is symmetrical. The distribution of the resultant field has, however, been altered, and the greater the current passed through a given armature, the greater will be the forward displacement or twisting round of the field. Any such displacement, however, implies a corresponding displacement of the neutral line, as is evident from figs. 243 and 244, where the line *n n* is seen to be shifted forwards in advance of the vertical line of symmetry between the poles. Provided that the area across the narrowest part of the pole-pieces be considerable, and the iron be therefore of high permeability both across the neck and at the pole-corners,

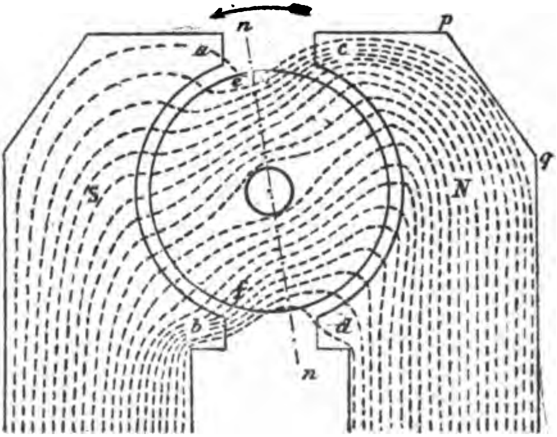


FIG. 244.—Magnetic field of dynamo reacted on by armature.

and provided also that the armature be far from saturation, almost the whole of the cross magneto-motive force may be regarded as expended in producing the cross flux in the two air-gaps traversed by each set of lines on either side of the armature. Over these two gaps the direction, although not the sense, of the lines of the cross flux exactly coincides with that of the main field lines, and it becomes permissible to add together algebraically the two M.M.F.'s in order to determine the resultant induction. The magneto-motive force impressed by the field-winding on each air-gap between the surfaces of a pole and the armature core is $1{\cdot}257\frac{X_g}{2}$. The magneto-motive force due to the cross turns acting on the single air-gap at any point k is $1{\cdot}257\frac{c\tau}{2}\cdot\frac{\theta}{360^\circ}$. The resultant induction at the point is therefore

$$1{\cdot}257\left(\frac{X_g}{2}\pm\frac{c\tau}{2}\cdot\frac{\theta}{360^\circ}\right)\div l_g.$$

The most important pair of symmetrical points are those at the leading and trailing pole-tips, or approximately between a,e and f,b. Here the polar angle ϕ may be substituted for θ, and we have for the resultant induction at the leading pole-tip

$$B_g{}'=\frac{1{\cdot}257\left(\frac{X_g}{2}-\frac{c\tau}{2}\cdot\frac{\phi}{360^\circ}\right)}{l_g}\quad . \quad . \quad . \quad . \quad (52)$$

and at the trailing pole-tip where the cross M.M.F. assists that of the field excitation,

$$B_g{}''=\frac{1{\cdot}257\left(\frac{X_g}{2}+\frac{c\tau}{2}\cdot\frac{\phi}{360^\circ}\right)}{l_g}\quad . \quad . \quad . \quad . \quad (53)$$

The two values gradually approximate as we approach the centre of the pole-face, where the induction again has its normal value $B_g=\frac{1{\cdot}257X_g}{2l_g}$.

§ 6. **Distortion of field in smooth-surface armature with iron of constant permeability.**—The above case closely corresponds to that of the smooth-surface armature in which the air-gaps are of considerable length. A numerical example will serve to show how in such cases the effect on the cross flux, when a high and constant value is assumed for the permeability of the iron, is practically inappreciable.

In a smooth-core armature with a normal induction in the air-gap $B_g = 6400$, let $l_g = 0{\cdot}84$ cm., and on each side consider a depth of, say, 4 cm. of iron both on the pole-face and on the armature core somewhat as indicated by the shaded bands of fig. 245. The difference of magnetic potential between the extremes of the shaded regions is then as follows:—

Over the double air-gap, $B_g \, 2l_g = 6400 \times 0{\cdot}84 \times 2 \qquad = 10{,}750$

Over the four shaded bands at the same density, $f(B) \, 2l = 2{\cdot}62 \times 8 \times 2 \qquad = 42$

Over the remainder of the armature path, say $\qquad 208$

Total C.G.S. units $= 11{,}000$.

Let the maximum M.M.F. of the armature cross ampère-turns acting on each pole reach the high value of 10,000, so that if the pole-breadth be 0·78 of the pole-pitch, as in our case of a two-pole dynamo with 140° polar arc (fig. 243), the cross M.M.F. acting between the pole-tips is $10{,}000 \times 0{\cdot}78 = 7800$. The permeability of the iron has above been given as $\mu = \frac{6400}{2{\cdot}62} = 2440$, and is now to be constant. Then neglecting for the sake of simplicity the small M.M.F. required to pass the cross flux upwards or downwards from the one half of the pole or armature to the other half, the maximum density of the cross flux at the pole-tips is obtained from the equation, $B \times 1{\cdot}68 + \frac{B \times 8 \times 2}{2440} = 7800$,

whence $\qquad B(1{\cdot}68 + 0{\cdot}00655) = 7800$

or $\qquad B = 4625$,

and for any point distant $x°$ from the centre of the pole, $B = 4625 \times \frac{x°}{70°}$. The small influence of the iron portion of the path is evident. The distortion is, however, very marked; B_g' at the leading pole-tip $= 6400 - 4625 = 1775$, B_g'' at the trailing pole-tip $= 6400 + 4625 = 11{,}025$, and the neutral line is displaced forwards 8° ahead of the centre of the interpolar gap (fig. 243). The brushes being assumed to be on the line of symmetry, all inductors within the 8° of lead of the neutral line are positively harmful, but their back E.M.F. is balanced by an equal increase of the E.M.F. within 8° behind the symmetrical line. There is in this case, therefore, owing to the symmetry of the two component curves, no net loss of E.M.F. in spite of the displacement of the resultant field.

There is, however, a disturbing cause present even in the smooth-core armature, since the trailing pole-tip and the armature iron beneath it become highly saturated, while at the leading pole-tip the flux-density is weakened. The same cause affects the toothed armature to a much greater degree, owing to its shorter air-gaps and the greater distortion of the flux; the problem is also further complicated by the teeth which are initially worked with a high value for their normal induction. The effect of the change of permeability of the iron needs then to be examined, and first let the smooth armature be considered. It is clear that with the brushes on the line of symmetry the total flux cannot be increased by the armature current, since when the circuit is closed and the current arises, the process is one of progressive distortion of the resultant flux, and no separate system of lines is actually called into being, which is linked solely with the armature turns (save around the end-connectors, which are not here under consideration). But as the displacement grows with a rising armature current, the permeabilities of the two corners alter, and even when the one falls and the other rises, the change may take place to different degrees in the two. Thus fig. 245 may be taken to represent the case of an increasing reluctivity as the trailing pole-corners are approached; the pole-face and armature surface are to be regarded as made up of a number of strips of different materials gradually increasing in reluctivity as indicated by the increasing depth of the shading when we pass from one quarter to another. The possibility therefore of a

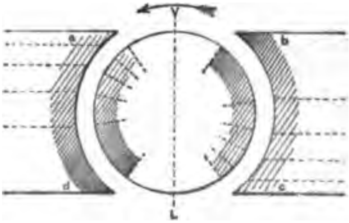


FIG. 245.

reduction in the total flux due to the permeability of the one corner being reduced by more than the amount of the rise in the other must be investigated. Or again the permeability of both corners might become reduced to the same or different degrees.

At any moment the actual resultant flux must contain combined within itself both the flux that would be produced by the field winding if acting alone on the circuit with its permeabilities in their resultant state as due to the resultant distribution, and likewise the cross flux that would be due to the cross armature turns if they also were acting alone on the circuit in its given resultant state. The distribution can therefore only be traced by assuming a certain change in the permeabilities, determining each component flux separately and subsequently combining them. By a series of approximations one could thus finally obtain such a resultant distribution as agrees with the resultant permeabilities that have been assumed.

§ 7. **Reduction of field in smooth armature with iron of varying permeability negligible.**—Taking first the case of the smooth-core armature, a peculiarity is rendered evident from fig. 130. When the normal density in the air-gap and in the pole-face or armature surface (each being of practically equal area with the air-gap) is in the neighbourhood of $B = 6000$, it is not far from corresponding with the point of maximum permeability of the iron or steel. Hence when distortion of the resultant field takes place, the new values of μ at the two pole-corners are in both cases lower than the original values for the undistorted field, and fig. 245 does not represent the true facts of such a case. Further, for considerable changes on either side of the maximum point, there is no great divergence between the reduced values. Roughly speaking, then, under the approximate assumption that the value of μ at each pole-corner is alike, both the main and the cross field remain symmetrical about the centre of a pole, but both are slightly reduced. A few trials show that for the same cross M.M.F. of 7800 the conditions are satisfied if the permeabilities at the two corners are reduced to 1600. At the extreme edges the density of the main flux is now 6385, and from the the new equation $B\left(1{\cdot}68 + \frac{16}{1600}\right) = B(1{\cdot}68 + 0{\cdot}01) = 7800$, the maximum density of the cross flux is $B = 4613$. The shape of the main component curve of fig. 243 now becomes slightly bowed, but owing to the preponderating influence of the two air-gaps the reduction of the total flux is practically negligible, since even at the two corners it is only reduced from 6400 to 6385. The resultant inductions become $B_g' = 1772$, and $B_g'' = 10{,}998$, as compared with the previous figures of 1775 and 11,025. As the curve of cross flux still remains symmetrical about VL, there is no further loss of E.M.F. due to the angle between the resultant neutral and the symmetrical lines.

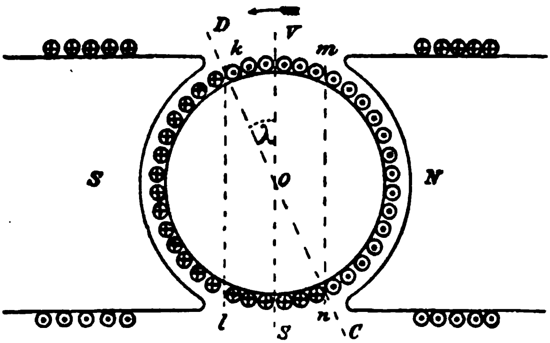


FIG. 246.—Ampère-turns of field and armature.

§ 8. **Effect of forward lead of the diameter of commutation.**—The effect of advancing the diameter of commutation forwards beyond the line of symmetry has now to be considered, and will be found to introduce an entirely new result. So long as the two coincided, the armature ampère-turns all tended to produce a cross-magnetisation, the general direction of which in the armature was at right angles to the direction of magnetisation due to the field alone. But now, when the brushes are given a forward lead, if the directions of the currents in the inductors on the armature be compared with the directions of the currents in the magnetising coils of the field, as shown by dotted and crossed circles in fig. 246, it will be seen that in all the inductors lying between the two vertical lines *k l*, *m n*, the direction of the current is opposed to the direction of the current in those field-magnet wires which are on the same side of the lines of the field, and further that the loops between *k m* practically embrace the entire magnetic circuit. The inductors may now be regarded as connected together across the ends of the armature in two sets of turns, one forming a coil in a vertical, and the other a coil in a horizontal plane. These are seen in fig. 247, the imaginary end-connections of the vertical coil being shown by full lines, and those of the horizontal coil by dotted lines. Thus, inductors 1 and 2 are equivalent to one turn or loop, round which a current of $\frac{C_a}{2}$ ampères flows, and similarly inductors 3 and 4 are equivalent to another turn, but in a plane at right angles to the former. The entire number of armature ampère-turns is divided into two sets, and we are justified in so dividing them, since the effects of the two are largely distinct. The two vertical lines of fig. 246 are fixed in position by the *angle of lead*, the angle *k o m* or *l o n* being equal to twice the angle of lead λ of the diameter of commutation, reckoned from the vertical diameter or line of symmetry. The belt of ampère-turns included between the lines *k l*, *m n* are in direct opposition to the ampère-turns of the field, and tend to demagnetise the armature and field; they are therefore known as the *back ampère-turns* of the armature. On the other hand, the belt of ampère-turns lying horizontally between the points *k l* or *m n* have the same effect as was previously considered when the diameter of commutation coincided with the line of symmetry; that is, they have a cross-magnetising tendency, which distorts the field round in the direction of rotation, weakening it at the leading pole-corners and strengthening it at the trailing pole-corners so that they are still 'cross ampère-turns.' Every conductor not situated on the neutral line or at right angles to it has both a cross and a direct magnetising tendency, but of those situated under one pole-piece the forward tendency of one half is balanced by the back effect of the other half, leaving them pure cross loops, while

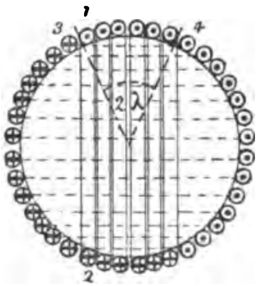


FIG. 247.—Back and cross ampère-turns of armature.

of those between the pole tips the cross tendency of the one half is balanced by the other half, which tend to lessen the distortion so that only their back effect is summed up.

Fig. 248 shows in the case of a multipolar dynamo the analogous division of the armature ampère-turns into the two belts of back and cross turns. The back ampère-turns acting on the magnetic circuit of a two-pole dynamo or on each magnetic circuit in a multipolar machine are equal to the product of the current in any one inductor multiplied by the number of inductors inclosed within twice the angle of lead, or if λ be expressed in degrees,

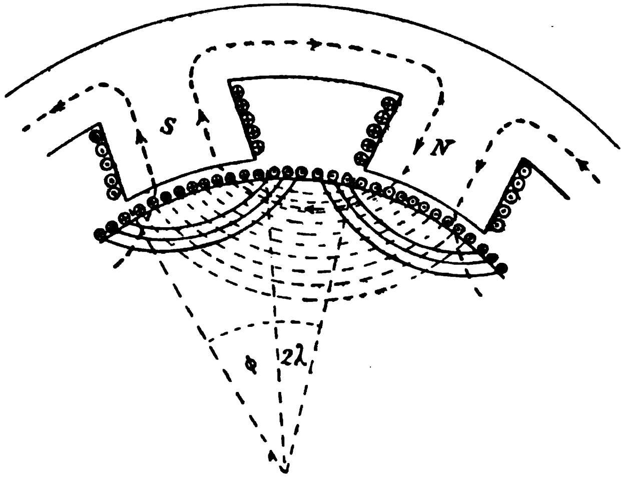


FIG. 248.—Back and cross ampère-turns of multipolar armature.

$$X_b = c\tau \cdot \frac{2\lambda}{360^\circ} \quad . \quad . \quad . \quad . \quad (54)$$

With the given angle of lead the maximum value of the cross ampère-turns is conversely

$$c\tau \cdot \frac{\theta'}{360^\circ} \quad . \quad . \quad . \quad . \quad (55)$$

where θ' is also expressed in degrees and is $= \frac{360^\circ}{2p} - 2\lambda$. The relative proportion of these two sets of ampère-turns will depend entirely upon the angle of lead. Together they form a total number of magnetising turns $\frac{cr}{2p}$, or the current-sheet corresponding to the one pole, and the number of such sheets is of course equal to the number of poles.

So long as the brushes were on the line of symmetry and the cross flux was alone present or was symmetrically distributed, then whatever its amount, whether in a smooth or toothed armature, it could in no wise affect the armature voltage either by way of increase or decrease; otherwise conductors carrying a steady current would in effect have been cutting their own lines. But now owing to the lead of the brushes the diameter of commutation is no longer symmetrical with respect to the cross flux which is itself symmetrical with respect to the poles. Considering the case from the basis of two superposed fields, the one due to the main field winding as weakened by the back ampère-turns, and the other due to the cross ampère-turns, it will be seen that the cross flux has such a direction that the back ampère-turns become forward inductors so far as their cutting of it is concerned. From the nature of the case, however, the cross flux within twice the angle of lead or *k m* (fig. 246) can never be more than equal to the back lines passing athwart the cross turns and causing the weakening of the field, and in most cases is much less. The extreme case of equality would hold if the angle of lead were as much as 45°, and the polar arc were practically equal to the pole-pitch, so that both sets of turns act upon circuits of equal area and of reluctance determined in the main by the length of the double air-gap. But at the same time the lines of the main field embraced within the arc D V (fig. 246) *i.e.*, between the lines of commutation and of symmetry, are now cut in the wrong direction so that they set up a back E.M.F., and the total E.M.F. of the armature is necessarily reduced. In the ordinary cases of practice, owing to the opening out of the pole-tips, the back lines of the back turns added under the pole-faces are considerably more than the weak fringe of the cross field outside the pole-faces which is cut by the back turns usefully. Thus the magnetic effect of the back turns always preponderates over what may be called their electrical effect as cutting the cross flux, and in addition the useful effect from a certain amount of the main symmetrical flux becomes lost. Thus, in the case of a smooth armature carrying current with a forward lead of the brushes, the resultant distribution of the flux can be determined by superposition of the symmetrical cross field on the symmetrical main field as weakened by the back ampère-turns, and the value of the E.M.F. follows from a consideration of the question as to what portions of either flux are cut usefully or harmfully.

§ 9. **Effect of backward trail of the diameter of commutation.**—If in a dynamo the brushes be shifted in the reverse direction and be given an *angle of trail* behind the line of symmetry, the effect of the direct magnetising turns on the armature is also reversed; instead of being back ampère-turns, they now become *forward ampère-turns*, assisting in the magnetisation of the armature. As regards the main symmetrical field, all the turns within the angle of trail are cutting the flux in the wrong direction and their electrical effect is harmful, and so far as the forward ampère-turns cut the cross field they are also generating a back E.M.F. proportional to the number of lines of cross-flux within twice the angle of trail. On the other hand, the forward lines which they add under the pole-faces are more than the lines which are cut in the wrong direction, and as before the magnetic effect preponderates over the electrical effect. Thus in consonance with the principle that the active inductors of the armature cannot increase its E.M.F. when the steady current which they are carrying is increased, it is seen that the conductors which assist in increasing the total flux are really back inductors themselves. It is only their magnetic effect which is useful and which has to be added to the field excitation acting between the poles. Indeed, if the brushes of a dynamo are shifted backwards, the armature may be made to excite its own field without the presence of any field winding proper; the exciting ampère-turns are in fact wound upon the rotating armature instead of upon the stationary magnet. With an initial field present, if the brushes are given a great angle of trail, the loss of E.M.F. from the cutting of the lines of the field within that angle in the wrong direction will eventually outweigh the useful lines which the forward ampère-turns add, but in ordinary cases for a comparatively small movement of the brushes backwards, the voltage added is greater than the back E.M.F. due to the cutting of the fringe lines, so that there is a net gain. There is also the additional advantage that the leakage is reduced for the same value of the armature flux, since the magnetising turns are now wound more uniformly over the magnetic circuit. Unless, however, special devices[1] are used in connection with the commutation, a backward position of the brushes in a dynamo is always accompanied by such excessive sparking that it cannot be used in practice.

§ 10. **Effect of varying permeability of teeth in a slotted armature.**—Returning to the case of a toothed armature with the brushes on the line of symmetry, when current flows through the winding, and distortion commences, the density in the trailing teeth increases and their permeability falls, while the reverse takes place in the leading teeth. If the density of either the main field or of the cross field for an equal width of arc at each end of the pole-face were alike, the new values of the resultant density for any two symmetrical points could be easily calculated. But since in reality (as contrasted with the case of § 8) the change of permeability of the teeth results in values differing from one another at the two corners or at any two symmetrical points, and the one is reduced more than the other is increased, the effect upon either the main field or the cross field considered separately is to concentrate them towards the leading corner where the permeability is higher, and to lessen the density on the trailing side where the permeability is lower. An increased portion of the main flux is, as it were, drawn into the leading corner, while at the same time the neutral line of the main field is displaced backwards; the curves of induction in the air-gap are thus no longer symmetrical about the centre of the pole-face. The cross flux is also displaced in an analogous manner as illustrated in fig. 249; its central point x on its own neutral line xy is shifted backwards, and its minimum point p on the line pq which separates the halves of the flux on the two sides is also displaced to some position behind VL or DC. The effect on either of the

---

[1] As in Sayers' regenerative dynamos, Brit. Pat. Nos. 16,572, 1891 and 10,298, 1893.

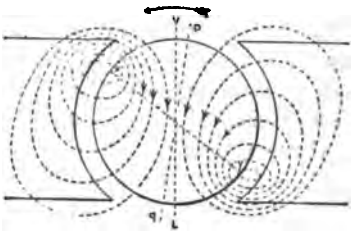


FIG. 249.

two component fluxes is analogous to that which would be produced if the air-gap was progressively shortened towards each leading corner and lengthened towards the trailing corners, so that the polar surfaces and armature were no longer concentric. Although the ordinates of the two curves taken separately are displaced backwards, yet their algebraic sum yields a resultant field which is throughout displaced forwards (fig. 250); for as will be seen later, while the main field is weakened, the cross field is strengthened as the armature current is increased, and their point of equality advances forwards.

§ 11. **Approximate analysis of distribution in a toothed armature.**—In order to estimate the effect of armature reaction upon the total main flux in a toothed armature, a case should be taken in which the teeth are long and highly saturated and vary but little in width; *i.e.*, the armature should have deep slots and be of fairly large diameter. The iron of the magnet and armature core apart from the teeth may be assumed to have a constant permeability with negligible effect as compared with the teeth. Let the armature be 150 cm. in diameter with six poles of 45°, and 220 slots each 4 cm. deep and 1·25 cm. wide. The widths of a tooth at the top and at the root are therefore respectively $k_{t1}=0{\cdot}89$ cm. and $k_{t2}=0{\cdot}77$, giving ratios $\frac{k_t}{k_s}$ of 0·71 and 0·615, or an average of 0·66 nearly. Let the normal density in the air-gap be as before 6400, and let the single air-gap $=0{\cdot}8$ cm. The breadth of the polar arc is 59·1 cm., embracing $27\frac{1}{2}$ teeth, or allowing a slight margin the number of lines corresponding to one tooth and 1 cm. of core length is $z=\frac{59{\cdot}1\times B_g}{28{\cdot}5}=2{\cdot}075B_g$. In the undistorted state, then, $z=2{\cdot}075\times 6400=13{,}250$. Allowing 15 per cent. for the insulation between the discs, $\eta=0{\cdot}85$, and the apparent inductions at the top and root of the teeth are $\frac{13{,}250}{\eta\,.\,k_t}$

= 17,500 and 20,250. The corrected inductions are for the given ratios of tooth and slot $B_1 = 17{,}400$ and $B_2 = 19{,}800$. Thence by eq. (40) of Chap. XV § 14 the magnetic potential lost over the 4 cm. of the teeth is $130 \times 4 = 520$. The equivalent length of the air-gap being a little over $1{\cdot}1 l_g$, say 0·882, the loss of potential over each air-gap is $0{\cdot}882 \times 6400 = 5650$, making a total of $5650 + 520$ on each side, or 12,340 C.G.S. units over the air-gaps and teeth on the two sides.

The surface of the armature may now be divided on each side into a number of compartments each of which carries the same flux; thus in fig. 245 the two compartments of different widths opposite to *a* and *b* are in series with respect to the main flux, while the two compartments opposite to *a* and *d* are in series with respect to the cross flux, and further *b* and *d* are alike. The loss of magnetic potential over the iron and air portions in either case are now dissimilar, and such proportions for the compartments must be chosen that the total flux in each case is a maximum within any pair of corresponding compartments. Assuming certain permeabilities or resultant inductions in the air-gaps, the two component fluxes must be estimated and their values corrected until their resultants at the two points agree with the assumed resultant distribution. Let it be assumed that the resultant density in the air-gap at the trailing corner *d* is $B_g'' = 7250$ and at the leading corner *a* is $B_g' = 4000$. By the same process as before we find that under these circumstances the loss of potential over the trailing teeth for $s = 2{\cdot}075$ $B_g'' = 15{,}000$ and for apparent densities at the top and bottom of the teeth of 19,800 and 23,000, or real densities of 19,500 and 21,500 is $453 \times 4 = 1812$. The reluctance per tooth is therefore $\frac{1812}{15{,}000} = 0{\cdot}121$, or since one tooth corresponds to an average pitch of 2·08 cm. this may be conveniently reduced to its equivalent per sq. cm. of path, viz. $R = 2{\cdot}08 \times 0{\cdot}121 = 0{\cdot}252$. On the leading side the resultant density is so low that practically all the lines pass through the teeth, the apparent or real densities being only 10,950 and 12,700; the average H therefore sinks as low as say 7, the reluctance per tooth being $\frac{7 \times 4}{4000 \times 2{\cdot}075} = 0{\cdot}00338$, or per sq. cm.

$R' = 0\cdot00338 \times 2\cdot08 = 0\cdot007$. The reluctances of air-gap and teeth per sq. cm. on either side are therefore

$$\begin{array}{r} 0\cdot252 \\ 0\cdot882 \\ \hline 1\cdot134 \end{array} \quad \text{and} \quad \begin{array}{r} 0\cdot007 \\ 0\cdot882 \\ \hline 0\cdot889 \end{array}$$

the former being 27½ per cent. higher; and these may be compared with the normal reluctance on one side of

$$\begin{array}{rr} \dfrac{520}{6400 \times 2\cdot075} \times 2\cdot08 = & 0\cdot0816 \\ ml_g = & 0\cdot882 \\ \hline & 0\cdot9636. \end{array}$$

The question then is by how much the strip at the trailing side must be widened in order to give the maximum number of lines or the maximum average density at the two sides in the case of the main field and cross field considered separately.

If $(1+m)$ be the multiplier by which the reluctance, R', per sq. cm. of path on the leading side must be multiplied to give the reluctance on the trailing side, and x be the fraction by which the width of the strip on the trailing side must be increased in order to make the total number of lines passing through the two strips in series a maximum; then the two reluctances in series $= R' + R'\left(\frac{1+m}{1+x}\right)$, and the total flux through the two is

$$\frac{1\cdot257 X_P}{R' + R'\left(\dfrac{1+m}{1+x}\right)} = \frac{1\cdot257 X_P(1+x)}{R'(2+m+x)}.$$

The average width of the two strips is $\frac{1+(1+x)}{2}$, and therefore the average density is

$$\frac{\dfrac{1\cdot257 X_P(1+x)}{R'(2+m+x)}}{\dfrac{2+x}{2}} = \frac{1\cdot257 X_P \,.\, 2(1+x)}{R'(4+4x+2m+xm+x^2)}.$$

Since X_P is a constant and R' and m are assumed to be fixed, the average density becomes a maximum when, with the known value of m, $\frac{x^2+x(m+4)+2m+4}{1+x}$ is a minimum. Equating the differential coefficient to zero, we find that this occurs when $x=\sqrt{1+m}-1$ or $\frac{1+m}{1+x}=\sqrt{1+m}$.

In our case $m=0{\cdot}275$, whence $\frac{1+m}{1+x}=1{\cdot}13$. The total flux through the strip 1 cm. wide on the leading side and 1·13 cm. wide on the trailing side is therefore $\frac{12340}{0{\cdot}889(1+1{\cdot}13)}=\frac{12340}{1{\cdot}894}=6520$ lines with an average density of 6120 and actual densities of 6520 and $\frac{6520}{1{\cdot}13}=5760$. At the trailing side, in order that the resultant density may agree with the value assumed for it, the density of the cross flux in the air-gap must be $7250-5760=1490$, and the total cross lines in the compartments considered $1490\times1{\cdot}13=1685$. The resultant density at the leading side would then be $6520-1685=4835$, which does not agree with the assumed $\text{B}'_g=4000$, so that the two assumed resultant densities are not possible corresponding values. It will, however, be seen that the permeability of the leading side is so high that its reluctance has but little effect on the whole, and an intermediate value of $\text{B}'_g=4750$ may be taken without appreciable alteration. With this change $z=9850$, and the densities at the top and bottom of the teeth on the leading side are 13,000 and 15,050, giving an average $\text{H}=17$, whence $\frac{17\times4}{9850}=0{\cdot}0069$, and $\text{R}'=0{\cdot}0069\times2{\cdot}08=0{\cdot}0143$. The two reluctances are now $(0{\cdot}882+0{\cdot}0143)=0{\cdot}8963$, and 1·134, the latter being 1·265 times the former. The coefficient $\sqrt{1+m}$ is now slightly reduced to 1·126, and the total main flux of the strip is also reduced to 6460, while the cross flux is $1515\times1{\cdot}126=1710$, giving a resultant density at the leading side of $\text{B}'_g=6460-1710=4750$. Thus two corresponding values of the density have been found, and their distance apart must be such that the cross M.M.F. between them is $1710\,(0{\cdot}8963+1{\cdot}13\times0{\cdot}8963)=1710\times1{\cdot}91=3270$ C.G.S. units.

With a larger amount of distortion and a greater resultant density in the air-gap at the trailing side, it is evident that the reluctance per sq. cm. on the leading side approximates more and more closely to that of the air-gap alone, or to 0·882, so that it becomes almost unnecessary to calculate it. Thus let the density on the trailing side in the air-gap be $B''_g = 8250$, corresponding to $B = 17{,}100$, to apparent teeth inductions of 22,600 and 26,100, and to real inductions of 21,450 and 22,800. The average H is then 1160, and the reluctance per. sq. cm. of path over the teeth is $\frac{1160 \times 4}{17100} \times 2{\cdot}08 = 0{\cdot}565$. The total reluctance on the trailing side is therefore $(0{\cdot}882 + 0{\cdot}565) = 1{\cdot}447$, or 1·635 times R', which may be assumed as $= 0{\cdot}885$. The maximum number of lines is reached with $\sqrt{1 + m} = 1{\cdot}28$, when the total reluctance on the two sides is $0{\cdot}885\,(1 + 1{\cdot}28) = 2{\cdot}02$; the flux in the main field is then 6115, giving a density on the trailing side of 4780, and an average density of 5360. The density of the cross flux on the trailing side will be $8250 - 4780 = 3470$, and its total amount $3470 \times 1{\cdot}28 = 4440$. When concentrated in one cm. width on the leading side the resultant density will be $B'_g = 6115 - 4440 = 1675$. Two more corresponding values of the densities in the air-gap have thus been found, and their distance apart must be such that the cross M.M.F. between the points is $4440 \times 2{\cdot}02 = 8960$. The resultant induction at the leading pole-tip is somewhat lower than that which was before found for the smooth-core armature, and it may be taken as approximating to the minimum value in a practical case. The total cross M.M.F. of the armature corresponding to the pole-pitch would therefore be $\frac{8960}{0{\cdot}75} = 12{,}000$, given by 9550 ampère-turns, or ampère-wires within the pole-pitch.

§ 12. **Curve of component and resultant inductions in air-gap of toothed armature.**—A series of corresponding densities with their distances apart may thus be approximately calculated, and when plotted enable a rough estimate to be made as to the nature and amount of the reduction of the total flux due to the armature reaction. As the centre of the pole is approached the difference in the widths of the corresponding compartments decreases, and the average density of the main flux rises; the value on the leading side may exceed the initial value of $B_g = 6400$, but the average of any two corresponding bundles is always less than the initial although it may be to a very slight extent. The centre of the two component fields where the cross flux is zero and the main field has its initial value is shifted slightly towards the leading side, and its location may be determined by remembering that the area of the cross flux on the one side XB′A′q (fig. 250) must be equal to the area XBCV plus

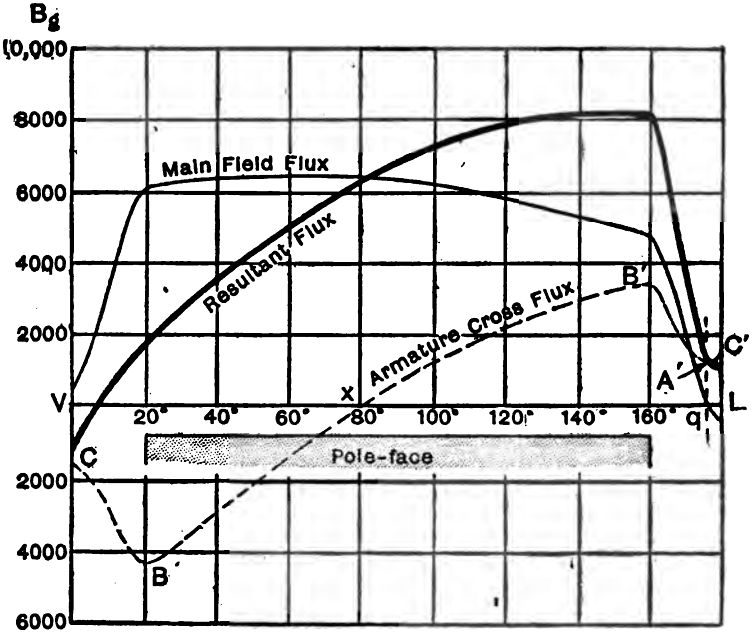


FIG. 250.—Flux distribution in toothed armature.

the area, qA′C′L, the latter corresponding to the small number of lines within the angle q L in fig. 249.

Under the pole-pieces the maximum flux in each of the two component fields was obtained by increasing the width of the compartments on the trailing side. If the same process is extended to the region beyond the pole-faces, the length of the air-path of the trailing fringe is increased more than its area is increased; in consequence an increasing air reluctance is thrown into series with the teeth of higher reluctance, and the required effect is not produced. When therefore the air-length is altered, a different procedure must be adopted. The maximum flux will now be obtained by increasing the length of path in the air on the leading side where the teeth are assumed to have the higher permeability, and shortening the length of the air-path on the trailing side where the teeth have the higher reluctance. Under the poles the loss of magnetic potential over the air-gap on the leading side is less than that over the air-gap on the trailing side, and continuously decreases as the leading pole-corner is approached; after passing the corner the difference is quickly reduced, and they become equal at or near the symmetrical line. The exact position of the neutral line of the component field it is not easy to predict, as it will depend upon the density and permeability of the teeth at various points; *e.g.*, on the resultant neutral line where the density in the teeth is practically zero the reluctance of a square centimetre of the path may well be higher than in a corresponding portion on the trailing side where the resultant density is much higher, since the permeability will increase with the higher density. There is, however, the tendency which has been described above for the lines on the leading side to extend backwards beyond the symmetrical line, and for those on the trailing side to be actually concentrated so that the higher reluctance of the teeth may find itself matched with a shorter length of path in the air. The point where the lines of the component fields change their direction is given when the loss of potential over the two sides becomes equal again for such lines as just dip into the teeth, the trailing half of the tooth being of slightly higher reluctance than the leading half. Thus the neutral line of the main field which also divides the cross flux into its two halves is displaced backwards. Now when the main and cross fields are unsymmetrically distributed, as in figs. 249 and 250, the displacement backwards of the line *pq* really implies the presence of back ampère-turns upon the armature, and the case becomes the equivalent of fig. 246. Although the brushes have not been moved the component fields have been shifted backwards, so that the brushes in effect have an angle of lead ahead of the neutral line of the main field. Considering one half of the armature as before, the flux of the main field within *q*L is now cut in the wrong direction and since it gives a back E.M.F. is harmful. On the other hand, since the lines of the cross flux within the arc *xq* which give a forward E.M.F. are greater than the number within the arc *x*V which give a back E.M.F. by the amount within *q*L and the lines within the arc *q*L are themselves cut usefully, the forward E.M.F. from the cross flux is proportional to twice the number of lines within *q*L, or twice the angle of trail of the field or of virtual lead of the brushes. But just as in the smooth armature, the magnetic effect of the back ampère-turns on the main field outweighs their electrical effect, and on the whole there is a reduction of the E.M.F., even apart from the loss of useful effect from a portion of the fringe of the main field. The question of the E.M.F. of the toothed armature when carrying current as compared with that on open circuit for the same excitation, thus depends in the first place on the value of the total flux due solely to the field winding as acting upon the altered permeances of the circuit and as weakened by the back ampère-turns, and in the second place, upon the relative amounts of the loss of useful effect from the lines of the main field between *q*L and the gain from the cross lines within twice *q*L. With the brushes on VL the difference between the smooth and toothed armatures arises solely from the change of permeability being considerable in the latter and from its consequent effect of making the two component curves unsymmetrical with respect to the line of symmetry. If the armature current of the toothed armature be gradually increased while the brushes are retained on the line of symmetry, there is a progressive reduction of the induced E.M.F. due partly to a reduction of the total flux of the main field and partly to a progressive displacement of the neutral line of the main field backwards. In order to put the toothed armature on the same footing as the smooth armature, the brushes would have to be shifted backwards in order to overtake the neutral line, but so far from this being possible, the brushes must be shifted forwards in order to overtake the resultant neutral line which is itself displaced forwards; the back ampère-turns are thus again increased and a further reduction of the E.M.F. ensues.

§ 13. **Amount of reduction of flux and of E.M.F.**— In the case treated above when plotted, as in fig. 250, the reduction in the total value of the main flux for a constant interpolar excitation amounts to about 8 per cent. The assumption of a constant loss of potential over the teeth and air-gaps does not, however, strictly represent the actual facts in practice. It is the total excitation that would really be maintained constant in such a case, and if the main flux were reduced by some 8 per cent. the reduced loss of potential over the magnet core and yoke would set free probably as much as 1000 additional ampère-turns to be expended between the poles. Against this increase in the air-gap ampère-turns must be set the fact that there may be a certain small number of back ampère-turns arising from the displacement backwards of the neutral line of the main field. Still after deducting these, there will still remain an increase in the ampère turns available for the air-gaps and teeth, amounting, say, to 400 AT. or 500 units of M.M.F. which will assist in maintaining the initial average density and total flux. Such a reduction, therefore, as 8 per cent. probably represents the utmost that may be anticipated in a fully loaded and saturated armature, since the case considered was an extreme one in so far that the teeth were assumed to have uniform density over their whole length.

The electrical loss of E.M.F. from a certain portion of the main fringe being cut in a harmful direction after deduction of the forward E.M.F. due to cutting a portion of the cross flux in a useful direction is probably but small, and in fig. 250 but little alteration in the forward lead of the resultant neutral line is shown as compared with the smooth armature.

While the resultant density at the leading pole-corner has been retained at approximately the same value in the two cases of the smooth and toothed armature, it will be seen that the permissible cross ampère-wires under a pole in the latter case exceed those of the former case by some 20 per cent., and this increase is practically proportionate to the increase in the ampère-turns required over the air-gap and teeth. For reasons which will be more apparent from the next chapter, it is advisable that the resultant B'_g at the leading pole-tip should not fall below a certain minimum. By the use of carbon brushes in the toothed machine, this minimum may be somewhat reduced, but in any case the induction at the leading pole-tip should not be reversed. (See Chap. XVIII, § 19.)

§ 14. **Back ampère-turns of smooth and toothed armatures.**—In the case of smooth-surface armatures which necessarily have comparatively long air-gaps, it has been shown that the reduction of the total flux even with maximum distortion of the field is practically negligible. In the calculation then of the ampère-turns required on the field it is only necessary to introduce in the third place the value x_b as explained in Chap. XV § 4. The correctness of the method is, however, limited to the ordinary cases of practice in which the brushes are seldom advanced further than up to the pole-corner; under such circumstances the ampère-turns within twice the angle of lead so nearly embrace the entire magnetic circuit through the armature, that they may be allowed their full weight, and must be counterbalanced by an equal number of forward ampère-turns on the field, $x_b = c\tau \cdot \frac{2\lambda}{360^\circ}$. If, however, the brushes are advanced beyond the pole-edge and the lead becomes abnormal, some allowance must be made for the fact that the back ampère-turns are not all of equal weight, and for x_b must be substituted a lower value, such as $c \cdot \tau \cdot \frac{2\lambda}{360^\circ} \cdot \cos\lambda$. With the symmetrical distribution of the component fluxes that holds in smooth armatures, any angle of advance of the diameter of commutation implies a reduction of the E.M.F. proportional to the number of lines of the main field that is embraced between the two lines of commutation and of symmetry. When as usual the angle of lead does not extend beyond the pole-corner, only the symmetrical fringe from the pole-edge is lost, and this is so small a proportion that its effect is in most cases nearly inappreciable. But with a very large angle of lead the proportion of the inductors which give a back E.M.F. due to cutting the lines of the symmetrical main field in the wrong direction quickly increases; even if the magnet be separately excited and the value of the symmetrical field be maintained against the back ampère-turns, the net E.M.F. diminishes until with the brushes under the centre of the pole it is reduced to zero.

In the case of a toothed armature when the brushes are retained on the line of symmetry even with a large current, x_b is apparently zero, but as explained in § 13, there is a reduction of the armature E.M.F. for the same excitation due to a reduction of the total main flux and also to a displacement backwards of its own neutral line pq, which gives a virtual angle of lead to the brushes. Correspondingly, when there is some actual angle of lead, the x_b deduced from its value does not express the full effect of the armature reaction in decreasing the E.M.F. When the effect of leakage is assumed to be constant, the horizontal distance between the no-current and full-current curves of flux may be expressed as a certain fraction of the armature ampère-turns, $= k.\text{AT}_a$, as in Chap XVI § 8. In the toothed machine, when compound-wound, since the cumulative effects of armature reaction increase faster than in proportion to the current, it is more difficult to secure an approximately straight-line rise of voltage than in the equivalent smooth-core machine.

Both in the smooth and toothed armatures it must also be borne in mind that any eddy-currents in the armature exert a demagnetising effect on the field exactly analogous to that of the actual back ampère-turns. In the toothed armature if the slots are wide, there is a further liability to eddy-currents in the pole-pieces, and these again must be counterbalanced by an increase[1] which must be added to the item x_b in reckoning out the field-winding.

For taking into account the whole effect of armature reaction in determining the field-winding, various methods have been proposed, mostly based on the composition of magneto-motive forces by a parallelogram law. They can, however, only be regarded at best as approximations to the actual facts, and in practice require so much correction that they are of but little use under different circumstances. An experimental method has been given by Prof. Arnold,[2] which may here be shortly discussed.

Let the brushes be placed in the position which it is desired that they should occupy at full load,—in most cases this will be the position which previous trial has shown to give sparklessness at full load—, let the armature be short-circuited and run at its normal speed, with the field separately excited by a weak current until the resulting current has its full normal value, or as much in excess of this as is considered safe. The apparent flux is then $\propto C_a R_a$. From the no-current flux-curve or from calculation the ampère-turns AT_o required to give the flux corresponding to the loss of volts over the resistance of the armature at the given load are known. It will now be found that the ampère-turns with which the field has had to be excited in order to give the short-circuit current or AT'_f are greater than AT_o. The difference is due partly to the presence of back ampère-turns on the armature if the brushes have been given a forward lead, and their effect upon the leakage, partly to any reduction of the total flux through distortion in a toothed armature and partly also to any back E.M.F. in certain inductors if the diameter of commutation is behind the neutral line of the main field. All these reactive effects may be summed up in an apparent number of back ampère-turns which are the difference between AT'_f and AT_o, or $k\, AT_a = AT'_f - AT_o$. If the sum of these effects remained constant when the field instead of being weakly excited is fully saturated, it would be legitimate to deduct the same number of apparent back ampère-turns from the field excitation in order to find the resultant turns to which the useful flux and induced E.M.F. correspond (cp. Chap. XVI §§ 8, 14). The mutual proportions of the effects certainly will vary when the condition of the field is thus altered, *e.g.*, the reduction of the main flux from distortion will be greater when the teeth of the toothed armature are more fully saturated, and the effect of the leakage will also be greater, while on the other hand the displacement of the resultant neutral line with a weak initial field will be reduced when the main field is increased. The method is not therefore free from certain objections, yet in practice it yields fairly accurate results.

---

[1] See Niethammer, *E.T.Z.* 1899, p. 768.

[2] *E.T.Z.*, Vol. xvii. p. 775.

A curve immediately relating the useful flux to the field ampère-turns with full current in the armature is, except in very large machines, as easily taken as the short-circuit characteristic which has been described above. But the method of Prof.

Arnold has an additional use. If two co-ordinate values are taken for the ampère-turns and volts at any point on the straight-line portion of the no-load characteristic, their quotient $\frac{\text{AT}}{\text{V}}$ gives the ampère-turns required to give 1 volt so long as the two are proportional to one another. The value of AT_0 when the field is far from saturation (as it is during the short-circuit readings) is therefore $\text{C}_a \text{ R}_a \left(\frac{\text{AT}}{\text{V}}\right)$. The armature ampère-turns per pair of poles corresponding to one magnetic circuit are $\frac{c\tau}{2p}$. Only a certain proportion of these are fully effective as back turns, so that they must be multiplied by some factor k less than unity to give the apparent number of back ampère-turns. Thus

$$k \, . \, \text{AT}_a = k \, . \, \frac{c\tau}{2p} = \text{AT}_f' - \text{C}_a \text{ R}_a \left(\frac{\text{AT}}{\text{V}}\right).$$

If a range of readings are taken for the short-circuit current increasing from zero to, say, $1\frac{1}{2}$ times the full current, the value of k is found to be nearly constant, and its value is at once given by the ratio

$$\frac{\text{AT}_f' - \text{C}_a \text{ R}_a \left(\frac{\text{AT}}{\text{V}}\right)}{\frac{c\tau}{2p}}.$$

The value thus found can then be used for any other machine similarly proportioned, and the back ampère-turns for any armature winding and current determined. In the machine experimented on by Prof. Arnold, $k = 0{\cdot}41$, and in a test made by the writer $k = 0{\cdot}385$.

A difficulty in the experimental determination of the short-circuit characteristic is found in the variable effect of residual magnetism. This and the previously mentioned objection that the field during the experiments is very weakly excited may to a certain extent be removed by running the machine at low speeds and so enabling a stronger excitation to be used without the short-circuit current becoming excessive. The ampère-turns AT_0 required to give the volts lost over the armature and brush resistance must then be calculated or read off from a no-current flux curve since the two will no longer be strictly proportional, and a better approximation is made to the actual condition of the field when in normal work.

CHAPTER XVIII

COMMUTATION AND SPARKING AT THE BRUSHES

§ 1. **Sparking at the brushes.**—All who have had practical experience of the working of dynamos will know that in bi- or multi-polar machines giving a continuous current, the brushes by which the current is collected are so mounted as to permit of their being shifted round the cylindrical surface of the commutator at least through some small angle. They will also be aware that if the position of the tips of the brushes, as they press on the commutator, be not properly adjusted, the result will be *sparking at the brushes*. The nature of these sparks is similar to the sparking which occurs on breaking circuit at a switch. As the edge of the blade leaves the jaws of the switch and their surface of contact diminishes, the current-density progressively rises; the local heat developed at the vanishing contact eventually raises the temperature of the metal to such a point that at the moment of rupture it becomes incandescent and is volatilised with the consequent formation of an arc in which the stored energy of the circuit is expended. In the same way sparking between the brushes and commutator sectors has its origin in an excessive current-density which continually recurs at their point of contact; and with the more or less sudden opening of the closed path in which a current is flowing, this culminates in a sucession of small arcs between the moving sectors and the stationary brush. As regards the detrimental effect of such sparking, a certain difference must be drawn between the cases of closed-coil and open-coil armatures: its presence, however, always implies a waste of energy, and tends to shorten the life of the commutator, so that, even where least injurious, its suppression, so far as possible, is in every way desirable. The presence or possibility of sparking at the brushes is, in truth, the peculiar bane of continuous-current bi- or multi-polar dynamos, as contrasted with alternators; the current of the latter may require to be collected by brushes or rubbing-contacts such as have been shown in many previous diagrams; but the nicety of adjustment required by the brushes of bi- or multi-polar continuous-current dynamos if sparking is to be minimised or entirely avoided, is a disagreeable characteristic of their whole class, and is entirely due to the presence of the commutator as opposed to the simple collecting rings of the alternator. The requirement that the brushes of the continuous-current dynamo should be properly adjusted by no means expresses the full extent of the disadvantage; more than this, the exact position which the tips of the brushes should have on the commutator in general depends on the amount of the current or load on the armature, and hence their setting may require to be varied to meet changes of the load. If the load on a dynamo were never altered, the position of the brushes might be accurately adjusted once and for all, and in that position they might be fixed; but the conditions of ordinary working are by no means those of constant load, and consequently it must be possible to shift the position of the brush-tips.

Leaving for the present the consideration of sparking in dynamos with open-coil armatures, let us first deal with the case of closed-coil armatures. In all such machines, if the position of the brushes be incorrect, a row of sparks will appear, leaping across between the commutator and the tips of the brushes. These sparks may be small, bluish-white in colour, and comparatively harmless, or if the inexactness of the adjustment be considerable, they may be of a reddish colour and extremely violent. But in either case, if allowed to continue, they will sooner or later pit the surface of the commutator sectors, destroy their smoothness and evenness, and heat the brushes. Once started, the effects are cumulative, and the mischief grows apace: the commutator becomes untrue and worn into deep and rugged grooves; increased sparking is caused by the 'jumping' of the brushes as they pass from sector to sector, and perhaps the tips of the brushes become partially fused: thus the commutator is gradually eaten away until its state is past all remedy. To check the evil, it is necessary to continually trim the brush-tips and 'true up' the surface of the commutator by turning it in a lathe or by grinding it with an emery wheel, and these often-repeated processes result in a greatly reduced life of both commutator and brushes.

§ 2. **Means for adjustment of the brush position.**—Sparking in closed-coil armatures is therefore an unmitigated evil, to be overcome almost at any cost, and the possibility of moving the brushes so that the position of their tips may be adjusted to suit the load on the armature becomes a first necessity. It should further be possible to do this easily and steadily while the machine is running and without in any way interrupting the passage of the current. In the two-pole dynamo this is effected by mounting the brushes on two arms projecting horizontally from a cast-iron 'rocking bar' or lever. This latter is made in two pieces, bolted together so as to fit into a groove turned in the surface of the bearing next to the commutator; it is then locked in position either by a set screw or more usually by a pinching screw which clamps the two halves together and causes them to closely embrace the turned portion of the bearing. From opposite ends of the rocking lever there project two gun-metal spindles, entirely insulated from the iron by means of ebonite or ambroin bushes, but rigidly held in position by means of nut and collar or by insulated screws: the brush boxes which hold the brushes are threaded on the spindles and firmly fixed in place. Fig. 251 shows the rocking-bar mounted in position on the bearing and carrying two sets of copper gauze brushes. Each brush box is fitted with a spring, by which the brushes are kept pressed down

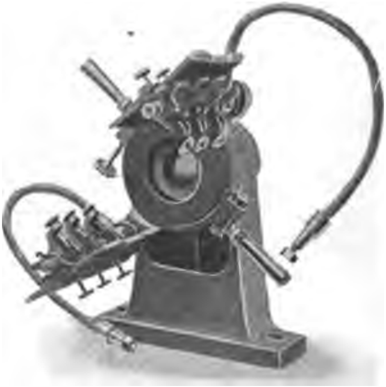


FIG. 251.—Two-pole brush gear.

on the commutator, and also with a 'hold-off' catch, by which the brushes are held when their tips are raised off the commutator surface. When the pinching screw of the rocker is slightly loosened,

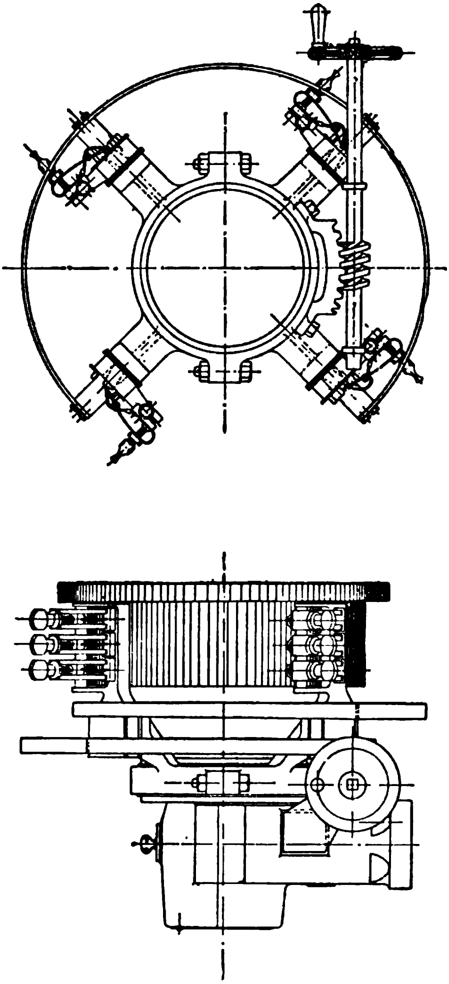


Fig. 252.—Four-pole brush gear.

the whole can be swung round concentrically with the cylindrical commutator by means of a handle or handles on the lever.

In multipolar machines, using as many sets of brushes as there are poles, the rocker takes the more complicated shape of a star frame, on the projecting rays of which the insulated brush spindles are fixed, and it is then usual for the shifting to be effected mechanically by means of a worm gearing into worm-wheel teeth on a sector of the star. Figs. 252 and 253 show the brush gear of a 4-pole dynamo in place on the plummer-block, with four sets of carbon brushes, opposite pairs being connected together by copper rings. In large multipolar machines, a cast-iron ring is frequently mounted on the face of the magnet yoke-ring and rotated within the circular groove which forms its seating by worm gearing and hand-wheel. From this ring project as many arms as there are poles, and to these are attached the brush spindles (cp. fig. 373).

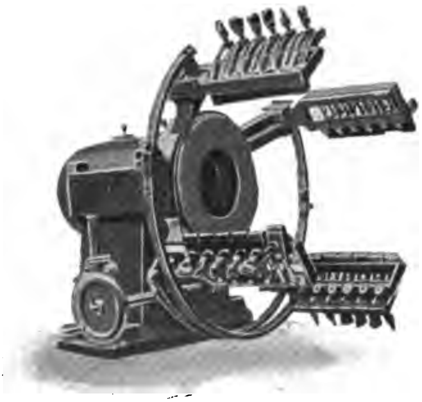


FIG. 253.—Four-pole brush gear.

§ 3. **The process of short-circuiting a section.**—The exact nature of the process by which a section of the winding is *short-circuited* in a closed-coil armature has hitherto only been generally described. It has been shown (Chap. XI § 2) that the brushes must be placed so that they sum up the E.M.F.'s generated in the sections forming one of the two or more parallels into which the winding is divided, and at the same time so that they short-circuit each separate section when it is passing approximately through the neutral gap between the poles, and has therefore little or no E.M.F. generated in it. Hence, if the field be bipolar, their position will be at opposite ends of a diameter, corresponding roughly with a position of the short-circuited coils between the poles or on a line of symmetry at right angles to the general direction of the field. This preliminary description now requires to be further amplified.

If the reader refers to diagrams fig. 106 or fig. 113, it will be evident that in any closed-coil armature, whether ring or drum-wound, in a bipolar or a multipolar field, the exact position relatively to the poles which the loops or coils of the winding occupy, at the time when they are short-circuited depends upon the position of the tips of the brushes as they rest upon the commutator, and that this position of the short-circuited coil can be altered if the brushes are shifted backwards or forwards round the the circle of the commutator sectors.

Considering any one section of the armature winding (whether a single loop, or a coil of many loops), terminated by connection to a commutator sector at either end, let us call that sector which first enters under the edge of the brush the 'leading' sector of the coil, just as that edge or corner of a pole-piece under which a coil first enters after passing through the gap between two pole-pieces has already been called the 'leading' edge, these being opposed respectively to the 'trailing' sector and the 'trailing' edge or corner.

Let the simplest case be taken in which the width of contact of the brushes on the circumference of the commutator is equal to the width of a sector, so that there is never more than one section short-circuited at a time at each set of brushes. Then at any time during short-circuit if i be the instantaneous value of the current in the short-circuited coil, and J be the full current then flowing in any one branch of the armature $\left(=\frac{C_a}{2}\right.$ or $\frac{C_a}{2p}$ according as the armature winding is series- or parallel-connected or in general $\left.=\frac{C_a}{q}\right)$, the current in the leading sector 1 (fig. 254) is $i_1 = J + i$, and in the trailing sector 2 is $i_2 = -J + i$, the direction of the current before commutation being taken as the positive direction round the short-circuit, and due regard being paid to the algebraic sign of i. Thus at the commencement of short-circuit, $i = J$, $i_1 = 2J$ and $i_2 = 0$. The total period of short-circuit, being the time that elapses between entrance of the trailing sector under the brush and emergence of the leading sector on the other side, is directly proportional to the thickness of the brushes less one strip of mica and inversely proportional to the peripheral speed of the commutator; or in seconds, $T = \frac{(b - m) \times 60}{\pi d N}$, where b = the width of the brush contact in the direction of rotation, m = the thickness of a mica strip, d = the diameter of the commutator, all expressed in the same units, and N = the number of revolutions per min. It is usually but a small fraction of a second; *e.g.*, with copper gauze brushes ¼″ thick and giving about ⅜″ width

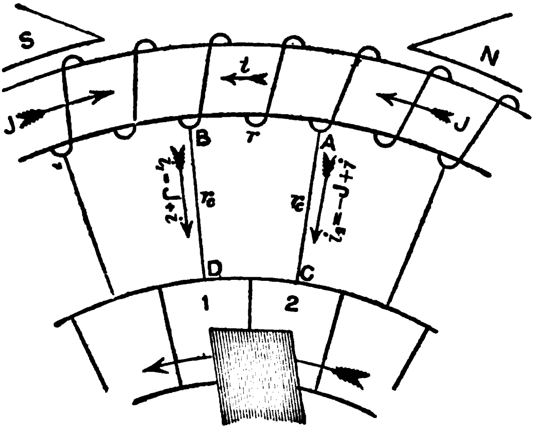


FIG. 254.—Commutation of current.

of contact on the circumference of a commutator 7½″ in diameter and running at 900 revs. per min., $T = \frac{0{\cdot}375 \times 60}{3{\cdot}14 \times 7\frac{1}{2} \times 900} = 0{\cdot}00106$ second. During this short time the current i must sink to zero and rise again in the reverse direction, *i.e.*, with changed sign. If the reversed current be raised to exactly the same value at the end of short-circuit as it had at the beginning, $i = -J$, $i_1 = 0$, and $i_2 = -2J$. The opening of the short-circuit will then find the coil carrying the exact current flowing in the coils of the branch of the armature which it is to join, the whole of the current originally carried by BD and the leading sector having been withdrawn therefrom. Thus in a bipolar machine with 100 sections in two parallels over the resistance of which 5 volts are expended, a net effective voltage of $\frac{1}{10}$th of a volt must be acting on the short-circuited coil as it leaves the brush, in order to correspond with the passage through it of the normal current. The commutation will then be effected without any violent change, and consequently without any sparking. It is evident that the transference of a section of the armature from one branch of the winding into another on the leading side of the brush can only be sparkless if the original current in it is stopped, reversed, and further is reversed to exactly the same value in the opposite direction within the period of short-circuit.

§ 4. **Apparent inductance of short-circuited section.**—Now the current-turns of the short-circuited coil since they surround a portion of the magnetic circuit react on the field system, so that the number or distribution of the lines of the resultant field is different from what it would be if the short-circuited coils were absent. For the consideration of the problem of commutation, then, the coils which are at any time under the brushes must be mentally isolated from the rest of the winding, and their magnetic effect on the system of armature and field-magnet may be accounted for by crediting them with a certain flux of their own. Since the coils are situated at or near to the line of symmetry between the poles, most of the lines of this self-induced flux cross the double air-gap and enter or leave the iron pole-faces; and thence pass onwards through the field-magnet bobbins to complete their circuit. The field-magnet bobbins are practically short-circuited by the armature winding, and as the double air-gap is the chief item in the magnetic reluctance, the system is so far roughly equivalent to a transformer with an air-core and a short-circuited secondary. Under these circumstances for rapid changes of current comparable with the frequency of commutation in a dynamo, the effect of the secondary exactly counterbalances the action of the short-circuited coil which is the primary, and the latter apparently has no self-inductance. Any change in the total flux linked with the exciting coils and the armature as a whole is in fact damped by the great mutual inductance existing between the short-circuited coil regarded as a primary and the field-coils as a secondary. There is, however, a certain smaller number of self-induced lines which do not pass through the field-coils, but which circle round through the air between the pole-tips. In virtue of these lines a certain "*apparent*" *self-inductance* may be attributed to the short-circuited coils, and the value of this is very much less than their true self-inductance.[1] The apparent inductance will vary with the position of the short-circuited coil, according as it is in the centre of the interpolar gap or nearer to one pole-tip than to the other, and thus will change somewhat during rotation. But as the arc through which the coil moves during the period of short-circuit is but small, the apparent self-inductance may approximately be regarded as constant[2] for a given position of the brushes and as having a certain value, L. Thus on arriving at the point when short-circuit begins, the coil is possessed of a certain amount of stored energy $= \frac{\text{L} \cdot \text{J}^2}{2}$, corresponding to its apparent self-inductance.

§ 5. **The equation of the short circuit.**—As the current J sinks to zero, this stored energy is liberated, and is again re-stored when the current is raised to the same value in the reverse direction, *i.e.*, to $-\text{J}$. The reappearance of this stored energy in an electrical form or its re-storage gives rise to a self-induced E.M.F. $-\text{L}\frac{di}{dt}$, and the direction of this E.M.F. will be throughout positive if the current i is simply falling to zero and then rising to value $-\text{J}$ without over-reversal; for then $\frac{di}{dt}$ is throughout negative. At the same time the coil is moving through an external field due to the fringe of lines within the interpolar gap; the value of the E.M.F. due to this movement will vary as short-circuit proceeds, and may be expressed as a function of the time, $= f(t)$. To complete the expression for the conditions during short-circuit, there ever their position and whatever the value of $f(t)$; since before the toe of the brush entirely leaves sector 1, the contact resistance between the two, and therefore also the resistance of the short-circuit as a whole becomes infinite. There is, however, a further condition to be considered, and this is the rate of change of the current i at the final moment. If this be infinitely great or $\left(\frac{di}{dt}\right)_{t=T} = \infty$, the current-density in the brush tip or heel is also infinite, and the physical result in nature must inevitably be sparking. Analysis of the mathematical equation[1] shows that the rate of change at the last moment is always (except in one special case which cannot be counted upon in practice) infinite if $\frac{R_1 T}{L}$ is $\leqq 1$. An indispensable condition then for sparkless running is that $\frac{R_1 T}{L}$ should be greater than unity. If the brush contact-surface be so great or the specific contact-resistance be so low that this condition cannot be fulfilled with a given winding and speed of armature, then either sparking takes place or the current-density towards the end of the time of short-circuit rises to such a high value that the brush-tip is momentarily overheated; in the latter case with copper brushes the resistance rises, so that in effect $\frac{R_1 T}{L}$ does become > 1. The two possibilities are in reality the same phenomenon expressed in different words, and produce the same damaging effect on the commutator. As a matter of fact with copper brushes, $\frac{R_1 T}{L}$ is frequently less than 1, and reliance must then be placed upon the rapid rise of resistance of the brush contact when even a small amount of sparking which is well nigh harmless takes place. In nearly every case of incorrect adjustment of the diameter of commutation, if the effect of the sparking on the commutator be carefully examined, it will be found that it is the trailing edges of the sectors which are first pitted and worn by the sparks; this shows that it is caused by the short-circuit current in the connector B D maintaining its path by small arcs between that edge of a leading sector which leaves the brush last and the brush itself.

---

[1] H. N. Allen, "Sparkless Reversal in Dynamos," *Journal Inst. Elect. Eng.* vol. xxvii. p. 209.

[2] Since the percentage of the lines which do not pass through the field-magnet bobbins is greater when the coil is moved away from the symmetrical line, the apparent inductance increases, but in Mr. Allen's experiments, the total increase when the coil was brought just up to the pole-tip was not more than about 15 per cent. and this would be less in a toothed armature.

---

[1] See the valuable paper by Prof. Arnold and Dr. G. Mie (*E.T.Z.*, Feb. 2, 16 and 23, 1899) on which the present sections are mainly based, and the results of which are further explained and illustrated in a pamphlet on *The Theory of Commutation* (Electrician Publishing Co.) by C. C. Hawkins.

It is not, however, enough merely that the final rate of change of the current and the density at the last moment should be finite. At no time throughout the whole process of commutation can the current-density in either the toe or heel of the brush be allowed to reach such a high value as to melt off particles of the brush or of the sectors under them. It is owing to this necessity of avoiding the effect of an abnormal current density in overheating some portion of the brush surface that the value of $f(t)$ and the position of the brushes become of material importance.

§ 8. **Need of a reversing field to reduce current-density of brush-tip.**—The effect of altering the position of the brushes on the external impressed E.M.F. has now therefore to be considered. First suppose that in fig. 254 the brushes are shifted slightly backwards away from the line of symmetry or against the direction of rotation. The loops at the commencement of their short-circuit will then be moving through the fringe of the trailing pole-tip; hence an E.M.F. is set up in them directed similarly to that of the coils from which they have just parted company, *i.e.*, in the original direction of the current prior to passage under the brushes. The initial value of the impressed E.M.F. $f(t)$ or E_0, is thus positive, and, as rotation continues, it falls and eventually, if short-circuit lasts long enough, will be reversed. If, however, the angle of trail of the brushes and the initial E_0 be large, it may even raise the ampères which the coil is carrying above the normal current being carried by the other armature coils. In any case it will largely increase the difficulty of commutation by causing an excessive current-density in the leading portion of the brush, and if short-circuit ends with the coil still behind the neutral line of zero field, the whole work of reversal will be thrown on to the electrical action of the brush contact-resistance. The leading edge of the brush, whatever its material, is then sure to be overheated, and sparking ensues between the trailing edge of the sector which has just emerged from under the brush and the tip of the brush itself. Thus the consequence if short-circuit ends while the coil is still moving on the trailing side of the line of symmetry is destructive sparking, so that any angle of trail is under ordinary conditions quite inadmissible.

Now the lines of the field entering into or leaving the armature core shade off more or less gradually as we pass out from beneath a pole-piece into the gap which separates it from a neighbouring pole-piece. The point at which the lines just dip into the core and immediately leave it determines the resultant neutral line, on passing which the direction of the impressed E.M.F. changes. Thus the fall and rise of the external E.M.F. during short-circuit may be expressed as $f(t) = \mathrm{E}_0 - \mathrm{H}t^x$, where E_0 is the initial value and is reckoned as positive or negative according as short-circuit begins behind or ahead of the neutral line. The power x to which t should be raised varies in most machines between the first and second powers, and is not strictly constant over any great part of the interpolar region. As, however, the time of short-circuit only coincides with a small angle of movement, the E.M.F. due to the external field may approximately be assumed to be a straight-line function of the time for any one position of the brushes, or $f(t) = \mathrm{E}_0 - \mathrm{H}t$, the values of E_0 and H being such as to suit the exact portion of the curve of impressed E.M.F. which corresponds to the position of the coils during short-circuit. If now the brushes are brought forward into such a position that they have neither lead nor trail, and the diameter of commutation coincides with the line of symmetry, they may be taken as coinciding with the neutral line at no load. When, however, current is taken out of the armature, then as explained in Chap. XVII § 4 the armature ampère-turns react upon the external field, and the resultant neutral line is displaced forwards or away from the line of symmetry in the direction of rotation.

On a line midway between the two poles in a smooth armature, the magnetic difference of potential between one pole and the armature core tending to make lines enter the armature is balanced by the magnetic potential of the other pole tending to cause lines to leave the core, so that the symmetrical flux is there reduced to zero. There still remains, however, the cross induction due to the total cross ampère-turns of the armature acting over a path of length $2(l_g + \xi c)$, and this has the same sign as the induction under the trailing pole-tip. The resultant neutral line is therefore shifted forwards, and, in order to overtake this, the brushes must be shifted forwards.

§ 9. **Angle of lead necessary to overtake the neutral line.**—Only, therefore, when the armature current is very small can its reaction on the field be neglected, and a position of the brushes on the symmetrical line be taken as coinciding with the neutral line. If the brushes are given no lead, then, with any appreciable amount of armature current, $f(t)$ may remain positive throughout the whole period of short-circuit owing to the displacement of the field forwards, and commutation is still dependent upon the action of the brush resistance. The corrective action of the varying brush surface in checking any divergence from a straight-line change of the current is increased by making $\frac{R_1T}{L}$ high, and this may be attained equally well by adopting a high normal current-density or by employing a material for the brushes having a high specific contact-resistance such as carbon. The two methods are simultaneously applicable, and how far they can be so applied in practice will be discussed later. If, however, they are to cope with the reversal of large armature currents, they must in general be supplemented by the additional precaution of giving the brushes a *forward lead* in the direction of rotation, and so shifting the diameter of commutation that it approaches, overtakes or passes the neutral line. The short-circuit is then not opened until $f(t)$ has become negative and there is a reversing E.M.F. acting on the coil and assisting in the production of a current in it, the same in direction as that which the coil will be called upon to carry as soon as it emerges from under the brush. Our case is now illustrated by fig. 113 or 114 where the coils short-circuited by either brush are slightly in advance of the vertical diameter, and are assumed to be just moving in the fringe of lines from the leading pole-edge when short-circuit ends. Although the E.M.F. impressed by the external field is now in the required new direction, it may not suffice by itself to raise the current to equality with the normal current of the other coils; yet much will have been done to prevent the rise of the current-density in any portion of the brush and at any time during short-circuit from reaching an excessive amount. Finally, if the brushes be so shifted that at the end of short-circuit the E.M.F. then generated in the coil is $e_T = -J\ (R + 2R_1)$, the rate of change of the current at the last moment becomes zero, and the transition from the varying to the steady state is rendered as gentle as possible. Such a condition is the most favourable when we consider that there must be slight differences between the various coils of the armature either in angular position, inductance, or resistance, and such dissymmetry will be then less likely to produce sparking; yet it may not always be attainable without necessitating such a large angle of lead as to involve too high a current-density at some earlier period.

§ 10. **Increase of angle of lead with increasing armature current.**—It is, however, now evident that if the electrical work thrown on the brushes is to be kept within reasonable limits or is to be reduced to its minimum, the amount by which the short-circuited coils must be advanced will require to be varied when the strength of the armature current is varied. As the armature current is increased, the new current which has to be started becomes greater, while at the same time the resultant neutral line is becoming more and more displaced from the line of symmetry. The ampères in the back turns are increased and the number of back turns is itself increased owing to the greater angle of lead required. In series- or compound-wound machines the weakening of the main field is partially or entirely neutralised by the corresponding increase of the magnetising turns of the field-winding; if the total flux is thus maintained or even increased, and the diameter of commutation be advanced right under the pole, the reversing induction could be made to approach the limiting value of the normal B_g, although at the cost of the E.M.F. of the machine as a whole. But it may still be asked how it is that in a simple shunt-wound dynamo a position can be found for sparkless commutation; since, as the brushes are shifted forwards to overtake the neutral line of the resultant field, the main component field is itself continuously weakened. The answer is that by the advance of the brushes, armature ampère-turns are progressively shifted from the belt of cross-turns to the belt of back-turns; the cross M.M.F. at the diameter of commutation is thus reduced, while at the same time, as we leave the line of symmetry and approach a pole-edge, the gradual decrease in the effect on the main field of the more distant pole and the shortening of the air-path together outweigh the increase in the back ampère-turns. It is true, therefore, that in a shunt machine even with constant excitation the reversing field very quickly reaches its maximum after the pole-edge is passed and then remains constant at some value[1] less than B_g; yet under ordinary conditions, if the brushes are shifted far enough forwards, a reversing induction of the requisite strength should be reached.

§ 11. **Limitation of output by sparking.**—If, however, an abnormally large current be passed through the armature of any machine, the reaction of the armature ampère-turns on the field will become excessive, a very large angle of lead must be given to the brushes, and the number of demagnetising ampère-turns, X_b. is therefore correspondingly great; even then the cross ampère-turns may remain so powerful that the short-circuited coils are not brought within the influence of a reversing field. As a consequence, no position can be found for the brushes where violent sparking does not occur. We are thus met with a condition which, entirely apart from any question of heating, limits the maximum current that can be passed through the armature, and therefore for a given speed of rotation and voltage limits the output of the machine. The armature ampère-turns at full load must not be so large that their reaction on the field renders sparking unavoidable; and it must be possible to bring the short-circuited coils at full load into a position where no sparking ensues, by shifting the brushes forwards.

When the armature current is increased from a very small value, and a lead has to be given to the brushes, the first result of shifting the diameter of commutation forwards away from the line of symmetry is to bring the short-circuited coil within the more clearly marked fringe of lines issuing out of or entering into the leading pole-tips. As the armature current is further increased, this fringe of lines is gradually driven inwards under the leading pole-tips by the increasing cross ampère-turns of the armature, and a further forward lead is necessitated, until with a still larger armature current the advance of the brushes to avoid sparking has brought the short-circuited coil close up under the pole-tips. The current might still be increased, and the brushes might be shifted forwards until the diameter of commutation falls some distance under the pole-pieces; there are, however, grave disadvantages attaching to such a large angle of lead, and these may now find mention.

---

[1] *E.g.*, with a smooth core, it cannot exceed the difference between the M.M.F.'s of the interpolar excitation X_P, and of the armature magnetising turns $\frac{c \cdot \tau}{2p}$, divided by $2\, l_g$.

§ 12. **Limiting value of angle of lead.**—The variation in the density of the field in the air-gap as we pass from the neutral line up to and under the pole-corners is always more or less abrupt, but when the diameter of commutation is almost or entirely under the leading pole-corners, and possibly has crossed the neutral line, the change of induction for small movements, especially on the leading sides, becomes very marked—much more so than when little lead is required, and the coils are short-circuited in the middle of the interpolar gap. If, therefore, the conditions for non-sparking require the diameter of commutation to be brought well under the leading pole-corners, a very small difference in the position of the short-circuited coils relatively to the poles means a large difference in the reversing E.M.F. set up in them. Hence the brushes must be very accurately placed to suit the exact load on the armature, or as the corollary of this, they are very sensitive to small changes of load, since a very slight alteration in the armature current materially alters the distribution of the field near the pole-corners. If the diameter of commutation at full load advances much beyond the neutral line, the machine further becomes inefficient as a generator of E.M.F. since some of the inductors are inducing a back E.M.F., and this is most likely to occur if the diameter of commutation be advanced under the pole-pieces. Hence it is decidedly inadvisable to load an armature with so much current that the diameter of commutation has to be brought under the leading pole-tips.

But more than this, as the short-circuited coils approach close to the leading pole-tips, their apparent inductance increases, owing to the proximity of the iron, and a stronger reversing E.M.F. is required. If, therefore, the diameter of commutation be advanced up to the maximum limit allowed, *i.e.*, up to the pole-tip, precautions must be taken that there should be an appreciable reversing field left in reserve at this point, since it is not permissible to advance further inwards to obtain a stronger reversing field.

§ 13. **Limiting value of ampère-wires per pole in smooth armature.**—Experience shows that in machines of good design with smooth-surface armatures the induction at the leading pole-edge should not be allowed to fall below 2500 in drum machines or 3000 in ring machines. The magneto-motive force of the cross ampère-turns acting under a pole between its extreme edges is $1{\cdot}257\, c\,.\,\tau\,.\,\frac{\phi}{360^\circ}$ the arc of embrace of the pole ϕ being substituted for θ as in Chap. XVII § 5. If β is the ratio of the polar arc to the pole-pitch or $= \frac{\phi}{\frac{360^\circ}{2p}}$, the M.M.F. of the cross ampère-turns may also be expressed as $= 1{\cdot}257 \frac{c\ \tau}{2p}\ \beta$ and on our previous assumption (Chap. XVII §§ 4-8) that this force in smooth-surface armatures may be practically regarded as expended in equal proportions over the two air-gaps, the cross flux is $1{\cdot}257 \frac{c\,.\,\tau}{2p}\,.\,\frac{\beta}{2l_g}$. Hence if B_g be the normal induction, the induction within the air-gap at the leading pole-tip being the difference between the main induction and the cross induction is

$$B_g{}' = B_g - 1{\cdot}257 \frac{c\ \tau}{2p}\,.\,\frac{\beta}{2l_g}$$

and this is to be not less than 2500 and 3000 in drum and ring machines respectively. Hence for smooth cores

$$\frac{c\tau}{2p} < \frac{0{\cdot}8(B_g - 2500)\,.\,2l_g}{\beta} \text{ for drum armatures} \qquad (57a)$$

or

$$< \frac{0{\cdot}8(B_g - 3000)\,.\,2l_g}{\beta} \text{ for ring armatures} \qquad (57b).$$

The limiting factor of the output of the armature as given by the above relation is in reality the cross ampère-turns acting athwart a pole, but as the same ampère-turns also act on the corresponding cross circuit of the other pole, the permissible number of cross armature ampère-turns *per pole* is the half of the number which may be allowed to act across a pole. There is, therefore, some ambiguity in the expression "the armature cross ampère-turns per pole" which might be taken to mean the number which in reality are acting across a pole. Now the product of the current c flowing in each of a number of inductors and that number of inductors τ may be called the *ampère-wires* of the dynamo and be symbolised by A_w. The ambiguity is then entirely avoided if we speak of the armature ampère-wires per pole instead of ampère-turns, the number of ampère-wires under a pole being the same as the number of ampère-turns, but their total number being twice the total number of ampère-turns. The left-hand side of the above equation gives in effect therefore ***the maximum permissible number of ampère-wires per pole*** that may be allowed in a smooth-core machine whether bi- or multi-polar.

It is evident that the maximum permissible number of ampère-wires on an armature is closely related to the normal induction and to the air-gap excitation $X_g = 0{\cdot}8\ B_g.2l_g$ although the relation is not one of simple proportionality. In, *e.g.*, a drum, if $B_g = 5000$, the induction B_g' at the leading pole-tip $= \frac{B_g}{2}$, but if the normal induction falls below 5000, the margin of difference after subtracting the required constant reversing field becomes so small that the possible number of ampère-wires rapidly diminishes. If, then, $B_g = 5000$ be taken as the lower limit we have the important relation that the maximum number of ampère-wires per pole is—

$$\frac{c\tau}{2p} = 2000 \cdot \frac{2l_g}{\beta} \text{ in drums} \quad . \quad . \quad . \quad . \quad . \quad (58a)$$

or

$$= 1600 \cdot \frac{2l_g}{\beta} \text{ in rings} \quad . \quad . \quad . \quad . \quad . \quad (58b);$$

while if $B_g = 6000$,

$$\frac{c\tau}{2p} = 2800 \cdot \frac{2l_g}{\beta} \text{ in drums} \quad . \quad . \quad . \quad . \quad . \quad (59a)$$

$$= 2400 \cdot \frac{2l_g}{\beta} \text{ in rings} \quad . \quad . \quad . \quad . \quad . \quad (59b).$$

It will be seen that the maximum number of ampère-wires that can be carried is entirely independent of the length of the armature. It varies inversely as the polar angle, so that if a large number of ampère-wires are to be permissible with a given density of lines and length of air-gap, the polar angle must be reduced. As a general rule, however, this does not vary much on either side of $0{\cdot}75 \times \frac{360^\circ}{2p}$, so that practically the chief determining factors are the values of B_g and l_g.

We have thus arrived at an expression for the maximum ampères that a smooth armature with a given winding can carry as determined solely by considerations of sparking; all questions of heating are for the present held over, but if the machine is not to be very sensitive to changes of load, and if it is to allow of a slight temporary increase of the armature current above its rated output without sparking, then the product $c\tau$ or A_w must not exceed the values given by the above equations. The rules thus laid down must of course be regarded as more or less elastic, but they will be found of great service in designing as indicating the chief elements of the sparking problem and the limits beyond which it is inadvisable to load the armature. Economy in manufacture would dictate as large a number of ampère-wires as possible, since out of the whole cost of the machine, from one third to one half must be credited to the armature, and to increase the output of a given dynamo it would appear an easy matter to increase the current taken out of the armature. It is now, however, seen that, apart from any question of heating, this is directly limited by the separate consideration of sparklessness.

§ 14. **Advantage of a number of poles.**—If the proportion of the polar arc to the pole-pitch be retained constant as in practice is permissible, it is evident that for the same air-gap and the same induction the maximum permissible number of ampère-wires on the armature as a whole is proportional to the number of pairs of poles, since, *e.g.*, in a drum

$$A_w = c \, . \, \tau = \frac{0{\cdot}8(B_g - 2500)2l_g}{\beta} \times 2p \quad . \quad . \quad . \quad . \quad (60).$$

Thus if the number of poles be increased from two to four or six, and the angle ϕ subtended by each is reduced in the same proportion, the armature ampère-wires may be doubled or trebled. Since $2p.\ \phi$ is then constant and the same induction B_g is assumed to be maintained, the total number of lines cut by each inductor in one revolution, viz., $2Z_a p$ upon which the voltage depends is constant. The output, therefore, can only be increased by increasing the ampères that flow through the inductors. In a great number of cases the heating of the armature will itself determine the permissible number of ampères, but if the limit set by sparking is reached sooner than the limit set by heating, it will at once be seen wherein the advantage of a multipolar field for large outputs lies. To increase the ampère-wires on the bipolar armature will involve a corresponding increase in the length of the air-gap, but if the machine be made four-pole instead of two-pole, the ampère-wires may be as much as doubled without any increase in l_g being necessitated. Or if the output of a machine when four-pole is limited by sparking, the ampère-wires may be raised in the proportion of 3 to 2 if it be made six-pole, with the same air-gap in both cases. While the above conclusions indicate the general nature of the advantage of the multipolar machine, it is needless to add that in any particular case many other considerations must be given their due weight, such as the alteration in the total number of inductors or in the number of loops per section which may be necessitated by an increase in the number of poles, and the possibility that the same result may be attained more cheaply by increasing the length of the air-gap of the machine with the smaller number of poles.

§ 15. **Limiting values of ampère-wires per unit of circumference of armature core.**—In bipolar machines it is as a matter of fact usual to increase the length of the air-gap nearly in proportion to the diameter of the armature core. The maximum permissible number of ampère-wires can then also be increased in direct proportion to the diameter. Thus the values of the single air-gap in a smooth-surface drum armature may be taken as 0·057 of the diameter of the core, or $l_g = 0{\cdot}057\,D$. Then with a normal induction of 5000 and a polar angle of 135°, or $\beta = 0{\cdot}75$

$$A_w = \frac{2 \times 2000 \times 0{\cdot}114 D_{cm}}{0{\cdot}75} = 610 D_{cm} \text{ nearly, or } 1545\,D'' \qquad . \quad (61).$$

This may also be expressed as about 195 ampère-wires per cm., or 495 ampère-wires per inch of circumference; while if $B_g = 6000$,

$$A_w = \frac{2 \times 2800 \times 0{\cdot}114 D_{cm}}{0{\cdot}75} = 850 D_{cm} \text{ or } 2150\,D'' \quad . \quad . \quad . \quad (62)$$

corresponding to 270 ampère-wires per cm. or 685 per inch of circumference. In the ring armature B_g seldom exceeds 5250, giving with a radial depth of core $= 0{\cdot}2D$, an induction in the armature core of

$$B_a = \frac{\pi D \frac{140^\circ}{360^\circ} \times L \times B_g}{0{\cdot}4D \times L \times 0{\cdot}9} = 17{,}800.$$

The average length of air-gap is also smaller, say $= 0{\cdot}0415 D_{cm}$, whence

$$A_w = \frac{2 \times 1800 \times 0{\cdot}083 D_{cm}}{0{\cdot}75} = 400 D_{cm} \text{ or } 1020 D'' \quad . \quad . \quad . \quad (63)$$

corresponding to about 125 ampère-wires per cm. and 320 per inch of circumference of the core. The ampère-wires per unit length of the circumference may be symbolised by a_w, and it will now be noticed that while the permissible number of ampère-wires per pole is but little else to the designer than a warning, its development in terms of a_w conveys some definite information as to the necessary dimensions of the armature, and that it is so far of much greater value. The figures given for a_w do in fact have considerable practical importance, although here derived solely from considerations of sparking. In small machines, the heating limit is reached first, and the actual values of a_w in practical cases are then found to be only one-half of those given above, but in large bipolar dynamos, the two limits are reached in general nearly simultaneously. The increase of the depth of the copper with the increase of the air-gap length keeps the watts generated per inch length of the core to a certain extent proportionate to the cooling surface. In large sizes, therefore, the above values of a_w correspond very generally to the limits imposed at once by heating and sparking considerations.

If a two-pole armature is not fully loaded from the sparking point of view but has a value of a_w suited to its permissible rise of temperature, its diameter can be increased, and the number of ampère-wires be simultaneously brought up to the limiting value which may be allowed to act across a pole without further change. After reaching this point, any further increase in its ampère-wires will necessitate an increase in l_g. At any stage in the process the armature might be bisected, and its diameter doubled or trebled to allow of its being made four- or six-pole with the same depth of air-gap as when two-pole. With large diameters of core in the two-pole stage, such a method would lead to such large air-gaps and such long field-magnet cores that the design becomes unpractical. There is therefore with multipolar machines a tendency towards the adoption of a certain maximum l_g and consequently a certain maximum number of ampère-wires per pole; beyond this any increase in the total number of ampère-wires required on the armature is obtained rather by increasing the number of poles. As considerations of heating then involve an increase of diameter roughly proportional to the number of poles, it follows that the practical values of a_w approximately hold for multipolar as well as bipolar machines and for toothed as well as smooth armatures.

§ 16. **Determination of angle of lead.**—So far although a maximum for the angle of lead of the brushes has been laid down, its exact value has not been definitely determined. The rule that the full-load cross ampère-wires under a pole must not weaken the field at the leading pole-tip below 2500–3000 merely secures the general fact that there is no abnormal amount of displacement of the field and that the resultant neutral line falls somewhere between the leading pole-edge and the line of symmetry. The exact lead which copper brushes will require for any particular armature current it is almost impossible to predict by calculation. To accurately map out the distribution of the displaced field in the interpolar space is itself a complex problem, although in the smooth armature a near approximation may be made by superposing a symmetrical cross flux on a symmetrical main field as weakened by the assumed number of back ampère-turns; the curve connecting the change of current in the short-circuited section with the time of commutation could then be plotted, and the conditions be adjusted until the required rate of change and current-density in the brush agree with the assumed angle of lead and have suitable values.[1] But in every case the apparent inductance of the section presents a difficulty since it is seldom accurately known. In practice, therefore, when it becomes necessary to know $x_b = c\tau \, . \, \frac{2\lambda}{360^\circ}$ in the calculation of the field-winding the designer must fall back on the evidence of similar machines already built and tested. A maximum limit to the angle of lead has, however, been implicitly laid down, since if the short-circuited coils at full load are brought nearly up to the pole-tips, and the normal polar angle is taken as $\phi^\circ = 0{\cdot}75 \, . \, \frac{360^\circ}{2p}$, the greatest angle of lead will not exceed $\lambda = \frac{1}{2}\left(\frac{360^\circ}{2p} - \phi^\circ\right) = 0{\cdot}125 \, \frac{360^\circ}{2p}$. Thus in a two-pole dynamo in which the angle subtended by the bored face of the pole-piece is 135°, and the effective field-angle after allowing for the fringe beyond the actual edges of the pole-face is about 140°, the angle of lead would be about 20°. This, however, implies that the armature is loaded to the maximum number of ampère-turns permitted by considerations of sparking; the angle at full load is usually not so great, and although no definite rule can be laid down, an average value may be set as falling within the limits of 8° and 13°. The above assumption of a constant ratio of polar angle to the pole-pitch requires also some caution. The fringe of lines at the leading pole-tips should shade off fairly gradually; if the variation of the induction be abrupt, a small error in the position of the brushes will mean a large error in the strength of the reversing field, the brushes will require excessive care in their adjustment, and the machine will be sensitive to any changes of the load. Hence even apart from the question of leakage across from one pole edge to the opposite edge, it is not practicable to bring the pole-edges very close together; the direct distance across from one pole-tip to the other should therefore bear a certain relation to the length of the air-gap, and a common proportion to find is that the former is about seven or eight times l_g. If therefore the air-gap of a multipolar machine is longer than usual in proportion to the diameter of the armature, the absolute value of the interpolar distance on the assumption of $\phi = 0{\cdot}75 \frac{360^\circ}{2p}$ may become too small in relation to l_r

---

[1] For such curves, see *Theory of Commutation* mentioned on p. 560.

§ 17. **Comparison of copper and carbon brushes.**—Although it may be assumed that in general the brushes will be given a certain amount of lead in any dynamo, a distinction requires to be drawn as to its amount in the two cases of carbon and copper brushes respectively. Apart from the increased sensitiveness of a machine which requires so much lead that the diameter of commutation falls under the pole-tip, a large angle of lead is attended with the additional disadvantage that if the load fluctuates between wide limits, and the brushes cannot be shifted quickly enough to meet the fluctuations, the inaccuracy of their adjustment must be greater and the sparking more severe. Further, the demagnetising turns of the armature with a large angle of lead increase the necessary weight of copper on the field, and in the case of compound-wound machines, as explained in Chap. XVI § 19, render the regulation for constant potential less perfect than it would otherwise be. It is therefore advisable to keep the necessary lead within small limits and to make as much use as possible of the action of the brush contact-resistance in effecting reversal. As stated in § 9, this is secured by making $\frac{R_1 T}{L}$ large, and for given values of T and L, R_1 must be high. If $b_1 =$ the length of the surface of one set of brushes at right angles to the direction of rotation, $b =$ their width of contact in the direction of rotation, and $R_k =$ the contact resistance of the material in ohms per square inch of surface, $R_1 = \frac{R_k}{b_1 \cdot b}$. Hence R_1 can be made large either by increasing R_k or by decreasing b_1 b, the latter alternative being equivalent to employing a high normal current-density at full load. With copper brushes it is not practicable to employ a higher normal current-density than about 175 to 200 ampères per square inch; if R_k be taken as $= 0{\cdot}0016$ ohms, this corresponds to a loss of pressure of about $0{\cdot}0016 \times 190 \times 2 = 0{\cdot}6$ volt over the two sets of brushes of opposite sign, and in practice the loss is more often nearer to 0·5 volt owing to the current-density being lower. If the normal current-density and loss of volts be increased beyond the above mentioned maximum, any further increase of the current-density due to such slight inaccuracy of adjustment as must in practice occur will be more likely to fuse the local portion whether brush-tip or heel, where the density is excessive. Thus with copper brushes a fixed position for all loads within the capacity of the machine cannot be attained; as the armature current varies, the brushes must be shifted in order that the reversing field may be of about the right strength to keep the local current-density fairly uniform, and the corrective action of the contact resistance can only be relied on to prevent sparking when the brushes are but little removed from the natural position for exact reversal. The proper setting is found in practice by shifting the rocking bar slightly backwards and forwards until a position is observable on either side of which the sparking becomes greater. Hence in all cases where the fluctuations of the armature current are large and rapid, as, *e.g.*, in traction generators, recourse must be had to a different material, viz., *carbon*. If some sparking does take place, the carbon brush, being non-metallic, has the advantage that it does not become fused and adhere to the surface of the commutator, so that the evil effects are minimised. Even, however, with carbon brushes, it is not possible to employ a very high current-density; not only would the normal loss of volts and the consequent heating be too large, but disintegration of the material is set up, so that the contact surface becomes eaten away. Further, if the very high current-density is localised in one part of the brush, *i.e.*, in the brush-tip or heel with under-reversal or over-reversal respectively, then this part becomes red-hot, and its resistance falls. The equalising action of the contact area is then defeated, and the carbon brush may be regarded as having failed to meet the requirements for which it was introduced. The two considerations of a reasonable loss of volts and avoidance of such a high normal current-density that there is but little margin to meet unequal current-densities without over-heating have led in practice to the adoption of a normal current-density not exceeding some 35 to 40 ampères per square inch. Assuming the specific contact-resistance at this density to be 0·03 ohms, the loss of pressure over the two sets of brushes of opposite sign is from 2 to 2·2 volts. Comparison of this loss with that for copper brushes gives a rough measure of the degree in which the superiority of the former can be utilised in practice and shows that it is necessarily purchased at the expense of the efficiency of the machine. Such sacrifice is, however, but small as compared with the advantage that the carbon brush offers; although it may not enable us to retain the brushes on the line of symmetry, yet with them an intermediate angle of lead can be found between the best possible positions for zero and full load, such that neither the too rapid reversal in the first case nor the insufficient reversing field in the second case cause such an excessive current-density as to over-heat the brush and produce serious sparking. The brushes can then be retained in this position through all changes of load perhaps up to an overload of 30 per cent.

§ 18. **Methods of reducing the field-displacement and angle of lead.**—For the purpose of reducing the angle of lead to a minimum or of attaining complete fixity of brush position under varying loads, several devices have been proposed or put into use. Thus in the first place small extra pole-pieces have been employed, these being so placed as to supply a field in which to reverse the armature sections. They are excited by magnetising coils in series with the armature, and are provided with sufficient ampère-turns to slightly more than counterbalance the effect of the armature turns, so that the reversing field increases nearly in proportion to the current to be reversed. Owing, however, to practical difficulties in the application of such *reversing pole-pieces*[1] they have not met with much favour.

In order to neutralise the cross ampère turns of the armature, a number of auxiliary turns in series with the main current may be wound in holes pierced through the poles close to their bored faces. Again, *chord-winding* (as already pointed out in Chap. XI p. 256) has the effect of neutralising what would otherwise be the back ampère-turns of the armature, and so of assisting in maintaining the strength of the main field. But more than this, when chord-wound, the two sides of the short-circuited section, so long as they are both on the same side of the neutral line (cp. fig. 266), act differentially instead of summationally as in the ordinary winding. At no-load the brushes do not stand precisely on the line of symmetry, and each side of the section is nearer to a pole-tip than would otherwise be the case. A slight movement forwards of the brushes thus suffices to bring one side nearly up to the leading pole-tip, and although the other side may still be acting against it, the difference of their E.M.F.'s rises very quickly in favour of the reversed direction of the current. Thus a small angle of lead is attained, but against this must be set the disadvantage that when the chord-winding is of any considerable amount, the machine is very sensitive to any changes of load, owing to the fact that the reversing E.M.F. is the difference of two E.M.F.'s which are nearly equal to one another.

---

[1] *Vide* Swinburne, *Journal Inst. Elec. Eng.*, vol. xix. No. 85, pp. 105-107; also *ibid.* No. 86, p. 227, and vol. xx. No. 93, p. 300. See also Sayers' Patents, Nos. 16,572, 1891, and 10,298, 1893.

Apart from such drastic remedies, much may be done by careful design to reduce the cross flux; a large displacement of the resultant neutral line, when the armature current is increased, can be thus prevented, and either the angle of lead is reduced, or the output of a given armature may be increased. In double horseshoe fields, such as figs. 192 and 193, the area of iron across the neck *c* on the line *ab* may be made small, or better still, the fields may be split along that line. The path of the cross flux is thereby contracted, and it becomes throttled by the saturation which results from any large number of lines crossing the narrow neck; the main flux is practically unaffected, and the resultant field still remains similarly distributed on either side of the line of symmetry bisecting the interpolar gaps. In the single-magnet horseshoe, prevention of the displacement of the field by narrowing the neck of the pole-piece as at N in fig. 244 would be in itself an advantage, but it would also reduce the main flux, and further it would not produce precisely the same effect on the other side at S. In such a case, therefore, the values of the induction at different points along the one polar arc from *c* to *d* would not correspond exactly with the values at diametrically opposite points of the air-gap under the other pole-piece from *b* to *a*; the resultant distribution of the field is then not symmetrical in accordance with the definition of Chap. XIV § 15, and this effect would be especially prejudicial in combination with a ring armature. Similarly, if the upper corners of the pole-pieces as at *pq* (fig. 244) be cut off too sharply, the increase of the induction at the trailing corner *c* will be throttled, although there will be no corresponding throttling at the other trailing pole-corner, *b*.

When, however, we pass to the symmetrical multipolar magnet such as fig. 197, analogous methods may be employed with impunity. Thus each pole may be divided down its centre by a slot passing right through its axial length and extending from the pole-face inwards to some little distance or even up to the yoke. The main path of the cross flux is thus bisected and an air-gap of say ¾" to 1" interposed therein, which will greatly reduce its amount. A combination of a split pole with an unsymmetrical shape for its two halves is employed in the Johnson-Lundell patent dynamo, and has the effect of still further reducing the distortion. Again, if in the ordinary machine the pole-tips are rapidly thinned off to a fine edge, the trailing pole-corner becomes highly saturated, and an appreciable portion of the total cross magneto-motive force is expended therein. The main flux may be slightly reduced thereby, but the distortion is decreased, and the advantage of keeping up the strength of the reversing field at the leading pole-corner may outweigh the disadvantage of the reduction. Especially in the case of short-air-space dynamos is saturation of the pole-tips likely to be of value, so that it is sometimes attained by special means; thus the poles may be built up of thin sheet metal, and at the pole-corner every alternate lamination may be cut away so that the area of metal is only half that of the solid pole-edge of similar shape.

§ 19. **Limiting value of ampère-wires per pole in toothed armatures.**—Exactly analogous is the employment of a high degree of saturation of the teeth in the slotted armature; the teeth under the trailing pole-corner then become still more highly saturated owing to the armature reaction, and the effect of the latter in displacing the resultant field is decreased. The extension of the foregoing formulæ for the maximum permissible number of ampère-wires per pole to the case of the toothed armature is by no means simple or readily made, but an approximation which indicates the nature of the phenomena may be given. As already explained in Chap. XVII § 12, two strips of the core which correspond to one another on the trailing and leading sides respectively are no longer of equal width, but the former must be widened by some fraction x, which depends upon the difference in the reluctivities on the two sides. The total reluctance of a strip of the cross circuit which is one cm. wide on the leading side and $(1+x)$ cm. wide on the trailing side and one cm. in length across the core is

$$ml_g + \frac{ml_g + \text{reluctance of trailing teeth per square cm.}}{1+x};$$

since by comparison with the air-gap and saturated teeth, the reluctance of the unsaturated leading teeth may be neglected. The cross M.M.F. between the pole-edges may thus be regarded as expended over the two air-gaps and the reluctance of the trailing teeth; and the value of the cross flux in the strip is

$$\frac{1\cdot257\frac{c\tau}{2p}\cdot\beta(1+x)}{(2+x)ml_g + \text{reluctance of trailing teeth per sq. cm. of path}}.$$

It will be seen that the denominator is greater than in the case of the smooth armature by the addition of the reluctance of the saturated trailing teeth to the air-gaps, which are themselves also slightly increased, yet that the apparent cross ampère-turns of the numerator are also increased, so that the advantage over the smooth-core depends only upon the proportion of the increments.

If B_t'' = the resultant induction in the trailing teeth immediately under the pole-edge, so that $f(B_t'')\,l_t$ is the loss of magnetic potential over their length of path, the reluctance of the trailing teeth per sq. cm. of cross circuit (not per sq. cm. of iron) is approximately $\frac{f(B_t'')l_t}{B_g''}$, so that it cannot be predicted until the whole case has been worked out. The instance analysed in Chap. XVII § 12 will, however, serve in some sort as a guide, and it is evident that for such a case the order of the figures for a normal induction B_g of about 6500 is given by the assumption of, say, $B_g'' = 8500$, $f(B_t'') = 1200$, and $x = 0\cdot3$.

At the leading edge there is but little reduction in the density of the main component flux, even though the average density of the two corners may be considerably reduced. We may therefore without much error still use the normal density of B_g as that of the main field at the leading corner. The resultant induction at the leading edge is therefore approximately

$$B_g' = B_g - \frac{1{\cdot}257\frac{c\tau}{2p}\beta\,.\,(1+x)}{(2+x)ml_g + \frac{f(B_t'')l_t}{B_g''}}\,.$$

Owing to the higher inductance of the short-circuited loops, carbon brushes are usually a necessity with the slotted armature, but the induction B_g' may then be allowed to fall lower than in the smooth-core machine with copper brushes, or say to a minimum of 1000. The maximum permissible value of the ampère-wires *under* the pole is then

$$\frac{c\tau}{2p}\cdot\beta = \frac{0{\cdot}8(B_g - 1000)\left\{(2+x)ml_g + l_t\frac{f(B_t'')}{B_g''}\right\}}{(1+x)}\,. \quad . \quad . \quad (64)$$

or in the case of Chap. XVII § 12 with our assumed values for B_g'' and x

$$= \frac{0{\cdot}8(5500)\left\{2{\cdot}3ml_g + \frac{l_t}{7}\right\}}{1{\cdot}3}$$

and with $ml_g = 0{\cdot}882$ and $l_t = 4$

$= 9000$ ampère-wires under the pole approximately.

If $\beta = 0{\cdot}75$, the maximum permissible value of the total ampère-wires within the pole-pitch is practically

$$\frac{c\tau}{2p} = 0{\cdot}8(5500)\left\{2{\cdot}3ml_g + \frac{l_t}{7}\right\}$$

$= 12{,}000$ ampère-wires per pole.

Such a limiting value as 12,000 ampère-wires per pole is frequently observed in the design of multipolar toothed armatures, but any such rule is only applicable for a more or less constant air-gap of say 0·7 to 1 cm. or $\frac{1}{4}''$ to $\frac{3}{8}''$, and also for some normal value of the average air-gap induction such as was assumed above. In large slow-speed machines the average B_g may be as high as 9000; the margin for reversing being still retained constant at 1000, the permissible armature load quickly rises with any increase of B_g, and in the above case becomes as much as 18,000 ampère-wires per pole. In practice subsidiary causes are also at work to reduce the cross flux, chief of which is the loss of potential over the remainder of the path in the pole-piece and armature which has been treated as negligible, and in especial any increased reluctance due to the saturated pole-tip. Thus, to sum up, the length of path over the saturated teeth of the slotted armature comes to play a part analogous to an increase in the length of the air-gap, and the chief difference between a toothed armature with a short air-gap and a smooth armature with a long air-gap is in the greater self-inductance of the wires when embedded between the teeth of the slotted core.

§ 20. **Inductance of section of (I) smooth armature-winding.**—We are thus led to a consideration of the inductance of armature coils in various types of machines and with different arrangements of their wires, and in the first place will take *smooth-core armatures.*

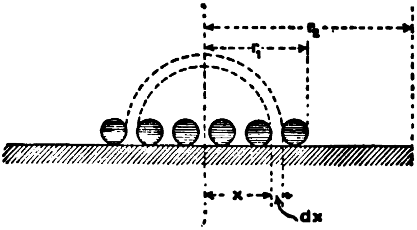


FIG. 255.

The *inductance* of a coil in absolute units is equal to the number of linkages between the turns of which it is composed and the lines which thread through them when the current carried is one C.G.S. unit. Let w be the number of wires in one side of the coil (whether in one or more layers) in a ring armature with smooth core of length l cm. Within the radius r_1 = half the width of the coil, consider a very small strip of breadth dx distant x cm. from the centre of the turns (fig. 255); the M.M.F. acting on it for a current of 1 C.G.S. unit per wire is $4\pi w \,.\, \frac{x}{r_1}$. On the assumption that the iron core is infinitely permeable, and that the paths of the lines in the air are semicircles springing out of a flat core, the permeance of the strip is $\frac{l.dx}{\pi x}$. Hence the number of lines in the strip is

$$4\pi w \,.\, \frac{x}{r_1} \,.\, \frac{l \,.\, dx}{\pi x} = 4\pi w l \,.\, \frac{dx}{r_1 \pi}$$

and these are linked with $w\frac{x}{r_1}$ wires. Therefore an element of the self-induction is $dL_1 = 4\pi w^2 \frac{l}{r_1^2 \pi} x dx$.

The integral $\int_0^{r_1} x dx$ being $\frac{r_1^2}{2}$,

$$L_1 = 4\pi w^2 l \,.\, \frac{r_1^2}{2 r_1^2 \pi} = 2 w^2 l.$$

Hence for the same number of wires, the inductance within the limits of the coil itself is constant, however close or far apart the wires are, since if the wires are spread out, the area and length of path increase equally, so that the same number of lines remains linked with the same number of turns.

Within the region $r_2 - r_1$, where r_2 = half the interpolar gap, the M.M.F. is constant, and the flux is linked with all the wires. The permeance of the region is by Chap. XV § 7 (II), $= l \frac{2 \cdot 3}{\pi} \log \frac{r_2}{r_1}$, and

$$L_2 = 4\pi w^2 l \,.\, \frac{2 \cdot 3}{\pi} \log \frac{r_2}{r_1} = w^2 l \times 4 \times 2 \cdot 3 \log \frac{r_2}{r_1}.$$

To the above has to be added the inductance L''' of the end and inside portions of the turns, which may practically be taken as at least equal to $L_1 + L_2$. The total apparent inductance of the ring coil is therefore

$$L = L_1 + L_2 + L''' = 2 w^2 \,.\, l \left(2 + 4 \times 2 \cdot 3 \log \frac{r_2}{r_1}\right).$$

If the width of the coil be only that of one wire, all the lines in the region of radius r_1 are within the copper of the conductor; other considerations then hold, the effect being to modify the distribution of the current within the wire and to render it non-uniform. Since r_1 is in such a case small, L_1 becomes of less importance and may be neglected by comparison with L_2.

If τ = the total number of wires counted round the periphery of the armature and N_2 = the number of commutator sectors, $w = \frac{\tau}{N_2}$, while if p = the number of pairs of poles, and p_1 = the number of pairs of brush arms, the number of coils in series undergoing short-circuit at a set of brushes or between two sets of brushes of the same sign in the wave-wound parallel-connected armature with as many sets of brushes as poles (Chap. XI § 20, ii. *b*) is $\frac{p}{p_1}$, so that the inductance of a short-circuited section in a *ring* armature is in general

$$L = \left(\frac{\tau}{N_2}\right)^2 \cdot \frac{p}{p_1} 2kl \times 10^{-9} \text{ henrys} \quad . \quad . \quad . \quad . \quad (65)$$

where $k = \left(2 + 9{\cdot}2 \ \log \frac{r_2}{r_1}\right)$.

If $\frac{r_2}{r_1} = 20$, *e.g.*, with two inductors per section in a single layer on an armature of 14" diam. and 140° polar arc, if each wire is $0{\cdot}125''$ diam. and the interpolar gap is $5''$, $k = (2 + 12) = 14$. If wound in two layers so that $r_1 = 0{\cdot}0625''$ and L_1 be neglected, the numerical constant k becomes $14{\cdot}7$.

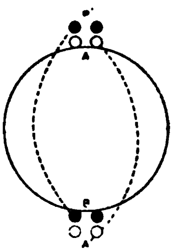


FIG. 256.

In a drum armature at least two sections are more or less simultaneously short-circuited, so that we have to take into account not only the apparent self-inductance of one section, but also the mutual inductance due to the adjacent wires of the second section. Let w = the number of wires in one side of a drum coil, r_1 = the half width of one side, and in the first place let the two sections which are simultaneously short-circuited be arranged one above the other in two layers (fig. 256).

Then the self-inductance of group A is $L_1 = w^2 l\left(2 + 9{\cdot}2 \log \frac{r_2}{r_1}\right)$, and neglecting the greater distance of the B′ group from the core, the mutual inductance due to B′ is the same, so that the total apparent inductance of A along the length of the core is

$$L_1 + M_1 = 2\, w^2 l\left(2 + 9{\cdot}2 \log \frac{r_2}{r_1}\right)$$

and of A + A′ is

$$2\,(L_1 + M_1) = 4\, w^2 l\left(2 + 9{\cdot}2 \log \frac{r_2}{r_1}\right).$$

It has now to be noted that the end-connections of the A and B groups do not coincide, but follow entirely different paths, so that the M.M.F. acting at the ends is only half of that acting on the core portions; in other words, only the self-inductance of the A coil has to be considered. The inductance of the end-connectors of a drum or barrel-wound armature is not strictly expressible per cm. length, since it depends on the shape of the coil and the area of the path which is traversed by the lines linked with the ends. In the case of a circular coil entirely in air or half embedded in iron throughout its length, so that every cm. length is similarly circumstanced, the area corresponding to a cm. length is a wedge-shaped sector; since the density of the lines decreases towards the centre, a sq. cm. near the periphery of the ring is of more account than one near the centre; hence as the diameter and length of a turn are increased, the lines per cm. length of the circumference and per C.G.S. current-turn rise very slowly. When surrounded entirely by air, the point where the curve of lines per unit length becomes nearly flat is reached when a diameter of 50 cm. is exceeded, and a figure of some 8 to 10 lines per cm. length is obtained. With a rectangular coil free in air or embedded uniformly in iron, the same effect is present; while the maximum number of lines per cm. length must be obtained when the coil is square, the reduction as the coil is narrowed is not very marked, until in the case of air one pair of parallel sides is less than 20 cm. apart. When the coil has a pair of parallel sides resting on or embedded in iron and a pair of sides in air, the iron core is so permeable by comparison with the air that the end-connectors may also be credited with any advantage to be obtained from the iron core as increasing the flux, and the number of lines due thereto is only limited by the distance of the iron from any cm. length of the end-connectors. It is therefore again practically legitimate to treat the end-connectors as linked with a certain flux per cm. length, when experience has shown within what limits of radius it must be integrated to give useful average results, and when it is borne in mind that the proximity or otherwise of the iron must be taken into account. With average conditions for the width of a loop in a barrel-wound armature, some such ratio as $\frac{r_2}{r_1}=20$ may be taken, leading to a representative figure of 8 lines per cm. length and per C.G.S. current-turn, as given by Mr Hobart in his paper on "Modern Commutating Dynamo Machinery" (*Journal Inst. Electr. Eng.*, vol. xxxi. pp. 185 ff.). Many of the tests described in this paper would seem to indicate an even greater value of 10 to 12, although it is difficult to establish so high a figure theoretically. In the case of a drum with evolute end-connectors fairly near to the iron core, such a figure would probably be reached. If therefore l' be the length of the end-connection on one side of the loop, $\text{L}'''=$ say, $2\times 8\,w^2l'$, or $=4\,w^2\times 4\,l'$. Thus

$$\text{L}=2\,(\text{L}_1+\text{M}_1)+\text{L}'''=4\,w^2\left\{\left(2+9{\cdot}2\log\frac{r_2}{r_1}\right)l+4\,l'\right\},$$

or in general since w is now $=\frac{\tau}{2\text{N}_2}$, the total apparent inductance of a short-circuited section in a *drum* armature with smooth core, in which the sections short-circuited at opposite brushes are wound immediately one over the other, is

$$\text{L}=\left(\frac{\tau}{\text{N}_2}\right)^2\cdot\frac{p}{p_1}\cdot(kl+k'l')\times 10^{-9}\text{ henrys}\quad .\quad .\quad .\quad (66)$$

where $k=\left(2+9{\cdot}2\log\frac{r_2}{r_1}\right)$, and $k'=4$ in a barrel armature with the end-connections well away from the iron, or $=6$ in a bar armature with end-connectors close up to the core.

If one of the two sections which are simultaneously short-circuited is to the side of the other (fig. 257), each coil is slightly chord-wound, but it is evident that no change is introduced if we imagine the B′ group replaced by the A′ group, and *vice versâ*. The inductance along the core of A and B′ or of a complete coil is therefore $L_1 = (2w)^2\, l\left(2 + 9{\cdot}2 \log \frac{r_2}{2r_1}\right)$, and since w is again $= \frac{\tau}{2N_2}$, while the end-inductance is practically unaltered, the apparent inductance of a short-circuited section in a smooth drum with opposite sections side by side is

$$L = \left(\frac{\tau}{N_2}\right)^2 \cdot \frac{p}{p_1}\,(kl + k'l') \times 10^{-9} \text{ henrys} \quad . \quad . \quad . \quad . \quad (67)$$

where $k = \left(2 + 9{\cdot}2 \log \frac{r_2}{2r_1}\right)$, and k' has the same value as above.

Thus for the same values of τ and N_2 the only difference between the inductance of a section on a drum and on a ring armature lies in the fact that the end-connection inductance of the former bears no particular relation to the length of the core, while in the ring the inductance of the end and inside lengths of wire is at least approximately equal to that of the external inductance. There is no further difference between the ring and the drum due to the position of the wires provided there is an even number of commutator sectors; but if there is an uneven number of sectors, and the armature be wound in a single layer, the ring coil of two turns is not the precise equivalent of the drum coil of one turn in relation to the reversing field.

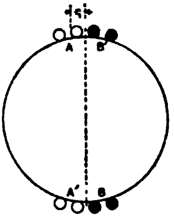


FIG. 257.

Since $k'l'$ is usually less than $= l\left(2 + 9{\cdot}2 \log \frac{r_2}{2r_1}\right)$, the inductance of the ring is the greater. Thus if $\frac{r_2}{2r_1}$ in the drum $= \frac{2{\cdot}5}{0{\cdot}125} = 20$, *i.e.*, in the same armature as before but wound with drum sections of one turn, the width of each bar being 0·125″ and the length of core 15″, $kl = 14 \times 38 = 532$; the end-connections of a two-pole drum will lie close to the core, or $k' = 6$, and l' will be about 33 cm., so that $k'l' = 198$. The total apparent inductance of the drum section will be therefore $\propto (532 + 198) = 730$, as against $2 \times 532 = 1064$ in the ring, the latter being 45 per cent. more. From experiments of Mr Allen,[1] the difference in the value of kl

---

[1] *Journal Inst. Electr. Eng.*, vol. xxvii. p. 217.

from that of $k'l'$ in a two-pole armature core of the same length led to the inductance of the ring coil being about $1\frac{1}{2}$ times that of the drum coil. But since $\frac{\tau}{N_2}$ can never be less than 2 in a drum and may be 1 in a ring, it is always theoretically possible to compensate for the higher inductance of the ring by employing twice as many sectors as in the drum.

§ 21. **Inductance of section of (II) toothed armature winding.**—Various shapes of slots and tunnels occur in practice, but they may be reduced to a few typical cases, out of which any given problem may at least approximately be solved.

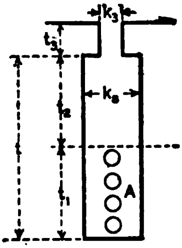


FIG. 258.

(a.) *Within the slot.*—(1.) Let w = the number of wires in the group A extending to a height t_1 from the bottom of the slot (fig. 258). If the iron be regarded as infinitely permeable, all the lines pass round through the iron at the root of the tooth. Taking any small strip across the air in the slot, distant x cm. from the bottom of the slot, and within the limit t_1, the M.M.F. acting across it with a current of 1 C.G.S. unit is $4\pi w \frac{x}{t_1}$, and the permeance is $\frac{l . dx}{k_s}$. Hence the lines in the strip are $\frac{4\pi w x . l . dx}{k_s t_1}$ and these are linked with $w \frac{x}{t_1}$ wires. An element of the inductance is therefore

$$dL_1 = \frac{4\pi w^2 l}{k_s t_1^2} \cdot x^2 dx.$$

The integral $\int_0^{t_1} x^2 dx$ being $\frac{t_1^3}{3}$,

$$L_1 = 4\pi w^2 l \cdot \frac{t_1}{3k_s} = 4{\cdot}19\, w^2 l \cdot \frac{t_1}{k_s}.$$

Above the level to which the wires reach, the M.M.F. is constant, $= 4\pi w$, the permeance $= \frac{l . t_2}{k_s}$, and the flux is linked with all the turns, whence $L_2 = 4\pi w^2 l \frac{t_2}{k_s} = 12{\cdot}56\, w^2 l \frac{t_2}{k_s}$.

Lastly, within the half-closed entrance of the slot,

$$L_3 = 12{\cdot}56\, w^2 l \frac{t_3}{k_3}.$$

Thus for the entire slot

$$L' = L_1 + L_2 + L_3 = w^2 l\left(4{\cdot}19 \frac{t_1}{k_s} + 12{\cdot}56 \frac{t_2}{k_s} + 12{\cdot}56 \frac{t_3}{k_3}\right) \quad . \quad (68)$$

In the most usual case of an open slot of total depth t half filled with wires, $t_1 = t_2$, and

$$L' = w^2 l\left(8{\cdot}37 \frac{t}{k_s}\right), \text{ or for a single bar } L' = l \,.\, 6{\cdot}28 \frac{t}{k_s}.$$

(2.) If w be the number of wires in the B group filling the upper portion of the slot (fig. 259), then

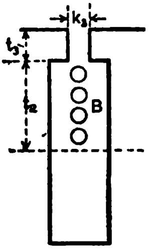


FIG. 259.

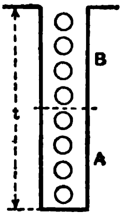


FIG. 260.

$$L' = L_1 + L_3 = w^2 l\left(4{\cdot}19 \frac{t_2}{k_s} + 12{\cdot}56 \frac{t_3}{k_3}\right) \quad . \quad . \quad (69)$$

and in an open slot half filled with wires

$$L' = w^2 l\left(2{\cdot}09 \frac{t}{k_s}\right).$$

(3.) If the wires fill the entire slot (fig. 260),

$$L' = L_1 + L_3 = w^2 l\left(4{\cdot}19 \frac{t}{k_s} + 12{\cdot}56 \frac{t_3}{k_3}\right) \quad . \quad (70)$$

With a single bar, L_1 always resolves itself into a question of current distribution in the bar and may be neglected. In every case the inductance within the slot for the same number of wires and the same width of slot varies directly as the depth, unless the slot becomes abnormally deep.

(b.) *From the tops of the teeth.*—Since the lines at the top of the slot have been assumed to pass straight across the opening, it is necessary in order to fill up the air-space above the slot that the paths should be taken as quadrants joined by a straight portion of length $= k_8$. On this assumption, the permeance between the limits $\frac{k_8}{2}$ and some assumed radius r cm. from the centre line of the slot is by eq. (33),

$$2{\cdot}3\,\frac{l}{\pi}\cdot\log\frac{\pi\left(r-\frac{k_8}{2}\right)+k_8}{k_8}$$

and this is acted upon by the M.M.F. $4\pi w$, the flux being linked with all the turns; hence

$$L'' = 4\pi w^2\,.\ \times 2{\cdot}3\frac{l}{\pi}\cdot\log\frac{\pi\left(r-\frac{k_8}{2}\right)+k_8}{k_8}$$

$$= 9{\cdot}2w^2l\,.\log\frac{\pi\left(r-\frac{k_8}{2}\right)+k_8}{k_8}\quad . \qquad (71)$$

Beyond and outside the slot, the permeance must be reckoned up to the limit of the interpolar gap, since the lines will spread out from the sides of the adjoining teeth, and fill practically the whole interpolar air-space just as if the core were smooth (fig. 261).

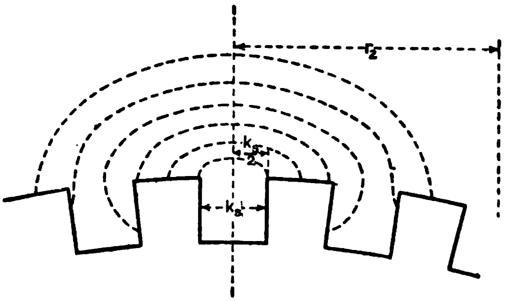


FIG. 261.

The inductance for any particular combination of wires can now be estimated. Take the case of a ring armature in which each section is composed of two turns lying one over the other in a single open slot, and completely filling it up to the top (fig. 262), and let $t = 3k_s$. The inductance within the slot reckoned by (3) is $L' = 4 . l \times 4{\cdot}19 \times 3 = 50{\cdot}28l$; or if the self-induced lines which pass through each wire are neglected, we have only to consider the flux passing across the slot above A, and by (1) $L' = l \times 6{\cdot}28 \times 3 = 18{\cdot}84l$. The discrepancy is due to the small number of wires, and an intermediate value of $25l$ approximating to the smaller figure will be taken as the more accurate. Outside the slot, if half the interpolar gap $= 2{\cdot}5''$, and there are five slots in it each $\frac{1}{4}''$ wide, $L'' = 4l . \times 9{\cdot}2 \log 30{\cdot}8 = 54{\cdot}8l$. The apparent inductance of the embedded portion is then $80l$, or since $\left(\frac{\tau}{N_2}\right) = 2$,

$$L' + L'' = \left(\frac{\tau}{N_2}\right)^2 \frac{p}{p_1} . \frac{80}{4} l.$$

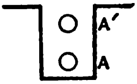


FIG. 262.

The inductance of the end and inside portions which form the free length may be taken as $L''' \propto 14l$, since they are unchanged from the case of the smooth-core armature. The total apparent inductance of the section short-circuited at one set of brushes is then

$$L' + L'' + L''' = \left(\frac{\tau}{N_2}\right)^2 . \frac{p}{p_1} . (20 + 14) l \times 10^{-9} \text{ henrys.}$$

Thus the value for the numerical constant, viz. 34, is about 20 per cent. greater than in the smooth-core armature, for which the value $2k = 28$ was previously obtained.

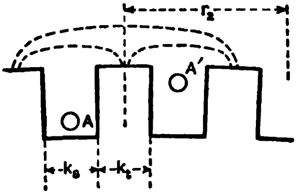


FIG. 263.

As, however, there are more turns than one in each section, one wire may be wound in the bottom of a slot and the other in the top of the adjacent slot (fig. 263). The inductance of A within the slot may then as before be taken $= 25l$. Outside the slots, the lines will gradually rise in density from the centre line, but approximately the permeance between the tops of the half teeth on either side of A or A′ may be reckoned as acted on by one wire, and the permeance between radii of (k_t+k^s) and r_2 as acted on by two wires. The inductance of A due to one of the two smaller groups of lines is then

$$9{\cdot}2\, l\,.\,\log \frac{\pi \dfrac{k_t}{2} + k_s}{k_s}$$

$$9{\cdot}2\, l\,.\,\log 2{\cdot}57 = 3{\cdot}77 l,$$

and the inductance of the two groups of A and A′ $= 7{\cdot}54\, l$.

Beyond and up to the limit $r_2 = 2{\cdot}5$, we have

$$4l\,.\,9{\cdot}2 \log 7{\cdot}28 = 31{\cdot}7 l.$$

Hence the total core-inductance of A and A′ is

$$(25 + 7{\cdot}54 + 31{\cdot}7)\, l = 64{\cdot}24 l,$$

and since $\dfrac{\tau}{N_2} = 2$,

$$L' + L'' = \left(\frac{\tau}{N_2}\right)^2 . \frac{p}{p_1} . l . \frac{64}{4}.$$

L‴ remains nearly unaltered, and the total apparent inductance is

$$L = \left(\frac{\tau}{N_2}\right)^2 . \frac{p}{p_1} . (16 + 14)\, l \times 10^{-9} \text{ henrys.}$$

The value of the numerical constant here is slightly too high, in so far as the lines only gradually rise in density from the centre line, so that on the whole the divided loops have but little higher inductance than on the smooth-core, viz., 30 as against 28. It will be seen what a marked effect is produced by distributing the wires in as many slots as possible.

With a number of small wires when the first term L_1 can be more satisfactorily introduced to allow for the inductance of each group due to lines which pass through some of its turns, the same calculations may be made. Thus if there are w wires in a section and these are all wound in the same slot and fill it, then with the same proportions of slot and tooth and interpolar gap,

$$L' \text{ is by case } (3) = w^2 l\,.\,12{\cdot}57$$

and $$L'' = w^2 l\,.\,9{\cdot}2 \log 30{\cdot}8 = w^2 l\,.\,13{\cdot}7.$$

The total apparent inductance is then $L' + L'' = w^2 l \,.\, 26{\cdot}27$,

or $$L = \left(\frac{T}{N_2}\right)^2 \frac{p}{p_1} \,.\, l \times (26{\cdot}27 + 14) \times 10^{-9} \text{ henrys,}$$

i.e., 42 per cent. more than that of the smooth-core armature. When, however, the w wires are divided between two slots, as in fig. 264,

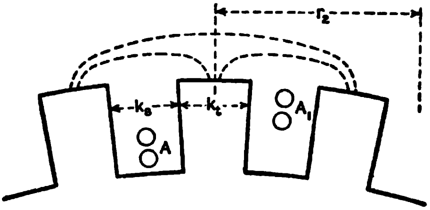


FIG. 264.

the inductance of A within the slot is $\left(\frac{w}{2}\right)^2 l \,.\, (8{\cdot}37 \times 3) = w^2 l \,.\; 6{\cdot}28$

and that of group A′ within itself is $\left(\frac{w}{2}\right)^2 l \,.\, 6{\cdot}28 = w^2 l \,.\; 1{\cdot}57$

The two small groups of lines from the tips of the teeth give $\left(\frac{w}{2}\right)^2 \,.\, l \,.\, 7{\cdot}54 = w^2 l \,.\; 1{\cdot}86$

while beyond and up to the limit $r_2 = 2{\cdot}5$, we have $w^2 l \,.\, 9{\cdot}2 \log 7{\cdot}28 = w^2 l \,.\; 7{\cdot}93$

The core-inductance is therefore $= w^2 l \,.\, 17{\cdot}64$

and the total $L = \left(\frac{T}{N_2}\right)^2 \,.\, \frac{p}{p_1} (17{\cdot}64 + 14)\, l \times 10^{-9}$ henrys.

The advantage of the subdivision between several slots, *i.e.*, the reduction of 40·3 to 31·6, is thus greater when there is a large number of wires in the coil.

In many cases, and especially with carbon brushes, the width of contact is such that at *each* brush two or more sections are simultaneously undergoing short-circuit. There has then to be added a further amount of mutual inductance due to the lines of the additional adjacent section which embrace the first. If the two or more sections are situated in the same slot, although at any moment they are passing through different stages of the process of commutation, there will be but little error in multiplying the previously calculated L by the number which are simultaneously short-circuited. But if the two sections under a brush are more widely spaced out, the L+M of each will be something less than twice L, say, 1·75L, and if there are three sections in question, the proportion will fall in a greater degree below 3L, say, to 2L.

If a drum armature be diametrically wound (fig. 265), it will be seen that the core-inductance, self and mutual, of coil A is precisely the same as the inductance of the ring coil filling one slot,

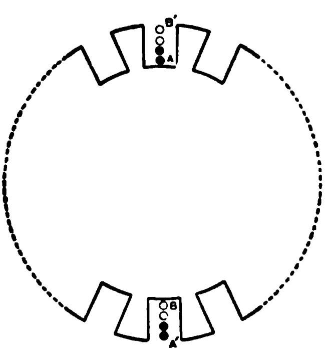


FIG. 265.

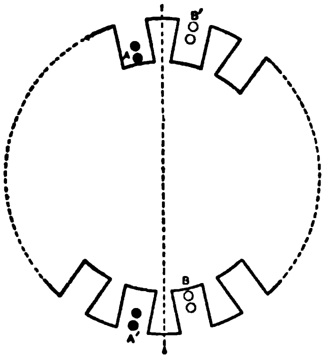


FIG. 266.

or with the same dimensions of slots, teeth, and interpolar gap as in our previous ring example,

$$L = \left(\frac{T}{N_2}\right)^2 \cdot \frac{p}{p_1} \cdot (20l + 4l') \times 10^{-9} \text{ henrys with one turn per section,}$$

and

$$= \left(\frac{T}{N_2}\right)^2 \cdot \frac{p}{p_1} \cdot (25l + 4l') \times 10^{-9} \text{ henrys with many turns per section.}$$

In a barrel-wound multipolar armature the length of the end-connections is high as compared with that of the core, and as pointed out by Mr Hobart (*Journal Inst. Electr. Eng.*, loc. cit.), the end-inductance is from 25 to 40 per cent. of the whole.

If, however, the drum be slightly chord-wound as shown in fig. 266, and there are an even number of commutator sectors, so that the two sections are simultaneously short-circuited, the portion A′ of the coil A A′ may be supposed to be removed to the position of B′, and *vice versâ*. The self and mutual inductance of A A′ has the same expression as the L of a ring coil composed of A and B′, so that, *e.g.*, with the same dimensions as before

$\text{L}=\left(\frac{\tau}{\text{N}_2}\right)^2 \cdot \frac{p}{p_1}(16l+4l') \times 10^{-9}$ henrys with one turn per section,

and

$=\left(\frac{\tau}{\text{N}_2}\right)^2 \cdot \frac{p}{p_1} \cdot (17{\cdot}65l+4l') \times 10^{-9}$ henrys with many turns per section. In both cases the reduction is greater than in the ring, since the constant inductance due to the end-connections is less.

A further possibility still remains if the chord-winding be extended so that the short-circuited coils are distributed between three slots in each interpolar gap (fig. 267). The self and mutual inductance of the coil A A′ B B′ will then be the same in its expression as the self-inductance of a ring coil containing C′D′ AB in series, and the reduction of core-inductance is such that the total approximates closely to the value for a smooth-armature.

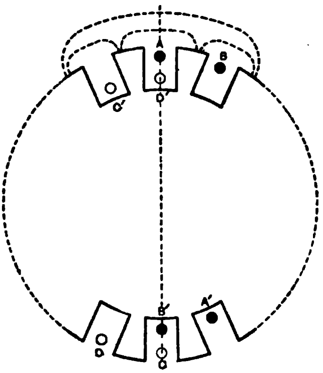


FIG. 267.

It may be pointed out that it is only permissible to divide the inductors of a section between two or more slots when they are in series; occasionally a coil is wound with several wires in parallel, and in such cases, if not placed within the same slots, eddy-currents would be set up within the coil itself, due to the different E.M.F.'s generated by the wires in the different slots.

§ 22. **Conditions favourable to sparkless commutation.**—It remains to add certain deductions as to the conditions which should be aimed at in order to secure sparkless commutation. The relation $\frac{\text{R}_1\text{T}}{\text{L}} > 1$ can always be obtained with carbon brushes, and the higher the value of the left-hand term, the better, provided that the current-density does not become too high. In practice it usually exceeds 2 with carbon brushes, but with copper the normal value often works out to less than 1, although it should not fall below 0·2. With a parallel-connected armature cross-connected at the commutator and only two sets of brushes, if R_1 is the joint contact-resistance of one set, the value pR_1 must be used in the above expression instead of R_1. With a multiplex or series-parallel winding and $2a$ parallel paths through the armature (cp. Chap. XI § 20), if only two wide sets of brushes are employed, aR_1 must be substituted for R_1.

With carbon brushes the width of contact is usually such as to cover more than one sector; under these circumstances the simple general equation (56) which was based on a width of brush equal to the width of a sector no longer holds good, since the current flowing along one of the commutator connectors now contains a further unknown quantity. The same principles will, however, still apply in the main; if s = the number of sectors simultaneously short-circuited at each brush, then for R_1 must be substituted sR_1, but at the same time if the sections are wound in the same slots, the new inductance of each section is nearly s times the old, so that on the whole but little difference is made.

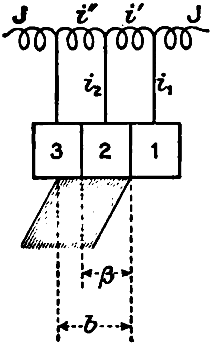


FIG. 268.

The second condition that e_T should be as nearly as possible $= -J\,(R+2R_1)$ may also be adapted to meet the case where two sectors are covered by each brush, if we make the approximate assumption that at the time T the current is divided between the two portions of the brush in proportion to their areas of contact. At the last moment the current-density in the leading sector is zero, when $i_1 = 0$ and $i'' = J$ (fig. 268); or since the contact resistance for i_2 at time T is $R_1\frac{b}{\beta}$, e_T must be

$$= -Jr - i_2\left(r_c + R_1\frac{b}{\beta}\right) \cdot$$

On our assumption

$$i_2 = 2J \cdot \frac{\beta}{b},$$

whence

$$e_T = -J\left(r + 2r_c\frac{\beta}{b} + 2R_1\right) \quad . \quad . \quad . \quad (72)$$

In place, therefore, of $R = r + 2r_c$ must be substituted $R = r + 2r_c\frac{\beta}{b}$.

It remains to combine the two conditions at which we are to aim, viz.:—

$$e_T = -J(R + 2R_1) \text{ and } \frac{R_1 T}{L} > 1,$$

and from the combination of the two it follows that

$$\frac{(-e_T - JR)T}{2LJ} > 1,$$

or

$$-e_T > \frac{2LJ}{T} + JR \quad . \quad . \quad . \quad . \quad (73)$$

If B_c = the flux-density in which the loop is moving at the end of short-circuit, then

$$B_c = \frac{p_1}{p} \cdot \frac{N_2 \cdot e_T \cdot 10^8}{\tau l v}.$$

If K be the specific numerical constant in the inductance expressed as

$$L = \left(\frac{\tau}{N_2}\right)^2 \cdot \frac{p}{p_1} \cdot l \cdot K \times 10^{-9} \text{ henrys},$$

we have by simple substitution, and remembering that the $-$ sign indicates a reversing field,

$$-B_c > \frac{2 \cdot \beta \cdot N \cdot J \cdot \tau \cdot K}{600\, b \cdot v} + \frac{p_1}{p} \cdot \frac{N_2 \cdot JR \cdot 10^8}{\tau l v}$$

Since

$$v = \frac{\pi D N}{60}, \text{ and } \frac{J\tau}{\pi D} = a_w,$$

where a_w is the number of ampère-wires per centimetre of armature circumference,

$$-B_c > \frac{2\beta \cdot K \cdot a_w}{10 \cdot b} + \frac{p_1}{p} \cdot \frac{N_2 \cdot JR \cdot 10^8}{\tau l v} \quad . \quad . \quad . \quad . \quad (74)$$

from which it is evident that the density of flux at the diameter of commutation which gives the best results is not only dependent upon the ampère-wires of the armature, but also upon the loss of volts over the resistance of a coil.

This conclusion may also be reached when it is noticed that of the two terms $\frac{2\text{LJ}}{\text{T}} + \text{JR}$, which e_T must exceed, the first deals with the reactance corresponding to the inductance of the section, and the second with the loss of volts over its ohmic resistance. The former is indirectly determined by the ampère-wires of the armature; but the latter, which must also be provided by the final impressed volts e_T, is independent of the armature ampère-wires, and although generally small as compared with the former, it is by no means negligible in every case.

§ 23. **Maximum permissible value of the quantity $\frac{2\text{LJ}}{\text{T}}$.**— It is, however, a favourable condition for sparklessness that the inductance should be as low as possible, and if we are to have $-e_\text{T} > \frac{2\text{LJ}}{\text{T}} + \text{JR}$, the fulfilment of the condition turns almost entirely upon the value of $\frac{2\text{LJ}}{\text{T}}$. This quantity, then, which should be as small as possible, has special importance, and its value in practical cases will afford a good clue as to the behaviour of a dynamo from the point of view of sparkless running. It corresponds to the reactance voltage discussed by Mr Hobart in his paper on "Modern Commutating Dynamo Machinery,"[1] but is free from the assumption of a sine-wave of current-change, which would seldom be realised in practice. Taking three volts as the maximum permissible value for the assumed reactance voltage on the sine-wave basis, or

$$2\pi \cdot \frac{1}{2\text{T}} \cdot \text{LJ} \lesseqgtr 3,$$

we have

$$\frac{2\text{LJ}}{\text{T}} \lesseqgtr \frac{6}{\pi} \lesseqgtr 1{\cdot}9.$$

The writers are, however, inclined to regard the inductance assigned to the embedded core-lengths by Mr Hobart as somewhat too high for the average case of a drum winding with the inductors well divided in open slots; in consequence, if the lower value of the inductance, L, as obtained by the above methods of reckoning, is employed, a smaller value should be taken for the maximum permissible value of $\frac{2LJ}{T}$, or, say,

---

[1] *Journal Inst. Electr. Eng.*, vol. xxxi. p. 206 ff.

$$\frac{2LJ}{T} \leqq 1{\cdot}5 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (75)$$

In small machines it is possible and advisable to secure even lower values, such as 0·8, but in large and slow-speed machines the value must rise, and in practice such cases as $\frac{2LJ}{T} = 1{\cdot}75$ will be found satisfactory in working.[1]

§ 24. **The self-induced field per cm. length of core as determining the number of sectors.**—With a given speed and time of commutation, if we consider armature cores of different lengths, e_T varies proportionally to their length, while, roughly speaking, R is more or less proportional to the core-length although the relative effect of the end-connections renders this not strictly true. There is further a certain value beyond which it is not easy to raise the commuting E.M.F. per cm. length of the core, owing to the effect of armature reaction in reducing the field at the leading pole-tip. If, therefore, different lengths of armature core are to be in the same position with respect to freedom from sparking, it is evident that LJ or the total self-induced field of the short-circuited coil must, approximately speaking, be kept proportional to the length; in other words, the self-induced field per cm. length of the core and per pair of inducing sides or $\frac{LJ}{l} \cdot \frac{p_1}{p}$ must not be allowed to exceed certain limits. Assuming that experience in practical cases gives such a value as

$$\frac{LJ}{l} \cdot \frac{p_1}{p} \leqq \frac{15{,}700}{10^9} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (76)$$

we obtain

$$N_2 \geqq 0{\cdot}008\, \tau \sqrt{JK} \quad . \quad . \quad . \quad . \quad . \qquad (77)$$

---

[1] Hobart, *Journal Inst. Electr. Eng.*, loc. cit.

The proportion of the core-length to the end-connection length, *i.e.*, of l' to l, varies widely in different designs, and ranges from 1·3 to 4 or more. On an average perhaps some such figure as 3 may be taken, so that $l' = 3l$. If, further, the inductors of the short-circuited loops are well distributed among a number of separate slots, the specific figure for the core-length inductance may be taken as, say, 15 in a drum armature with the usual proportions of slots open at the top, then

$$\mathrm{L} = \left(\frac{\tau}{\mathrm{N}_2}\right)^2 \cdot \frac{p}{p_1} \cdot (15l + 4 \times 3l) \times 10^{-9},$$

or $\mathrm{K} = 15 + 12 = 27$.

We thus finally obtain as a rule for the number of commutator sectors in a drum armature

$$\mathrm{N}_2 \geqq 0{\cdot}04\,\tau\sqrt{\mathrm{J}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (78),$$

and this expression, first published by Prof. Arnold,[1] has been found to yield good service over a wide range of practical instances. Any such rule is, however, liable to a number of disturbing conditions, and must be regarded as an empirical check rather than as superseding a calculation, however approximate, of $\frac{2\mathrm{LJ}}{\mathrm{T}}$. This latter quantity would be the average value of the E.M.F. of self-induction if the current changed from $+\mathrm{J}$ to $-\mathrm{J}$ in a straight line in time T, and it may therefore be called the average E_r. Its importance is, however, in no way dependent upon any such assumption, which can seldom, if ever, hold in practice, and its calculation is always to be recommended. Further, the condition $\frac{s\mathrm{R}_1\mathrm{T}}{\mathrm{L}} > 1$ may be thrown into the form $\dfrac{\frac{2\mathrm{JR}_1 s}{1}}{\frac{2\mathrm{LJ}}{\mathrm{T}}} > 1$, *i.e.*, the ratio of the loss of volts over one set of brushes, multiplied by the number of sectors covered by them to the average E_s, must be greater than 1, and as the loss of volts over the brushes has always to be calculated, a convenient combination of both expressions is obtained.[2]

---

[1] *Die Ankerwicklungen und Ankerkonstructionen der Gleichstrom-Dynamomaschinen*, 3rd ed. p. 278.

[2] Pichelmayer, *E.T.Z.*, vol. xxii. p. 967.

§ 25. **Importance of a large number of sectors.**—The subdivision of the armature winding into a large number of small sections is evidently an essential requirement for the avoidance of sparking, and the extent to which it may advantageously be carried is only limited by the question of expense in manufacture and the difficulty of dealing with very thin commutator sectors. The armatures of closed-coil machines for high pressures of from 500 to 1500 volts necessarily have a considerable number of turns per section, since they are wound with a large number of inductors; hence, even though the current of such machines may be comparatively small, special care is required to render them sparkless in working. In the case of ring-machines for moderate pressures of from 50 to 200 volts, the number of turns per commutator sector seldom exceeds four if the current in each inductor be greater than 10 ampères; while if this current be greater than 30 to 40 ampères, it is generally reduced to three or two turns per section, until finally, with over 70 ampères, it is advisable to adopt as many commutator sectors as there are turns. In drum armatures, with two loops per section, containing four inductors, the ampères in an inductor may be raised to about 50; for currents above this, each section seldom consists of more than one loop containing two inductors. In designing large bipolar armatures, especially if for comparatively low voltages, we are met by the further fact that there is a minimum limit beyond which it is inadvisable to reduce the total number of commutator sectors—a limit which may be set at about 50. This difficulty is especially felt in the case of large drum armatures, where the total number of inductors required may work out to less than 100; the maximum possible number of sectors is thus less than 50, even when one section of the winding is limited to a single loop of two inductors. The required armature current may be as much as 1000 ampères, and therefore each section when short-circuited is carrying 500 ampères with a correspondingly large value of the stored energy $\frac{LJ^2}{2}$. Under such circumstances two courses are open to the designer. The one is to subdivide the armature winding into more than two parallel circuits from brush to brush, the sectors of the two or more windings being formed into one commutator, and each brush having sufficient width to simultaneously cover as many sectors as there are pairs of paths from brush to brush, *i.e.*, the adoption of a multiplex winding (Chap. XI § 20, i. *b*). The advantage thereby secured is that the commutation of the two or more subdivisions is not contemporaneous, but one is always in advance of the other: the self and mutual inductance of the turns is then somewhat less than that of the simple undivided loop, yet against this must be set the fact that the time of commutation is reduced.[1] The alternative course is to entirely change the design, and adopt a multipolar armature with its winding arranged in as many parallels as there are poles.

In a toothed armature the winding of two or more sections with a corresponding number of commutator sectors in the same slots is, theoretically speaking, wrong, since the spacial displacement of the sectors is not matched by an equal spacial displacement of the coils. But in practice the use of a number of slots equal to the number of sectors may involve too great a loss of space in insulation, and the theoretical objection that the several sections are not quite symmetrical, the one with the other, in relation to the commutator, is outweighed by the advantage of subdividing the winding as much as possible in order to reduce the inductance of each section.

In ring armatures an uneven number of commutator sectors is not to be recommended. With such an arrangement in, *e.g.*, a two-pole machine, if the brushes are set exactly opposite to each other, the coils at the opposite brushes are short-circuited alternately; when only one coil is undergoing commutation, that part of its current which is not balanced by an equal current on the opposite side is tending to send lines round through the armature core, and when the opposite section is in turn alone short-circuited, a similar action, but in the reverse direction, takes place. Hence the field will pulsate with a periodicity $= \mathrm{N}_2 \cdot \frac{\mathrm{N}}{60}$, and this pulsation may injuriously affect the commutation. When the two opposite sections are both under the brushes and withdrawn from the main winding, one side of the commutator must include one sector more than the opposite side, and this odd section so included causes an alternating E.M.F. to act round the entire armature winding in series; the current is therefore divided unequally between the two halves, first one side and then the other carrying the larger part. The effect last described is also present in the drum armature with an uneven number of sectors, but with a fairly large number of sections the see-saw action is reduced to a negligible amount. Since the drum loops embrace the entire magnetic circuit, the former or magnetic effect is symmetrical, and results in a diminution of any fluctuation of the field, so that with the drum an uneven number of sectors is, if anything, better than an even number.

---

[1] C. C. Hawkins, "Armature Reaction," *Electrician*, vol. xxxix. p. 212.

If the armature be multipolar and parallel-connected, the number of sectors is preferably a multiple of the number of poles, even in the drum machine, if only for the reason that it is then possible to attach equalising cross-connections.

§ 26. **Greater tendency to sparking in ring machines.**—The tendency to sparking at the brushes is considerably greater in ring-wound than in drum-wound armatures, and on this account the maximum cross ampère-turns of a ring have in our previous equations been maintained at a lower value as compared with x_g than in the drum. The reasons for this inferiority are closely allied to the fact that a greater angle of lead is required in the ring armature than in a drum of similar dimensions, wound with the same number of active inductors and divided into the same number of sections. The primary cause of this must be sought in the greater resistance and inductance of ring loops as compared with drum loops, when both comprise the same number of inductors. Unless the armature be of a very large diameter as compared with its length, the resistance of a pair of ring loops is greater than the resistance of the equivalent single loop on the drum armature, and in consequence the voltage and strength of reversing field required for a given current round the ring loops are greater. A further cause lies in the disturbing influence of the internal field of lines produced within the ring by the armature ampère-turns. Fig. 239 shows that the current-turns on the two parallel halves of the ring unite to propel a certain number of lines across the air from the one side to the other of the interior of the ring along the diameter of commutation. Any such lines form an entirely separate self-induced field which does not blend with the main field, but exists in nature within the ring in addition to any reaction of the armature conductors on the main field. The cutting of this internal field by the inner connecting wires of the short-circuited sections will be found to set up in them an E.M.F. opposing the direction of the E.M.F. induced in the external wires, and therefore again a stronger reversing field is required in the ring than in the drum. To a certain extent, the tendency towards greater inductance in the ring method of winding is counteracted by the fact that the armature core has usually less radial depth than in the drum, and is worked at a much higher induction. The armature in duction has its highest value at the diameter of commutation, and in a ring is usually over 17,000; hence the short-circuiting of one section produces but a small variation in the number of lines through it, owing to the saturation of the iron core; it is, in fact, inadvisable to work with a lower induction in the ring armature owing to the increased self-induced field which would result.

An additional tendency to sparking in the case of single magnet dynamos with ring armatures, is due to a want of symmetry in their fields, produced by the causes described in Chap. XIV § 15. When the field distribution is unsymmetrical on the two sides of the armature, the correct position of the two brushes will not be at opposite ends of a diameter, but one will require more lead than the other: the prejudicial effect of this, in so far as it divides the armature winding into unequal portions, has been described previously, and with an even number of commutator sectors it may further produce sparking owing to the brushes not short-circuiting sections simultaneously. Apart, however, from any such reason, if the field be unsymmetrical at full load, a very slight change in the armature current produces considerable alteration in its distribution, destroys the balance required for non-sparking, and leaves the brushes in an incorrect position.[1]

---

[1] *Vide* Swinburne, "The Theory of Armature Reactions in Dynamos and Motors," *Journal Inst. Electr. Eng.*, vol. xix. No. 85, pp. 95, 96.

Again, some of the devices mentioned in Chap. XIV § 15 for neutralising the magnetic pull on armatures may effect that object fairly well, but are somewhat apt to produce a different distribution of the field in the top and bottom interpolar gaps when the cross magnetising forces of a large armature current are called into play.[1]

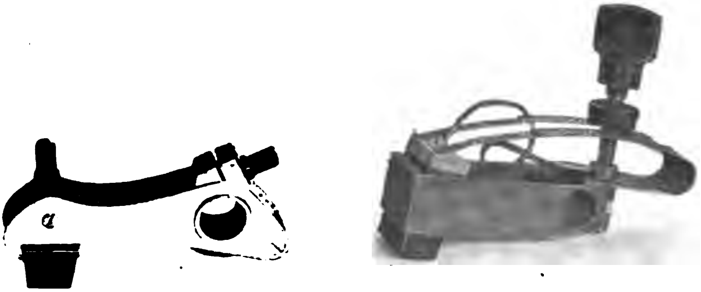
§ 27. **Brushes and brush-holders.**—*Copper brushes* for closed-coil dynamos are now most commonly made of gauze of a fine mesh: this is folded over on itself, stitched together with fine copper wire, and pressed flat. The one end is then lightly soldered together, while the other, which is cut diagonally across the mesh to avoid fraying at the tip, is trimmed to an angle so as to bed evenly on the commutator surface over its entire length and thickness. Less frequently brushes are made of fine copper wires or thin sheets of copper soldered together at one end. An average thickness for the insulating strips between the sectors in closed-coil armatures is $\frac{1}{25}$ of an inch, and the amount by which the width of contact of the brush-tip should exceed this depends upon the length of time during which it is advisable to short-circuit each section in order to minimise the sparking and upon the number of sections which it is permissible to short-circuit simultaneously. As a general rule, with gauze brushes it is best not to short-circuit more than one section at a time, unless it be for a mere instant, and therefore (except in the case of duplex- or triplex-wound armatures) the width of brush contact should not exceed the width of one sector and one insulating strip by more than a fraction of a sector. In ordinary cases this is obtained by using a brush of $\frac{1}{4}''$ or $\frac{5}{16}''$ thickness, so set that it makes an angle of about 45° with a line drawn tangentially to the commutator surface. The width in which gauze brushes are usually made varies from $\frac{3}{4}''$ to $2''$, but the latter dimension is seldom exceeded, since it then becomes troublesome to maintain proper contact along its entire bearing surface. Hence, to carry any considerable current, two or more brushes are mounted in line on each arm of the rocking-bar, forming in effect one wide brush. This arrangement further renders it possible to adjust each brush separately, or even to remove one temporarily, without interrupting the current; and this advantage is so great that every dynamo which is more than a toy should invariably be furnished with at least two brushes on each arm, each brush being of such width that, if one be removed, the other can temporarily carry the whole current.

---

[1] *Vide* Esson, *Journal Inst. Electr. Eng.*, vol. xix. No. 86, p. 280.

Carbon brushes are usually from 1" to 1½" wide, and in order to obtain sufficient contact-surface without unduly increasing the length of the commutator, are from ½" to 1" thick. They may further be allowed to cover more than one sector, but when this is the case, a hard variety of carbon is to be recommended rather than a soft electrographitic quality. The latter approximates in conductivity to a metal brush, and should therefore have a somewhat analogous width of, say, 1½ sectors.

Carbon brush-holders may be classified under one or other of two leading types. In the first, the more or less wedge-shaped block of carbon is fixed rigidly within its box, which forms the further end of a pair of stamped or cast brass or aluminium cheeks; these latter are pivoted on the brush spindle so as to be free to turn round it were it not for the constraining action of the pressure spring (fig. 269). In the second type, the brush is a rectangular slab, free to slide radially up or down in a guiding box, but pressed down by a spring bearing on the top of the brush (fig. 270). In both cases the carbon brush is nearly radial, although it may have a slight rake with respect to the commutator, and this rake in machines by different makers is sometimes against and sometimes with the



FIGS. 269 and 270.—Carbon Brush-holders.

direction of rotation. The second type is perhaps the simplest and best, but has the disadvantage that it is not so easy to arrange for the brushes to be held off the commutator, although with carbon brushes this requirement is not so necessary as with metal brushes. In all cases a good electrical connection between the carbon and the brush spindle which collects the current is of vital importance. In the first type, the brush may be wedged into the box with an interposed layer of copper gauze to form a good contact between the two; the box is then joined by a flexible copper connector to the central part of the brush-holder, which is either clamped or screwed to the brush spindle. In the second type, the flexible copper conductor is soldered to a clamp sprung over the top of the brush, which is itself coppered; or it may be sweated on to a plate fastened by screws to a small rod driven tightly into a hole through the brush; in either case so that it forms a twisted pig-tail with enough slack to allow of the normal amount of brush movement.

A combination of a gauze brush with a carbon tip ahead of the copper,[1] has been used in many cases where the current is heavy and at the same time variable; the carbon to some extent renders any slight sparking innocuous, but the arrangement has the objection that it is difficult to obtain a proper division of the current between the two heterogeneous substances, owing to the much higher conductivity of the metal. Still the economy in the cost of the commutator and the higher efficiency renders the device of considerable value.

§ 28. **Causes of local sparking on particular sectors.**— The pressure of the brush-tips on the commutator may be adjusted by altering the tension or pressure of the 'hold-on' spring. 'Jumping' of the brushes, due to vibration of the machine when running, must be carefully avoided, since it will give rise to sparking, and on this account a substantial rocking-bar with strong but light brush-holders, capable of being firmly fastened, is an essential part of a well-designed and well-built dynamo. The brushes should then bear lightly and evenly on the commutator. Any pressure beyond this should be avoided, since it will cause increased friction and wear. Occasionally, one or two sectors in a commutator wear down below the general cylindrical surface of the rest, and form what is known as a '*flat*'; as the brushes pass over the faulty spot, the circuit is momentarily broken, and sparking occurs, which rapidly increases the evil. The development of a flat may be caused by inequality in the wear-resisting properties of the sectors, but it is more often due in the first instance to sparking. Owing to a want of uniformity in the spacing of the winding on the armature surface, a particular section may be short-circuited when in an incorrect position; its passage under the brushes is then accompanied by sparking, and the sector to which it is attached becomes worn. The sections in a ring armature in close proximity to the gun-metal arms of the hub, partly owing to their want of symmetry and partly owing to their different inductance, are particularly subject to flats. With carbon brushes, it is especially important to employ a soft quality of mica having approximately the same rate of wear as that of the metal sectors. Irregularity of the turning moment of the prime mover, if considerable, occasionally causes flats corresponding to the dead centres of the crank-shaft. If an armature wire is broken or its connection to the commutator becomes loose, violent sparking may be set up, and the faulty coil may then be located by running the machine until one sector becomes pitted by the sparks.

---

[1] Crompton, Brit. Pat. No. 24,790. 1893.

§ 29. **Brushes of open-coil dynamos.**—Passing to the case of open-coil armatures, we are met by many of the same phenomena as occur in closed-coil armatures, save that the brushes must now be placed near to the line of maximum field, and that there is no short-circuiting of a coil as it passes from one field to another. The conditions for non-sparking will be readily apparent from Chap. X. Each coil should cease to be connected in parallel with its more active neighbour at the exact moment when the current flowing in the former has been stopped by the higher E.M.F. of the latter. We are now, however, able to see why, in the diagrams of that chapter, the line *m m* is shown displaced forwards in the direction of rotation, and is not exactly at right angles to a line of symmetry drawn vertically between the two poles. The reaction of the armature current in open-coil dynamos produces effects of exactly the same nature as in closed-coil dynamos; the field is displaced forwards, and consequently, when the armature current is considerable, the line of maximum action, while still approximately at right angles to the neutral line *n n*, no longer coincides with a line passing through the centres of the poles. It will readily be understood that in such machines, owing to the small number of coils, the abruptness of the changes made in their mutual connections, and the high voltages induced per coil, a certain amount of sparking is almost unavoidable. Such sparking, however, is not by any means so detrimental as might be thought; the commutator sectors being insulated on the surface by air-gaps, the sparks pass from the overhanging edge of the sector to the brush, and do not, therefore, destroy the surface of the commutator. Hence the apparently violent sparking of open-coil armatures is not nearly so injurious to their commutators as very much less sparking would be when acting on the smooth-surface commutators of closed-coil armatures.

The brushes of high-tension open-coil dynamos are usually made of thin sheet copper of considerable springiness. Each brush is slit up longitudinally for some distance from its bearing end, so as to form three or four teeth, and is then set almost tangentially to the commutator surface.

CHAPTER XIX

THE HEATING OF DYNAMOS

§ 1. **Rise of temperature in dynamo at work.**—With the exception of 'sparking,' probably no subject is of such importance, alike to the designer, the purchaser, and the attendant, as the question of the heating of dynamos. The continuous generation of heat in the armature and magnet-windings of all dynamos, so long as they are at work, is a necessary consequence of the passage of the current through their coils, and the appearance of this heat implies that a corresponding amount of energy is 'lost,' in so far as no useful work is derived therefrom. All that can be done from the point of view of economy is to minimise the amount of the heat which is thus generated, so as to obtain a reasonably high efficiency, such as is suited to the circumstances of any given case. Apart, however, from the question of the amount of heat produced every second, or its rate of generation in watts, there is the further and equally important question of the temperature to which any part of the dynamo is thereby raised. Whether it be the field-magnet coils or the armature which is the source of heat in question, when the machine is set to work the temperature of their mass gradually and continuously rises above the temperature of the surrounding air, until, finally, the rate at which the heat is generated is balanced by the rate at which it is carried off by radiation, convection, and conduction: after attaining the temperature which satisfies this condition, no further rise takes place. Evidently, therefore, the *rise* of temperature depends essentially upon the amount of cooling surface provided, and its actual effectiveness in dissipating heat, and, this being so, it follows that it may be regulated so as not to exceed a certain maximum, if the amount of cooling surface be duly proportioned to the watts expended. We have, however, first to consider in what way a high temperature is actually injurious to a dynamo, and also the closely connected question, in what way a large range of temperature is detrimental to its working: it will be found that they are so in three ways through their effect on the efficiency, the regulation of voltage, and the durability of the insulation.

§ 2. **Disadvantages of high temperatures. Increase of electrical resistance.**—In the first place, the higher the temperature of any portion of the electrical circuit of a dynamo, the greater is the loss of energy due to the passage of a given current through it. The limits of temperature within which dynamos are worked under average conditions may be taken as 70° F. and 140° F., the former corresponding with an average value for the temperature of surrounding air in the engine or dynamo room, and the latter to an ultimate temperature which it is frequent for the coils to attain when the dynamo is worked continuously, or for many hours together, at its normal output. Hence the increase in the resistance of the copper wire on an armature which rises 70° F. is as much as $70 \times 0{\cdot}21 = 14{\cdot}7$ per cent. of its initial resistance when at the temperature of the dynamo room. Or if R_a be calculated during the process of design from a table of resistances at a standard temperature of 60°F., R_a hot $= 1{\cdot}168$ R_a cold; and when a given current is passed through the armature, the loss of volts over its resistance when hot is, of course, increased by the same percentage. Not only must this increase be allowed for in the design, but it further involves an equally increased loss of energy. Again, the heating of the series coils on the field-magnets of series- or compound-wound dynamos increases the rate of loss in them for the same output at the terminals; while in the case of separately-excited or shunt machines the E.M.F. at the ends of the exciting coils must be raised if the same number of ampères is to be passed through their turns when hot as when cold, so that, again, energy is expended at a greater rate after the machine has become heated.

Armatures are seldom wound with more than one or two layers of wire, and therefore the temperature of the outside, as measured by a thermometer placed in contact with the outer insulating covering of the conductors, may be taken as indicating approximately the actual temperature of the conductors themselves. But in the case of coils with a large number of layers, as already explained in Chap. XV § 16, the mean temperature as deduced from measurement of the resistance is considerably higher than that of the surface. Thus, if the depth of winding in a field-magnet coil be 2½ inches, and the temperature of its surface rises 55° F., the actual increase in its resistance will be about $0{\cdot}21 \times 55^\circ \times 1{\cdot}5 = 17{\cdot}3$ per cent. of its resistance when at the temperature of the surrounding air;[1] and in our previous calculation (Chap. XVI § 7) for a depth of winding of 1½″, the mean rise of temperature of the coil was taken as 1·39 times the surface rise. Such considerable percentages show that the effects of heating must on no account be neglected in designing machines or in estimating their efficiency. Even in armatures with a single layer of conductors, if they are the rotating portion, there may be a divergence of some 30 per cent. between their actual temperature as deduced from measurements of their resistance immediately after stopping and the temperature measured by a thermometer laid on their exterior.[2] In all cases, therefore, measurements of the rise of resistance are to be preferred as giving more information than the temperature of the exterior as measured by the thermometer, although in the case of armatures their very low resistance may necessitate the use of a Thomson double bridge or other suitable method.

§ 3. **Less accuracy of self-regulation.**—In the second place, the rise in temperature of a dynamo when at work produces a disadvantageous effect upon the regulation of its voltage. As already mentioned, the separately-excited machine requires the E.M.F. applied to its exciting coils to be raised, if the same number of ampère-turns is to be maintained when the field-winding is hot as when it is cold; while if the terminal E.M.F. of the dynamo is to be kept constant, its internal E.M.F. must be increased in order to compensate for the increased loss of volts over the heated armature coils, and this necessitates either a further increase in the exciting E.M.F. or a higher speed of rotation. Similarly, the self-regulation of the compound-wound machine for constant potential is injuriously affected by the differences in the resistances of its shunt, series, and armature coils when hot and when cold: if correctly compounded when cold, the constancy of the potential must necessarily be inferior when it is hot, or *vice versâ*. In fact, in designing compound-wound machines it is especially important that the rise of temperature of the field-magnet winding be not so great as to seriously affect the compounding action of the two sets of coils, and it should preferably be limited to about 55° F. on the surface at the most.

---

[1] Compare *Electrician*, October 11, 1895, 'The Heating of Dynamos,' (Wilson). Even higher ratios were obtained in the case upon which Mr Brown experimented (*Journal Inst. Electr. Eng.*, vol. xxx. p. 1159, *qu. v.*).

[2] Wilson, *loc. cit.*

§ 4. **Deterioration of insulating materials.**—Thirdly, and of chief importance,—if the temperature of any coil becomes very high, the cotton or other fibrous material commonly used for the insulating covering of the copper wires will be burnt or charred; the insulation between neighbouring turns is thus broken down, and the short-circuiting which ensues is only terminated by complete collapse. A 'burnt-out' armature may be the result of an accidental short-circuiting of the machine, the heat from the excessive current almost instantaneously raising the temperature so much as to literally burn the insulation. Quite apart, however, from such accidental heating, the result of continually working a machine at a high temperature is a gradual deterioration in the toughness and mechanical strength of the insulating coverings of the wires. All materials in ordinary use for insulating purposes are alike subject to this gradual decay. Slowly but surely they become charred and rotten, the cotton or calico crumbling away when touched, and the blackened paper and press-spahn becoming excessively brittle; so that, although the insulation resistances may still remain very high, the liability to a breakdown is enormously increased. When old ring armatures are taken to pieces for repairs, the cotton covering of the internal layers of wire which have been continuously subjected to high temperatures may be found to be a mere charred powder, which can be wiped away with the finger.

Finally, among the drawbacks incidental to a great variation in temperature of the armature may be mentioned the expansion and contraction which take place in its metal framework as it heats or cools; for example, if the rise be very great, heat is conducted along the armature shaft, and its expansion, either within the bearings, or lengthwise between the collars that limit the end-play, may set up undue friction, or even cause it to seize. Under ordinary circumstances, however, such troubles are rather to be charged to bad workmanship, and hardly require to be separately considered here.

§ 5. **Maximum permissible temperature.**—It will now be sufficiently evident that the ultimate temperature attained by a dynamo when at work is, of all questions, the most important. If its wires are insulated with the usual materials, the frequent attainment of a very high temperature is incompatible with a long life, and this consideration of durability leads us to fix a maximum temperature, which no part of the machine should exceed in continuous working.

Some inventors have endeavoured to make use of insulating materials such as mica and asbestos which may by contrast with cotton and paper be called fireproof; the wires may then be allowed to reach very high temperatures, such as 250° F., provided that considerations of efficiency allow of it. But such attempts have not met with any great success; even if the insulation be satisfactory as regards its mechanical and heat-resisting qualities, it may occupy too much room; further, the heat gradually spreads to the commutator and brushes, while, lastly, the second consideration of § 3, viz. the great difference in the electrical resistances when hot and when cold introduces difficulties in the regulation of the voltage. Returning, therefore, to the more usual case of armatures and rotating coils insulated with cotton, silk, tape, fibre, or paper, the maximum temperature which they should be allowed to attain in continuous work cannot be set higher than about 190° F. at the most, and even then it must be remembered that, if a coil consist of a large number of layers, the limiting temperature of the surface must be appreciably lower.

§ 6. **Maximum permissible rise of temperature as limiting output.**—Having thus fixed upon a maximum permissible temperature, it is at once evident that the number of degrees by which a dynamo may be allowed to rise in temperature is entirely dependent upon the starting-point from which this rise takes place; in other words, upon the temperature of the surrounding air during the working of the dynamo. Thus, in the case of dynamos working in hot atmospheres, for instance, in the engine-room of a steamer in the tropics, where the normal temperature may be, and frequently is, as much as 115° F., the permissible rise is much smaller than in the case of a dynamo working in a well-ventilated central station on land, where the temperature will seldom exceed 70° F. Since the maximum current of a dynamo is dependent upon the *rise* in temperature which is permitted, it follows that the output is indirectly limited by the normal temperature in which it is to work, and from which the rise is reckoned. It is seldom, however, that in specifications of dynamos the actual limits of temperature are stated, inasmuch as they must necessarily be somewhat vague, and it is more frequent to find the maximum *rise* of temperature alone specified, this rise being such as will not be likely to endanger the insulation under ordinary conditions. It is generally stipulated that the machine must run satisfactorily during a test of six hours' duration at full load without undue heating of any part, and that at the end of the run the temperature of the armature or field-winding must not exceed the temperature of the surrounding air by more than 70° F. For short periods of say 1 to 2 hours, the normal full-load current may usually be exceeded by some 20 to 30 per cent. without raising the temperature of an armature in an excessive degree, and such a permissible overload enables the dynamo to deal with a large demand for current lasting a comparatively short time. The rise of temperature is usually obtained by comparison of the readings of two thermometers, the one registering the temperature of the room within a few feet of the dynamo, and the other placed in contact with the exterior of the winding, and therefore indicating the temperature of the surface.

Assuming a normal temperature of 70° F. for the surrounding air, it will be seen that the maximum temperature which the surface of the coils may attain in continuous work is 140° F., and this limit is found to give thoroughly satisfactory results in practice.[1] It further results from such a rule that the temperature of an armature as measured by rise of resistance may differ by 35 per cent. from its temperature as measured by thermometer, before our limit of 190° F. is exceeded. In the case of stationary field-magnet coils, the rotating armature causes a circulation of air which spreads to the magnet bobbins and cools their outer surfaces. The centre layers about one quarter of the total depth away from the inside [2] then attain the highest temperature, which may be 15 or 20 per cent. above even the average temperature, and 70 or 90 per cent. above that of the outermost layer. The innermost layers next to the iron core part with their heat by conduction to the magnet, and fall in temperature midway between the outer and centre layers. Hence with stationary field-magnet coils having considerable depth of winding, it might be advisable to fix an even lower limit of surface rise such as 50° F. in order that the centre layers may not exceed an actual temperature of 190° F., but the absence therein of the vibration and mechanical stresses to which the rotating armature is subjected permits of a somewhat higher local temperature, say 210° F., without impairing the durability of their insulation.

Thus the great importance of the safe *rise* of temperature lies in the fact that it determines the output of any dynamo, and this it does chiefly by limiting the maximum current that may be passed through a given armature in continuous working. Of course, the permissible armature current may be determined in the case of continuous-current dynamos by the question of sparking as explained in Chap. XVII, or in the case of alternators by the self-induction of the armature causing too great a drop in the terminal volts (Chap. XXIII); but in the majority of continuous-current dynamos the full normal output is decisively fixed by the serious heating which would occur in everyday working if the armature current were increased.

---

[1] In the specifications of the British Admiralty, from which the rise of 70° F. has largely been copied, the temperature of the room during the test run is further given as 120° F., and hence the permitted final temperature of the surface would be as much as 190°. Such a surface temperature would, however, not often be reached, although on the Continent and in the United States dynamos are frequently worked at higher temperatures than are usual in Great Britain.

[2] Brown, *Journal Inst. Electr. Eng.*, loc. cit.

§ 7. **Testing dynamos for rise of temperature.**—The time taken for the armature and field-coils of a dynamo to attain their ultimate temperatures is dependent upon their size. At the commencement of the run the rate at which heat is produced is almost as great as when the machine has attained its maximum temperature, but part of this heat is absorbed in raising the temperature, not only of the winding, but also of the core or magnet on which it is wound. The larger this mass which has to be heated, or the greater its specific capacity for heat, the longer will be the time taken in raising its temperature, until the final state is reached in which the heat has to be dissipated almost entirely by radiation and convection-currents in the surrounding air. A certain difference of temperature between the cooling surfaces and the surrounding air must then have been established sufficient to enable the heated masses to part with their heat as fast as it is generated. Strictly speaking, the temperature approaches its final value asymptotically, the rate of increase being a maximum at starting and thence gradually falling off; but if a machine be run with constant load for several hours, and the rise of temperature of armature or field-winding, as taken at intervals of, say, one hour, be plotted as ordinates to a horizontal axis of time, the curve so obtained will be found to gradually bend over and become more and more flat; finally, the readings will fall almost in a straight and horizontal line, showing that a steady temperature has then practically been attained. Such an experiment enables us to be certain that the final state has been reached, and the time which we find that a dynamo of given size takes to attain its maximum temperature will serve as a clue to the number of hours for which a machine of similar size should be run at full load in order to test it thoroughly. While a ring armature, of which the core dimensions are 9″ diameter × 12″ length, will attain its final temperature after about four hours' run at full load, a drum armature 15″ diameter × 19″ long will barely reach its maximum rise in six hours, and larger machines will require to be run for still longer periods. Even, however, in large machines, since they are multipolar and are usually barrel-wound or in other ways have their windings well exposed to the cooling effect of the air, there is but little rise of temperature after the first 8 or 10 hours. The thermometer employed to measure the temperature of the surfaces should preferably be of a sensitive chemical type, the graduated glass stem having a very fine bore, and the small cylindrical bulb containing but little mercury; after being laid or held in close contact with the winding, it should be covered with some material (such as a piece of rag) which is a bad conductor of heat, and then allowed to remain undisturbed for several minutes until the mercury entirely ceases to rise. When a rotating armature is stopped at the end of a run or at any time for the purpose of taking thermometer readings, the temperature of its exposed surface continues to rise for some minutes after the rotation has ceased. The generation of heat ceases with the rotation, but the simultaneous cessation of the air-currents set up by the revolving parts virtually amounts to a large reduction in the cooling power of the surfaces, and in consequence the fall of temperature between the inner or hottest parts and the external surface is reduced; in other words, the outside rises by conduction to a higher temperature, more nearly the same as that of the centre of heat. Both the cotton insulation of the wires and the shellac or other varnish with which they are coated are bad thermal conductors, and it therefore takes an appreciable time for the heat to penetrate through them to the outside of the armature. It may further be mentioned that, owing to the low thermal conductivity of insulating materials in general, if a heated armature be cursorily felt with the hand the bare metal of the binding-wires, or even of the commutator, will appear hotter than the insulated wires, and these latter may seem comparatively cool to the touch: any such conclusion is, however, entirely illusory, and the continued application of the hand to the conductors will usually suffice to correct the error.

§ 8. **Predetermination of rise of temperature in field-magnet coils.**—We have now to consider the predetermination of the rise of temperature in a dynamo—a subject which is naturally divisible into two portions, dealing respectively with the field-magnet coils and the armature.

In the case of *the exciting-coils* the question is fairly easy of solution. The heat generated in them is simply that due to the passage of the current through their resistance, and the watts expended over them are therefore easily calculated as the product of the square of the current into the resistance. Further, the coils are usually stationary, and are therefore by no means so much affected by currents in the surrounding air as is the rotating armature. Hence their rise of temperature is closely dependent on the ratio which the watts bear to their cooling surface, and apart from external draughts can be very approximately predicted. We have already, in Chap. XIV § 5, given a basis for calculating the cooling surface of a coil, or at all events a surface to which the cooling effect may approximately be regarded as proportional.

The increase of temperature per watt per sq. inch is, however, considerably affected by the shape of the machine and the position of its magnet coils. With close-fitting coils of double-cotton-covered wire varnished with shellac on the outside and wound on sheet-metal spools with wooden end-flanges, the final rise of temperature of the exterior will be in the case of two-pole single-horseshoe magnets such as figs. 2 and 190 approximately 100° Fahrenheit per watt per sq. inch, or

$$t^\circ \mathrm{F} = \frac{100 \times \mathrm{w}}{\mathrm{s}_c}$$

where w = the rate of generation of heat in the coil in watts, and s_c = the cooling surface of the coil in sq. inches measured over the external surface only of the wire. Thus in the case of the design of Chap. XVI § 7 we have $t^\circ = \frac{100 \times 460}{816} = 56{\cdot}5^\circ$F., agreeing sufficiently well with the assumed rise of 58° F. If, however, the two coils approach one another very closely on the inside between the magnet limbs, and the cooling surface is still reckoned after the same fashion, the inner portion is not equally effective in dissipating the heat, and the numerical coefficient may then require to be increased to as much as 140, or, say, on the average,

$$t^\circ \mathrm{F} = \frac{120 \times \mathrm{w}}{\mathrm{s}_c} \quad . \qquad . \quad . \ (79a)$$

On the other hand, in the double-horseshoe Manchester machine the bobbins are somewhat better exposed, the rise per watt per sq. inch is reduced, and

$$t^\circ \mathrm{F} = \frac{80 \times \mathrm{w}}{\mathrm{S}_c} \quad . \quad . \quad . \quad . \quad (79b)$$

In the above cases the bobbins are, to a certain extent, cooled by the currents which the armature revolving in close proximity sets up in the surrounding air, and this fanning effect is much intensified in multipolar machines with armatures of large diameter. The end-flanges have so far not been reckoned in the cooling surface, but their influence becomes more important the greater the depth of the winding and the shorter the coil. Wooden flanges act as partial heat insulators, while brass or iron end-cheeks assist considerably in conveying away the heat. Hence in multipolar dynamos with open magnet frames and well-exposed coils, short and deep, and in which metal end-cheeks are always advisable in order to economise valuable room in the length of the magnet-cores, their effect will approximately be taken into account by reducing the constant to 70, so that

$$t^\circ \mathrm{F} = \frac{70 \times \mathrm{w}}{\mathrm{S}_c} \quad . \quad . \quad . \quad (79c)$$

If the permissible rise of temperature be taken as about 50°–60° F., it will be seen that from 1·2 to 1·5 sq. inches of cooling surface must be allowed per watt, and such values of the ratio may frequently be found in practice. A greater rise, although perhaps permissible on the score of the final temperature reached, is seldom advisable owing to the difference which it causes in the exciting power according to the time during which the machine has been at work. If a number of separate bobbins are revolved at a high peripheral speed as in alternators of the type shown in figs. 76 and 77, a much higher ratio of watts to square inches is permissible, owing to the great cooling effect due to their rapid movement through the air.

§ 9. **Sources of heat in the armature.**—When we pass to the question of the final rise of temperature in an *armature*, the problem before us is far more complex, and this for two reasons. In the first place, it is difficult to calculate with accuracy the watts actually expended in heating the armature.

The sources of the heat are threefold, viz., (1) the loss of electrical energy due to the passage of the total current through the armature resistance; (2) eddy or Foucault currents set up in the copper winding, and in the armature core; and (3) the magnetic loss due to hysteresis. The first and third of these can be calculated with a close degree of approximation, but the second loss is very largely indeterminate, in as much as it is dependent on a number of variable conditions, such as the width of the armature inductors, the strength of the field, and the speed.

§ 10. **Importance of the eddy-current loss.**—In order that the reader may judge of the importance of the eddy-current loss the following table is given, which embodies the results of tests made on ordinary commercial machines, constructed by different well-known makers: the losses are those which occur when the dynamos are run at their rated outputs and speeds, and in each case the eddy-current loss has been determined by actual experiment.

No.	—	Dimensions of Core in Inches		Output in Kilo-watts.	Volts.	Amps.	Revs. per min.	Periph. Speed in ft. per min.	Watts.			Z_a in C.G.S. lines × 10⁸.	B_a	B_g
		Dia.	Length						C_aR_a	Hyst.	Ed-dies.			
1	Drum	20	36	120	135	900	375	2100	2300	694	2,730	30·6	11,400	5300
2	Drum	10	12	15	130	115	865	2420	520	187	530	4·65	17,500	4470
3	Ring	7¼	7	3·5	130	27	1250	2600	167	75	265	1·9	18,600	4200
4	4-pole Drum, slotted	21¼	11·8	31·4	114	275	520	2900	845	675	1,895	..	10,600	7500

To these may be added [1] the three following, of which full

No.	—	Dimensions of Core in Inches.		Output in Kilo-watts.	Volts.	Amps.	Revs. per min.	Periph. Speed in ft. per min.	Watts.			Z_a in C.G.S. lines × 10⁸.	B_a	B_g
		Dia.	Length						$C_a^2R_a$	Hyst.	Ed-dies.			
5	Drum	9⅝	20	35	110	320	780	2150	1160	215	585	10·95	13,500	6800
6	Ring	9½	9½	13	100	130	1050	2750	900	142	148	3·85	17,400	4600
7	Drum	7¼	15	11·2	140	80	880	1830	330	100	400	4·7	9,100	5200

---

[1] In the case of a discoidal-ring continuous-current armature, described by Mordey, *Journal Inst. Electr. Eng.*, xviii. 1889, p. 620, the watts expended

accounts have been published by Dr Hopkinson (reprinted in 'Original Papers on Dynamo Machinery,' pp. 79-133 and pp. 135-139); but in these cases the eddy-current loss is obtained by subtracting from the measured waste power in the armature the calculated hysteresis loss.

Thus the relative proportion of the three armature losses varies considerably in machines of different construction and size. As compared with the watts expended over the resistance of the armature, the hysteresis loss is relatively most important in the case of small machines driven at a high speed, and therefore having a high periodicity; in the case of large drum armatures coupled directly to the crank-shaft of the driving engine, not only is the periodicity low, but the induction is seldom pressed so high as in the case of smaller ring machines, and therefore the loss per cubic centimetre per cycle is less.[1]

But further, the figures above cited show that the eddy current loss is usually of considerable importance and as often as not is greater than the C_a^2 R_a loss; the precautions taken to minimise eddies have already been described in Chap. XIII, but their complete elimination is altogether exceptional; and it is clear that no formula can be of much value which does not specifically take them into account when calculating the rate at which heat is developed in the armature. If only the watts lost over the armature resistance and the hysteresis loss are calculated, a formula expressing the rise of temperature in terms of these watts can at best only apply to armatures of very similar construction in which the eddy-current loss may be treated as approximately proportional to the other two losses. This, however, is very far from being the case if the width of the inductor, or the value of the induction in the air-gap or armature, or the degree of lamination of the core, or the peripheral speed, be

---

in hysteresis and eddy-currents at 900 revs. per min. were respectively 291 and 1240. The designed output was 18 kilowatts, but it is not mentioned at what speed the dynamo was intended to be run.

[1] Alternators are not here under consideration, and in these the hysteresis loss is more important owing to the high periodicity. As a general rule, however, the armature is not worked at a high induction, and the effect of hysteresis is thereby lessened.

largely altered. Many of the published formulæ are either implicitly or explicitly applicable only to some more or less standard peripheral speed, such as 2000 ft. per minute; or, again, to smooth-surface armatures, and are therefore entirely unsuited to toothed armatures. In fact, an alteration which might at first sight seem unimportant in any one of the conditions above mentioned may radically affect the amount of the eddy-currents, and may therefore entirely vitiate any results calculated from an incomplete formula.

§ 11. **Effect of peripheral speed.**—In the second place, the rate at which a given armature can part with the heat generated within it is most materially affected by its speed of rotation, and also by the shape and number of the pole-pieces which surround it, in so far as they allow of or actively promote a more or less free circulation of the air over the surface of the winding. The exact effect of either of these causes does not admit of very accurate generalisation, since it necessarily varies under different conditions. How important a part is played by the peripheral speed of a rotating armature in increasing its cooling power is at once evident from a comparison of the watts which it can dissipate with the watts which a stationary field-coil can dissipate: if their cooling surfaces be reckoned on a similar basis, and the rotating armature have a peripheral speed of 2000 ft. per minute, roughly speaking, it will be found that the latter can dissipate at least twice as many watts per square inch of cooling surface for the same rise of temperature in the two cases. But even the increase in the cooling power, due to a given peripheral speed, depends largely upon the nature of the winding, the construction of the armature core, the disposition of the pole-pieces, and the ratio of their area to the total cooling surface.[1] If the surface of the armature winding be broken, so that the air can play upon more than the outer face of the inductors, or even reach freely to the core, the cooling action is increased. Again, the numerous poles of large multipolar machines break up the currents of air, and this effect is very marked in the case of discoidal-ring alternators, revolving between a number of poles on either side of the armature. It is therefore impossible to lay down any general formula, expressing the cooling power of a given surface at different speeds, which will meet the case of entirely different types of machines, and all equations connecting rise of temperature with a certain ratio of watts per square inch must necessarily be only approximate.

---

[1] See *Electrical Engineer*, August 10, 1894.

§ 12. **The loss over the ohmic resistance of the armature.**—Considering, in detail, the three sources of heating in an armature, the first, or the electrical loss over the copper resistance of the winding, is entirely independent of the speed, and is simply equal to the product of the square of the armature current and the resistance of the armature from brush to brush, *i.e.*, $C_a^2R_a$ watts. In the process of designing a dynamo the resistance of the armature can easily be calculated, and from it the loss of watts due to the passage of the normal armature current. It is only necessary to remark that if the resistance of an armature from brush to brush be actually measured, it will usually be found to be slightly higher than the resistance as calculated from the length and area of copper used. This discrepancy is due to the inferior conductivity of the soldered joints, even when carefully made. In this respect, armatures wound with former-shaped coils have the advantage that in them the number of soldered joints is reduced to a minimum; indeed with small wires or a multipolar machine joints need only occur at the unions with the commutator sectors. Owing to the low resistance of a large armature with few inductors, the most convenient method of measuring its actual value is by passing through it a known current of considerable strength and measuring the difference of potential between opposite commutator sectors, where the current leaves and enters. The quotient obtained by dividing the measured difference of potential by the known current is, of course, the resistance of the armature, and from our knowledge of this resistance, after making due allowance for its increased value when hot, the $C_a^2R_a$ loss can be accurately determined.

§ 13. **The eddy-current loss as depending on speed.**—It is in the second or eddy-current loss that most difficulty lies. With a given core, a given winding, and a given field, this loss will be directly proportional to the square of the speed; since if the speed be increased x times, the E.M.F. of an eddy will be equally increased, and this will increase the current of the eddy x times, so that the product of eddy E.M.F. × eddy current will be increased x^2 times. It may therefore be expressed by a coefficient multiplied by the square of the speed, the value of the coefficient being different in different cases. A brief description of the experimental methods by which the eddy-current loss in a given dynamo when run at different speeds can be measured with considerable accuracy will be found in Chap. XX §§ 8, 9. Two points alone require to be further mentioned. In the first place, the value of the eddy-current loss, as thus determined, includes any eddies set up in the pole-pieces of the magnets by the rotation of the armature. Such loss of power in the pole-pieces will be reserved for consideration in a later part of the present chapter, and does not here concern us. In continuous-current machines, having conductors wound uniformly over a smooth core, the loss in the pole-pieces is quite inappreciable, and it may therefore be assumed that in such cases all the eddy-currents which the above method measures are generated in the armature itself: but in toothed armatures the loss of watts in solid pole-pieces may require to be allowed for. In the second place, during the experiments, the distribution of the field is approximately the same as that at no load, and it has already been shown, in Chap. XVII, that this distribution may be, and usually is, considerably modified at full load. Hence the eddy-current loss at full load may be greater than the experiment would show, and when the results of such tests have been checked by other methods, it has been found in practice that this difference in the distribution of the field does sensibly alter the amount of the eddy-currents. We are, however, justified in assuming that the method above alluded to furnishes us with a ready means for determining, at least approximately, the eddy-current loss, and that we can thence calculate under given conditions the value of the coefficient, which, when multiplied by the square of the number of revolutions per minute, gives the loss in watts. Let F be the value of this coefficient for a given armature with a particular excitation; then the rate of loss by eddy-currents in the armature when run at N revolutions per minute is equal to FN^2 watts. There still remains the difficulty that, in calculating the watts expended in an armature, we must, during the process of designing, estimate the value of F from the coefficients of machines previously tested. In so doing, not only must any alteration in the length and volume of the core be taken into account, but, as previously stated, it is equally important to estimate the probable effects of alteration in the value of the induction in the air-gap or armature, the degree of lamination of the core, or the width of the inductors; in especial, the question of whether the inductors are stranded or twisted in order to eliminate Foucault currents will materially modify the value that should be assigned to the eddy-current coefficient. The subject needs therefore to be discussed in further detail.

§ 14. **Eddy-currents in the armature core.**—The eddy-current loss may be divided into (*a*) that within the armature core, (*b*) that in the copper winding, and (*c*) that in the binding wire on the circumference of the armature. To allocate to these sources their respective shares in the total loss is an extremely difficult matter, as any change in one of the conditions such as can be tried in a practical machine produces but a small change in the total which can scarcely be isolated; yet the combined effect of all the losses adds up to an amount which seriously reduces the possible output of an armature from the heating point of view.

Taking them seriatim, we have first the armature core with its supporting framework and surroundings. The eddy-current loss produced by an alternating field in thin iron plates such as are used in transformers and laminated armature cores is proportional to the square of the maximum induction and also to the square of the thickness of the plates, and the same law has been experimentally established by Mr. Holden[1] when the field rotates about a stationary armature or *vice versâ*. The energy consumed in an alternating field per cycle and per cubic cm. of iron in thin plates or discs is $\frac{1{\cdot}645 t^2 . f . B_a^2}{\rho} \times 10^{-16}$ joules, where t is the thickness of the plates in cm., f is the periodicity or frequency, and ρ is the resistivity of the iron. The latter quantity may be taken as from 1×10^{-5} to $1{\cdot}2 \times 10^{-5}$ ohms with wrought iron or thin sheet steel, whence the loss of energy per cycle and per cubic cm. is approximately $1{\cdot}645\, t^2 . f . \mathrm{B}_a^2 . 10^{-11}$ joules. The loss of power at any frequency $f = \frac{p\mathrm{N}}{60}$ in a total volume of v cubic centimetres of iron is therefore

---

[1] *Electrician*, vol. xxxv. p. 327.

$$1{\cdot}645\, t^2 . f^2 . \mathrm{B}_a^2 . \mathrm{V} \times 10^{-11} \text{ watts.} \qquad (80)$$

The same expression may also be applied to the case of a rotating field, but owing to the average E.M.F. being larger than in an alternating field, and the average length of path and its resistance being smaller, the first numerical constant requires to be increased to 3; its exact value being dependent to a certain extent upon the distribution of the field in the air-gap.[1] When the above formula is worked out, the watts, even in the case of a high-speed multipolar dynamo, are found to be so small that they may be neglected in comparison with other more serious losses, and in practice the actual loss from this cause would be even smaller, owing to any magnetic screening. The reduction in the eddy-loss through the increase of resistance when the iron becomes heated is about 5·5 per cent. for each 20° F. rise of temperature.

It must not, however, be assumed that the loss in the armature core as a whole is by any means negligible. The figures of Table I, Chap. XV § 6, show that the end-fringe from the flanks of the pole-pieces is quite appreciable, and unless the end-plates which clamp together the discs are kept well outside the edges of the poles the lines which curve round into the ends of the armature will set up considerable eddy currents. Again, in barrel-wound armatures the cylindrical structure which supports the end-connections of the bars should not be a solid sheet of metal, but as far as possible should be cut away near to the armature core so as to remove the iron into which the lines of the fringe might stray (compare figs. 140 and 141). Even the cast-iron end-plates where they approach to the surface of the core may advantageously be designed so that their edges are scalloped out into projecting teeth at frequent intervals, instead of forming a smooth circle of nearly the same diameter as the core. Next, the operation of turning the surface of a smooth armature-core increases the eddy loss by burring over the edges of the discs, and requires to be carried out with considerable judgment. Analogous to this in a toothed armature core is the operation of drifting or filing out the slots so as to remove any roughness or inequality of their sides. Any such mechanical treatment requires to be both lightly and cleanly executed with the minimum amount of injury to the laminations. On this account it is not advisable to mill the longitudinal grooves on the assembled core, and even when the notches are stamped in the discs before assembling they must be carefully inspected to see that they are free from burrs, by reason of which the rough edges of neighbouring discs would be driven into good contact with one another.

---

[1] See Prof. Baily, *Phil. Trans.*, 1896, vol. 187. A pp. 715–746; and Prof. E. Wilson, *Proc. R. S.*, vol. 70. p, 359.

§ 15. **Eddy-currents in copper winding of armature.**—(*b.*) Passing to the copper winding of the armature, and taking first the case of a smooth-surface core, the eddy-current loss within the inductors, when the machine is run on light load, is chiefly to be located in those which at any moment are moving between the poles, since it is only here that the field varies much in density in the absence of armature reaction. Considering one inductor, the eddy-current E.M.F. in it at any moment as it passes through the interpolar gap is proportional to the linear speed, to the length of the armature, and to the gradient of the field as it declines in value from B_g to zero on the line of symmetry. For a rough approximation on the assumption of $\frac{c}{l_g} = 4$ (Chap. XV § 6), the gradient may be assumed to be a straight line, starting with the full value B_g at a distance l_g within the pole-edge and ending in zero at a distance short of the symmetrical line by the same amount l_g; thus the slope for the same initial value of B_g varies inversely as the distance c between the pole-edge and the symmetrical line. The loss in the bar, being equal to $\frac{e^2}{r}$, is then proportional to the square of $\frac{B_g}{c}$, to the square of $\frac{\pi DN}{60}$, and to the first power of the core-length, since the resistance will also increase as the length of the armature; *i.e*, it is $\propto \frac{B_g^2 \times \pi^2 D^2 N^2}{c^2} \times L$. A little calculation then shows that if θ_2 be the interpolar angle in degrees, or $= \frac{360^\circ}{2p} - \phi^\circ$, the total number of bars at any one moment within all the interpolar gaps is $\tau \times \frac{2p}{360^\circ} \times \theta_2$, and since $c = \frac{\pi D \theta_2}{2 \times 360^\circ}$, the total loss in all the bars is $\propto \frac{B_g^2 . N^2 . \tau . 2p}{\theta_2} \times L$. If β is the ratio of ϕ° to $\frac{360^\circ}{2p}$, $\theta_2 = \frac{360^\circ}{2p}(1 - \beta)$, and the expression becomes reduced to $\propto \frac{B_g^2 N^2 . \tau . p^2 . L}{(1 - \beta)}$ where β is practically a constant. Thus the gradient varying inversely as the diameter, and the linear speed directly as the diameter, the two partially cancel one another, and the present loss is seen to be independent of the diameter of the armature. It is, however, proportional to the square of the number of poles; hence if a given two-pole armature is made multipolar, the armature bars being retained of the same width, and the winding being series-connected to give the same voltage as before, the eddy-current loss in the bars themselves will be much increased. If, however, the new armature be parallel-connected, although the number of bars must be increased to give the same voltage, the thickness of each is correspondingly decreased, and the problem becomes much more complicated. If the inductors were in all cases solid and rectangular, the loss in each bar should be proportional to the product of the eddy-current E.M.F. (which is itself $\propto t^2$) and the sectional area through which the current flows ($t h$); in other words, to the cube of the thickness and to the height, or $t^3 h$. Under these circumstances the loss in the multipolar armature when parallel-connected to give the same voltage would be precisely the same, whatever the number of the poles. It may, however, be taken for granted that the bars will be stranded, if their width exceeds the limit of size named in Chap. XIII § 20. We are then met with the difficulty that with lightly insulated and twisted strands it is almost impossible to determine the law connecting the dimension of the bar with the amount of loss due to the small eddy-currents which still persist in it. Apart from the loss in the separate strands, the nature of the insulation and the degree of compression as affecting the number of contact-points renders the result largely indeterminate. We can, in fact, only fall back upon a comparison of armatures wound with bars of different thickness, and make some approximate assumption as to the effect of varying dimensions. A round section is certainly less subject to eddy-currents than a rectangular section of equal width, and if comparison be made between a round wire and a solid square bar of the same height on the assumption that the loss in the former is $= k' t^4$, and in the latter is $= k t^3 h = k t^4$, the relative values of k' and k are as 1 to 1·69. It must then be remembered that the section of the square wire stands to that of the round as 1·27 : 1, so that its current-carrying capacity is greater, and a larger loss by eddy-currents in it may be expected. Simple division of the rectangular bar into a number of thin laminæ without any twist at the centre has some effect in decreasing the loss, although the reason is not very apparent; it is true that the eddy-current is forced thereby to flow along the whole length of the bar, but the E.M.F. acts summationally along this length, so that but little is thereby gained, and any reduction which there may be is probably to be connected with the fact that the field also varies somewhat in the opposite plane to that above considered.

It is not, however, only along the length of the armature core that eddy-currents are set up. Where the solid bars project beyond the core or where the stranded bars are soldered to their end-connectors in the bar-wound drum armature with separate connectors, the solid ends become the seat of eddies set up by the end-fringe from the poles. Further, when the end-connectors are broad strips of copper lying concentrically with the axis of rotation, they are subjected to the action of a field of density varying with their distance from the edge of the pole. The loss due to these two causes is perhaps more than is usually recognised, and furnishes an additional argument for making the axial length of the armature appreciably greater than that of the pole-face. All losses due to the end-fringe from the flanks of the poles are proportional not simply to B_g^2 but to $B_g^2 \cdot l_g$ or X_g^2, since the stray lines are caused by the difference of potential between pole and core; they are also proportional to D^2N^2, or the square of the linear speed, but are independent of the length of the armature core. Herein they are sharply distinguished from the eddies within the bars acted on by the interpolar fringe, and owing to this difference the increase in the loss which results from the adoption of a multipolar field with a given thickness of bar depends entirely upon the proportion of the total loss which is due to incomplete elimination of the eddies in the bars within the limits of the core-length. The above conclusions in the case of the smooth armature are amply borne out by experiment, which shows that the loss in the multipolar machine is greater than in the two-pole, but not so much as in proportion to the square of the number of poles.

§ 16. **Increase of eddy-current loss under full load.**—The distortion of the field by armature reaction under load has the effect of adding a further loss in the bars which are at any time under the poles. If B_0 is the armature cross induction at the pole-tips, the resultant field now varies from $B_g + B_0$ to $B_g - B_0$. The gradient of the field between the pole-tips is but little altered, and the loss in the interpolar bars is unaffected. There is, however, the additional loss proportional to $\frac{(B_0)^2 . \tau . 2p}{\phi}$, and the ratio of this quantity to the no-load watts is

$$\left(\frac{B_0}{B_g}\right)^2 \times \frac{\theta_2}{\phi}; \text{ or if } \frac{B_0}{B_g} = \frac{1}{2} \text{ and } \frac{\theta_2}{\phi} = \frac{0·25}{0·75},$$

the increase is of the order of about 10 per cent. Comparison of direct eddy-current tests with the efficiency measured by Hopkinson's method indicates that in smooth-core armatures such an increase allows a sufficient margin in practical working, but that in toothed cores the increase is probably larger.

§ 17. **Eddy-currents in toothed armatures.**—The case of the toothed armature is one of equal, if not greater, complexity. The concentration of the lines within the teeth enables solid bars of considerable width to be employed even in open slots, and the difference of density within any one slot in the interpolar gap is very much decreased. If, as is usually the case, the density of the lines at the bottom of a tooth is much greater than that at the top, some of the lines are forced out again into the air and pass through the lower layers of the winding. A deep solid bar may therefore not only be subjected to a varying field within the interpolar gaps, but may also be passing through different densities of field at the top and bottom of the slot. This is probably, however, to be discounted by the consideration that the lines will move across the slot with a velocity varying inversely as their density, so that the rate of cutting and the E.M.F. produced along any layer across the bar will remain the same as that of any other layer.

The interpolar loss in the bars appears in fact to be so much reduced that in the toothed armature there is but little effect directly traceable to any alteration in the number of poles. There is further a reduction in the value of l_g; but on the other hand, the end-fringe is now due to the combined effect of the magnetic difference of potential over both the air-gap and the length of path in the teeth, so that the end-loss is $\propto (X_g + X_{at})^2$ instead of X_g^2. The bars being solid, it results that the loss from the end-fringe is greater than in the stranded bar armature with smooth core. There is, too, a greater distortion of the field owing to the shorter air-gap and increased armature reaction, and this acts, if not upon the bars, at least upon the core. The eddy-currents in the core are probably also increased considerably by the greater surface exposed by the slots, along which the discs are in partial contact. Analysis of a number of experiments shows that with the same armature and winding the eddy loss in the toothed armature does not increase so rapidly as in proportion to B_g^2, and this is probably due to a certain amount of magnetic screening set up by currents in the teeth as well as in the pole-pieces, about which more will be said in § 27. On the whole, the loss in the slotted armature with solid bars is not far different from that in the equivalent smooth-surface armature with stranded bars.

§ 18. **Approximate formula for eddy-current loss in smooth armatures.**—Enough has been said to show the great difficulty, if not impossibility, of expressing in any simple formula the combined effect of the numerous variables which enter into the question of the amount of the eddy-current loss. The law which governs any one constituent may be unknown, or if known, the magnitude of its effect may be quite uncertain owing to its being dependent upon varying conditions in practice such as the turning of the core or the stamping of the discs. By some writers the total eddy-current loss has been placed on the same footing as that by hysteresis and has been expressed as a multiple of the latter. Or again, it is regarded as determined by the lbs. of iron in the armature core, the flux-density and the number of cycles per second, just as is the hysteresis, although not necessarily following the same law of rate of increase with increase of the induction. The basis of either method is, however, so far from the truth that any correspondence of their results with facts must remain entirely empirical, and when applied under changed conditions they can lay no claim to reliability. Yet the need of some such formula to the dynamo designer is so imperative that the required form of expression may at least be tentatively suggested as a starting-point for purposes of comparison, even if the constants may need to be afterwards corrected as experience accumulates.

The problem, therefore, is to construct a formula for F or the watts lost in eddy-currents per rev. per min. which will in some degree differentiate between the different localities of eddy-currents and the different causes of the loss of energy by them. In the case of smooth-core armatures, experiment shows that with given dimensions of machine every source of eddy-currents is nearly equally affected by an increase of the induction in the air-gap and that the loss of power practically varies as the square of B_g. The losses in the end-connections and solid ends of the bars and in the ends of the core due to the fringe issuing sideways from the flanks of the poles may be grouped together; since both will vary as $B_g^2 . l_g^2$ or as X_g^2. Further, they will vary as the bulk of the metal in question, which may roughly be regarded as proportional to the circumference or to D, and lastly, they will vary as the square of the linear speed or as D^2, so that they may be expressed as $\propto D^3 . X_g^2$. Next, the loss in the core itself and binding wires under the pole-pieces may be taken roughly as proportional to the bulk of the iron affected, *i.e.*, to the surface and to the radial depth within which any current-sheet under one pole-piece can pass to complete its circuit. The former will vary as DL and the latter as bD, where b is a constant decreasing from, say, 1 for two poles, to 0·6 for four, 0·435 for six, 0·333 for eight, down to 0·16 for twenty poles. Thus the core loss per rev. will vary as $b\,\mathrm{D}^2\mathrm{L}$, and in addition as the squares of the linear speed and the flux-density, or as $\mathrm{B}_g{}^2\mathrm{D}^2$. Both the above-considered losses are practically independent of the number of poles, save so far as the radial depth is concerned which decreases with an increase in the number of the poles although not quite so rapidly. The further loss in the armature bars in the interpolar gaps has been shown to be independent of the diameter, but to be proportional to the square of the number of poles, and in any one solid bar to vary as the length of the core, the cube of the thickness and the height. Combining the three terms, we have,

$$\mathrm{F} = \left\{\mathrm{D}^2(k_1\mathrm{X}_g{}^2 \,.\, \mathrm{D} + k_2 \,.\, \mathrm{B}_g{}^2 b\mathrm{D}^2\mathrm{L}) + k_8\mathrm{B}_g{}^2 . p^2 \,.\, \tau \,.\, \mathrm{L} \,.\, t^3 h\right\} \times 10^{-14}. \quad (81)$$

If the dimensions are reckoned in inches with the flux-density in C.G.S. lines per sq. cm. and it is assumed that θ_2 is about $0{\cdot}25\,\frac{360^\circ}{2p}$, the constants with solid bars have some such values as $k_1 = 0{\cdot}15$, $k_2 = 0{\cdot}008$, and $k_3 = 5100$, or the latter must be proportionately increased if θ_2 be reduced. A considerable influence is here assigned to the dimensions of the solid bars, but while the variation of the loss with their height and the cube of their thickness may be justified by theory and experiment, it is difficult to establish on theoretical grounds the reason for the magnitude of the loss in the bars at no load if strictly confined within the interpolar region. When, however, we pass to stranded bars, their nature forbids any direct dependence of the loss within them upon their outside dimensions, and the influence of these must be reduced by taking some such smaller expression as $t\sqrt{th}$; the equation then becomes

$$\mathrm{F} = \left\{\mathrm{D}^2(k_1\mathrm{X}_g{}^2\ \mathrm{D} + k_2 \,.\, \mathrm{B}_g{}^2 b\ \mathrm{D}^2\mathrm{L}) + k_8\mathrm{B}_g{}^2 p^2 \,.\, \tau \,.\, \mathrm{L} \,.\, t\sqrt{th}\right\} \times 10^{-14} \quad . \quad (82)$$

where $k_8 = 11$, while k_1 and k_2 remain unaltered.

§ 19. **Approximate formula for eddy-current loss in toothed armatures.**—In the case of toothed armatures when barrel-wound with solid wires or bars, the extent to which the latter will project beyond the core and be affected by the end-fringe will decrease with the number of poles, and may be taken as $\propto b\,\mathrm{D}$ where b has the same values as before. The sectional area of copper th will also come into play, and apart from the linear speed, the end-loss may be made $\propto (X_g + X_{at})^2 \, b\text{D}\, th$. The loss in the core and slots will be proportional to the length of the armature and its radial depth, and will also be affected by the cross section of the copper as to some extent determining the depth and proportions of the slots or, say, $\propto \text{B}_g{}^2 b\text{D} \,.\, th \,.\, \text{L}$. To these may be added a further term which varies as the length and as the first power of the induction, and which has the effect of rendering the total loss less than proportional to the square of the induction. An approximate expression therefore will take such a form as

$$\text{F} = \left[\text{D}^2\left\{k_1.(X_g + X_{at})^2.b\text{D}th + k_2 \,.\, \text{B}_g{}^2 \,.\, b\text{D}th\text{L}\right\} + k_3 \,.\, \text{B}_g \,.\, \text{L}\right] \times 10^{-14} \quad (83)$$

where $k_1 = 4$, $k_2 = 2$, and $k_3 = 7000$, all linear dimensions being expressed in inches as before.

In conclusion, it may again be repeated that direct laboratory experiment as to the various sources of eddy-currents and their suppression is much needed, and the lack of such a firm experimental basis must be the excuse for the approximate nature of the formulæ which have been above hazarded, and which can lay no pretence to any scientific accuracy.

In commercial work machines can seldom be designed so as to test separately the influence of the several varying factors, yet it is only to the more complete elimination of eddy-currents that we must look, if the efficiency and output of dynamos for a given mass of iron and copper are to be appreciably increased in the future.

§ 20. **Hysteresis loss in armatures.**—Returning to the third source of heat or the 'magnetic' loss by hysteresis, the amount of power spent in changing the direction of magnetisation of the core must be calculated, as explained in Chapter XII § 12. For a given maximum induction in the core it is simply proportional to the number of complete cycles per second, and may therefore be expressed by a coefficient, H, dependent on the volume and induction of the core, multiplied by the number of revolutions per minute, or in terms of the symbols of eq. 18, $\text{H} = h \,.\, \frac{p}{60} \, \text{V}_c$ and $\text{H}\,\text{N} = \text{H}_w$. The cross-magnetisation of the armature under load here also increases the loss as compared with that at no load. In toothed armatures the volume of the teeth must be calculated separately from the rest of the core and the maximum induction to which they are subjected under the influence of the armature cross ampère-turns be taken into account.

§ 21. **Predetermination of rise of temperature in armatures.**—Combining the three losses together, we obtain, as the total watts expended in heating the armature,

$$\text{W} = \text{C}_a^2\text{R}_a + \text{FN}^2 + \text{HN}\,; \quad . \quad . \quad . \quad (84)$$

and it is the ratio of this value to the cooling surface of the rotating armature that determines its rise of temperature. When the total watts are thus taken into account, the authors have found the following formula to give fairly accurate results for ordinary bi- or multi-polar continuous-current dynamos with ring or drum armature having a smooth exterior surface, and also for toothed armatures with barrel-winding up to peripheral speeds of 4000 ft. per min. The final increase of temperature in degrees Fahrenheit per watt per square inch or

$$t^\circ\text{F.} \times \frac{\text{S}_c}{\text{W}} = \frac{125}{1 + 0{\cdot}7\left(\dfrac{v}{1000}\right)^{1\cdot8}},$$

whence the maximum rise of temperature of the outside of the armature is

$$t^\circ\text{F.} = \frac{125 \times \text{W}}{\text{S}_c\left\{1 + 0{\cdot}7\left(\dfrac{v}{1000}\right)^{1\cdot8}\right\}} \quad . \quad . \quad . \quad (85)$$

where W = the total watts of the preceding equation, S_c = the cooling surface of the armature in sq. inches, and v = the peripheral velocity of the exterior of the armature in feet per minute. Fig. 271 shows the curve connecting together the varying numerical coefficient $t^\circ\text{F.} \times \frac{\text{S}_c}{\text{W}}$ with the value of the peripheral speed. The constant value of this when $v = 0$, or the armature is at rest, *i.e.*, 125, is higher than the average figure for stationary field-magnet bobbins, owing to the fact that the armature is more or less shut in by the pole-pieces which embrace the centre of the core; for any given type of machine it may be determined by measuring the rise of temperature for a given number of watts when the armature is placed in its appropriate field-system and is stationary.[1]

§ 22. **Method of reckoning the cooling surface.**—In bipolar machines the cooling surface, S_c, is reckoned for rings and drums alike as the external cylindrical surface ($\pi D_1 L_1$) where L_1 is taken from the outer edge of the commutator lug to the extreme opposite end of the armature, together with the area of the two ends $\left(2 \times \frac{\pi D_1^2}{4}\right)$. In the case of small bipolar rings in which the wire practically fills the internal space, and also in

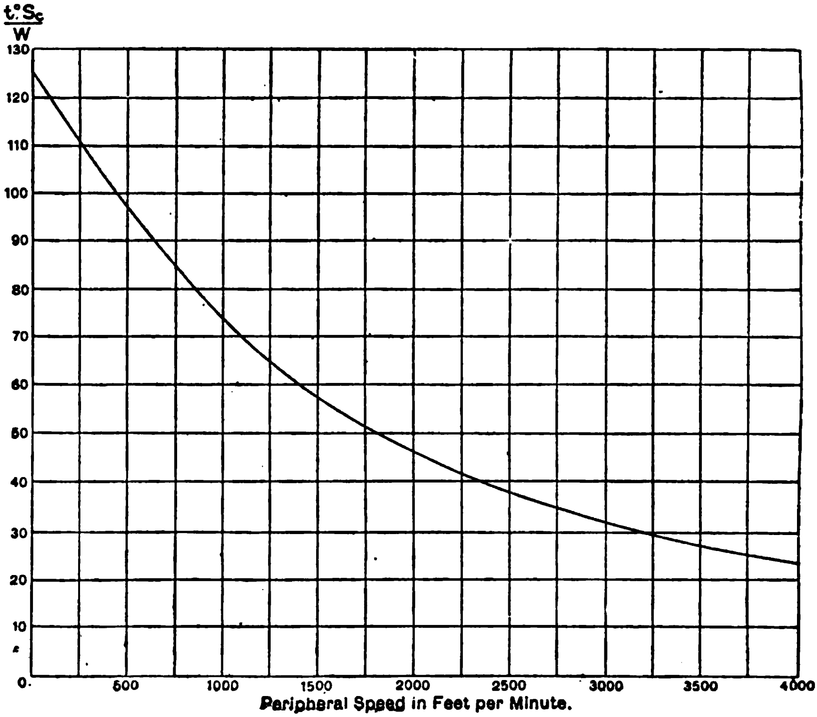


FIG. 271.

---

[1] See *Electrician*, October 11, 1895 (Wilson), 'The Heating of Dynamos.'

small hand-wound drums in which the winding is wrapped closely round the core without any means of ventilation, the cooling surface must not be reckoned as so great, and the area of only one end should be taken. In large multipolar drums with smooth core and evolute end-connectors, the area of one entire end and the annular ring occupied by the connectors at the other end $\left(=\frac{\pi D_1^2}{4}+\frac{\pi(D_1-D_2)^2}{4}\right)$ should be added to the external cylindrical surface. In barrel-wound multipolar armatures, there is but little difference between the external and internal diameters of the winding; the external cylindrical surface may then be taken together with the annular ring occupied by the radial commutator connection at one end. These latter have a very important effect on the cooling, since they are well exposed, and, if numerous, produce a considerable fanning action. In all cases, the question of the heating of the commutator must be also considered in relation to that of the armature. The above figures assume that the temperature of the commutator is kept well below that of the armature, and this is in practice always secured with a smooth-core and copper gauze brushes, so that the temperature of the far end of the armature is usually some 5° to 10° F. higher than that of the commutator end. With carbon brushes, however, the final temperature of the commutator often exceeds that of the armature. Hence the commutator, so far from helping to dissipate the heat of the armature, positively assists in raising the temperature of the near end. Again, in estimating the relative cooling value of any surface the general principle must be to allow approximately for differences in their degree of exposure to the air,—*e.g.*, the end-portions of the drum cylinder are of more importance than the centre which is under the poles,—and also for differences in their surface speed. Thus for different types of machines different methods of reckoning the cooling surface may be employed so as to agree with any particular curve, such as fig. 271, yet in the long run there is some one method which will give the most consistent results for a given line of machines, and this method can only be determined by practical experience.

For a peripheral speed of 2000 ft. per min. a rise of 65° F. is approximately obtained if the watts per sq. inch or $\frac{w}{s_c} = 1{\cdot}41$, or *vice-versâ*, if $s_c = 0{\cdot}71w$; in other words, nearly three-fourths or 0·75 of a sq. inch of cooling surface must be allowed per watt expended over the armature.

§ 23. **Effect of internal ventilation.**—In the above formula the cooling surface which has been taken into account has been confined almost entirely to the outside of the armature; yet in many dynamos the inside of the armature permits of the free passage of air into and through it. Especially is this the case with large ring armatures, in which the radial depth of the core is comparatively shallow, in order to allow of the requisite connecting wires passing through the interior; while in the case of multipolar drum armatures, the discs being supported on the arms of a hub, a clear air-space is secured between the internal circumference of the core and the cylindrical hub. The end-connections, especially if the armature be barrel-wound, become comparatively shallow, and being arranged at some distance from the shaft, the free circulation of the air through the inside renders its cooling effect more comparable with that of the outside. Again, in most cases, at intervals of some 3 to 5 inches along the length of the armature, distance-pieces of brass are arranged between neighbouring core-discs, or projecting bosses are stamped on the surface of certain discs so as to keep them entirely apart; free passages, say, from ⅜″ to ¾″ wide, are thus obtained from the inner air-chamber to the outside through openings on the surface of the core between groups of inductors, so that the air may be drawn out by centrifugal force. Owing, however, to the confined nature of such internal apertures and passages, and the low peripheral velocity of ordinary armatures running at moderate speeds, the draught of air which is thus set up through the interior openings is but small unless the diameter of the armature exceeds some 15 inches with a proportionately high peripheral speed, and unless the ventilating ducts are of considerable width. In toothed armatures it is especially important to keep the air-way through the teeth as free as possible, either by thin metal distance-pieces of channel or L section, or by twisting round the teeth of certain special discs until they stand at right angles to the plane of lamination. In some designs, the ventilating ducts are so numerous and so wide, that the effective length of the iron is only two-thirds of the gross length of the core. The general effect, however, of taking closer account of any internal cooling surface in the s_c of eq. (85) will merely be to raise the value which must be assigned to the coefficient when the armature is stationary.

It may here be remarked that a cool machine is by no means necessarily efficient; although in most cases these two desirable qualities are attained by the same means, still it should be remembered that, while the efficiency is dependent on the ratio of the lost watts to the useful output, the rise of temperature is determined by the ratio which the lost watts bear to the cooling power of the surfaces.

Finally, with a given armature, since the eddy-current loss is dependent upon the square of the revolutions, the amount of current that can be taken out of it at different speeds for a fixed rise of temperature depends largely upon the proportion of the copper to the eddy loss and upon the way in which the effectiveness of the cooling surface is modified by alterations of the peripheral speed.

§ 24. **Heating of the commutator.**—The heating of the commutator with carbon brushes has an importance second only to that of the armature. The sources of heat within the commutator itself are fourfold, viz.:

(1.) the loss of energy due to the passage of the armature current over the contact resistance of the brushes, this current being assumed to be divided between the several sets of brushes of the same sign and between the portions of any one brush in strict proportion to their areas; *i.e.*, on the supposition of a uniform current-density under the brushes.

(2.) the additional loss due to the unequal division of the current over the surface of the brush contact, and to sparking if commutation is not properly performed.

(3.) the loss from the mechanical friction of the brushes.

And (4.) the loss from eddy-currents in the sectors adjacent to those which are at any moment carrying the armature current. As the current flows along the sectors which are undergoing commutation and is gradually tapped off into the brushes, the neighbouring sectors find themselves situated in a stationary magnetic field, while they themselves are moving forwards. Eddy-currents are thereby set up in the mass of the copper plates forming the commutator.

When copper brushes are employed, these several losses are not sufficient to cause any great rise of temperature in the commutator, even though there may be undue sparking. But with carbon brushes the great increase which is possible in the first and second items renders it imperative for the designer to carefully consider the heating due to the combined effect of the four causes.

§ 25. **Conditions affecting the contact-resistance of carbon brushes.**—For a given peripheral speed of the commutator, the contact-resistance of the brushes per square inch of bearing surface depends upon the pressure, the current-density, and the condition of the commutator surface. The effect of these has been investigated by several experimenters, and especially by Prof. Arnold,[1] from whom the following results are mainly derived. With normal brush pressures and conditions of surface, the contact-resistance decreases rapidly as the current-density is raised from zero, but the rate of decrease falls off with higher current-densities, and the resistance approaches a more or less constant value under good conditions of working. This effect of an increasing current-density is much less marked when the commutator is at rest than when it is running, and it increases with the speed of rotation. Although the resistance when the commutator is stationary may be regarded as the real specific resistance of the contact, it has little value in practice, and the actual values under ordinary conditions of speed have alone to be considered. The reduction of the contact-resistance when the commutator is running and the current-density is increased, is then to be explained as due to the small carbon particles which are worn off the brushes, especially under a high current-density, when a blackening of the commutator results; a more intimate contact between the brush and the commutator surface is thereby obtained. For peripheral speeds from 1000 to 2000 ft. per min. and for average values of the brush pressure such as 1 to 2 lbs. per sq.

---

[1] *Die Ankerwicklungen und Ankerkonstructionen*, 3rd edit. pp. 281–287.

inch, fig. 272 shows approximately how the contact-resistance may be expected to decrease as the current-density is increased. With copper brushes, as might be anticipated, there is much less difference, and for any density over the very small value of, say, 40 ampères per sq. inch, the contact-resistance for a given speed is practically constant.

The influence of peripheral speed has next to be considered, and for a given pressure this in all cases increases the resistance as compared with a state of rest, owing to the vibration of the commutator and brush-holders which impairs the contact. If there is any want of complete smoothness of surface in the commutator (and such there must be in practice), the brushes are periodically subjected to momentary jerks as they pass the divisions of the sectors. There is therefore for any given commutator and brush-holders one particular speed at which the natural period of vibration of the brush-holders and the periodicity of the shocks imparted by unevenness in the commutator or by the strokes of the driving engine combine to produce the most unfavourable condition, and the contact-resistance reaches a maximum for the particular current-density in use. Whether this maximum is

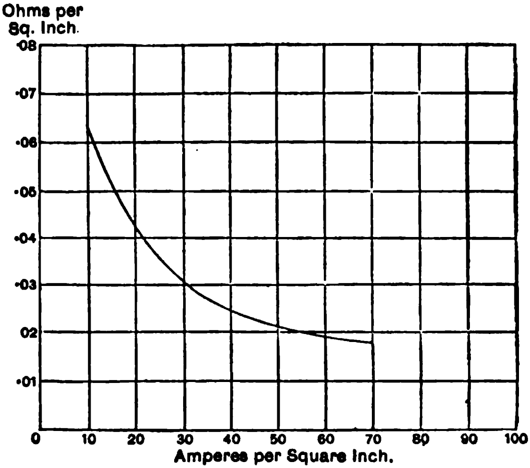


FIG. 272.—Contact-resistance of carbon brushes.

reached at high or at low speeds is entirely dependent upon the circumstances of the case, and admits of no law. It can only be stated that in general the heavier the brushes or their holders, the lower the speed at which the maximum value of the contact-resistance for a given current-density occurs. Yet in practice it is found that the lighter the moving parts of the brush-holders are, the better for all ordinary conditions of speed. Even with the same peripheral speeds, the actual number of revs. per min. have an influence, since the greater their number, the more likelihood is there of vibration. Owing to this, small high-speed machines are more liable to give trouble than large machines with commutators of large diameter running at a comparatively small number of revs. per min.

Increase of the brush pressure for any given speed causes better contact and decreases the resistance. As might be expected, this effect is more marked at high speeds since the pressure is then more useful in counteracting the vibration of the commutator. There is, however, a limit to the pressure that can be advantageously employed owing to the increase of the mechanical friction that results therefrom.

Lastly, the condition of the commutator surface has considerable effect in the case of carbon brushes. If the surface be bright, the contact is less intimate owing to the absence of the small conducting particles of carbon, while if the surface be oily, the resistance may be more than doubled especially if the current-density be low. The surface should therefore be free from oil, and though blackening is to be avoided, it should be allowed to retain the dull brown colour which continuous running will produce in the absence of sparking.

§ 26. **Data relative to copper and carbon brushes.**—Coming now to numerical figures for the various conditions named above, the brush pressure may be considered first. In a good dynamo running under ordinary conditions, the pressure per square inch with carbon brushes should range from 1¼ to 1¾ lbs.; and this may be tested by noting the pull required to lift the brush from the commutator with a small spring balance. In cases where the vibration is great it may become necessary to increase the pressure to 2 lbs. per square inch; except, however, at very high speeds there is but little reduction in the resistance if

2 lbs. be exceeded, and if possible the lower limit named above should be used. For copper brushes the limits of 1¼ to 1¾ lbs. suffice. The peripheral speed of the commutator may reach 3000 ft. per min., but preferably should not exceed 2500 ft., and with carbon brushes it is advisable to keep the speed below 2000 ft. per min. With copper brushes a current-density of 40 ampères per sq. inch is almost always exceeded, and under these circumstances the contact-resistance is dependent upon the speed and is practically constant for any particular speed. The specific running contact-resistance of copper brushes then varies between such wide limits as from 0·0007 to 0·0025 ohms per sq. inch, the latter value occurring when the periods of vibration of the commutator and brush happen to coincide. The average value of R_k for copper brushes may therefore be taken as 0·0016, and in no case is it likely to exceed 0·003 ohms per sq. inch.

Carbon brushes are made of various grades, and the selection of the best grade to suit given conditions of peripheral speed, current-density, and voltage, is largely a matter of experience proceeding empirically by trial and error. For general use the coarser and harder varieties are the most suitable for densities of 30 to 40 ampères per sq. inch. Where a greater conductivity is imperative, a graphite brush produced in an electric furnace is used with higher current-densities up to 60 ampères per sq. inch; the softer material is, however, more readily disintegrated, and the commutator surface becomes blackened and requires to be more frequently cleaned, while the contact-resistance to oppose sparking is less. Such brushes may therefore be regarded as to a certain extent intermediate between copper and the harder varieties of carbon which allow of a more fixed position for commutation. Further, they should not be used when several sectors are to be covered by the brush simultaneously. With carbon brushes the effect of peripheral speed becomes reduced as the current density is increased up to or beyond 60 ampères per sq. inch. Thus while the contact-resistance for a current-density of 10 ampères per sq. inch may vary from 0·014 to 0·05 for peripheral speeds within the limits of practice, the variation for a current density of 65 ampères per sq. inch will only be from 0·0116 to 0·016. It is not, however, possible in most cases to employ so high a current-density even as 60 ampères per sq. inch, and on the other hand, it is not economical to employ a lower density than 20 ampères. The limits of variation with different speeds are thus much reduced with normal current-densities such as from 30 to 40 ampères per sq. inch, and for these the contact-resistance may be taken as varying between 0·04 and 0·02 ohms. The average value of R_k for ordinary peripheral speeds, a brush pressure of 1½ lbs., and a current-density of 35 ampères per sq. inch, will therefore be 0·03 ohms per sq. inch.

§ 27. **Calculation of commutator losses.**—The commutator losses can now be calculated as follows:—

(1.) The contact-resistance of one set of brushes on one arm is $R_1 = \frac{R_k}{b . b_1}$, where b_1 is the length of the brush surface in one set measured parallel to the axis of rotation, and b is the width of their contact in the direction of rotation. If C_a be the total armature current, the loss in watts is $2C_a{}^2R_1$ in the bipolar machine, in the series-connected multipolar with two sets of brushes, and also in the parallel-connected multipolar with cross-connected commutator and two sets of brushes. If, however, there are as many sets of brushes as there are poles, whether the armature be series- or parallel-connected, the loss is $2C_a^2 . \frac{R_1}{p}$, since the contact-resistance of all the positive or all the negative brushes is $\frac{R_1}{p}$.

(2.) While the above gives the loss over the contact-resistance on the assumption that the current-density is strictly uniform, it must be remembered that in practice this condition will seldom hold. The additional loss due to want of uniformity of the density throughout the whole period of commutation can only be determined in each individual case, and then only if all the necessary data as to the shape of the short-circuit current curve are known. It may be remarked, however, that such want of uniformity cannot reach any great amount with copper brushes owing to the sparking that would result, but with carbon brushes it may form a very considerable item in the total without the production of prohibitive sparking or overheating of the brush edges. The advantage of carbon in reducing the sparking is in fact secured at the expense of the efficiency of the machine.

(3.) If p be the pressure on the brushes in lbs. per sq. inch, and a be the total area of surface of all the brushes on the machine, the total pressure is $\mathrm{P} = pa$, and the loss in watts from the mechanical friction is

$$\frac{\mu \mathrm{P} \times v \times 746}{33{,}000} \quad . \quad . \quad . \quad . \quad . \quad (86)$$

where v is the peripheral speed in ft. per min., and the coefficient of friction, μ, may be taken for both hard carbon and copper brushes as 0·3 when under good conditions of working. With softer graphitic brushes the coefficient of friction is probably somewhat higher, and may amount to 0·45.

(4.) Definite experiments as to the magnitude of the fourth loss from eddy-currents are wanting, but probably they are but small, although increasing with increased thickness of the commutator plates.

§ 28. **Temperature rise of commutator.**—The rise of temperature of the commutator surface should preferably not exceed 70° F. or as a maximum may be allowed to reach 80° F. in continuous running at full load. It may be calculated by a formula similar in its construction to that for the rotating armature. The constant of the numerator which determines the rise per watt per sq. inch when the commutator is at rest is lower than in the armature owing to the better exposure of the former to the air. On the other hand, owing to its smooth surface the influence of the peripheral speed is not so marked as in the case of armatures, although still considerable if there are separate connectors to the armature winding. If W = the total watts expended over the commutator, and the cooling surface be reckoned in sq. inches as the external cylindrical surface plus the area of one side of the radial connectors up to a limiting length of say 4″ from the commutator surface

$$t^\circ \mathrm{F.} = \frac{100\,\mathrm{W}}{\mathrm{S}_c \left\{ 1 + 0{\cdot}3\left(\frac{v}{1000}\right)^{1\cdot3} \right\}} \quad . \quad . \qquad (87)$$

where v is the mean peripheral speed in feet per minute.

Any such formula is, however, liable to many disturbing conditions, among which especial importance must be given to the number and shape of the connections which lead from the armature winding to the sectors. If these are numerous and are thin but wide blades of copper, they have a powerful fanning action, which very greatly assists in dissipating the heat of both the commutator and the armature.

Lastly, in some cases with large commutators the inside can be arranged so as also to be exposed to the air, with a consequent reduction in the rise of temperature.

§ 29. **Eddy-currents in pole-pieces.**—It remains to consider the question of eddy-currents as set up in the pole-pieces when a rotating toothed armature causes the density of the field over their bored face to be rhythmically varied. Such currents do not spread to any great depth within the metal mass, but whirl round near the surface facing the armature. One complete cycle of varying induction corresponds to the passage of one tooth and one slot past a fixed point on the pole-face; or, in other words, a period corresponds to the time taken by the armature in moving through the pitch of the teeth ($=k_{t_1}+k_s$). If $N_t =$ the total number of teeth, the periodicity is $f=\frac{N}{60}\cdot N_t$. If the local paths had no inductance, the currents would run along the pole-face opposite to each projecting tooth, and then dividing would curve round to complete their circuit opposite the slots. Owing, however, to the action of the inductance which causes the current to lag behind the impressed E.M.F., the positions of the eddies are displaced relatively to the teeth; if the reactance ($2\pi f L$) were very high or the resistance very low, the angle of lag would approach a quarter of a period, so that the currents would embrace the teeth and openings of the slots. The self-induced flux would then act to reduce the density opposite the teeth, and to raise it opposite to the slots. Thus by the effect of magnetic screening the distribution is rendered more uniform, and the eddy-currents are prevented by their own inductance from reaching any great amount.

With ordinary speeds and a considerable number of teeth as in practical cases, the periodicity of the alternating currents set up is so high that the screening action has the effect of rendering the general law of the proportionality of the eddy-current E.M.F. to the speed, and of the loss to the square of the speed no longer true in the case of solid pole-pieces. The case of a magnetic brake in which all the work is expended in producing eddy-currents in a solid mass then becomes nearly analogous, and in such a brake it is found[1] that the loss or the energy absorbed is proportional to a low power of the speed such as the 1·2 power.

The amount of the eddy-current loss depends entirely upon the nature of the curve of the non-uniform distribution. The extreme limit is reached if we assume a uniform current to flow across the pole through a strip of width equal to a quarter of the pitch or $\frac{1}{4}(k_{t1}+k_s)$ cm., and to complete its circuit by returning through another adjacent strip of the same width, and further, if we assume the maximum B_g to hold throughout the first strip and the minimum B_g throughout the second strip. The E.M.F. in the small strip is proportional to its peripheral speed, to the difference in the inductions and to the length of the pole-piece, L. The peripheral speed $\frac{\pi DN}{60}$ may also be expressed as $N_t(k_{t1}+k_s)\cdot\frac{N}{60}$, and the E.M.F. acting round the entire circuit is then

$$N_t(k_{t1}+k_s)\frac{N}{60}\cdot(B_{max}-B_{min})L\times 10^{-8} \text{ volts}$$

or half this value acting along the one strip. The resistance of the single strip to a distance h cm. within the pole-piece is $\frac{L\cdot\rho}{\frac{1}{4}(k_{t1}+k_s)\cdot h}$, where ρ is the resistivity in ohms of the material of which it is composed; the resistance of the end or cross portion of the circuit is regarded as negligible as compared with that of the length. The loss of power in the one strip is therefore in watts $\frac{E^2}{R}=$

$$N_t^2(k_{t1}+k_s)^2\left(\frac{N}{60}\right)^2\left(\frac{B_{max}-B_{min}}{2}\right)^2 L^2\times\frac{\frac{1}{4}(k_{t1}+k_s)\cdot h}{L\cdot\rho}\times 10^{-16}$$

and as there are $\frac{A}{\frac{1}{4}(k_{t1}+k_s)}$ strips in the width of the pole, where

---

[1] Dettmar, *E.T.Z.*, vol. xxi. pp. 947, 948.

A is the pole-width in cm., the total loss in the pole for the depth h is

$$\mathrm{N}_t^2(k_{t_1} + k_s)^2\left(\frac{\mathrm{N}}{60}\right)^2\left(\frac{\mathrm{B}_{max} - \mathrm{B}_{min}}{2}\right)^2\frac{\mathrm{AL}h}{\rho} \times 10^{-16} \text{ watts,}$$

ALh being the volume of the portion under consideration (fig. 273). Further inside the pole-piece the flux will rapidly become more uniform, and the watts, being proportional to the square of the difference of the induction, will diminish still more rapidly. As a rough approximation, a second pair of values for B_{max} and B_{min} might be taken on the assumption that the air-gap is increased half as much again; then another pair with the air-gap assumed to be doubled, and so on. Following a formula given by Mr. Carter (*Electr. World and Engineer*, vol. xxxviii. p. 886), the amplitude of the variation of the induction or $\frac{\mathrm{B}_{max} - \mathrm{B}_{min}}{2}$ may be expressed as $= m\left(\frac{1}{2} - \frac{1}{\sqrt{4 + \left(\frac{k_s}{l_g}\right)^2}}\right)$ times the average induction, where m has the same meaning as in Chap. XV § 13. The depth of each strip must then be made equal to half the single air-gap, and the total loss in all the poles will be

$$\mathrm{N}_t^2(k_{t_1} + k_s)^2\left(\frac{\mathrm{N}}{60}\right)^2\frac{\mathrm{V}}{\rho}\sum\left(\frac{\mathrm{B}_{max} - \mathrm{B}_{min}}{2}\right)^2 \times 10^{-16} \text{ watts} \quad . \quad (88)$$

where V is the volume of a layer in every pole of depth $\frac{1}{2}\ l_g$ cm. While the value of ρ may be taken as 1×10^{-5} for annealed wrought iron, it rises to, say, 2×10^{-5} for cast steel, and to 10×10^{-5} for cast iron. The inferior electrical conductivity of cast iron is therefore decidedly advantageous in reducing the eddy loss to one-fifth of its value with solid cast steel.

While the above may be regarded as giving an extreme limit for the eddy-current E.M.F. and loss, the formula is easily extended to the case where the fluctuation of the density is assumed to be sinusoidal. The range of induction being as before, from B_{max} to B_{min}, the maximum E.M.F. acting round the edges of the circuit corresponding to half the pitch of the teeth is $\propto \mathrm{B}_{max} - \mathrm{B}_{min}$, or along the one edge $\propto \frac{\mathrm{B}_{max} - \mathrm{B}_{min}}{2}$; thence it gradually diminishes after a sine law to zero midway between the two edges. The loss of power is proportional to the square of the E.M.F., and the average value of the squares of the ordinates of a sine curve is half the square of the maximum value (Chap. VI § 17). Thus the total loss is exactly half of that given by eq. (88). Finally, a still closer approximation may be made by again returning to the actual curve of distribution and drawing through it a straight line (shown in fig. 273 ii.) corresponding to the average value of B_g as calculated from the formula at the end of Chap. XV § 13. The actual flux may then be regarded as produced by the superposition upon the straight line of an alternating flux. This alternating flux may be replaced by an equivalent sine-wave (fig. 273 iii.) having the same effective value B_e and giving a certain maximum value B' which is not quite the same as $\frac{B_{max} - B_{min}}{2}$; the value of B_e^2 or,

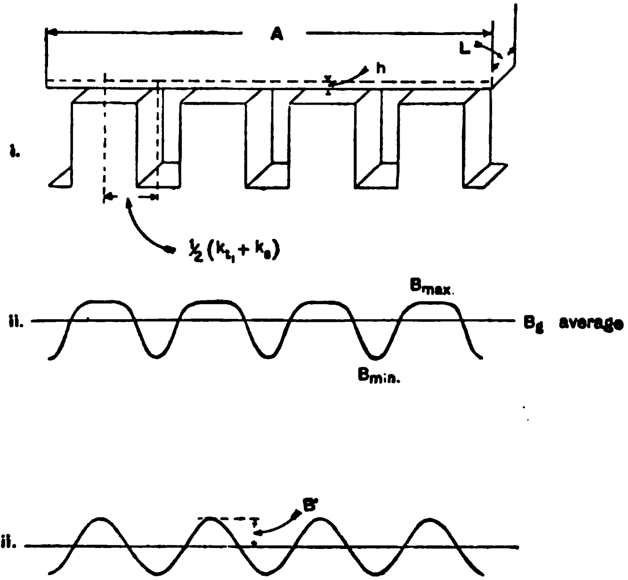


FIG. 273.—Eddy loss in pole-pieces.

which amounts to the same, of $\frac{1}{2}$ $(B')^2$, calculated for each strip as before, must then be substituted in eq. (88) for $\left(\frac{B_{max}-B_{min}}{2}\right)^2$.

Since the eddy-currents curve round in the mass of the pole and especially at the ends of the pole-face, it is evident that they may be partially reduced by axial slits along the pole-face, provided the thickness of the subdivisions is less than $\frac{1}{2}$ $(k_{t1}+k_s)$; but any such subdivision cannot be so effective as the more usual plan of laminating the pole in a direction at right angles to the axis of rotation, just as the armature is itself laminated. Taking two zones in the thickness of the lamination, each distant x cm. from the centre of the lamination, and itself dx wide, the E.M.F. acting across between them in the one half of the circuit is proportional to $2x$ and to $B' \sin \alpha$, where α is the phase angle of the assumed sinusoidal induction at the spot in question. Neglecting the resistance of the cross path by comparison with the lengths along the sides, the resistance of the half rectangular circuit is itself dependent upon the angle α and is

$$\frac{\frac{1}{2}(k_{t1}+k_s)\times\frac{\alpha}{\pi/2}\times\rho}{dx\,.\,h}.$$

The watts in the elementary half rectangular circuit are therefore

$$N_t^2(k_{t1}+k_s)^2\left(\frac{N}{60}\right)^2(B')^2\sin^2\alpha\,.\,4x^2\times\frac{dx\,.\,h\,.\,\pi}{(k_{t1}+k_s)\rho\,.\,\alpha}.$$

The integral $\int x^2dx$ between the limits $x=0$ and $x=\frac{t}{2}$, being $\frac{t^3}{24}$, and the integral $\int\frac{\sin^2\alpha}{\alpha}$ between the limits $\alpha=0$ and $\alpha=\frac{\pi}{2}$ being $0{\cdot}835$, the loss in the thickness of one lamination and in one strip corresponding to a quarter of the pitch is

$$0{\cdot}835\,.\,\frac{\pi}{6}\,.\,N_t^2(k_{t1}+k_s)\left(\frac{N}{60}\right)^2(B')^2\,.\,t^3\,.\,\frac{h}{\rho}.$$

Multiplying by the number of laminæ $=\frac{L}{t}$, and by the number of strips $\frac{A}{\frac{1}{4}(k_{t1}+k_s)}$, the total loss in a layer h cm. deep is

$$1{\cdot}75\left(N_t\,.\,\frac{N}{60}\right)^2(B')^2\,.\,t^2\,.\,\frac{V}{\rho}\times10^{-16}\text{ watts,}$$

or nearly the same as the expression (eq. 80) for the loss in an alternating field.

With solid pole-pieces, magnetic screening so largely reduces the loss, that calculations based on the preceding formulæ usually give values far higher than are found in practice, and they can only be regarded as illustrating some of the elements of the problem. The introduction of screening renders the formula so complex, that it can hardly be taken into account in an elementary treatment of the subject.[1] Its general effect is to render the loss proportional, say, to $\left(N_t \cdot \frac{N}{60}\right)^{1.5}$ rather than to the square of the frequency.

In order that the eddy-current loss in solid poles may be kept within reasonable limits, it has often been stated that the ratio of the width of the opening of the slot to the single air-gap or $\frac{k_s}{l_g}$ must be $\leqq 2$. The eddy E.M.F. is certainly dependent upon the variation of the induction, and so varies more or less proportionally with the ratio $\frac{k_s}{l_g}$. But if the zones of iron through which the currents flow past the slot are small as compared with the widths of the teeth over which the induction is practically uniform, the total eddy loss is but small over the pole-face as a whole. Hence if the proportion of the widths of the teeth to the total pole-face or $\frac{k_{t1}}{k_{t1} + k_s}$ is large, and approaches its limiting value of unity, l_g can be small or the ratio $\frac{k_{t1}}{l_g}$ may be large. There is therefore for each value of the ratio $\frac{\text{width of tooth}}{\text{pitch}}$ a maximum limit that the ratio $\frac{k_{t1}}{l_g}$ should not be allowed to exceed, and a curve connecting the two has been published by Dr. F. Niethammer,[2] which is here reproduced in a slightly modified form (fig. 274). Thus with a given pitch and width of tooth, l_g must be increased until $\frac{k_{t1}}{l_g}$ falls below the limiting value of the curve. Or with a given diameter of armature and a given l_g, the number of teeth must exceed a certain value; and if the number of slots thus obtained is inconveniently high, l_g must itself be increased.

---

[1] See Niethammer, *E.T.Z.*, vol. xx. p. 767.

[2] *E.T.Z.*, vol. xxi. p. 549.

The influence of a change of the normal induction in the air-gap admits of no general statement, since it depends upon the ratios of widths of slot and teeth. In the case of a magnetic brake the total work with a constant speed and varying induction is proportional to the 1·6th or 1·8th power of the induction or of the total flux, and since the hysteresis loss is but a small proportion of the whole, it is evident that the loss from eddy-currents in a solid pole increases by some power of the induction less than the square. Experiment[1] may be made on this point by winding

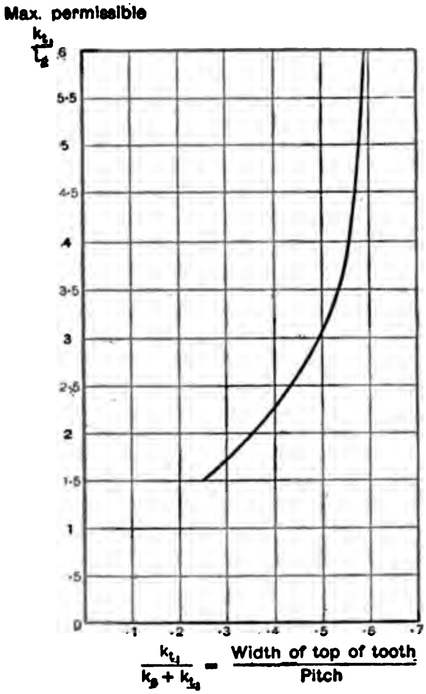


FIG. 274.

---

[1] Dettmar, *E.T.Z.*, vol. xxi. p. 948.

a number of coils across the pole-face at such distances apart as to correspond to the pitch of the teeth; the alternating voltage set up in the exploring coils for different values of the induction can then be measured. When tested in this manner, a machine in which the air-gap was very little smaller than the width of opening of the slot showed that the eddy-current loss in the pole varied as the 1·7th power of the induction, while in the case of a machine in which the air-gap was 1·86 times the width of the slot, the distribution, as might be expected, was so nearly uniform that no effect could be detected except by a telephone.[1]

In every case when the field is distorted by armature reaction under full load, the eddy-current loss is increased, not only in the armature but also in the pole-pieces; the amplitude of the variation of the field is increased at the trailing edge, and decreased at the leading corner, but as the loss is proportional to the square of the amplitude, there must be a net increase in the loss. With a short air-gap and wide open slots as in the Brush arc-lighting machine, the effect is so great that the increased heating of the trailing corners is distinctly marked.

[1] An additional source of loss by eddy-currents in the field-magnet has been suggested by Dr. W. M. Thornton in a paper republished in *Electr. Eng.*, Supplement to Dec. 5th, 1902; the subject deserves further investigation as a possible explanation of some part of the difference between the actual and the estimated losses by eddy-currents.

CHAPTER XX

THE DESIGN OF CONTINUOUS-CURRENT DYNAMOS

§ 1. **Range of speeds in practical use.** — The various parts of the dynamo having now been considered in detail, our results may be brought to a focus in the complete design of a machine. The practical art of designing is a matter of striking a balance between a variety of conflicting considerations, all of which are of importance in different degrees, and each of which will vitally affect the entire design of the machine. Thus a dynamo must be efficient, yet at the same time it must not be too costly to manufacture; it must be compact, yet well ventilated; thoroughly strong, yet not too heavy. Any one feature, however desirable in itself, will, if carried to excess, have some disadvantageous consequence in another direction, and he is the best designer who can effect a series of compromises, such that, while, each consideration is given its proper weight, none are forced into undue prominence, and a design well-balanced as a whole results from his practised judgment.

Since the dynamo is primarily a generator of electric pressure, the first consideration must be the production of the requisite volts. This, being a question of the rate of cutting lines, will depend on the speed, the number of inductors, and the number of lines passing through the armature. In the majority of cases, for a given output, the speed may be taken as fixed: either it is directly specified, or it is to a great extent settled by recognised practice or questions of mechanical strength and durability. Thus, to take the case of an ordinary continuous-current dynamo, either it will be belt-driven, or it will be engine driven. In the former case it will usually fall into one or other of two series, running at speeds which may by comparison be regarded as respectively high and low. In the latter case its speed may again be classified as high or low, according as the engine is of the enclosed class, single-acting or double-acting with forced lubrication as the case may be, or of the open type so largely used for marine or mill work. Hence, although makers vary considerably in the speeds which they select for different outputs, yet there is sufficient agreement to enable a table to be drawn up giving average values for the speeds of the different classes.

Output in Kilowatts.	Speed in revs. per min.				
	Belt-driven.		Engine-driven.		
	High.	Low.	Enclosed.		Open.
1·5	1500	850			
5	1200	750			330
10	1000	700	550	Single cylinder	300
15	900	650	535		275
20	800	600	520		250
30	700	500	575	Two-crank	330
40	600	450	550		300
60	550	420	520		275
100	480		475		250
150	430		450		225
200			375		200
300			320	Three-crank	150
500			275		120
750			230		100
1000			200		90
1500			180		85

§ 2. **Determination of length and diameter of armature core.**—Suppose that it is desired to design a small high-speed machine to give an external current of 140 ampères with a pressure of 103 volts at its terminals when running at 950 revs. per min. For such an output a 2-pole drum-wound machine would be suitable; the exact type will depend on the class of machine constructed by the maker, but we will here assume that a 2-pole drum-armature with over-type horseshoe field has been decided upon.

The determination of the best diameter and length of armature core, without reference to other designs or machines previously built, can only be effected by a method of trial and error. The speed being fixed, the peripheral velocity which is usually regarded as permissible in the desired type of machine gives a rough idea of a reasonable diameter of core; although, as a matter of fact, the speed has in the first instance become more or less settled by the peripheral velocity which results from the diameter of core that is best employed in ordinary machines. Assuming, then, a certain diameter of core, D, and an average value for the polar angle, ϕ, we can estimate the polar arc, and adopting a usual proportion of length of core, say, 1¼ or 1½ times D, we can calculate the effective area of the air-gap a_g, making due allowance for the fringe (Chap. XV § 6). Taking a normal value of B_g, we thus arrive at $Z_g = B_g\, a_g$, and thence the requisite value of τ for the given speed follows of necessity. Dividing the available circumference of the armature by τ, the size of wire that can be employed is found, and the resistance of the armature from brush to brush can be calculated; the question of one or more layers on the outside, as allowed by the air-gap, and of room on the inside in the ring armature, must also be considered. At this point the full-load current is a decisive factor, the output of any continuous-current dynamo being limited by the two considerations of heating and sparking, either or both. If, therefore, we find either the total watts lost in the armature or the ampère-wires per pole too great, the only course open to us is to increase either the length or diameter of core, or both, the value of Z_g or Z_a being increased, and the value of τ being proportionately reduced. By a series of trials such a mutual relation between τ and Z_a is reached as will give a suitable size of armature and a winding of practicable nature which will not overheat nor give rise to sparking.

§ 3. **The output per rev. per min. as proportional to the length of core and the square of its diameter.**—The calculations can, however, be simplified and shortened by an approximation which at once gives a preliminary idea of the size as fixed by the two leading dimensions. It has been shown in Chap. XVIII § 15 that in practice there is a permissible number of ampère-wires per unit length of armature circumference beyond which the limits imposed by heating and sparking are exceeded. If this value per inch of circumference be a_w, and the current in any one wire is $\frac{c}{q}$, $a_w . \pi D_{''} = \tau . \frac{c}{q}$. The E.M.F. is by eq. (14) $E_a = \frac{2}{q} . z_a . \frac{pN}{60} . \tau \times 10^{-8}$, whence the total watts per rev. per min. are

$$\frac{E_a . C_a}{N} = \frac{2}{q} . z_a . \frac{p}{60} . \tau \times 10^{-8} \times \frac{q . a_w \pi D_{''}}{\tau}.$$

Substituting for z_a the equivalent expression $\frac{\pi D_{''}}{2p} . \beta . L . B_g \times 6{\cdot}45$, we have

$$\frac{E_a . C_a}{N} = \frac{\pi^2 D_{''}^2 L_{''} a_w . \beta . B_g}{60 \times 10^8} \times 6{\cdot}45 \quad . \quad (89)$$

Since β, B_g, and a_w are all more or less constant, the total watts per rev. per min. are proportional to the square of the diameter and to the length of the core, or $\frac{E_a C_a}{N} \propto D^2 L$. This important result requires, however, some further explanation. The constant value of a_w for armatures of somewhat similar size was really derived from the assumption of a certain ratio of air-gap to the diameter as required to avoid sparking; it assumes also a certain depth of copper which practically fills up the air-gap in order to avoid overheating. The watts per rev. per min. for a given type of armature and number of poles are therefore only proportional to the square of the diameter and the length of the core if the depth of the copper envelope in the smooth-core machine approximates to the length of the air-gap, and both are increased with the increase of diameter. The result is then easily followed from first principles. If the length of the core and pole are increased, while the thickness of the copper envelope and air-gap are unaltered, the volts are increased in the same proportion, while the ratio of the heat generated per unit length of the core to the cooling surface along that length is unaffected for the same current. Yet as the end-connections which are exposed to the air and play the greatest part in the cooling have not been increased, the ampères must be somewhat decreased for the same rise of temperature, and the increase of output is a little less than in proportion to the increased length. If the diameter of the core be increased, more inductors of the same size may be wound on, without affecting the ratio of the heat generated in them to the cooling surface; at the same time the flux may be equally increased, so that the voltage is proportional to the square of the diameter. Or if the number of inductors be kept the same, and their width be increased, the volts are increased simply in proportion to the diameter, and the ampères can be increased until the heating again bears the same ratio to the new cooling surface. In both cases the larger diameter gives increased cooling effect, but this is offset by the greater eddy loss. But now either of the last two alterations have caused the ampère-wires per pole to be raised, possibly beyond the sparking limit; if so, they must then be accompanied by a further change in the length of the air-gap. This will allow the depth of the copper to be increased, and the ampères to be raised until approximately they have the same value as before the original change of length, so that finally the output becomes closely proportional to the length of core and to the square of the diameter. *Mutatis mutandis*, the same general principles apply to the toothed armature; as the length of the air-gap is increased, it may also be possible to increase the depth of the slots and the volume of the copper, although other considerations limit the possibilities in this direction. Whether the armature be smooth or toothed, if it is not thought advisable to increase the length of the air-gap, the number of poles must be increased, and as the diameter is then increased, the watts per rev. per min. may be raised in proportion to the square of the diameter. Thus as a preliminary generalisation it may be held that in all cases the watts per rev. per min. are $\propto \mathrm{D}^2\mathrm{L}$.

§ 4. **Formulæ for approximate size of armature core.**— If the values for the constants of eq. (89) are $\beta = 0{\cdot}75$, $\mathrm{B}_g = 5250$, and $a_w = 255$ per inch or 100 per cm. of circumference as in the case of a ring armature, we have

$$\frac{\mathrm{E}_a \cdot \mathrm{C}_a}{\mathrm{N}} = 0{\cdot}01065\ \mathrm{D}^2_{''}\ \mathrm{L}_{''} \text{ or in centimetres} = 0{\cdot}00065\ \mathrm{D}_{cm}\ \mathrm{L}_{cm}.$$

If then λ be the ratio $\frac{\mathrm{L}}{\mathrm{D}}$, for which suitable values have been given in Chap. XIV § 17, $\frac{E_a C_a}{N} = 0{\cdot}01065\lambda \,.\, D^3_{''}$, whence in round numbers

$$D_{''} = 4{\cdot}5 \sqrt[3]{\frac{E_a \,.\, C_a}{N} \,.\, \frac{1}{\lambda}} \quad . \quad . \quad . \quad (90)$$

If the ring is loaded to its maximum, and $a_w = 320$

$\frac{E_a C_a}{N} = 0{\cdot}0133\ D^2_{''}\ L_{''}$ or $0{\cdot}00081\ D^2_{cm}\ L_{cm}$, whence

$$D_{''} = 4{\cdot}25 \sqrt[3]{\frac{E_a \,.\, C_a}{N} \,.\, \frac{1}{\lambda}} \quad . \quad . \quad . \quad (91)$$

In the case of a drum armature of moderate size or of high voltage, a_w will average about 380 per inch of circumference, whether it be smooth or toothed, and with $B_g = 5600$ in the former case and 6000 in the latter, $\frac{E_a \,.\, C_a}{N} =$ from $0{\cdot}017$ to $0{\cdot}0182\ D^2_{''}\ L_{''}$, or from $0{\cdot}00104$ to $\cdot 00111\ D^2_{cm}\ L_{cm}$, whence

$$D_{''} = 3{\cdot}9 \text{ to } 3{\cdot}8 \sqrt[3]{\frac{E_a \,.\, C_a}{N} \,.\, \frac{1}{\lambda}} \quad . \quad . \quad . \quad (92)$$

while if $B_g = 6000$, and $a_w = 480$ as in machines of low voltage or of large size $\frac{E_a \,.\, C_a}{N} = 0{\cdot}023\ D^2_{''}L_{''}$ or $0{\cdot}0014\ D^2_{cm}L_{cm}$, whence

$$D_{''} = 3{\cdot}5 \sqrt[3]{\frac{E_a \,.\, C_a}{N} \,.\, \frac{1}{\lambda}} \quad . \quad . \quad . \quad (93)$$

Lastly, in very large machines even if of high voltage, with toothed armatures, B_g may be 8000 and a_w as a maximum with specially good ventilation = 685 per inch; the constants then become 0·041 and 2·9 in inch measure.

Assuming in our case a ratio of $\frac{L}{D} = 1{\cdot}33$ as suitable for a two-pole dynamo, and in view of its small size taking eq. (92), we find

$$D_{''} = 3{\cdot}9 \sqrt[3]{\frac{15{,}000}{950}} \times 0{\cdot}75 = 8{\cdot}88'',$$

or say 9″ diameter of core × 12″ length.

In thus determining the size of armature core from first principles, considerable latitude may be permitted to the judgment and discretion of the designer; for within certain limits the relation between D and L may be varied without greatly affecting the cost or efficiency of the machine, although, as a general rule, as small a diameter as is practicable is to be preferred. In the technical office of a dynamo factory, however, the result is in most cases quickly arrived at by reference to a list of standard armatures, or at least of standard diameters of core-discs, from which armatures of different lengths may be built up. After a number of machines have been standardised, a table can be drawn out showing the maximum value which the quantity $\frac{\text{watts of output}}{\text{revs. per min.}}$ has for each of the standard armatures, in continuous working under average conditions of voltage and speed.

§ 5. **Watts per rev. per min. for each size of armature core.**—In the use of any such table care is required to guard against the disturbing effect of certain secondary conditions which prevent the output per rev. per min. of any size of core from being strictly constant. These conditions are firstly the speed, secondly the permissible temperature, and lastly the voltage. With regard to the first, it is evident that if the revolutions are exceptionally high, the eddy loss will assume such large proportions as to decisively limit the possible current that can be carried by a given winding on the armature. Although this is to a certain extent counter-balanced by the increased cooling due to the high peripheral speed, yet on the whole if the speed be high the watts per rev. per min. must be slightly reduced, or conversely with very low speeds the output, *ceteris paribus*, may be appreciably increased. Next the permissible rise of temperature must be taken into consideration, and this rise may itself be greatly affected by an alteration of the nature or disposition of the winding, and its relative exposure to the air; or by a change from copper to carbon brushes on the commutator. But most important of all is the influence of the E.M.F.; a high voltage implies a large number of inductors, each with an increased thickness of insulation. The percentage loss of space in insulation is therefore much greater than in low-voltage machines, and the watts per rev. per min. are reduced. The ratio of the copper area of the inductors to the area of the cylinder which they occupy on a smooth core rises from 0·3 with small high-voltage dynamos to

0·5 in larger machines for 500 volts or even to 0·7 at 60 volts. Especially is the output of toothed armatures affected by the E.M.F. If the amperage is small, round wires may be necessitated, by which the ratio of the copper to the available area is very largely reduced as compared with the same armature wound with rectangular bars. In a small multipolar toothed machine with round inductors the ratio of the copper area to the area of the slot may sink to as low as 0·25. In a low-voltage machine, say for 110 volts with rectangular bars, the ratio rises from 0·35 for small outputs at low speeds to 0·55 for large outputs, but here again if the bars become very thick, it may be necessary to limit the number of bars per slot to two only in order that the width of opening may not be too great, when the ratio again sinks to 0·45. At 250 volts the ratio for normal speeds and outputs of 40 to 200 kilowatts ranges from 0·4 to 0·52; while at 500 volts, it rises from 0·25 in very low-speed small machines with round wire to 0·3 in small machines of 50–80 kilowatts at moderate speeds and to 0·5 in machines of 500-1500 kilowatts.

Precisely the same three conditions affect any rules as to the output of a machine such as those of eqs. (90–93), so that they can at best only be rough guides. Still within wide limits, each size of armature core may be regarded as corresponding to a certain number of watts of output per rev. per min., and by this number its cost to manufacture is almost immediately determined with very fair accuracy. The value which the output watts per rev. per min. or the kilowatts at an assumed speed of 1000 revs. per min. have in any particular machine may therefore be regarded as its 'specific output' determining its weight, cost, and commercial value in general.

§ 6. **Design of small two-pole dynamo with smooth armature.**—Thus in the present case the designer would look out in his table the standard size of core corresponding most closely to $\frac{14{,}000}{950} = 14{\cdot}75$ watts per rev. per min., and we will assume that this leads him to a drum-wound core of discs 9″ external diameter, 4¼″ internal diameter, and 12″ long. The effective area of the air-gap in such a machine is calculated, as explained in Chap. XV § 6, and is found to be 885 sq. cm. A suitable induction in the air-gap will be 5700 lines per sq. cm., whence $Z_a = 5700 \times 885 = 5{,}045{,}000$.

The loss of volts over the armature will be roughly about 3·5, so that $E_a = 106{\cdot}5$ volts; and therefore from equation (15)

$$\tau = \frac{E_a \times 60 \times 10^8}{Z_a \times N} = \frac{106{\cdot}5 \times 60 \times 10^8}{5{,}045{,}000 \times 950} = 133.$$

The number of inductors in a drum armature must, however, be a multiple of 2, so that they may form a whole number of loops, and we must therefore choose between $\frac{132}{2} = 66$, involving a slightly increased induction, or $\frac{134}{2} = 67$, with a slightly reduced induction. In drawing-office practice the number of sections may have to be arranged to suit a standard list of commutators, but, apart from such a list, the question is simply as to the number of sections which is most suitable, and will not exceed the maximum that is commercially practicable.

An uneven number of sectors does not in the present case allow of equal division of the inductors between the driving strips if these are equally spaced, and on this account 66 sectors and 132 inductors should be decided on, as being slightly preferable. The flux must then be raised to 5,100,000 and $B_g = 5750$. The ampère-wires on the armature are then $A_w = c\tau = \frac{144}{2} \cdot 132 = 9500$, or 337 per inch of circumference. With small machines, as already pointed out, a_w is limited rather by considerations of heating than of sparking, and as the proposed value falls below the average figure of 380 given in the preceding section, and this latter is itself considerably below the limiting figures of Chap. XVIII § 15 for drum armatures, ample reversing field will be left at the leading pole-tip. The product of the ampère-wires on the armature and the normal air-gap induction, or $A_w \cdot B_g = 9500 \times 5750$, and this, divided between six hard-wood drivers, gives $9{\cdot}1 \times 10^6$ per strip (Chap. XIII § 17), which will lead to a thickness of driving-strip of, say, 0·150″, or the nearest standard width which may be in use.

The size of inductor has next to be determined, and in order to arrive at this the first step must be to divide the available space on the circumference by τ. Allowing 0·040″ for the insulation of the core (Chap. XIII § 16), its external circumference before winding is 28·5″. From this must be deducted the space occupied by the six hornbeam driving-strips, each 0·150″ thick, and the available circumference is thereby reduced to 27·6″, giving 0·209″ when divided between 132 inductors in a single layer. The thickness of the braiding on the bars will be 0·025″ (Chap. XIII § 19), but some margin must be allowed thereon in order to take into account any slight want of straightness in the bars or unevenness in their insulated width. This will necessitate our deducting 0·045″ to 0·05″ from 0·209″, leaving for the thickness of the bare copper, say, 0·209″ − 0·049″ = 0·160″. The length of the single air-gap by the formula of Chap. XVIII § 15 = 0·057D may be taken as $=\frac{1}{2}$″. The clearance between the binding-wire and the iron of the pole-piece should not be less than $\frac{3}{32}$″ for all diameters up to 9″, and $\frac{1}{8}$″ for diameters from 9″ to 13″, while above these sizes it should be increased to $\frac{5}{32}$″. These clearances together with the thickness of the insulation on core and wire and of the binding-wire with its backing (fig. 181) lead to a difference between the length of the air-gap and the height of the bare bar which will rise from 0·185″ in a 6″ armature to 0·280″ in a 12″ armature, and 0·340″ in an 18″ armature. In our 9″ armature with 0·040″ for the insulation on the core, and 0·025″ for that of the bar, the addition of 0·060″ for the total thickness of the binding-wire (Chap. XIII § 27) and of 0·125″ for the clearance makes a total of 0·25″ in all and leaves 0·25″ for the height of the copper.

The bars will therefore be $\frac{1}{4}$″ high × 0·160″ thick, and as they must be stranded to avoid eddy-currents, their net area may be taken as 84 per cent. of their gross area, or 0·0336 sq. inch, with a resistance per 1000 ft. at 60° F. of $\omega' = \frac{0·00813}{0·0336} = 0·242$ ohm.

The end-connections are to be flat strips of copper $1\frac{1}{4}$″ wide, bent to a double evolute curve or butterfly shape, with a gap of $\frac{1}{4}$″ between the two whorls at either end (fig. 275). The mean length of a bar to the centre of its end-connectors, with an allowance of $\frac{9}{16}$″ for each insulated end-plate, will therefore be $15\frac{7}{8}$″, say 16″. The length of path of the current in an end-connector may be

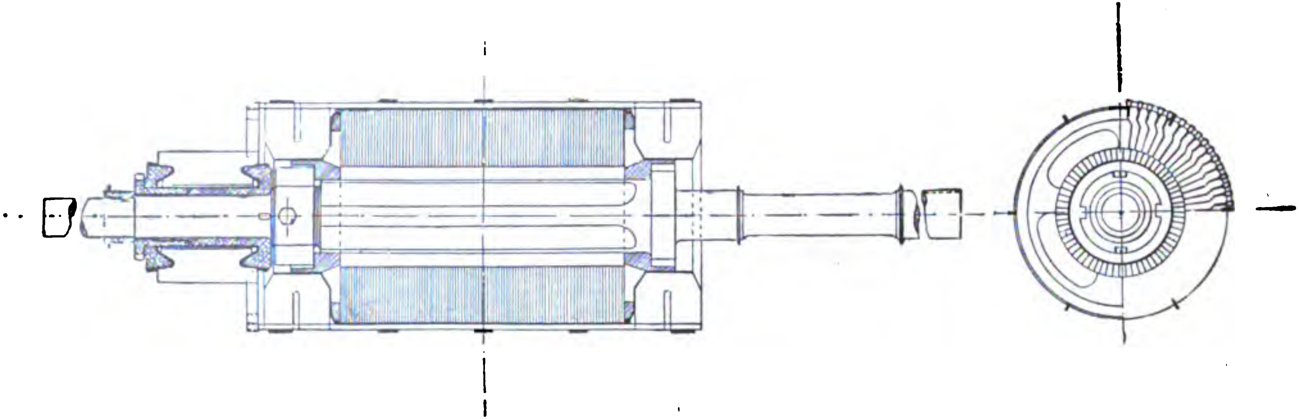


FIG. 275.—Two-pole bar-wound drum armature (core 9″ diam. × 12″ long).

taken as equal to half the circumference of the armature core, or 14·1"; if the ratio of the area of the connector to the area of the stranded bar is 1·3, and the thickness of the connector is thus 0·035", the equivalent length of the end-connector is $\frac{14·1}{1·3} = 10\frac{7}{8}$". The total length of a half loop corresponding to one inductor will thus be 26⅞" = 2·24 ft., and the resistance of the armature at 60° F. (Chap. XIII. § 31) will be

$$R_a = \frac{132}{4} \times 2·24 \times \frac{0·242}{1000} = 0·0179 \text{ ohm.}$$

The thickness of the insulated end-connector will be 0·035" + 0·030" = 0·065", and their radial depth $\frac{1}{8} \times 0·065" = 1·07"$. The insulated circle on which they are to rest is to have a diameter of 4⅞", so that the available radial depth below the diameter of the core is $\frac{9 - 4·875}{2} = 2·0625"$. The straight radial portion at the centre of the end-connector is to be made by folding over the sheet copper on itself, so that its dimensions are ¾" deep × 0·070". The total radial depth of the end-connectors will therefore be 1·07 + 0·75 = 1·82", leaving a margin of nearly ¼" to allow for bending the connector end straight up immediately under its junction with the slotted end of the bar. The space round the insulated circle of 4·875" diameter has still to be checked. The circumference demanded by the connectors will be $\frac{132}{2} \times (0·07 + 0·04) = 7\frac{1}{4}"$, while the available circumference is 15·2", so that there is more than sufficient room.

Assuming that the maximum temperature reached in continuous working is about 70° F. above the temperature of the room, and that the latter averages 70° F., 16·8 per cent. must be added to the resistance of the armature at 60° F., or R_a hot = 1·168 × 0·0179 = 0·0209 ohm. The total armature current of 144·2 ampères, obtained by adding a shunt current of about 3 per cent. to the external current, will thus give a loss of pressure over the armature resistance, or $C_a R_a = 144·2 \times 0·0209 = 3·02$ volts, and a loss of power, $C_a^2 R_a = 144·2 \times 3·02 = 437$ watts.

In order to determine the loss by hysteresis in the armature core, its net sectional area is calculated as in Chap. XV § 6 and is found to be 165 sq. cm. on a radial section. The mean diameter of the core being $\frac{9+4{\cdot}25}{2}=6{\cdot}625''$, the volume of iron in it is $165\times 6{\cdot}625\times\pi\times 2{\cdot}54=8720$ cubic cm. The cycles per second are $\frac{950}{60}=15{\cdot}8$, and the maximum induction is $B_a=\frac{5{,}100{,}000}{330}=15{,}450$. From the curve of fig. 136 the joules expended in hysteresis per cubic cm. per cycle are 0·00133. Hence the rate of loss in watts is $0{\cdot}00133\times 15{\cdot}8\times 8720=183$. The eddy-current loss by the formula of Chap XIX § 18 is calculated as follows:—

$$
\begin{aligned}
&k_1\,.\,X_g{}^2D=0{\cdot}15\times(0{\cdot}8\times 2{\cdot}54\times 5750)^2\times 9 &&=18{\cdot}4\times 10^7\\
&k_2\,.\,B_g{}^2\,.\,b\,.\,D^2\,.\,L=0{\cdot}008\times(5750)^2\times 1\times 81\times 12 &&=\underline{25{\cdot}7\times 10^7}\\
& &&\;\;44{\cdot}1\times 10^7\times D^2\times 10^{-14}=0{\cdot}000358\\
&k_3\,.\,B_g{}^2\,.\,p^2\,.\,\tau\,.\,L\,.\,t\sqrt{th}\times 10^{-14}=11\times(5750)^2\times 1\times 132\times 12\times 0{\cdot}16\times 0{\cdot}2 &&\\
& &&\times 10^{-14}=\underline{0{\cdot}000184}\\
& &&=0{\cdot}000542
\end{aligned}
$$

$FN^2=0{\cdot}000542\times(950)^2=490$ watts at no load, and increasing this by 10 per cent. to allow for the effect of the armature reaction, the loss under full load is 540 watts. Such an approximate estimate may be checked by the results obtained from previous machines, if such are to hand with similar construction and similar winding. The total loss of watts over the armature is therefore $437+183+540=1160$. The finished diameter of the armature winding is $9\frac{5}{8}''$, and its over-all length from end to end including the commutator lugs is about 19″. The cooling surface calculated as explained in Chap XIX § 22 is therefore

$$
\begin{aligned}
9\tfrac{5}{8}''\times\pi\times 19&=575\\
\frac{\pi\times(9\frac{5}{8})^2}{2}&=\underline{145}\\
&\,720=S_c.
\end{aligned}
$$

The peripheral velocity is $\frac{9\frac{5}{8}''\times\pi}{12}\times 950=2390$ feet per min., whence by the curve of fig. 271 the heating coefficient is 39·4.

The final rise of temperature of the armature is therefore

$$t = \frac{39{\cdot}4 \times 1160}{720} = 64{\cdot}5^\circ \text{ F.}$$

Our original assumption therefore of 70° F rise of temperature is on the safe side, and the winding may be allowed to stand without alteration.

To collect 140 ampères two copper gauze brushes will suffice, each $1\frac{3}{4}''$ wide by $\frac{1}{4}''$ thick; their joint area of contact surface when inclined at an angle to the commutator will be about 1·32 sq. in. giving a current-density of 110 ampères per sq. in. and their contact-resistance will be approximately $\frac{0{\cdot}0025}{1{\cdot}32} = 0{\cdot}0019$ ohm. The loss of volts over the two sets of brushes will be $2C_aR_1 = 2 \times 144 \times 0{\cdot}0019 = 0{\cdot}55$. Since the rise of temperature of the armature is not so high as at first estimated, its resistance when hot will be reduced to $0{\cdot}0179\ (1 + 0{\cdot}0021 \times 74{\cdot}5) = 0{\cdot}0206$ ohm, and the loss of volts will be correspondingly reduced to 2·98, which with the loss over the brushes makes a total of nearly 3·5 volts and leaves a terminal voltage of 103 as desired.

By equation (23), the stress on the binding wire due to the bars of average length $17\frac{1}{4}''$ is

$$P = 7{\cdot}5 \times 17\tfrac{1}{4} \times 132 \times 0{\cdot}0336 \times 9\tfrac{3}{8} \times 950^2 \times 10^{-7} = 485 \text{ lbs.}$$

To this must be added the stress due to the end-connectors; in a section at right angles to the shaft, the area of each of the 66 pieces of copper which are cut may be taken as equal to half the length of path in the end-connector multiplied by their thickness or $7{\cdot}05 \times 0{\cdot}035 = 0{\cdot}246$ square inches, and their radius of gyration may roughly be taken as $7''$. Hence by the same formula, for the two ends each of axial length about $2{\cdot}75''$

$$P = 7{\cdot}5 \times 5{\cdot}5 \times 66 \times 0{\cdot}246 \times 7 \times 950^2 \times 10^{-7} = 425 \text{ lbs.,}$$

or nearly as much as that due to the bars by themselves. It will therefore be advisable to place a wide band of one inch on each set of end-connectors and three smaller bands, each $\frac{3}{4}''$ wide, along the centre of the core at equal intervals. The total width is then 4·25, containing $\frac{4{\cdot}25}{0{\cdot}042} = 100$ wires of total area 0·138 square inches. With an ultimate breaking strength of 90,000 lbs. per sq. inch, the factor of safety[1] is $\frac{90,000 \times 0{\cdot}1385}{910} = 13{\cdot}7$. In addition the centre of the end-connector whorls is held down by a band of string wound within their dividing gap of $\frac{1}{4}''$.

The design of the armature is now complete, and the size of the field-magnet is the next point to be decided. The length of the pole-piece parallel with the shaft should be somewhat less than the length of the armature core in order to prevent lines from curving round into the flat surfaces of the end discs of the core, and there generating eddy-currents (Chap. XIII § 3), and in this case it will be made $11\frac{1}{4}''$, or three-quarters of an inch less than the length of the core. On the assumption that $Z_m = 1{\cdot}25\ Z_a$(Chap. XV § 11), the number of lines passing through the magnet will be 6,370,000; or from previous designs we may even know more definitely that P will be between 70 and 80 (Chap. XV § 8), while $X_P = X_a + X_g + X_b$ can already be calculated to be $275 + 11,700 + 515 = 12,490$ (*vide* Chap. XV § 6); any such preliminary estimate of P or Z_m may, however, require to be subsequently verified when the design is further advanced. Assuming the magnet limb to be rectangular in section, the same dimension of $11\frac{1}{4}''$ is best retained throughout the length of the limb, since then magnet-core and pole-piece are conveniently forged and machined as one slab.

Taking an average value for B_m such as 16,000 (Chap. XIV § 1), we obtain for the wrought-iron limb an area of $\frac{6,370,000}{16,000} = 398$ or say 400 sq. cm. = 62 sq. in., and thence the thickness works out to $\frac{62}{11{\cdot}25} = 5\frac{1}{2}''$. Taking the length of bobbin as about equal to the diameter of armature (Chap. XIV § 6), say 10″, a rough sketch of the magnetic circuit such as fig. 208 shows us that the length of path in the wrought iron will be 104 cm. and in the cast-iron yoke 16 cm. A final revision will lead us to slightly lower our value for P, bringing Z_m to 6,275,000, whence $B_m = 15,700$ and $B_y = 8260$. The final excitation can then be calculated as in Chap. XV § 6. If the field is to be shunt-wound, the size of wire is determined as has already been done in Chap. XVI § 7; or if it is to be compound-wound to give 100 volts on open circuit and 103 volts on full load, the field-winding is that given in Chap. XVI § 18. The rise of the temperature of the coils for a given winding is obtainable from Chap. XIX § 8 and by a process of trial and error a certain number of complete layers is fixed upon, such that a due ratio obtains between the watts expended and the cooling surface.

---

[1] If the initial tension under which the bands are put on be neglected.

The over-all efficiency in either case is nearly the same, viz., $\frac{14,420}{16,420} = 88$ per cent.; the losses as a shunt machine being :—

	Watts.	Percentage of Output.
Field-winding, . .	460	3·19
$C_a^2(R_a+R_b)$ . . .	505	3·5
Hysteresis, . . .	183	1·27
Eddies, ...	540	3·74
Friction, say, . . .	312	2·16
Total losses, . . .	2000	13·86
Output, ...	14,420	
Input, ...	16,420	

§ 7. **Design of four-pole dynamo with toothed armature.**—Next may be taken the design of a multipolar dynamo having a normal output of 165 kilowatts and suitable for direct coupling to a high-speed enclosed engine running at 450 revs. per min. The dynamo is to be used both for lighting and for traction purposes, being shunt-wound in the former case, and compound-wound in the latter. It has further to comply with the following conditions; when working as a shunt machine, it must give its full output of 165 kilowatts for any load between the limits of 480 volts, 344 ampères and 520 volts, 317 ampères; it must also be capable of giving 460 volts and 360 ampères, but in this case and and for any smaller load at 460 volts, the speed may be reduced by 5 per cent. or say to 425 revs. per min., in order to maintain the stability of the magnetism. On a traction load, the machine is to give 500 volts at no load, and to be over-compounded to give 550 volts with the full load of 300 ampères, while it must further be capable of giving an overload of 20 per cent. or 198 kilowatts made up of 500 volts, 395 ampères for at least 15 minutes without sparking at the brushes or any undue heating. After a continuous run of six hours with either of the two normal full loads, the temperature of the surface of the armature or field when measured on bare copper must not exceed that of the surrounding air by more than 70° F. The shunt regulating resistance and switch is to be large enough to enable all the above conditions of load to be obtained by alteration of the field strength under any normal conditions of temperature of the engine room and of the dynamo itself; but when the switch is set to give 500 volts at no load, it must not be used to assist in obtaining the required compounding effect. Lastly, in parallel with the series winding is to be placed a regulating switch and resistance, by means of which the amount of the over-compounding may be raised in four or five steps while the machine is working. A machine to comply with such varied conditions and to work over so large a range of voltage will introduce to us several of the nicer problems in the art of dynamo design.

The type of dynamo most suitable for the present output, especially in view of the rapid fluctuations of a traction load, will be one having a toothed armature with barrel winding and as many sets of brushes as there are poles. The maximum current being practically 400 ampères, four sets of brushes will give 200 ampères to be collected per brush arm, which will lead to a reasonable length of commutator. Four poles may therefore be decided upon with a parallel-connected lap winding and a maximum of 100 ampères per armature path. The watts per rev. per min. at the normal full load are $\frac{165}{450} = 366$, and assuming a ratio $\frac{L}{D} = \lambda = 0{\cdot}62$ as suitable for four poles (Chap. XIV § 17), and $a_w = 480$ per inch of circumference, eq. (93) would give $D_{\prime\prime} = 3{\cdot}5 \sqrt[3]{366 \times 1{\cdot}61} = 29{\cdot}4$ inches, and $L_{\prime\prime} = 18{\cdot}2$. Since with four poles and barrel winding the end connections project considerably and the over-all length of armature with commutator is high, it would be advisable to shorten the armature if possible. With good ventilation at the ends, the armature wires per inch of circumference may be raised to, say, 620, if at the same time the density of the field is raised above $B_g = 6000$, and a slightly smaller ratio of polar arc to pole-pitch is taken. The former change will render the magnetism more stable at the lowest voltage of the wide range which is to be covered, and the latter change will assist in securing an economical section of magnet core with the shorter length of armature. The leading dimensions of the armature may therefore be taken as 30″ diameter × 15″ length of core between the end-plates. If $a_w = 620$ with the normal lighting current in the armature of 348 ampères, $a_w \times 94{\cdot}2 = 58{,}400 = c\tau$, whence $\tau = 58{,}400 \div \frac{348}{4} = 672$. The maximum flux will correspond to the full traction load, and allowing a loss of 2½ per cent. over the armature, brushes, and series-winding, or 13·7 volts on 550, with a slight drop in speed to, say, 445 revs. necessitated by the range of the governor, eq. (14) becomes

$$z_a \cdot \frac{445}{60} \cdot 672 \times 10^{-8} = 563{\cdot}7 \text{ volts,}$$

whence the maximum $z_a = 11{\cdot}3 \times 10^6$ C.G.S. lines.

If $\beta = 0{\cdot}7$, or the polar arc be 63°, and the air-gap be assumed for the present to be $l_g = \frac{1}{2}''$, the breadth of the polar area at the mean radius of the air-gap is $A' = \pi(30{\cdot}5'') \times \frac{63^\circ}{360^\circ} = 16{\cdot}75''$, and a preliminary calculation of the area gives 262 sq. inches, or 1690 sq. cm., and $B_g = 6700$

The 672 inductors will best be distributed among 112 slots, six per slot in two layers. An average depth of slot may be taken as 1½″, and the width must be such that the maximum density at the root of the teeth is about 20,000 to 21,000 under full load, so that the armature re-action may be kept within bounds. One ventilating duct will be arranged in the centre of the core, with a width of ½″. The number of teeth under a pole is $112 \times \frac{63^\circ}{360^\circ} = 19{\cdot}6$, or to allow for a little spreading of the field say 20½. The area of the iron at the bottom of the teeth through which the lines of one field pass is therefore $\frac{11{,}300{,}000}{20{,}500} = 550$ sq. cm. The necessary width of the tooth at its root is then given by the equation $k_{t2} \times 14\tfrac{1}{2}'' \times 0{\cdot}9 \times 20\tfrac{1}{2} \times 6{\cdot}45 = 550$, whence $k_{t2} = 0{\cdot}319''$. The diameter at the bottom of the slots being 27″, the width of a tooth and a slot on this diameter is $\frac{27 \times 3{\cdot}14}{112} = 0{\cdot}756''$, so that with a slightly higher value for B_{t2} we may allow 0·316′ for the root of the tooth, and obtain an even dimension of 0·440″ for the width of the slot.

On the external diameter the ratio of the width of tooth to the pitch is $\frac{k_{t1}}{k_{t1} + k_s} = \frac{0{\cdot}4}{0{\cdot}84} = 0{\cdot}476$, so that our ratio for $\frac{k_{t1}}{l_g} = \frac{0{\cdot}4}{0{\cdot}5}$ is well within the limits of the maximum permissible as given in fig. 274. Under the overload the loss over the armature, brushes, and series-winding will be rather less than $13{\cdot}7 \times \frac{400}{304}$, = say 17½ volts, since with the increased current-density in the brushes the loss will not be increased in quite the same proportion. The flux at a slightly lowered speed will therefore be

$$Z_a = \frac{517{\cdot}5 \times 10^8}{672 \times \frac{440}{60}} = 10{\cdot}5 \times 10^6$$

and the air-gap induction $B_g = 6210$. By equation (64), we have as the maximum permissible value for the ampère-wires under the pole

$$\frac{c\tau}{2p} \cdot \beta = \frac{0{\cdot}8(5210)\left\{2{\cdot}3 \times 1{\cdot}08 \times 1{\cdot}27 + \frac{3{\cdot}8}{7}\right\}}{1{\cdot}3} = 11{,}750$$

or since $\beta = 0{\cdot}7$, the ampère-wires per pole may be 16,800. The actual value is $\frac{100 \times 672}{4} = 16{,}800$, so that the armature is fully loaded up to the limit of safety, but will still be able to cope with the overload without any detrimental sparking during the short time for which it will in practice last.

The final design of the armature may now be proceeded with. The inside diameter of the armature discs may be made 18¾″

p. 673.

giving a radial depth below the teeth of $4\frac{1}{8}''$, and an effective iron area of $4\frac{1}{8}'' \times 14\frac{1}{2}'' \times 0{\cdot}9 \times 2 \times 6{\cdot}45 = 695$ cm., whence the maximum $B_a = 16{,}260$. The discs will be carried on a cast-iron hub with 6 arms, and, in order to relieve the shaft from the torsional strains due to rapid changes of load, the hub is to have a cylindrical prolongation cast on it by which it is bolted directly to the flywheel of the engine (fig. 276) with eight coupling bolts on a large pitch circle. For 550 volts the slot may be lined with a trough of flexible micanite 0·020″ thick and for additional safety with a thin layer of press-spahn 0·010″ thick (cp. fig. 158). Each group of three bars in one layer should be wrapped round with a covering of press-spahn 0·010″ thick, the total thickness of such insulation reducing the width of slot by 0·080″. A width $\frac{0{\cdot}440'' - 0{\cdot}080''}{3} = 0{\cdot}120''$ is thus obtained for each inductor, and, finally, allowing 0·035″ for the double thickness of the cotton braiding on the bars and a margin of 0·015″ for slight inequalities in their straightness, the width of the bare copper of the bar may be made 0·070″. Between the two layers of bars a strip of press-spahn 0·010″ thick is inserted; the thickness of the micanite trough and of the folded coverings of press-spahn is then $0{\cdot}020'' + 0{\cdot}010'' + (0{\cdot}010'' \times 5) = 0{\cdot}08''$, leaving 1·42″ for the bars, or 0·71 for each layer. The insulation of the bars, together with a margin to ensure the bars lying easily within the slot and below the level of the top, may be taken as 0·06″, so that the height of the bare copper is 0·65″. The ratio of the copper area to the area of the slot is thus $\frac{0{\cdot}65 \times {\cdot}070 \times 6}{1{\cdot}5 \times 0{\cdot}44} = 0{\cdot}413$, and the current-density with the full normal lighting current is $\frac{87}{0{\cdot}0455} = 1900$ ampères per sq. inch. The resistance of 1000 ft. of the given section is $\omega' = \frac{0{\cdot}0081}{0{\cdot}0455} = 0{\cdot}178$ ohm. Next must be calculated the mean length of a half loop, and the amount by which the end-connections will project beyond the core at either end. In order to economise in the length of copper and to introduce a small amount of chord-winding effect, the pitch at the engine end may be made two slots short of the pole-pitch, so that the number of bars in one layer in the half pitch which are crossed by the bent end of the bar is $n = \frac{\tau}{4pl} - 3 = \frac{672}{16} - 3 = 39$. The thickness of the insulated bar with margin for slack may be taken as $0{\cdot}07'' + 0{\cdot}065'' = 0{\cdot}135,''$ whence $nt = 39 \times 0{\cdot}135 = 5{\cdot}26''$. At the same time the distance through which the lower bar must be bent measured along the bottom of the slots or on the smallest diameter is $m = 27'' \times \pi \times \frac{13}{112} = 9{\cdot}85''$. Hence by the formula of Chap. XIII § 25, $L = \frac{(nt)m}{\sqrt{m^2 - (nt)^2}} = 6{\cdot}25''$. In order that there may be free entrance for the air between the adjacent bars, they may be slightly opened out, and an allowance of 7″ be made for the axial length of the bent portions at each end. On the mean diameter of 28·5″, the circumferential space through which the bar is bent will be $28{\cdot}5'' \times \pi \times \frac{39}{336} = 10{\cdot}4''$ at the engine end, and $28{\cdot}5'' \times \pi \times \frac{38{\cdot}5}{336} = 10{\cdot}25''$ at the commutator end. The actual spiral length of the bent-ends of a half loop will be the hypothenuse of the right-angled triangle having for its two sides the circumferential distance and the axial length, or at the engine end $\sqrt{10{\cdot}4^2 + 7^2} = 12{\cdot}5''$, and at the commutator end $\sqrt{10{\cdot}25^2 + 7^2} = 12{\cdot}4''$. To the length of the core must be added at each end a space of, say, 0·44″ to allow of the bars being brought well out of the slot before the bend begins, and a similar space before the straight portions of the bars are soldered together, while for the length within the soldered junctions must be allowed $\frac{5}{8}''$ at each end. The total length of path in a half loop is therefore

$$15 + (0{\cdot}44 \times 4) + 12{\cdot}5 + 12{\cdot}4 + (0{\cdot}625 \times 2) = 42{\cdot}9'' = 3{\cdot}57 \text{ ft.}$$

The resistance of the armature at 60°F. is then by eq. (24)

$$\frac{672}{16} \times 3{\cdot}57 \times \frac{0{\cdot}178}{1000} = 0{\cdot}0267 \text{ ohm,}$$

and allowing a temperature of 75° F. in the engine-room with a rise of 70° F., the heating coefficient will be $85 \times 0{\cdot}21 = 0{\cdot}178$, or when hot

$$R_a = 0{\cdot}0267 \times 1{\cdot}178 = 0{\cdot}0314 \text{ ohm.}$$

To collect the current, six carbon brushes in each arm, each $\frac{5}{8}''$ wide × $1\frac{1}{2}''$ in axial length, will give 5·6 sq. inches, and current-densities of $\frac{348}{2 \times 5{\cdot}6} = 31$ on the normal lighting load, of $\frac{304}{11{\cdot}2} = 27{\cdot}2$ on the traction load, and a maximum of $\frac{400}{11{\cdot}2} = 35{\cdot}7$ ampères per sq. inch on the overload—sufficiently low values to ensure cool and sparkless collection. The specific contact-resistance R_k will vary (fig. 272) from a normal of 0·03 ohm per sq. inch, to a maximum of 0·034 on the traction load and a minimum of 0·027 on the overload; and $\text{R}_1 = \frac{\text{R}_k}{b \cdot b_1}$ will correspondingly vary from a normal of 0·00535 to a maximum of 0·0061 and a minimum of 0·0049. The loss of volts over the two sets of brushes of opposite sign will be $2\text{C}_a \cdot \text{R}_1 \times \frac{1}{p} = \text{C}_a \cdot \text{R}_1$, and a table may now be drawn up of the watts lost in the armature and brushes under different loads.

	C_a	Loss of Volts over Brushes.	Loss of Volts over Armature.	$\text{C}_a(\text{R}_a+\text{R}_b)$	Loss of Watts over Brushes.	Loss of Watts over Armature.
Normal lighting loads	348	1·86	10·9	12·76	646	3800
	321	1·83	10·1	11·93	588	3240
Maximum ,, ,,	364	1 85	11·4	13·25	673	4150
Normal traction ,,	304	1·85	9·55	11·4	562	2900
Overload . .	400	1·92	12·5	14·42	767	5000

A closer calculation of the air-gap and teeth excitation must now be made. The axial length of the pole-face being made one inch shorter than that of the armature core, $\frac{a}{l_g} = \frac{0{\cdot}5}{0{\cdot}5} = 1$, and from Table I. (Chap. XV § 6) $\text{K}_1 = 1{\cdot}2$. The interpolar edges may be slightly rounded so as to give a finely graduated field (fig. 277), the mean breadth being such as to give a polar angle of 63°; the average angle made by the sloping edge of the pole with the armature core is about 135°, and $\frac{c}{l_g} = \frac{3{\cdot}5}{5} = 7$, whence $\text{K}_2 = 1{\cdot}95$

The effective area of the air-gap is then by eq. (30)

$$a_g = (14 + 1{\cdot}2 \times 0{\cdot}5)(16{\cdot}75 + 1{\cdot}95 \times 0{\cdot}5) = 14{\cdot}6 \times 17{\cdot}72 \text{ sq. inches}$$

or 1670 sq. cm. The ratio of width of slot to air-gap is $\frac{k_s}{l_g} = \frac{0{\cdot}44}{0{\cdot}5} = 0{\cdot}88$, and the ratio of the top of the tooth to the slot is $\frac{k_{t1}}{k_s} = \frac{0{\cdot}4}{0{\cdot}44} = 0{\cdot}91$, whence by fig. 216 $m = 1{\cdot}08$. The single air-gap reluctance is therefore by eq. (38)

$$\frac{1{\cdot}08 \times 0{\cdot}5}{(14 + 1{\cdot}08 \times 0{\cdot}6)(16{\cdot}75 + 1{\cdot}08 \times 0{\cdot}97) \times 2{\cdot}54} = 0{\cdot}000812.$$

and $\frac{X_g}{2} = 0{\cdot}8 \times 11{\cdot}3 \times 10^6 \times 0{\cdot}000812 = 7350$ ampère-turns.

The ampère-turns required over the teeth must next be estimated by the methods of Chap. XV § 14. The number of lines to be carried by one tooth and slot per cm. length of core is $z = \frac{11{,}300{,}000}{20{\cdot}5 \times 15 \times 2{\cdot}54} = 14{,}420$. Taking into account the central air-duct of ½″ and the insulation between the discs, $\eta = 0{\cdot}85$; the apparent densities at the top and bottom of the teeth $= \frac{z}{\eta \, . \, k_t}$

are
$$B_{t1}' = \frac{14{,}420}{0{\cdot}85 \times 0{\cdot}4 \times 2{\cdot}54} = 16{,}700$$

and
$$B_{t2}' = \frac{14{,}420}{0{\cdot}85 \times 0{\cdot}316 \times 2{\cdot}54} = 21{,}100.$$

To these correspond the real densities $B_1 = 16{,}600$ and $B_2 = 20{,}500$ (fig. 220), the ratio $\frac{k_{t1}}{k_s}$ being $\frac{0{\cdot}4}{0{\cdot}44} = 0{\cdot}91$ and $\frac{k_{t2}}{k_s} = \frac{0{\cdot}316}{0{\cdot}44} = 0{\cdot}72$.

By figs. 222 and 223 $\int_{B_2}^{B_1} H d\left(\frac{1}{B}\right) = -0{\cdot}00105 + 0{\cdot}00275 = 0{\cdot}0017$

$$\frac{z}{\eta(k_{t1} - k_{t2})} = \frac{14{,}420}{0{\cdot}85 \times 0{\cdot}084 \times 2{\cdot}54} = \frac{14{,}420}{0{\cdot}181} = 79{,}600$$

$$\int_{B_1}^{B_2} H d\left(\frac{H}{B}\right) = 2{\cdot}75$$

$$\frac{k_s + (1 - \eta)k_{t2}}{\eta(k_{t1} - k_{t2})} = \frac{(0{\cdot}44 + 0{\cdot}047) \times 2{\cdot}54}{0{\cdot}181} = 6{\cdot}83.$$

Thence by eq. (40)

$\frac{X_t}{2} = \frac{1{\cdot}5 \times 2{\cdot}54}{1{\cdot}257}\{(79{,}600 \times 0{\cdot}0017) + (6{\cdot}83 \times 2{\cdot}75)\} = 465$ ampère-turns. Thus with the maximum flux, $B_g = 6760$ and $(X_g + X_t) = 15{,}600$. On the normal lighting load when running as a shunt machine 12·7 volts are lost over the armature and brushes, and the necessary flux corresponding to an armature E.M.F. of 492·7 volts at 446 revs. is $z_a = \frac{492{\cdot}7 \times 10^8}{672 \times \frac{445}{60}} = 9{\cdot}85 \times 10^6$ lines; on carrying out the same calculations as above, we find

$$B_g = \frac{9{\cdot}85 \times 10^6}{1670} = 5900,$$

$$B_a = \frac{9{\cdot}85 \times 10^6}{695} = 14{,}200,$$

$$\frac{X_g}{2} = 0{\cdot}8 \times 9{\cdot}85 \times 10^6 \times 0{\cdot}000812 = 6400,$$

$$B_{t_1}' = 14{,}550,$$

$$B_{t_2}' = 18{,}400, \text{ and } B_2 = 18{,}250,$$

$$\frac{X_t}{2} = 100, \text{ and } (X_g + X_t) = 13{,}000 \text{ ampère turns.}$$

We are now in a position to calculate the eddy and hysteresis loss in the two cases. Under the traction load, by eq. (83)

$$k_1 . (X_g + X_t)^2 b D^3 t h \times 10^{-14} = 4 \times 15{,}600^2 \times 0{\cdot}6 \times 30^3 \times 0{\cdot}65 \times 0{\cdot}07 \times 10^{-14} = 0{\cdot}00715$$

$$k_2 . B_g^2 . b D^3 t h L \times 10^{-14} = 2 \times (6760)^2 \times 735 \times 10^{-14} = 0{\cdot}01$$

$$k_3 . B_g . D^2 L \times 10^{-14} = 7000 \times 6760 \times 900 \times 15 \times 10^{-14} = 0{\cdot}0064$$

$$F = 0{\cdot}0236$$

The eddy watts are therefore $FN^2 = 0{\cdot}0236 \times 446^2 = 4700$.

The mean diameter of the core below the teeth being 22·875″, the volume of the iron is $22{\cdot}875 \times \pi \times 2{\cdot}54 \times 347 = 63{,}500$ cubic centimetres, and $B_a = 16{,}300$. Since this induction corresponds nearly to the maximum hysteretic coefficient 0·0013 in fig. 136 for a rotating field, the iron teeth may also be assigned the same coefficient and their volume be added to that of the core. The volume of the tapering teeth of average width 0·358″ is $1\frac{1}{2} \times 0{\cdot}358 \times 112 \times 14\frac{1}{2} \times 0{\cdot}9 \times 2{\cdot}54^3 = 12{,}850$, so that the total volume is 76,350 c.cm. The frequency is $\frac{pN}{60} = \frac{2 \times 446}{60} = 14{\cdot}9$, and the total hysteresis loss is by eq. (18)

$$H_w = 0{\cdot}0013 \times 14{\cdot}9 \times 76{,}350 = 1480 \text{ watts.}$$

The sum of the armature losses on the traction load is therefore

$$C_a^2R_a + H_w + FN^2 = 2900 + 1480 + 4700 = 9080 \text{ watts.}$$

On the normal lighting load, the copper loss is considerably increased, but the eddy loss is decreased to a greater degree, so that the sum is rather less. Substituting the new values for B_g and $(X_g + X_{at})$ in the eddy-current expression, the three terms become

$$\begin{array}{r} 0{\cdot}00496 \\ 0{\cdot}0076 \\ 0{\cdot}00545 \\ \hline F = 0{\cdot}018 \end{array}$$

whence $FN^2 = 3560$ watts.

For the hysteresis coefficient may be taken 0·00125, and $H_w = 1425$. The total watts are thus $3800 + 1425 + 3560 = 8785$.

The peripheral speed of the armature being 3500 ft. per min. the heating coefficient from fig. 271 is 27. The cooling surface reckoned by the method of Chap. XIX § 22 and with a diameter d for the commutator of 19·7″ is

$$\begin{array}{lr} \pi DL = 3{\cdot}14 \times 30'' \times 33'' = & 3110 \\ \frac{\pi}{4}(D^2 - d^2) = & 400 \\ \hline & S_c = 3510 \end{array}$$

The permissible watts for a rise of temperature of 70° F. are then

$$W = \frac{3510 \times 70^\circ}{27} = 9100,$$

which agrees sufficiently well with the maximum loss that may be anticipated.

Since the maximum current in each armature path is close upon 100 ampères, one section of the armature should not consist of more than a single loop or two inductors. There will thus be 336 sectors in the commutator, or three times as many as there are slots. With a commutator diameter of 19·7″, and mica insulating strips 0·030″ thick, the thickness of each commutator sector at the top will be $0{\cdot}184 - 0{\cdot}030 = 0{\cdot}154''$, which is not unreasonably thin, but will necessitate the commutator connection to the armature winding being riveted on to the side of the sector. It will not be advisable for more than three sections to be undergoing short-circuit at a time, and on this account a thickness of $\frac{5}{8}''$ has been chosen for the brushes, which exceeds but little the width of three sectors and four mica strips. The inductance of a section may next be calculated, and it should be noticed that owing to the slight amount of chord-winding that has been introduced, the two wires that form the elements of a group within one interpolar gap are separated by two teeth and a slot. As explained in Chap. XVIII § 21 the inductance of a single bar A within the slot is by eq. (68) $6{\cdot}28\,\frac{t}{k_s}\cdot l$, but in order to make some little allowance for the lines that pass through the copper of bar A and through bar B at the top of a slot, an intermediate value $=8{\cdot}3\frac{t}{k_s}\cdot l$ will be taken, *i.e.*, $8{\cdot}3\times\frac{1{\cdot}5}{0{\cdot}44}\,l=28{\cdot}2\,l$. The inductance due to each separate group of lines spreading from a tooth on either side of a bar will be by eq. (71)

$$9{\cdot}2\,l\cdot\log\frac{\pi\left(k_{t_1}+\frac{k_s}{2}\right)+k_s}{k_s}$$

or twice this amount for the two bars $=13{\cdot}4\,l$. The inductance due to the common set of lines which embrace both bars up to the limit of the pole-edge ($r_2=c=3{\cdot}5$) will be

$$4\times 9{\cdot}2\,l\log\frac{\pi\{r_2-2(k_{t_1}+k_s)\}+4(k_{t_1}+k_s)}{4(k_{t_1}+k_s)}=15{\cdot}9\,l.$$

Thus the core-inductance of a single section $=(28{\cdot}2+13{\cdot}4+15{\cdot}9)\,l$ or since l must be expressed in cm. $=57{\cdot}5\times15\times2{\cdot}54=2200$. To this must be added the end inductance $=16\,l'$, where l' is the length of the bent portions of each half loop $=42{\cdot}9''-15''=27{\cdot}9''$ or 71 cm. The total apparent inductance of a section is therefore $(2200+1140)\times10^{-9}=0{\cdot}00000334$ henrys. Since the three sections simultaneously short-circuited may be within one and the same slot, this value must be multiplied by 3, or L = say 0·00001 henrys. The time of short-circuit is

$$\mathrm{T}=\frac{(b-m)\times60}{\pi d\mathrm{N}}=\frac{(0{\cdot}625-0{\cdot}03)\times60}{\pi\times19{\cdot}7\times450}=0{\cdot}00128\text{ second}$$

With the maximum overload current, $J = 100$ ampères, and we obtain for the quantity $\frac{2LJ}{T}$ a value of $\frac{2 \times 0{\cdot}00001 \times 100}{0{\cdot}00128} = 1{\cdot}56$, which is sufficiently close to the limit given by eq. (75). The corresponding value of $\frac{R_1 T}{L}$ for the maximum overload, as explained in Chap. XVIII § 22, is $\frac{0{\cdot}0049 \times 0{\cdot}00128 \times 3}{0{\cdot}00001} = 1{\cdot}88$.

In the barrel-wound armature the centrifugal force per inch length is considerably greater at the ends where the copper bars are brought close together than over the length of the core. Hence the binding wire bands should be given greater width at the ends. Taking the length of the core first, the force acting on the cross-section of the wires within the length of the core is by eq. (23),

$$P = 7{\cdot}5 \times 15 \times 672 \times 0{\cdot}0455 \times 28{\cdot}5 \times 455^2 \times 10^{-7} \text{ lbs.}$$
$$= 2025 \text{ lbs.}$$

If steel wire 0·056″ in diameter be employed, this force may be taken up by three bands each 1″ wide, giving a total of $\frac{3}{0{\cdot}056} = 53$ wires, and an area of $53 \times 0{\cdot}00246 = 0{\cdot}13$ sq. inch. Taking the breaking strength as 200,000 lbs. per sq. inch, the apparent factor of safety is $\frac{200{,}000 \times 0{\cdot}13}{2025} = 12{\cdot}8$. Along the bent ends of the winding the sectional area of the bars in a plane at right angles to the axis of rotation is increased in the proportion $\frac{t'}{t} = \frac{1}{\cos\theta} = \frac{m}{nt} = \frac{9{\cdot}85}{5{\cdot}26}$. The axial length of the two ends beyond the core is approximately 18″; hence $P = 2025 \times \frac{18}{15} \times \frac{9{\cdot}85}{5{\cdot}26} = 4550$ lbs. This may be divided at each end between one band 1¼″ wide close to the edge of the core, and one band 1½″ wide on the outer cylindrical ring supporting the end-connections. The total number of wires in the four bands is thus $\frac{5{\cdot}5}{0{\cdot}056} = 98$ with an area of $98 \times 0{\cdot}00246 = 0{\cdot}241$ sq. inch. The apparent factor of safety is therefore

$$\frac{200{,}000 \times 0{\cdot}241}{4550} = 10{\cdot}6\,,$$

the real value being in both cases reduced by the initial tension.

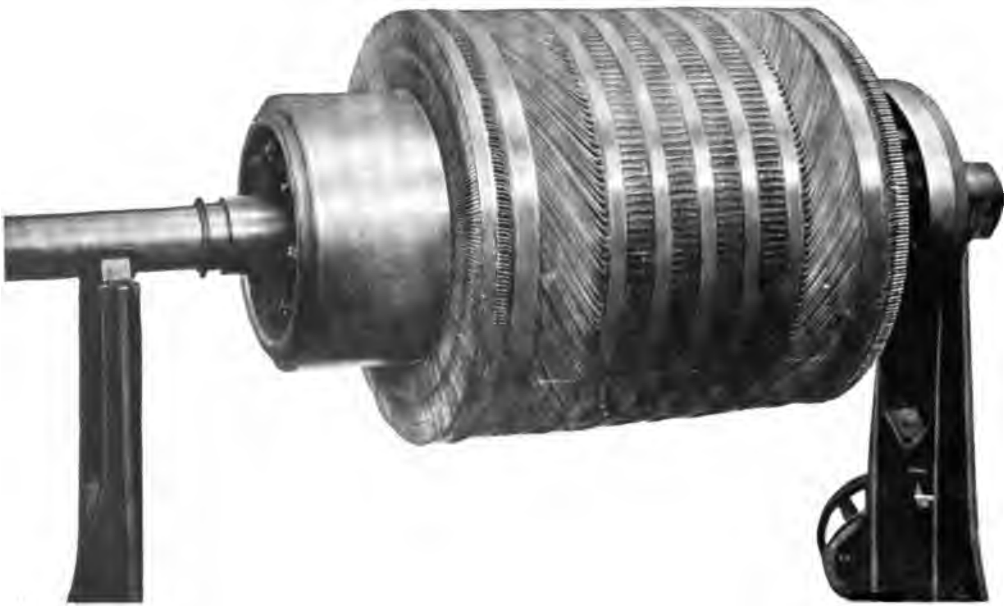


FIG. 276A —Slotted armature of 150 K W. dynamo. (W. H. Allen, Son & Co., Ltd.)

[*To face p.* 680.

The necessary cross-section for the magnet which is to be of cast steel will require careful consideration owing to the long range of volts over which the dynamo is to work. On the one hand, at the lowest flux the magnetism must not become unstable; on the other hand, at the upper limit the exciting power must not be unreasonably great and the proportion of the series to the shunt turns must not be too great. With the maximum flux of $11 \cdot 3 \times 10^6$ lines, and following the order of equation (44), we have $B_a = 16{,}260$, and $f'(B_a)$ from the lower curve of fig. 128 = 44 ampère-turns per cm. length; for $\frac{l_a}{2}$ may be taken the same value as c or half the distance between the poles measured on the armature core, *i.e.*, $3 \cdot 5'' \times 2 \cdot 54 = 8 \cdot 9$ cm.; hence $\frac{X_a}{2} = 390$. The ampère-turns required over the teeth and air-gap on one side have already been calculated to be $\frac{X_t}{2} + \frac{X_g}{2} = 465 + 7350 = 7815$. Assuming an angle of lead of 7°, by eq. (54) $\frac{X_b}{2} = \frac{76 \times 672 \times 7^\circ}{360^\circ} =$ say, 1000, and the sum of the four items is $\frac{X_P}{2} = 9205$.

From eq. (37*a*) the leakage permeance is reckoned as

$$P = 5 \cdot 85 \times 15 + 0 \cdot 82 \times 30 + 21 = 133 \cdot 6$$

The leakage lines are therefore $\zeta = 1 \cdot 257 \times 9205 \times 2 \times 133 \cdot 6 = 3 \cdot 1 \times 10^6$, making with the useful lines a total $Z_m = (11 \cdot 3 + 3 \cdot 1) \times 10^6 = 14 \cdot 4 \times 10^6$. The densities in the pole and yoke are required to be high in order to keep the magnetism stable and may approximately be made 17,000 and 16,000. We are thus led to a sectional area in the pole of $12'' \times 11''$, or $a_m = 850$ sq. cm., the two dimensions giving a reasonable contraction as compared respectively with the chord of the polar arc ($= 16 \cdot 2''$) and the axial length of the pole-face ($= 14''$), and further giving an economical section approximating to a square. The yoke may have a section of 70 sq. in. or $a_y = 452$ sq. cm. To assist the stability of the magnetism the magnet core may be given a somewhat greater length than usual, leaving a net winding length (after deducting the thickness of the end-flanges of the bobbin) of 9″. A rough sketch then shows for the length of path in the pole and through half of the yoke, $\frac{l_m}{2} = 32{\cdot}2$ cm. and $\frac{l_y}{2} = 58{\cdot}5$ cm. The densities are $B_m = 16{,}930$ and $B_y = 15{,}930$, and from the lowest curves of fig. 127, $f'(B_m) = 56$ and $f'(B_y) = 28{\cdot}2$, whence

$$f'\left(\frac{Z_m}{a_m}\right)\frac{l_m}{2} = 56 \times 32{\cdot}2 = 1800,$$

$$f'\left(\frac{Z_m}{2a_y}\right)\frac{l_y}{2} = 28{\cdot}2 \times 58{\cdot}5 = 1645.$$

Thus $AT_c = 9205 + 1800 + 1645 = 12{,}650$, which can be secured within the designed length of bobbin.

The smallest flux is given by the condition of 480 volts at no load and a speed of, say, 455 revs. per min. to allow for the range of the governor, and is then

$$Z_a = \frac{480 \times 10^8}{672 \times \frac{455}{60}} = 9{\cdot}43 \times 10^6.$$

When the volts are lowered to 460 on open circuit, the speed may also be lowered to, say, 430, and the flux is slightly higher, so that the first-named condition is the worst as regards the question of stability, which must now be definitely tested. We then have

$$B_a = \frac{9{\cdot}43 \times 10^6}{695} = 13{,}550\,;\ f'(B_a)\frac{l_a}{2} = 11 \times 8{\cdot}9 = \qquad 98$$

$$\frac{X_t}{2} = \qquad 70$$

$$\frac{X_g}{2} = 0{\cdot}8 \times 9{\cdot}43 \times 10^6 \times 0{\cdot}000812 = 6110$$

$$\frac{X_r}{2} = 6278$$

$$\zeta = 1{\cdot}257 \times 6278 \times 2 \times 133{\cdot}6 = 2{\cdot}1 \times 10^6$$

$$Z_m = (9{\cdot}43 + 2{\cdot}1) \times 10^6 = 11{\cdot}53 \times 10^6$$

$$B_m = 13{,}600 \qquad f'(B_m)\frac{l_m}{2} = 11 \times 32{\cdot}2 = \qquad 354$$

$$B_y = 12{,}800 \qquad f'(B_y)\frac{l_y}{2} = 8 \times 58{\cdot}5 = \qquad 468$$

$$AT_c = 7100$$

A tangent to the flux-curve at this point shows a ratio $\frac{z}{z_1} = 3$, so that on the question of stability the machine just falls within the limit named in Chap. XVI § 10.

The shunt winding may now be determined, as follows: taking the highest voltage of 520 at 445 revs. with a loss of 11·93 volts over the resistance of armature and brushes,

$$Z_a = \frac{531{\cdot}93 \times 10^8}{672 \times \frac{445}{60}} = 10{\cdot}65 \times 10^6.$$

$$B_a = \frac{10{\cdot}65 \times 10^6}{695} = 15{,}310;\ f'(B_a)\frac{l_a}{2} = 25 \times 8{\cdot}9 = \qquad 222$$

$$\frac{X_t}{2} = \qquad 250$$

$$\frac{X_g}{2} = \qquad 6920$$

The angle of lead will be increased, say, to 9° with the increased armature current of 320 ampères and weaker field, and

$$\frac{X_b}{2} = \frac{80 \times 672 \times 9^\circ}{360^\circ} = \qquad 1345$$

$$\frac{X_r}{2} = 8737$$

$$Z_m = (10{\cdot}65 + 2{\cdot}93) \times 10^6$$

$$B_m = 16{,}020 \qquad f'(B_m)\frac{l_m}{2} = 33 \times 32{\cdot}2 \qquad 1060$$

$$B_y = 15{,}050 \qquad f'(B_y)\frac{l_y}{2} = 16 \times 58{\cdot}5 \qquad 935$$

$$AT_c = 10{,}732$$

The previous calculations for the normal shunt voltage of 480 give

$$B_a = 14{,}200 \qquad f'(B_a)\cdot\frac{l_a}{2} = 15 \times 8{\cdot}9 = \qquad 133$$

$$\frac{X_t}{2} = \qquad 100$$

$$\frac{X_g}{2} = \qquad 6400$$

With the increased armature current of 348 ampères, weaker field and increased angle of lead,

$$\frac{X_b}{2} = \frac{87 \times 672 \times 11^\circ}{360^\circ} = \qquad 1790$$

$$\frac{X_P}{2} = 8423$$

$$Z_m = (9{\cdot}85 + 2{\cdot}83) \times 10^6$$

$$B_m = 14{,}900 \qquad f'(B_m)\frac{l_m}{2} = 15{\cdot}5 \times 32{\cdot}2 = \qquad 500$$

$$B_y = 14{,}020 \qquad f'(B_y)\frac{l_y}{2} = 12{\cdot}5 \times 58{\cdot}5 = \qquad 730$$

$$AT_c = 9653$$

It is evident, therefore, that the last-considered condition will not require quite so many shunt ampère-turns, and that the rheostat resistance will require but little alteration between the two cases. The maximum lighting current of 360 ampères, necessitating a flux of nearly $9{\cdot}95 \times 10^6$ lines at the lowered speed of 425 revs., will again require more ampère-turns.

$$B_a = \frac{9{\cdot}95 \times 10^6}{695} = 14{,}300 \qquad f'(B_a)\frac{l_a}{2} = 16 \times 8{\cdot}9 \qquad = 142$$

$$\frac{X_t}{2} = \qquad 120$$

$$\frac{X_g}{2} = \qquad 6450$$

$$\frac{X_b}{2} = \frac{91 \times 672 \times 11^\circ}{360^\circ} = 1870$$

$$\frac{X_P}{2} = 8582$$

$$Z_m = (9{\cdot}95 + 2{\cdot}88) \times 10^6$$

$$B_m = 15{,}100 \qquad f'(B_m)\frac{l_m}{2} = 16{\cdot}5 \times 32{\cdot}2 \qquad = 533$$

$$B_y = 14{,}200 \qquad f'(B_y)\frac{l_y}{2} = 13 \times 58{\cdot}5 \qquad = 760$$

$$AT_c = 9875$$

The total number of shunt ampère-turns is then $\text{AT}_c \times \text{P} = 9875 \times 4 = 39{,}500$, and as the exciting voltage is now only 460, this combination will determine the necessary size of wire. Allowing $\frac{3}{4}''$ for the insulation and necessary clearance between the bobbin-case and the pole on either side, the length of the innermost turn will be $2\{(12+0{\cdot}75)+(11+0{\cdot}75)\}=49''$, and after a preliminary trial and adjustment, a suitable depth of winding is found to be $t=2{\cdot}1''$. The mean length of a shunt turn is therefore $l_s=49''+\pi t=55{\cdot}6''=4{\cdot}63$ ft. The average rise of temperature of the whole coil will be for a depth of $2{\cdot}1''$ about $1{\cdot}43$ times the surface rise (Chap. XIX § 2), and if this latter is under the present conditions rather less than 70° F., say 68° F., while the temperature of the engine-room itself is 75° F., the average temperature of the wire when hot will be $75°+97°=112°$ above the standard temperature of 60° F. Thus the temperature coefficient, $k=1+(0{\cdot}021\times 112)=1{\cdot}236$. By eq. (46),

$$\omega' = \frac{460\times 1000}{39{,}500\times 4{\cdot}63\times 1{\cdot}236} = 2{\cdot}04 \text{ ohms,}$$

and by eq. (48), diameter of wire $=\dfrac{0{\cdot}1015}{\sqrt{2{\cdot}04}}=0{\cdot}071''$, *d.c.c.* to an over-all diameter of $0{\cdot}086''$.

The net winding-length of the bobbin, being $9''$, will admit of 104 turns per layer, and 26 layers will give practically the assumed depth of $2{\cdot}1''$ viz., $0{\cdot}9\times 26\times 0{\cdot}086''$ (Chap. XV § 17). The total number of turns on the four bobbins will be $104\times 26\times 4=10{,}816$ with a length of 50,000 ft., and a resistance at 60° F. of 102 ohms; or when hot $\text{R}_s=102\times 1{\cdot}236=126$. The shunt current is then $\text{C}_s=\dfrac{460}{126}=3{\cdot}65$, and the loss in excitation $\text{C}_s^2\text{R}_s=1680$ watts. When, however, the speed is raised to 445 revs. per min., and the armature current is reduced, the exciting voltage and shunt current will increase. By means of the regulating resistance in series with the shunt, the current must with 520 volts at the brushes be maintained at $\dfrac{10{,}732\times 4}{10{,}816}=3{\cdot}97$ ampères, in which case the watts apart from those in the rheostat are raised to $3{\cdot}97^2\times 126=1980$. The external perimeter of the shunt will be $49''+2\pi t=61{\cdot}6''$, and the total cooling surface over the shunt-winding of the four bobbins will be $s_c = 61{\cdot}6'' \times 9'' \times 4 = 2220$ sq. inches. As the shunt will be to a certain extent covered by the series-winding, the heating coefficient will be rather higher than in eq. 79c, say, 75, and

$$t^\circ = \frac{75 \times 1980}{2220} = 67^\circ\text{F}$$

Since the resistance of the shunt when hot will not be quite so high as has been assumed, there will be a little margin of control by the rheostat even when the machine is fully heated.

It remains to determine the compounding and the series winding. At no load with 500 volts at the terminals,

$$Z_a = \frac{500 \times 10^8}{672 \times \frac{455}{60}} = 9{\cdot}85 \times 10^6 \text{ lines.}$$

As this happens to be identical with the normal flux as a shunt machine, we have again

$$\begin{array}{lllr}
\frac{X_a + X_t + X_g}{2} = 133 + 100 + 6400 = \frac{X_P}{2} = & & & 6633 \\
Z_m = (9{\cdot}85 + 2{\cdot}22) \times 10^6 & & & \\
B_m = 14{,}200 & f'(B_m)\frac{l_m}{2} = 12{\cdot}8 \times 32{\cdot}2 & = & 412 \\
B_y = 13{,}350 & f'(B_y)\frac{l_y}{2} = 10 \times 58{\cdot}5 & = & 585 \\
\hline
 & & & AT_c = 7630.
\end{array}$$

When the rheostat is set to give $\frac{7630 \times 4}{10{,}816} = 2{\cdot}82$ ampères at no load with 500 volts at the brushes, the shunt current on the full traction load will be $2{\cdot}82 \times \frac{550}{500} = 3{\cdot}1$ ampères, and the shunt ampère-turns per pole will be 8400, or a little more if arranged as a short shunt. The necessary series ampère-turns per coil will therefore be $12{,}650 - 8400 = 4250$. To obtain a thickness of copper strip easy to wind, two bobbins may be placed in parallel and the two pairs in series. The number of series turns per bobbin will then be $\frac{4250}{150} = 28\frac{1}{2}$, which may be arranged in 7 layers of 4 turns each with one half-turn to carry the winding from the front to the back of the machine (fig. 277). The series coils may

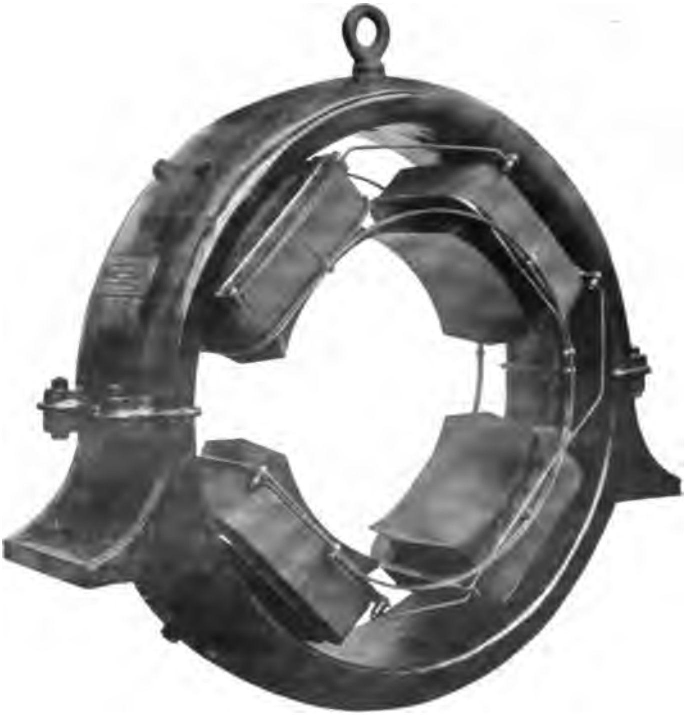


FIG. 277A.—Magnet of 150 K W. four-pole dynamo.
(W. H. Allen, Son & Co., Ltd.)

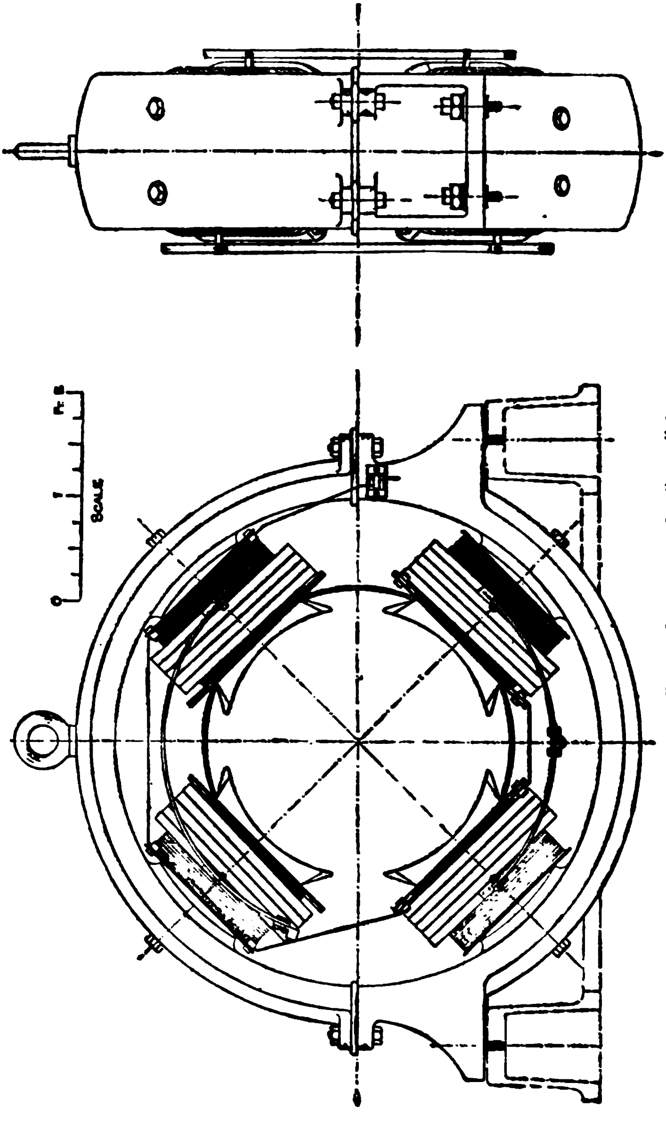


FIG. 277.—Four-pole magnet of 30" × 15" dynamo.

be wound separately and slipped on over the shunt coils with an air-space of ⅝″ between the two; they may be then held in place by curved wooden wedges at the corners, so as to allow of free circulation of the air between the series- and shunt-windings. The half thickness of the series-winding being, say, a little over ¾″, the mean length of a series turn will be $l_s = 49'' + 2\pi\ (2'' + 0{\cdot}625 + 0{\cdot}775'') = 70{\cdot}3'' = 5{\cdot}86$ ft. From the assumed induced E.M.F. and the known loss of volts over the resistance of the armature and brushes, we have as the permissible loss of volts over the series-winding $13{\cdot}7 - 11{\cdot}4 = 2{\cdot}3$, or 1·15 over each coil. Thence

$$\omega' = \frac{1{\cdot}15 \times 1000}{4250 \times 5{\cdot}86 \times 1{\cdot}2} = 0{\cdot}0386 \text{ ohms}$$

a lower value for *k* being taken owing to the greater exposure and lesser depth of the series turns. The necessary area of $\frac{0{\cdot}00813}{0{\cdot}0386} = 0{\cdot}21$ will be given by a rectangular strip 1⅛″ wide × $\frac{3}{16}$″ thick. The current-density is then $\frac{150}{0{\cdot}21} = 715$ ampères per sq. inch, and the total $R_m = 0{\cdot}00645$ at 60° F. or 0·00774 when hot. The total watts lost in the bobbin are now $C_s^2R_s + C_s^2R_m = 1210 + 700 = 1910$, dissipated over a larger surface than before, while the watts to be dissipated in the shunt rheostat are 490. The width of the taped strip being say $1{\cdot}125 + {\cdot}035 = 1{\cdot}16''$, the height of the series-winding as explained in Chap. XVI § 13 will be $5\frac{1}{2} \times 1{\cdot}16'' = 6{\cdot}4''$ out of the total winding length of 9″ on the bobbin.

The maximum number of ohms that the shunt rheostat must furnish is given by the condition of 480 volts on open external circuit when the shunt is quite cold and has its resistance at 60° F. of 102 ohms. The required shunt current is then $\frac{7100 \times 4}{10{,}816} = 2{\cdot}62$ ampères, and the total resistance of shunt and rheostat must be $\frac{480}{2{\cdot}62} = 183$ ohms. The difference of 81 ohms must be supplied by the rheostat, and in order to give a margin in either direction it should be designed for 90 ohms divided between fifty contacts. The series regulator will be further explained in Chap XXI § 6.

The efficiency under different conditions and for various pro-

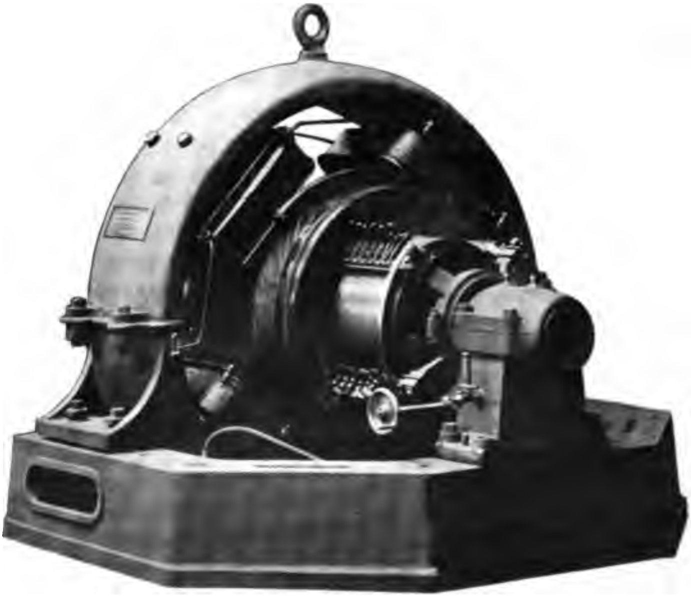


FIG. 277B.—Four-pole 150 K W. dynamo. (W. H. Allen, Son & Co., Ltd.)

[*To face p.* 688.

portions of the full lighting or traction loads may now be calculated as follows.

The loss over the resistance of the armature will decrease from full load in proportion to c_a^2, while over the brushes the decrease will be slightly less rapid owing to the rise of the contact resistance as the current-density diminishes. For a more or less constant terminal voltage the loss in the field-winding will decrease as the necessary excitation becomes reduced. On the other hand, the loss from eddy-currents and from hysteresis will remain more nearly constant, although even here there is some decrease as the necessary flux is decreased. The loss from the mechanical friction of the brushes is by eq. (86)

$$\frac{0{\cdot}3 \times 1\tfrac{1}{2} \times 22{\cdot}4 \times 2320 \times 746}{33{,}000} = 530 \text{ watts,}$$

the peripheral speed of the commutator being 2320 ft. per min. and the total brush area 22·4 sq. inches with a brush pressure of 1½ lbs. per sq. inch. The loss from the friction of the single outer bearing of the dynamo and from windage may be estimated as a constant quantity of 500 watts.

Taking the normal lighting load of 480 volts and 344 ampères, the efficiency as a simple shunt machine is calculated as follows:—

	Full Load.	¾	½	¼
$C_a^2R_a$	3800	2140	950	250
$C_a^2R_1$	646	410	210	60
Eddies, . . .	3560	3310	3150	3000
Hysteresis, . . .	1425	1410	1405	1400
Brush friction, . .	530	530	530	530
Bearing and air friction,	500	500	500	500
$C_s^2R_s$	1600	1330	1140	990
Rheostat, . . .	110	230	305	360
Total losses, . .	12,171	9860	8190	7090
Output, . . .	165,000	123,750	82,500	41,250
Input, ...	177,171	133,610	90,690	48,340
Efficiency, . . .	93%	92·5%	91%	85·5%

On the full load as a compound-wound traction generator[1]:—

$C_a^2 R_a$	2900
$C_a^2 R^1$	562
Eddies,	4700
Hysteresis,	1480
Brush friction,	530
Bearing and air friction,	500
$C_s^2 R_s$	1210
$C_e^2 R_m$	700
Rheostat,	490
Total losses,	13,072
Output,	165,000
Input,	178,072
Efficiency,	92·7%

§ 8. **Motor current method of measuring losses.**—The methods of testing the eddy-current loss in a continuous-current machine may here be introduced as bearing directly on the design of dynamos. The first, independently devised by Mr. Kapp and Mr. Housman,[2] consists in running the machine as a motor without load at various speeds, and noting the current through its armature, the E.M.F. applied to it, and the speed, the excitation being kept constant at the desired value throughout the test. The motor is not considered in the present book, but it is here necessary to state that the power absorbed by a motor (exclusive of field-excitation and loss over armature resistance) = $(E_e - C_a R_a)\,C_a$, where E_e = E.M.F. impressed on the motor armature, C_a = the current through it, and R_a = its resistance. When the motor is running light, C_a is very small, and with a fairly large armature of low resistance, $C_a R_a$ is practically negligible as compared with E_e; *i.e.*, the back E.M.F. of the motor is closely equal to the E.M.F. impressed on it. In a motor, as in a dynamo, the E.M.F. induced in the armature is by eq. (14)

---

[1] For design of large generators, see H. M. Hobart, *E. T. Z.* 1901, Aug. 15th p. 650.

[2] *Electrician*, vol. xxvi. pp. 699 and 700.

$= \frac{2}{q} z_a . \tau . \frac{pN}{60} \times 10^{-8}$ volts; and since z_a is in our case constant, it follows that the speed is proportional to the E.M.F. impressed on the armature.

Now the losses in hysteresis and mechanical friction are nearly proportional to the speed, while the eddy-current loss is nearly proportional to the square of the speed (Chap. XIX § 13); hence the total power absorbed by the motor may be divided into two portions, viz., the watts spent in hysteresis and friction $= (H + f)N$, and those spent in eddies $= F.N^2$, where $N =$ speed in revolutions per minute, and $(H + f)$ and F are two constants for the machine with a given excitation. If, therefore, the speed of the motor is increased, so also are the watts lost in hysteresis and friction; but, as we have said, the back E.M.F. of the armature is increased in equal proportion. Hence, if we mentally regard C_a as split up into two portions, one of which is employed in overcoming the torque from hysteresis and friction, and the other in overcoming the torque from the eddy-currents, the value of the former portion must remain constant in order that its product with the back E.M.F. may vary simply as the speed, and the curve showing its relation to the speed will be a horizontal straight line at some height above the horizontal axis, as shown at A C in fig. 278. On the other hand, as the speed is increased, and the watts lost in eddy-currents increase as the square of the speed, the second portion of C_a which is spent in overcoming the eddy-current torque must itself increase equally with the speed, in order that its product with the back E.M.F. may vary as the square of the speed. Hence the curve showing the

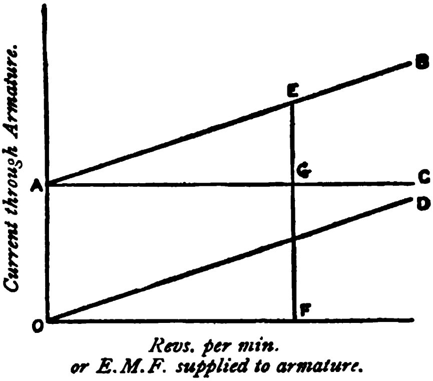


FIG. 278.—Separation of hysteresis and friction from eddy-current losses.

relation of this second portion of C_a to the speed must be a straight line inclined at some angle to the horizontal axis, as O D. Adding together the ordinates of the two curves A C and O D for each abscissa, we obtain a third curve, A B, which represents the total current through the armature: this will start from A and rise in an inclined straight line; and such is the actual form of curve obtained from an ordinarily good machine when tested in the manner described. If, therefore, we experimentally determine A B and draw A C parallel to the horizontal axis, we can deduce the required values of the constants F and $(H+f)$. To obtain the power in watts absorbed at any point of the curve, it is only necessary to multiply the abscissa (in volts) by the corresponding ordinate: thus for the point E the total watts absorbed $= OF \times FE$, and of these the watts absorbed by hysteresis and friction are $W_{(H+f)} = OF \times FG$, and the watts absorbed by eddy-currents, $W_F = OF \times GE$. Thence $(H+f) = \frac{W_{(H+f)}}{N_1}$, and $F = \frac{W_F}{N_1^2}$, where N_1 is the speed corresponding to OF volts.[1]

Fig. 279 shows the observed results of a test on a small dynamo, the values of the E.M.F. being given by the inclined straight line passing through the origin, and the corresponding values of the armature current by the inclined straight line which starts from an initial value of 2·85 ampères. At 600 revs. per min. with an E.M.F. of 85 volts the total current taken is 6·85 ampères; since the current required to balance the torque from hysteresis and friction is 2·85 ampères, the difference of 4 ampères multiplied by 85 volts gives the loss by eddy-currents in the armature, viz., 340 watts.

If two machines are coupled together rigidly, one can be run as a motor, and the change of current in its armature may then be used to determine the several losses in the other when excited or not excited, and in this way an unwound core can be tested.

Although on the whole much useful information may be obtained by a careful application of the above method, it is open to the theoretical objection that the loss from the mechanical friction of the bearings and by windage is not strictly proportional to, but increases faster than, the speed; in other words, the coefficient of friction is not strictly a constant quantity, and in consequence the readings of current may not fall exactly on an inclined straight line. A greater objection arises from the fact that in practice there is considerable difficulty in keeping the bearings in a steady normal state under the changes of speed which are a necessary part of the test. In any case, the machine should be run for some time prior to taking the readings in order to bring the bearings into good condition; but even then a change from a high speed to a low speed temporarily reduces the friction loss to a value below that which would obtain in steady running, and *vice versâ* if the change be made from a low to a high speed, so that some minutes must be allowed to elapse before the current is

---

[1] For certain cautions as to the application of these values in designing armatures *vide* Chap. XIX § 13.

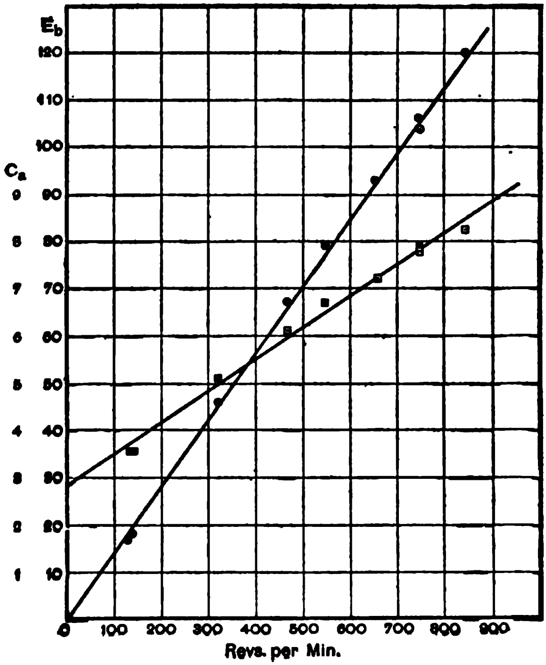


FIG. 279.—Eddy-current test.

measured after a change of speed.[1] It is further advisable to take two sets of readings in reverse order, the one with ascending and the other with descending speeds, and to note their exact sequence. Slight changes in the amount of oil passing through the journals or in the pressure on the shaft from the magnetic pull causing a minute alteration in its alignment are quite sufficient to alter the small friction loss appreciably.[2] At very low speeds the friction may increase owing to the lesser amount of oil that is swept into the bearing by the rotating shaft, or with ring lubricators owing to their then failing to throw the proper amount of oil.

§ 9. **Retardation method of measuring losses.**—The second, or as it may be called the *retardation method*, first applied by M. Routin,[3] necessitates a knowledge of the moment of inertia of the rotating armature, but possesses several incidental advantages. No assumption is made as to the law which the friction obeys, and its actual amount can be very accurately measured; since the changes of speed are automatically made in a definite sequence and with perfect regularity in each repetition of the test. The armature is first run up to speed, and the driving power is cut off; it is then allowed to come to rest, while at small intervals of time the speed is measured either directly or indirectly. The speeds when converted into angular velocities $\left(\omega = \frac{2\pi N}{60}\right)$ in radians, are plotted in relation to time At any given moment let ω be the angular velocity, and let a tangent be drawn to the curve at the point corresponding to the moment in question. The tangent then measures the time-rate of change of the angular velocity, or $\frac{d\omega}{dt}$. Let I be the moment of inertia of the revolving part in C.G.S. units. At any point of time as the rotor comes to rest, the rate of change of its angular momentum is equal to the retarding torque acting on it in C.G.S units; or

---

[1] *Electrician*, vol. xxvi. p. 740 and vol. xxvii. p. 162, and *Journal Inst. Electr. Eng.*, vol. xxxi. pp. 641, 642.

[2] *Electrician*, vol. xliv. p. 323.

[3] *L'Eclairage Electrique*, vol. ix. p 169. A full description of the method and its various modifications has been given by Dr. Alfred Hay, *Electr. Review*, vol. xlvii. p. 287, from which the following account has largely been drawn and to which the reader is referred for a more detailed discussion

$$\mathrm{T} = -\mathrm{I}\frac{d\omega}{dt} \tag{94}$$

and the rate at which the stored kinetic energy is expended in overcoming the retarding torque is

$$\mathrm{T}\omega = -\mathrm{I}\omega\frac{d\omega}{dt} \text{ ergs per second,}$$

or

$$= -\mathrm{I}\omega\frac{d\omega}{dt} \times 10^{-7} \text{ watts} \tag{95}$$

The latter expression is at any moment equal to the rate at which energy is dissipated in friction of bearings and brushes, and by hysteresis and eddy-currents. Thus if the angular-velocity curve be determined and plotted, first with the field unexcited and without brushes, then with brushes down, and lastly with the field excited to various strengths, a complete analysis of the various losses may be made.

The retardation curve by which the bearing friction may be determined is best taken by the following method: the machine is run up to or rather above its full speed by means of a small motor; the revolutions are taken on a speed counter, and the voltage due simply to residual magnetism is read on a low-reading voltmeter with a pair only of small brushes resting on the commutator; the driving belt is then thrown off, and at intervals of, say, three seconds, the voltage is read as it gradually dies away. The speed of the armature as it comes to rest is then simply proportional to its voltage, and by means of the residual magnetism can be measured indirectly, and more accurately than by a tachometer which in itself adds an indeterminate amount of friction and inertia, comparable in small machines with that which is to be measured. Next the field is excited, and the machine is run at its full speed as a motor. The armature circuit is broken while the voltmeter leads are left attached to the brushes; the same readings are taken of the voltage as it dies away, and a second curve is obtained. If the time of coming to rest is too short to allow of accurate readings being taken at successive intervals of a few seconds, the process must be continually repeated, a stop watch being started each time that the driving power is cut off and again stopped when the voltmeter needle passes a prearranged point. The difference between the two values of $\frac{d\omega}{dt}$ on the two curves for the same angular velocity ω of the armature, when multiplied by the angular momentum $I\omega$ measures the power absorbed by the hysteresis and eddy-currents in the excited field, and the latter may be given any desired value in order to test the effect of the flux in a given armature. The subsequent separation of the hysteresis loss from that by eddy-currents must be made on the approximate assumption that the former loss is proportional to the speed, and the latter to the square of the speed. Or graphically, by the principles of § 8 if the watts due to hysteresis and eddies are divided by the corresponding voltage, and the current so derived is plotted with the voltage as abscissæ, an inclined straight line is obtained, of which the intersection with the axis of ordinates measures the current required to overcome the torque due to hysteresis, while the line drawn parallel to the straight line of total current but passing through zero measures the current required to overcome the torque due to eddy-currents.

The determination of the moment of inertia I is the chief difficulty in the practical application of the method. In the case of a small armature it can be directly measured by determining the period of a complete oscillation when it is hung vertically by a bifilar suspension.[1]

Or if the periodic times T_1 and T_2 be determined, first for the armature alone, and secondly with the addition of a flywheel or of symmetrical weights of which the moment of inertia I_w can be easily and accurately calculated, the moment of inertia of the armature alone is

$$I = I_w \frac{T_1^2}{T_2^2 - T_1^2} \quad . \quad . \quad . \quad . \quad (96)$$

Or again, two retardation curves may be taken with and without a flywheel for which I_w can be calculated, when

$$I = I_w \frac{d\omega_1/dt}{d\omega_2/dt - d\omega_1/dt} \quad . \quad . \quad . \quad (97)$$

---

[1] Cp. *Journal Inst. Electr. Eng.*, vol. xxxi. p. 664, 'The Breaking of Shafts,' by Messrs. Frith and Lamb.

$\frac{d\omega_1}{dt}$ and $\frac{d\omega_2}{dt}$ being respectively the rate of change of the speed for the same value of ω in the two cases. Thus in fig. 280 a flywheel of known moment of inertia $I_w = 1{\cdot}8 \times 10^7$ was substituted for the usual pulley of the same dynamo for which the curves of fig. 279 were found. The upper retardation curve

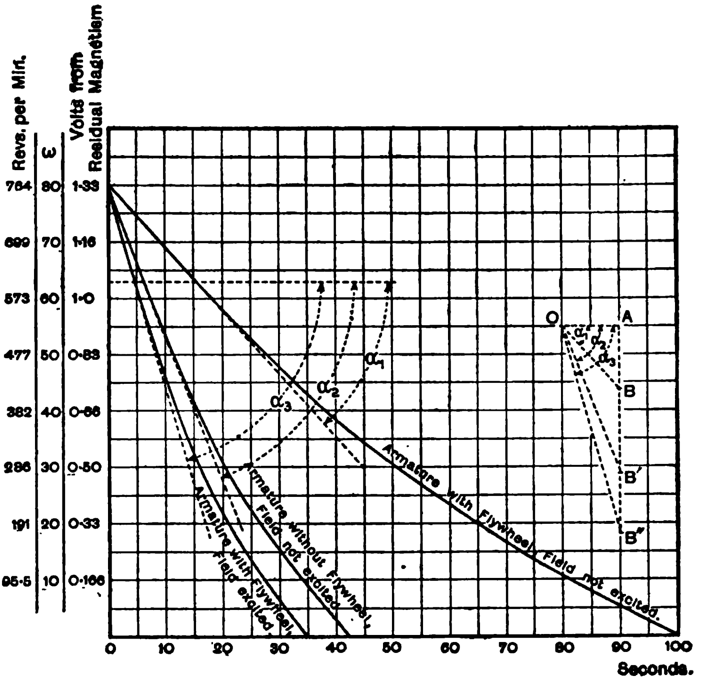


FIG. 280.—Retardation test.

was then obtained with the field unexcited, the speed being measured by means of the volts from residual magnetism as given in the third scale of ordinates. At 600 revs. per min. or $\omega = 62{\cdot}8$, $d\omega_1/dt = -1{\cdot}1$, angular velocities and seconds of time being plotted to the same scale and the angle a_1 being $47{\cdot}75^\circ$; at the same speed and under the same conditions but without the flywheel $d\omega_2/dt$ was $-2{\cdot}62$, the angle a_2 being $67{\cdot}1^\circ$.

Hence $$\text{I} = 1{\cdot}8 \times 10^7 \times \frac{-1{\cdot}1}{-2{\cdot}62 + 1{\cdot}1} = 1{\cdot}3 \times 10^7$$

and the total moment of inertia of armature and flywheel $= 3{\cdot}1 \times 10^7$. In the two preceding methods since the denominator is the difference between two quantities, accurate readings are necessary to avoid considerable error. With larger armatures having considerable moment of inertia, a retarding torque may be applied by means of a mechanical brake formed by a band or cord passed over the shaft or pulley, with one end carrying a weight and the other end anchored to a spring balance. Thus the second curve of fig. 280 might equally well have been obtained by an additional mechanical torque T_0; if the difference between the reading of the spring balance and the weight had been 4·16 lbs. = 1885 grammes at a speed of 600 revs. per min. and the radius of the flywheel or pulley to which the brake was applied was 10 in. = 25·4 cm.

$$\text{T}_0 = -1885 \times 981 \times 25{\cdot}4 = -4{\cdot}7 \times 10^7 \text{ C.G.S. units.}$$

The moment of inertia of the armature and flywheel is then

$$\text{I} = \frac{\text{T}_0}{d\omega_2/dt - d\omega_1/dt} \quad . \quad . \quad . \quad . \quad (98)$$

$$= \frac{-4{\cdot}7 \times 10^7}{-2{\cdot}62 + 1{\cdot}1} = 3{\cdot}1 \times 10^7.$$

When fully excited and with the flywheel in place in order to prolong the time of coming to rest, the lowest curve of fig. 280 was obtained, whence at 600 revs. $d\omega_3/dt = -3{\cdot}27$, the angle α_3 being 73°. The watts expended in overcoming all losses are at 600 revs. $3{\cdot}1 \times 62{\cdot}8 \times 3{\cdot}27 = 636$, while the rate of loss by friction and windage is $3{\cdot}1 \times 62{\cdot}8 \times 1{\cdot}1 = 214$, the difference of 422 watts being the loss by eddy-currents and hysteresis, which may be compared with the figure of 340 obtained for the eddy-loss alone from fig. 279. Graphically, as shown at the side of fig. 280, if the watts absorbed by the brake are $w_1 = \text{T}_0\omega \times 10^{-7}$, the watts absorbed by eddies and hysteresis are $w_1 \times \frac{\text{BB}''}{\text{BB}'}$. The readings of the brake may in fact be checked by comparison with the additional watts taken to drive the armature when the brake is applied. The curves of fig. 279 may also be themselves used to determine the moment of inertia of the armature; if at any speed the watts taken to drive the armature as a motor are *w*, and the rate of change of the angular velocity at the same speed is $d\omega_3/dt$ from the retardation curve, $\text{I}\frac{d\omega_3}{dt} = -\frac{w \times 10^7}{\omega}$. Indeed the two methods of the preceding and the present sections when combined afford a useful check upon the readings taken by the motor-current method, since for all speeds the determination of I from the retardation curve must give the same result if the readings are correct.[1]

If the armature has a very large moment of inertia, the additional torque required to calibrate the readings of the retardation curve may also be applied directly by electrical means; a circuit of suitable resistance is closed on the terminals of the armature when it is in process of coming to rest, and the additional watts thus added are calculated from the volts and ampères given off at the different speeds.[2]

§ 10. **Efficiency test by Hopkinson's method.**—The *efficiency* of a dynamo, or the ratio—

$$\frac{\text{power supplied from the terminals of the dynamo to the external circuit}}{\text{power given to the shaft of dynamo by the prime mover (engine or belt, \&c.)}}$$

may be calculated as in §§ 6 and 7 by adding the various losses in field, armature, and friction to the output, and dividing the output by the sum of output + losses. Since the constants F and (H + *f*) as determined for no load may be slightly too low for full load, the efficiency thence found may prove somewhat too high, the true value at full load being perhaps 1 or ½ per cent. less. There are, however, various ways of measuring the efficiency more directly. In cases where the dynamo is coupled directly to the engine, the power indicated in the engine cylinders, less the portion of this power which is wasted in the engine itself, gives the brake horse-power supplied to the dynamo; the results, however, of such a method of calculation cannot be regarded as very accurate, owing to the difficulty of ascertaining the exact value of the waste in the engine. Another method is to transmit the power from the prime mover to the dynamo through a transmission dynamometer which registers the power passing through it; unfortunately, such a dynamometer when of large size is both costly and difficult to manage, and, further, does not admit of very accurate readings being obtained from it. A much more exact method is that due to Dr. Hopkinson,[1] by which two similar machines are so coupled together that the one acts as a motor driving the other as a dynamo. The output from the latter, being returned to the armature of the motor, supplies the greater part of the power required to drive it; and it is thus only necessary to supply to the motor from some external source the amount of power expended in the losses within the two machines. As this is but a small fraction of the total power developed, it is more easily measured on a transmission dynamometer, and even a large error in its determination, since it only affects the comparatively small item of the waste power, produces but slight error in the result. The only objection to the method is that it requires two machines of exactly similar size and output. These are driven at their normal speed and approximately at their normal voltage; the field of one, M, is, however, slightly weakened by a rheostat in its magnet circuit, so that its internal E.M.F. (E_2) is less than the terminal E.M.F. (E_1) of the other machine (D); hence D sends a current through M as a motor, and by means of the rheostat the amount of this current is regulated until it corresponds to the normal armature current of either machine. Let $W =$ the total mechanical power in watts supplied from an external source to the motor armature, and $C =$ the dynamo armature current which is also passed through the motor armature, the fields of both being separately excited. Neglecting the loss over the connecting leads, which can be made as small as desired, if we deduct from W the losses in the armature resistances of D and M, the remainder, $W - C^2(R_{aD} + R_{aM})$ is the total loss by eddy-currents, hysteresis, and friction in the two armatures. The field of the one is slightly stronger than that of the other, but if the E.M.F. of D is made slightly greater, and the E.M.F. of M slightly less than the normal voltage of either machine, the error will be very slight if we assume this loss to be equally divided between the two. Let

---

[1] Cp. Prof. Peukert, 'Measurement of Losses in Dynamos,' *E.T.Z.*, vol. xxii. p. 393.

[2] See Dr. Sumpner, 'The Testing of Motor Losses,' *Journal Inst. Electr. Eng.*, vol. xxxi. p. 632.

---

[1] A description of this method is to be found in Dr. Hopkinson's paper on 'Dynamo Electric Machinery,' *Phil. Trans.* 1886, reprinted among his *Original Papers on Dynamo Machinery* (Whittaker and Co.), p. 112, ff.

$$\frac{W - C^2(R_{aD} + R_{aM})}{2} = \frac{L}{2};$$

then the commercial efficiency of the dynamo is

$$\eta = \frac{E_1 C}{E_1 C + C^2 R_{aD} + \frac{L}{2} + E_{f1} C_{f1}} \quad . \quad . \quad . \quad (99)$$

$E_{f1}C_{f1}$ being the watts expended in magnetising the field of the dynamo. Where it is not necessary to obtain such great accuracy, part of the armature current of the dynamo, D, may also be used to excite the field-magnets of both machines; in this case, assuming the machines to be shunt-wound, if C_1 = the sum of the currents C_{aM} and c_{sM} supplied respectively to the armature and field of the motor, the efficiency of the dynamo is

$$\frac{E_1 C_1}{E_1 C_1 + C_{aD}^2 R_{aD} + \frac{L}{2} + E_1 c_{sD}}, \quad . \quad . \quad (100)$$

where $$L = W - C_{aD}^2 R_{aD} - C_{aM}^2 R_{aM} - E_1(c_{sD} + c_{sM}).$$

It is convenient in carrying out these tests to couple the two shafts of motor and dynamo rigidly together, in one line; but it should be remarked that when this is done in the case of belt-driven machines, the loss by friction in the bearings may not reach its normal amount, since the pull of the belt corresponds to the transmission of but a small fraction of the normal power. On the other hand, when the motor and dynamo are coupled together by belt, an extraneous loss of power in bending the belt is introduced.

A further improvement of the above method consists in the use of a third dynamo as the external source whence the waste of energy in the system is supplied. This auxiliary dynamo need be of but small size, and may be coupled either in series or preferably in parallel with the two machines which are to be tested. When the method is thus modified, all the measurements can be made electrically by one voltmeter and one ammeter, and further, as regards the efficiency of the two armatures, great accuracy in the calibration of the instruments is not a matter of vital importance.

Fig. 281 (A) shows the series arrangement, in which the auxiliary dynamo A adds volts to the terminal E.M.F. of the dynamo, and must be capable of carrying the full current of the machines to be tested. The voltage of the auxiliary machine, being approximately $=\frac{\text{total losses in the two machines}}{\text{armature current}}$, will be from 20 to 40 per cent. of the voltage of either dynamo or motor according to their efficiency. The field of the dynamo D must be weakened by the rheostat *r*, and the two-way switch enables the observer to read in quick succession either the combined voltage on the motor (E_m) when the arm is placed on contact *a* or the terminal voltage of the dynamo (E_1) when it is placed on *b*. If the dynamo and motor are both separately excited from a fourth dynamo, and C be the ampères passing through the system as read on the ammeter, the power supplied to M is CE_m, while the power obtained from D is CE_1. The voltage of the auxiliary dynamo is $E_m - E_1$, and the power added by it is C ($E_m - E_1$). The combined

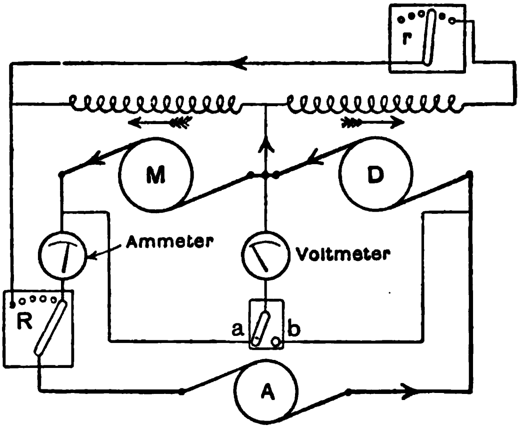


FIG. 281A.—Series electrical arrangement of Hopkinson efficiency test.

efficiency of the two armatures of M and D is thus $\frac{CE_1}{CE_m} = \frac{E_1}{E_m}$, and the efficiency of each separately is very approximately $\eta = \sqrt{\frac{E_1}{E_m}}$. A small percentage error in the voltmeter produces but little error in the result, since the efficiency is only proportional to the square root of the voltage ratio. The fourth dynamo employed for separate excitation of the fields may also be dispensed with by the arrangement shown in the diagram, but in this case the system must be started by means of the switch and resistance marked R. When the arm of the switch is placed on the contact furthest to the left at starting, resistance is inserted in series with the armatures, and sufficient fall of potential is obtained to pass a small shunt current through the two fields in order to supply an initial excitation and start the motor. As the speed rises, the arm is brought over to the right until all the resistance is cut out of the armature circuit, while its presence in the shunt circuits will produce little effect. In calculating the efficiency, allowance must then be made for the fact that the dynamo armature is carrying not only its own shunt current but also that of the motor; hence as in expression (100), if $C_1 = C + c_{sm}$, the combined efficiency of dynamo and motor is $\frac{E_1C_1}{CE_m + c_{sM}^2 r_{sM}}$, and the square root of this gives approximately the efficiency of either machine; or, separately, the efficiency of the dynamo

$$= \frac{E_1C_1}{\text{output of motor}} = \frac{E_1C_1}{CE_m - C^2R_{aM} - \frac{L}{2}}$$

and the efficiency of the motor

$$= \frac{CE_m - C^2R_{aM} - \frac{L}{2}}{CE_m + c_{sM}^2 r_{sM}}$$

where L = the watts added by the auxiliary machine minus the electrical losses over the armatures and fields of the dynamo and motor.

The parallel arrangement is shown in fig. 281 (B), and it will be seen that the auxiliary dynamo adds ampères and must be capable of giving the full E.M.F. of the machines to be tested. It is now the field of the motor M which must be weakened by means of the rheostat *r*, and readings are taken of the ampères in quick succession for the two positions of the arm of the two-way switch. When this is placed on the contact *a*, the dynamo current is read, exclusive of its own shunt current, and when multiplied by the volts on the voltmeter gives the nett output of the dynamo. When the two-way switch is placed on contact *b*, the ammeter reads the motor current inclusive of its own shunt current, or its total input. The combined efficiency of the two machines is then $\frac{C_d}{C_m}$, and the efficiency of each machine is very closely

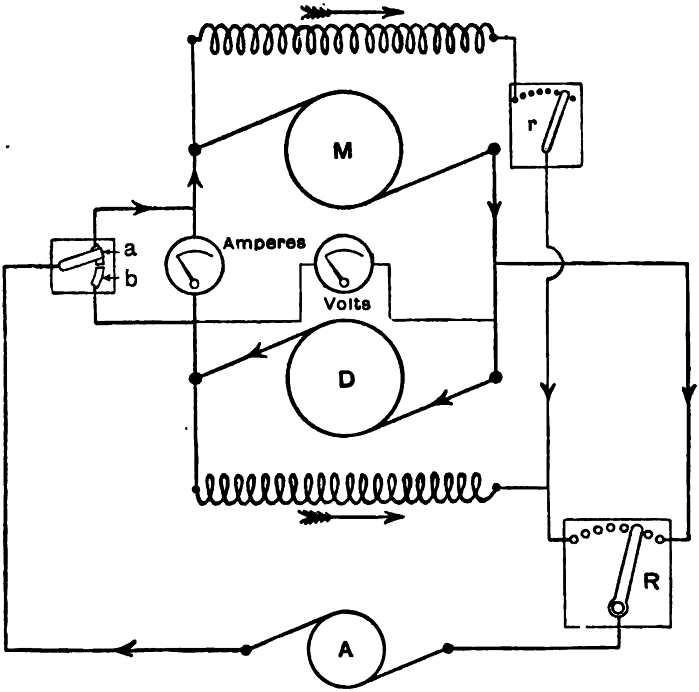


FIG. 281B.—Parallel electrical arrangement of Hopkinson efficiency test.

$\eta=\sqrt{\frac{C_a}{C_m}}$, the voltage being maintained constant during the short time necessary to take the two readings of ampères. The system is started by means of the switch and resistance R, with the arm thrown over to the left, and the same switch will also enable the voltage to be regulated to the right amount if that of the auxiliary dynamo is slightly higher than that of the machines to be tested. As before, a small percentage error in the ammeter produces an almost negligible error in the result.

In both cases care must be taken to ensure the connections to the fields being such as to cause the one machine to act as a motor and the other as a dynamo, and any change of the rheostats must be made gradually so that its effect on the system may not be masked by the inertia of the revolving armatures.[1]

§ 11. **Efficiencies of continuous-current dynamos.**—In fig. 282 are given the efficiency curves of two typical dynamos for various proportions of their full loads. The exact shape of the curve depends upon the relative proportions of the constant and variable losses. Thus in a 1000-kilowatt traction generator, the two are nearly equal at full load, each being about 3 per cent. of the output, and giving efficiencies—

at	full load	of	94·5	per cent.
„	$\frac{3}{4}$	„	94·1	„
„	$\frac{1}{2}$	„	93	„
„	$\frac{1}{4}$	„	87·7	„

Fig. 283 shows the efficiency at full load that may be obtained under ordinary commercial conditions with machines of different outputs. From the latter curve it will be seen that the rise in efficiency is but little after an output of some 75 kilowatts is reached. Any such curves are, however, greatly affected by conditions of the design, and in especial by the speed, a low-speed machine being in general less efficient for the same output than one which runs at a high speed. For a given carcase the loss in excitation remains practically constant for any speed. The loss from hysteresis, friction, and windage bears a nearly constant ratio to the output, and the only loss that increases faster than the speed is that from eddy-currents in the armature or pole-pieces. On the other hand, the output rises nearly proportionally with the speed, so that, on the whole, the higher speed is favourable to the efficiency. There is, however, for

---

[1] For a full description of these two methods, see a paper by G. Kapp on 'The Determination of the Efficiency of Dynamos,' *Electrical Engineer*, January 22 and 29, 1892.

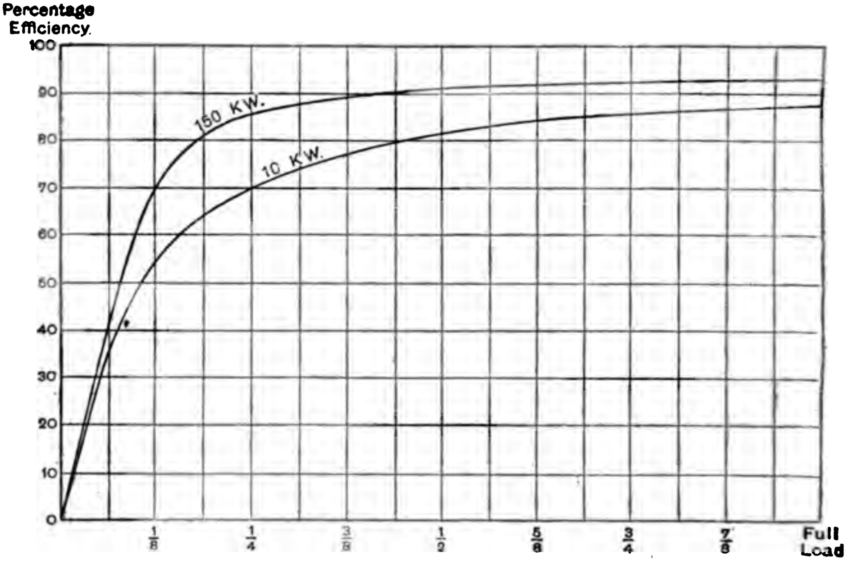


FIG. 282.—Efficiency at various loads.

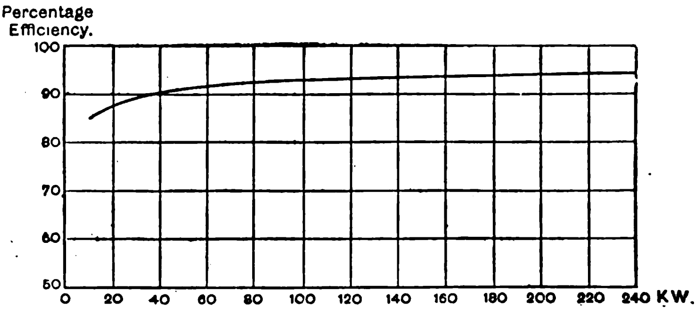


FIG. 283—Efficiencies at full load.

a given armature with a given winding a certain speed at which the efficiency reaches a maximum, the assumption being made that the increase of the output with increase of the speed is simply due to an increase in the volts; this speed is reached when the loss by eddy-currents is equal to the constant losses in the copper resistance of the armature and field-windings.[1] Such a fact is, however, of little assistance in the process of designing a machine in the first instance, since it presupposes a fixed copper loss. In the case of a shunt dynamo run at a constant speed, if the losses in friction, hysteresis, and eddy-currents be regarded as constant, then so long as the loss in excitation can also be regarded as constant, the maximum efficiency is reached when the variable loss over the resistance of the armature is equal to the sum of all the other losses supposed constant, but whether such a value for the armature current can be reached depends upon the slope of the external characteristic curve. In plating dynamos, owing to the large proportion of the total voltage which is lost over the contact-resistance of the brushes and other connections, and also owing to the considerable friction loss from the numerous brushes, the efficiency is necessarily low; *e.g.*, in a 10-kilowatt machine giving 4 volts and 2500 ampères, the efficiency may fall between 60 and 70 per cent. Very small dynamos are unable to excite themselves owing to the comparatively large air-gap required for mechanical reasons, and if the excitation becomes equal to the output, the efficiency is zero; hence, in the smallest sizes, a permanent magnet is generally employed.

§ 12. **Weights of continuous-current dynamos.**—Owing to the great diversity of the types and designs of dynamos, no expression even approximately accurate can be given for the weight of a machine in terms of its output per rev. per min.; yet for purposes of rough calculation, and with a given type of dynamo the weight may often be taken as proportional to the two-thirds power of the watts per rev. per min., or $\mathrm{W} = c\left(\frac{w}{\mathrm{N}}\right)^{\frac{2}{3}}$ lbs., and c has then such values as 350 in the case of bipolar machines with drum armatures, and 210 in the case of multipolar machines with slotted armatures, the weights in both cases being exclusive of that of the bedplate. The weight of the armature is usually from 20 to 40 per cent. of that of the entire machine; it bears the higher proportion in the case of multipolar machines, and correspondingly its cost bears a higher ratio to that of the machine as a whole. Analysis of a complete line of large multipolar direct-connected generators with slotted armatures for traction or power work shows the ratio of armature weight to total weight of machine rising from 0·22 with 6 poles through 0·28 for 8 poles, 0·32 for 10 poles, 0·35 for 12 poles, up to 0·42 for 22 poles; at the same time, while the watts per rev. per min. increase from 350 to 20,000, the value of the constant c falls gradually from 390 to 210.

---

[1] A. G. Hansard, *Electrician*, vol. xxxviii. p. 401.

CHAPTER XXI

THE WORKING AND MANAGEMENT OF CONTINUOUS-CURRENT DYNAMOS

§ 1. **The interconnection of dynamos.**—In many cases it is required to connect two or more dynamos to the same pair of mains in order to increase the amount of electrical energy that can be supplied to their common circuit. If the dynamos are coupled together in series, although the maximum current that may be passed through them is no more than the current permissible in the smaller of the two, yet the available voltage is increased. On the other hand, if two dynamos of the same voltage are coupled in parallel, the total amount of current can be increased, although the terminal voltage remains unchanged. Any such coupling together of dynamos must necessarily be so arranged that the working of one dynamo does not interfere with the proper working of the other or others; and as this requires certain precautions in their interconnections, the more usual cases which occur in practice will here be shortly considered.

§ 2. **The coupling of series-wound dynamos in series.**—The simplest case is the coupling of two series-wound continuous-current machines in series. To effect this, it is only necessary to connect the + terminal of one machine with the – terminal of the other, the external circuit being then connected to the remaining terminals of the pair. Such an arrangement is not unusual in cases of transmission of power over long distances, where it is necessary to work with high pressures in order to combine economy in the first cost of the copper leads with a high efficiency of transmission. It has been already mentioned that the delicate nature of the commutator hardly permits of more than 3000 volts being generated in any one continuous-current dynamo; but by the use of two or more similar dynamos of the closed-coil class coupled in series, and each giving, say, 2000 volts, a combined E.M.F. of 4000 or more volts is obtained on the external circuit.

§ 3. **The coupling of shunt-wound dynamos in parallel.** —The coupling of two or more dynamos in parallel is even more frequent. In all large installations and central electric works, as the load increases, more dynamos have to be brought into use, without interruption to the supply from the machines already running, and in such cases, in order to obtain the greatest economy, each dynamo should be worked as long and as closely as possible at its maximum output: this is best attained if they are all capable of being worked in parallel, an additional machine being switched on to the same mains as soon as the load exceeds the combined output of those already at work. The simplest case is that of two continuous-current shunt-wound dynamos, connected in parallel by joining their like terminals to form a common + and a common − terminal. If, as is usually the case, one machine is already running and excited, care must be taken that it is not joined in parallel with a second machine while the latter is at rest or is unexcited. The armature of the second machine would then form a short-circuit to the first, and would present no E.M.F. opposing an excessive rush of current through its low resistance. Hence the second machine, B, must be run up to its normal speed, and before it is thrown into parallel it must either be allowed to excite itself to approximately the same voltage as that of machine A, or B's shunt circuit must be closed on the first machine so as to excite B's field before its armature circuit is closed: the machines may then be safely thrown into parallel, and each will supply a certain part of the load, the exact proportion in which the total current is divided between them depending on their respective internal E.M.F.'s and armature resistances. The condition which determines this division is that after deducting the volts lost by the passage of the current over the armature resistance of either machine from its internal E.M.F., the remainder or the terminal voltage must be alike in both machines. Thus, if two similar machines, similarly excited and run at the same speed, be coupled in parallel, each will take half of the total current. If the speed of one be now lowered or its field weakened, its current will gradually pass over to the other machine; when its internal E.M.F. falls to equality with the terminal voltage of the second machine, it supplies no current at all, the whole of the load being thrown on to the one machine. To take a numerical example, suppose that each machine runs normally at 1000 revolutions, and is then excited with 13,000 ampère-turns giving $z_a = 4{,}300{,}000$, and $E_a = 103$ volts, further, that the loss of volts over the armature at the full load of 100 ampères reduces the terminal voltage E_b to 100 volts. If the speed of machine B falls, a larger portion of the current passes over to machine A. The increased loss over the armature resistance of A and the increased reaction of the armature current on its field combine to reduce both its internal and terminal voltage. When it takes the whole of the current, let $z_a = 4{,}100{,}000$ be the number of lines that are produced by the 12,000 ampère-turns due to the terminal E.M.F. of 92 volts; in other words, $C_a = 200$ ampères, and $E_b = 92$ volts, give a point on its characteristic curve for a constant speed of 1000 revolutions. This same terminal voltage will, however, in the case of machine B, which is carrying no current, give, say, $z_a = 4{,}250{,}000$, and this flux will give an internal E.M.F. of 92 volts when B is running at 900 revolutions. Thus the speed of B may be reduced by 10 per cent. before the whole of its load passes on to machine A, provided that the speed of the latter is kept strictly constant. On a further reduction in the speed or strength of field of B, its internal E.M.F. falls below the terminal voltage of A, and the latter then drives a reverse current through B's armature: the internal E.M.F. of B + the volts lost over its armature resistance due to the reverse current are then equal to the terminal voltage of A. The effect of the reverse current through B's armature is to assist in turning it as a motor with the same direction of rotation as before, without mechanically damaging the brushes, and thereby it tends to keep up B's speed. The electrical interaction of the two machines with drooping characteristics thus exerts a considerable inherent influence, tending to equalise their speeds and loads; it is therefore easy to work shunt-wound dynamos in parallel, and the share of the current which each machine takes is easily regulated by altering its speed or its field-excitation. In practice, when, as is most often the case, the machines are driven by separate prime movers, the equalising tendency is much assisted by the mechanical action of the prime movers themselves. Thus, when the load of one machine increases, the speed of its engine or other prime mover falls and tends to check the increase, while, for a similar reason, when a machine fails to maintain its fair share of the load, the speed of its prime mover rises, and tends to keep up its E.M.F.

The use of shunt-wound dynamos in parallel is common in central stations for electric lighting: as soon as the load becomes too great for a single machine, a second is run up to speed, excited and switched into parallel with the first, the same process being repeated with other dynamos as often as required. When so connected, they all supply current to a common pair of 'omnibus bars,' whence the feeders are run to the network of mains. As a precautionary measure, a magnetic switch is frequently inserted between each dynamo and the omnibus bars: this automatically flies off and breaks contact if the armature current falls below a certain minimum; and hence, if for any reason a dynamo begins to slow down, it is cut out of circuit before a reverse current passes through it, while its load is taken up and divided among the other dynamos at work, without interruption to the general supply of current.

§ 4. **The coupling of series-wound dynamos in parallel.**—When series-wound dynamos are connected together in parallel after the same fashion by joining their like terminals, we are met by the difficulty that if, for any reason, the E.M.F. produced by one machine, B, falls considerably below that of the other machine, A, then the current through B is reversed; and this, since the machine is series-wound, reverses its polarity. The direction of B's E.M.F. is consequently reversed, with the result that both dynamos simply act in series round their own internal resistances, and in a short time would be damaged by the excessive current. This difficulty is, however, at once overcome by the addition of an *equalising* wire connecting the ends of the series coils adjoining the brushes (fig. 284). It is especially important that this equalising lead, *b b′*, should be of such large area that its resistance is practically negligible as compared with the resistance of either of the series windings. By it the extremities of the two field-windings are joined in parallel, and the current supplied to the external circuit flows in the same direction through both and magnetises each equally. Should the internal E.M.F. of machine B fall below the voltage at the brushes of A, a reverse current passes through B driving it as a motor, but as this motor current flows through the lead *b b′*, and does not pass in a reversed direction through the series winding of B, it does not reverse B's polarity. If the fall of volts over *b b′* is considerable, a certain proportion of the motor current may pass through the alternative path of *b d d′ b′*, but this is prevented by making *b b′* of sufficiently low resistance, as mentioned above. If it be required to switch B into parallel with A while the latter is running, it is necessary, as in the case of shunt machines, to excite B to approximately the same voltage: this is effected by closing the switch K while B is running, and a few seconds later switch S′ may be closed.

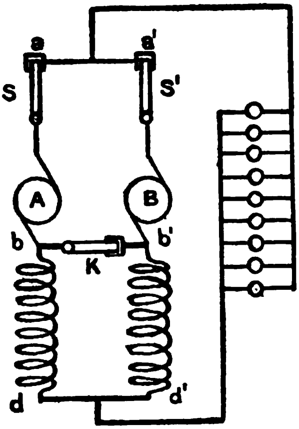


FIG. 284.—Two series-wound dynamos coupled in parallel.

§ 5. **The coupling of compound-wound dynamos in parallel.**—Series-wound dynamos are, however, but seldom required to work in parallel, and the above remarks are merely introduced owing to their applying equally well to the series windings of compound machines when worked in parallel. The arrangement usually adopted for these is shown in fig. 285, from which it will be seen that when the switches are closed, not only are like terminals connected together, but also like brushes, exactly as in the case of series machines and for the same reason; while, again, the equalising wire *b b′* must be of negligible resistance as compared with the series windings. In order to throw the one machine, say B, into parallel with the other, A, when the latter is running, B must be run up to approximately the same voltage as A, the excitation being provided by the shunt alone; switch K is then closed, and finally switch s′, while if it be required to withdraw machine B from parallel connection, its speed should be slightly reduced until it is supplying little or no current, after which switch s′ is opened, and lastly switch K. The opening or closing of the two connections in due order may be conveniently effected by a composite switch by which the contact at K is made first on closing the switch, and broken last on opening it.

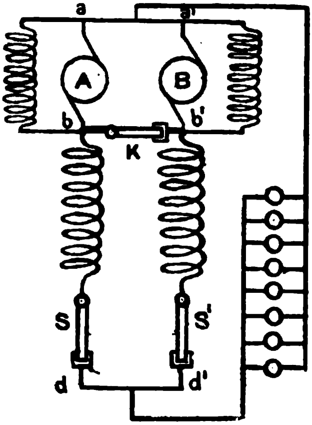


FIG. 285.—Two compound-wound dynamos coupled in parallel.

If the dynamo ammeters are placed in the leads to the bus bars and on the same side as the equalising connections, the sum of the currents gives the total ampères supplied to the external circuit, but the separate readings of the instruments in no way indicate how the load is divided between the several machines, and one or more might be running as motors with current supplied by the other machines. This may be obviated by placing near the equalising leads small polarised needles which indicate in which direction the current if any is flowing in the equalising leads. It is, however, better to place the ammeter of each dynamo in the lead to the bus bar which is on the side furthest from the equalising connections, when it will measure the current actually passing through the machine to which it is connected. A small polarised needle will then indicate when the current is in the wrong direction and is driving the machine as a motor. Still better is it to employ moving-coil ammeters with a zero position and a short scale on one side so that the readings of the needle to one or the other side of the zero render the state of affairs evident at a glance. In every case, either the hand-wheel governing the speed of the engine or the shunt-regulating switch must be employed to secure equal divisions of the load.

§ 6. **Regulation of load between compound-wound dynamos in parallel.**—In the case of over-compounded dynamos, it is not easy to secure an equal division of the load between several machines owing to small differences in their characteristic curves. Thus suppose that two dynamos of equal size are running in parallel with the load equally divided between them and that for some reason the speed of one falls slightly; its load decreases, while that of the other machine rises, and this change is accompanied by a similar change in their voltages, so that the load tends to pass completely over to the machine which has maintained its speed, and this tendency is only held in check by the mechanical governor and not by any inherent electrical effect. The first requirement for running compound-wound machines of different make or of different output in parallel is that the resistance of their series windings and connecting leads between the equalising and bus bars should vary inversely as their normal full loads, so that when carrying their maximum currents the drop of volts over the series windings should be the same for all machines. The amount of the compounding action of a machine is frequently regulated by a *diverting switch and resistance*, which shunts a portion of the current away from the series turns through a byepass. When this is the case, any alteration of the diverting switch of one machine also affects the current through the series turns of the other machines. This may be corrected by the employment of a *compensated series regulator*, of which the principle is shown in fig. 286 in connection with a dynamo which is used as a shunt machine on a lighting load and as a compound-wound machine on a traction load. In the position of the switch shown in the diagram, the current passes immediately through the arm from *a* to *b*, and there is no compounding action at all. But when the arm is moved in a clockwise direction, a shunting resistance r_2 is inserted in parallel with the series turns R_m, and as the resistance r_2 increases step by step, more and more of the total current C passes through the series turns. At the same time with a movement of the switch from its starting position a compensating resistance r_1 is inserted in series with the parallel branches of the series turns and shunting resistance, and this resistance r_1 is gradually cut out

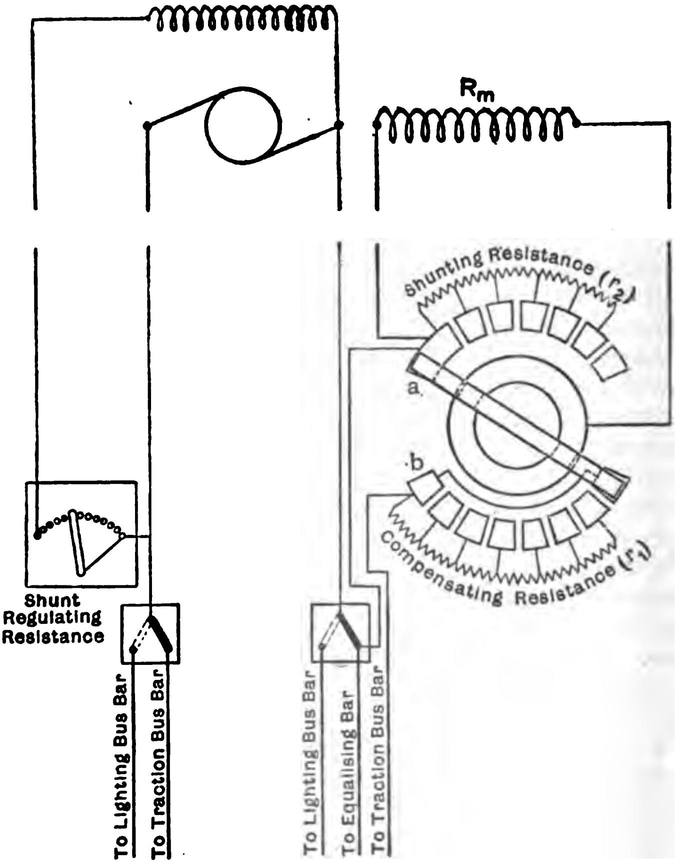


FIG. 286.—Series regulator for compound-wound dynamos in parallel.

until with the arm on contact *b* all the current passes through the series turns, and the maximum compounding action is obtained. On any contact (except the starting position) $r_2 = \frac{C_m}{C - C_m} \cdot R_m$, and

$r_1 = \frac{C - C_m}{C} \cdot R_m$, where R_m is the resistance of the series turns and of the leads therefrom to the regulating switch. By this means the resistance between the points *a* and *b* is maintained perfectly constant at a value $= R_m$, and an increase of C_m from 0 to C is obtained by six equal steps without in any way affecting the other machines which are in parallel.

§ 7. **Foundations and erection of dynamos.**—It remains to add a few remarks on the fixing and working of dynamos in general. In the first place, good foundations are as much an essential for successful working as a good dynamo, inasmuch as any vibration is most detrimental to the life of the armature, commutator, and brushes, and if transmitted to the surrounding floor and walls will prove a constant source of annoyance. The ground should be excavated until a sound and solid bottom is reached, on which may be built up a foundation of concrete, or of concrete and brickwork, sufficiently massive to absorb and damp the vibration of a fast-running dynamo. With small dynamos, Lewis bolts having a tapering shank of rectangular section with jagged edges are used to hold down the bed-plate or slide-rails: the square holes required for these bolts are cut into the concrete, their position being marked off usually from a wooden template of the slide-rails, and when these latter are ready to be laid down, the bolts are dropped in and fixed by lead or sulphur cement run in round them. With larger dynamos, long holding-down bolts are used, passing right through to the bottom of the concrete and terminating at their lower ends in large square plates. The holes for such bolts are formed in the concrete by inserting long tapering wooden boxes of square section through which pass the bolts; when the bed-plate is in position, thin cement is run in round the bolts, until the holes are full, and is then allowed to set without disturbance. In all cases, the upper surface of the foundation should be carefully levelled with a straight-edge and spirit-level, or a smooth slab of York stone may be used as a seating on the top of the concrete.

When driven by belt, it is essential that the dynamo should be set square with the driving engine or shafting, in order that the belt may run fairly on the centres of the two pulleys. This is ascertained by taking a length of string and stretching it tightly from the far edge of the driving pulley or flywheel to the dynamo pulley; when the string just touches the near edge of the driving pulley, its transverse distance from a centre line drawn round the dynamo pulley should be the same both on the side nearer to the driving pulley and on the side farther from it.

The belt itself should be soft and flexible, and joined into an endless band either by a cemented and riveted joint, or by belt fasteners with a butt joint; a lapped or laced joint causes a jerk each time that it passes on to the dynamo pulley, which gives rise to vibration and fluctuation of the E.M.F., together with sparking. Wherever possible, the direction of rotation should be such that the under side of the belt is the tight or driving side.

Perfect steadiness of driving is of great importance, especially for direct incandescent lighting; a very slight fluctuation in the speed of the dynamo, even though it be not great enough to cause sparking, is immediately discernible as a pulsation in the light of the lamps, owing to the slight change of E.M.F. to which it gives rise. On this account many types of gas- or oil-engines are inadmissible for direct lighting owing to their great fluctuation of speed during each cycle, even when fitted with heavy flywheels. When such prime movers are used, the dynamo is usually worked in conjunction with an accumulator battery, and even then it is advisable for the dynamo to be itself fitted with a heavy disc flywheel. In such cases, the secondary cells should never be placed in the same room with the dynamo, since the cotton or other fibrous insulation of the dynamo wires is rapidly attacked and eaten away by the acid spray given off during the process of charging.

After the machine has been fixed in position, the armature should be turned round by hand to see that it revolves freely and that nothing is loose; it should then be run for some time with the brushes raised from the commutator, in order to test the alignment of the bearings and of the machine generally. The adjustment of the brushes when lowered on to the commutator requires careful attention: usually lines are cut on the collar or sectors of the commutator next to the outer bearing (these lines being in a 2-pole machine diametrically opposite to each other), and to them the brushes must be set, the tips of each set of brushes carried by one arm of the rocker being in line with one setting mark.

All screw contacts should be firmly screwed up, and any dirt or lacquer (if such there be) on the points of binding-screws should be cleaned off. Want of contact of the brushes on the commutator or elsewhere may cause a failure of the dynamo to excite. The electrical connections should be carefully examined and verified; and on starting, it should be borne in mind that a shunt-wound dynamo will not excite on a very low resistance, or a series-wound dynamo on a very high resistance. If, therefore, a shunt-wound dynamo fails to excite even when the main switch is open, a short-circuit in the leads may be suspected. Or if a series-wound dynamo will not excite with its normal external circuit closed by the main switch, there may be a disconnection either within or outside the machine. Any such difficulty will usually be solved by testing with an ordinary linesman's detector; or in default of a solution by this means, trial may be made with the connection of the brush leads to the field winding transposed, in case it is through misconnection that the field will not excite.

§ 8. **Care of machine in working.**—As the brushes wear, they must be fed forward through their holders; and if they do not then bed properly on the commutator over their entire thickness, their tips must be filed to the correct angle. To trim a gauze brush, it should be clamped in a vice, with a plate to support it at the back, and filed with a smooth file, the strokes being made towards the extreme tip. When a new set of brushes requires to be bedded, it may be conveniently done by tying a piece of glass paper accurately round the commutator, and running the armature slowly while the brushes are pressed on to the glass paper. After any such operation the tips of the brushes must be cleaned from any adherent copper dust.

For the filling of the lubricators, copper oil-cans should invariably be used, since iron cans are liable to be drawn to the magnet, and thereby, perhaps, cause damage by catching in the armature. All oil-pipes and waste oil-chambers require occasional attention to see that they are not clogged. If from any neglect in this respect, or from original defective construction, oil creeps from the bearings on to the surface of the commutator, it becomes carbonised by any sparking at the brush tips, and forms a thin conducting film which bridges across the strips of insulation between the sectors of the commutator, the result being a loss of E.M.F., due to the local leakage which ensues between neighbouring coils; while if oil further makes its way on to the armature winding, it has a deteriorating effect on the protecting varnish, and causes adherent deposits of dirt and copper dust. Every dynamo should therefore be kept clean, and in especial its commutator requires scrupulous care on this point. Lubrication of the surface of the commutator is to be avoided—except at occasional intervals, when with gauze brushes a little vaseline smeared on a piece of clean rag may be rubbed on while the armature is rotating.[1] With carbon brushes paraffin wax may be sparingly used as a lubricant. If carbon brushes show signs of copper dust working into their bearing surface, any such coppering must be carefully scraped away. The dark burnished lustre which may be seen on the commutator of a good non-sparking dynamo is the sure evidence of a careful attendant. Occasionally, before stopping, fine emery cloth may be applied if the surface shows signs of wearing into grooves and becoming uneven. If a flat begins to develop on one or more sectors (Chap. XVIII § 27) it should be filed out, the sectors on each side being carefully rounded so as to retain as far as possible the cylindrical surface of the commutator. But if the disease has gone too far for such remedy, the armature must be put in a lathe, and the commutator surface turned up true; the tool should be sharp and fine-pointed and the feed should be light, so as not to drag the copper over the insulating strips of mica: after turning, it should be lightly filed with a smooth file, and finally examined to see that no particles of copper are embedded in the mica, and bridging adjacent sectors. Still better is it to grind the commutator true in place by means of a small motor-driven emery wheel.

---

[1] In high-tension open-coil machines with air-insulation dividing the sectors of the commutator, oil may be more freely used.

CHAPTER XXII

ARMATURE WINDING OF ALTERNATORS.

§ 1. **Suitability of the alternator for high pressures**—While the continuous-current constant-potential dynamo is limited to pressures not exceeding some 3000 volts[1] owing to the difficulty of commutation at the brushes and partly also of insulating the wires of its rotating armature, the alternator is by contrast well adapted on every score for the production of high voltages. Its armature coils may be stationary and heavily insulated; there is no commutator with its liability to trouble from sparking; lastly, the alternating-current transformer with its stationary windings affords a simple, economical, and thoroughly reliable means for transforming electrical energy from a high to a low pressure or *vice versâ* just as the circumstances demand. The special field therefore of the alternator is the transmission of electrical energy over long distances, and for this purpose it is practically without a rival. In the early days of electric lighting, single-phase alternators were largely employed in central stations for the smaller cities and suburbs which could not be served by the low-tension continuous-current system; at pressures of 1000 or 2000 volts step-down transformers dotted over large scattered areas could be fed without an excessive amount of copper in the transmitting lines. As the demand for lighting increased and also became more concentrated, the spread of the three-wire system and the raising of the pressure at the consumer's terminals to 200 volts enabled the continuous-current system to reach a wider area of dense supply with reasonable economy of copper in the mains; at the same time for want of a satisfactory single-phase motor, energy could not be supplied for motive purposes on the alternating-current system. On both counts therefore the latter found less favour. With the introduction, however, of the induction motor and polyphase alternator, the last-named objection was removed, and the alternating current once again recommended itself from the ease with which the energy could be transmitted at high pressures up to as much as 40,000 volts, and from the flexibility with which it could be adapted to meet every kind of demand. The generating station could now be placed on some convenient and economical site, perhaps near to a waterfall or to a coalfield, at a considerable distance from the area to be supplied, the alternator could itself be built directly for pressures of 5000 or in large sizes for as much as 10,000 to 15,000 volts, while the energy could be supplied at high pressure and transformed down at a substation, or again converted into direct-current form for traction purposes by means of the motor-generator or rotary converter. The latter serves as the connecting link between the alternating and continuous systems, and is especially useful for short-distance traction or railroad work, for which the continuous-current series-wound motor is unrivalled from its great starting torque and accelerating power.

---

[1] A special dynamo with stationary armature giving 25,000 volts has lately (1902) been built to the design of M. Thury for the St. Maurice-Lausanne transmission.

§ 2. **Drum and disc windings more common than ring armatures.**—Owing to the absence of the commutator, the winding of the alternator armature is somewhat simpler than that of the analogous continuous-current machine. Of the four kinds of windings, the ring method is seldom now used except occasionally in its discoidal form. The advantage of the drum winding is the same as in the case of continuous-current dynamos, viz., that with a properly proportioned core it requires less wire for a given number of turns or of inductors, and further with the same class of armature, either toothed or smooth, the drum loops have less inductance than the ring; in virtue therefore of its lower resistance and inductance, it gives a better regulation for constant potential. Disc winding is precisely analogous in its general principles to drum winding, and the diagrams (figs. 287–293) of the ensuing sections may be taken as applying equally to either drum or disc armatures.

Again, the ring, drum, and disc methods are in theory all equally applicable to polyphase alternators, if the widths of the coils or of their sides are arranged to suit the number of separate phases of the same frequency which the machine is to give. Yet, for practical reasons connected with the ease of manufacture, the most usual type is the drum, and the disc method is practically confined to single-phase armatures, which have now first to be considered.

§ 3. **Single-phase drum armature winding.**—In the drum armature of a single-phase alternator, there are usually as many coils as there are poles; each of the coils then has its outside end connected to the outside end of the adjacent coil on one side, and its inside end connected to the inside end of the adjacent coil on the other side. In fig. 287 the radial portions of each coil in the left-hand side of the diagram mark the inductors which lie on the surface or in the slots of the armature, and the rounded ends are assumed to lie outside the influence of the field. If the winding be connected in series right round the armature, one group consisting of the adjacent halves of the first and last coils is subjected to a strain on the insulation of contiguous wires amounting very nearly to the total voltage of the machine. The inside ends of two neighbouring coils are best made the terminals of the winding, but if the E.M.F. of the machine is very high, it may be advisable to interpose a dummy coil so as to more effectually separate the ends of the armature. Or the winding may be divided into two halves in parallel (Chap. VIII § 8); the coils which are in series are then wound in the same direction, and this direction is reversed at the points where the two halves are united in parallel. When the number of pairs of coils is even, the junctions are made by connecting together contiguous inside ends of one pair of coils and the distant inside ends of the opposite coils. With an uneven number of pairs of coils, contiguous inside and contiguous outside ends are joined together. The maximum difference of potential between any neighbouring wires is then equal to that generated by a pair of coils, and so in a machine giving, *e.g.*, 2000 volts of effective E.M.F. with 12 poles does not amount to more than about 950 volts at the moment of maximum induced E.M.F.

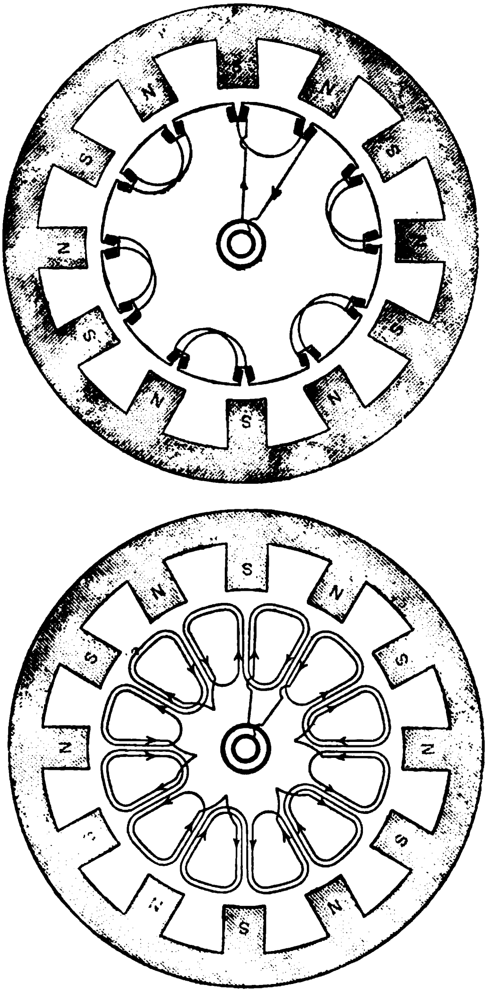


FIG. 287.—Single-phase drum or disc armature with divided coils.

The method of parallel division is, however, accompanied by the disadvantage that there are then twice as many inductors, each of half the area, and consequently there is more space wasted in insulation, while further it is difficult to secure exact equality of the E.M.F. generated at each instant in the two parallels, from which there may result a loss of efficiency and greater heating of the armature. On the above score a single large coil per pair of poles corresponding to fig. 71 has the advantage; it will be seen from fig. 288 that with this arrangement which in the toothed armature gives only half the teeth wound, the beginning and the end of the winding when connected in one series are kept well apart, and there is little difficulty in insulating the coils at the point of maximum strain. The method is therefore well suited for high voltages, but has the disadvantages already mentioned in Chap. VIII § 6, viz., that the space at the ends of the core is not so well utilised, less cooling effect is obtained, and the inductance of the winding is greater.

While the windings on the left-hand side of figs. 287 and 288 are distributed, the corresponding windings of the toothed armatures on the right-hand side are completely concentrated within a single slot per pole. It is evident that whether the armature be smooth or toothed, there are numberless intermediate cases of grouped windings, the side of each coil being divided into two or more groups, and in the toothed armature the separate divisions being placed in as many separate slots or tunnels. Although not often used for single-phase machines, such arrangements are of importance in connection with polyphase machines. In the single-phase case the slots might be uniformly distributed over the periphery, although with some loss of E.M.F., but usually they would be grouped more or less together. Under these circumstances the periphery of the armature would be made with a number of uniformly distributed slots in order to give a more constant reluctance to the magnetic circuit, and some of the slots would be unwound. Fig. 289 shows the form of the windings of fig. 288 when the side of each coil is divided into two groups situated closely together.

In many cases step-up transformers are employed where transmission over long distances necessitates the use of very high voltages. The generating machines can then be built for low voltages with few large conductors, and little loss of space in insulation.

Thus instead of the wire coils of figs. 286–9, which may consist of a great number of turns, bar-winding with few inductors per group can be employed.

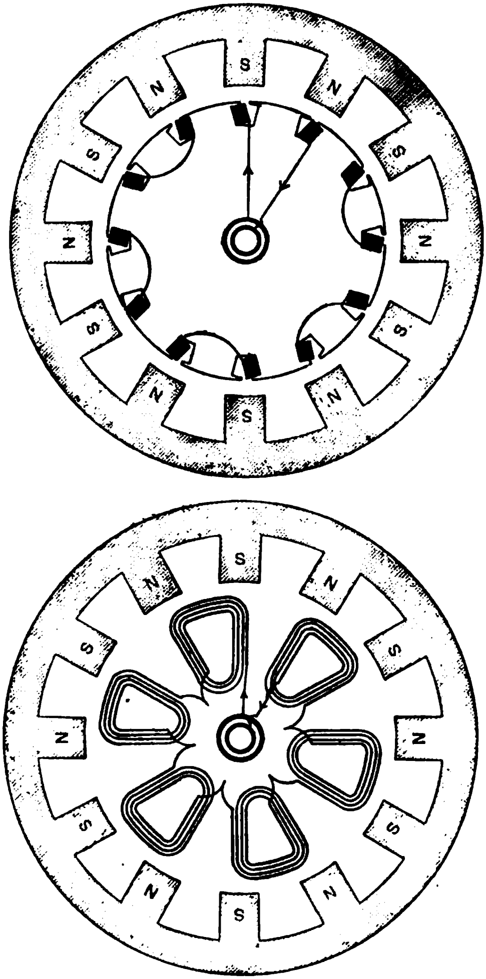
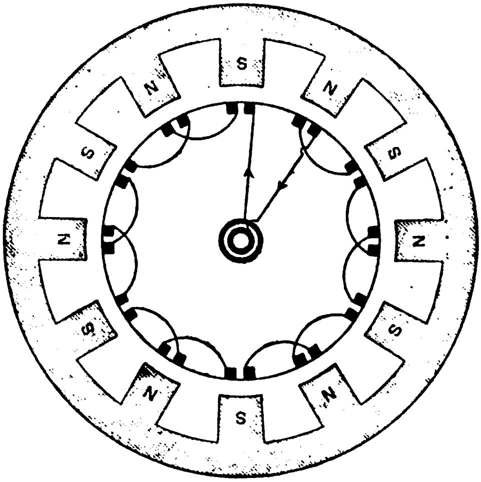


Fig. 288.—Single-phase drum or disc armature with undivided coils.

Each of the coils of fig. 287 can then be lap-wound, the electrical effect as shown in fig. 290 being precisely the same as in fig. 287, save that the terminals of the winding are now formed by



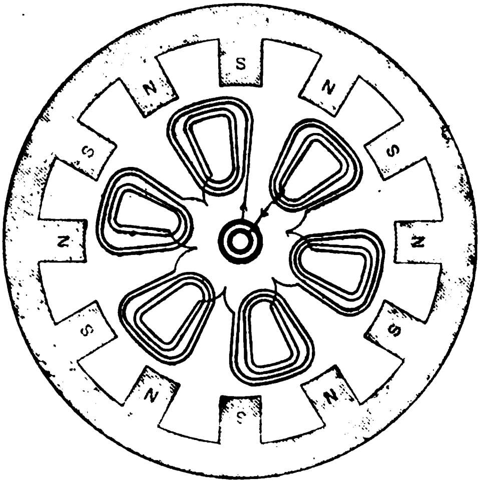


FIG. 289.—Single-phase drum armature with two slots per pole.

the inside and outside ends of a pair of adjacent coils. The bars are bent or are joined by end-connectors very similar to those employed in continuous-current dynamos, and the several groups are connected in series in the same progression as before. By a further rearrangement we pass to wave-winding analogous to that employed in the multipolar series-connected continuous-current machine. At one point where the winding has completed a tour of the armature and starts off again on another tour, the end-connectors are shorter than elsewhere (fig. 291). The larger coils of fig. 288 may also be lap-wound, and are then connected in series by a wave method of progression round the armature (fig. 292). The end-connectors are of course longer than in the case of the smaller divided coils of fig. 291, and have three different lengths. Finally, with only one inductor per pole the two diagrams of figs. 291 and 292 become identical, and the winding is a simple undulatory zig-zag (fig. 293).

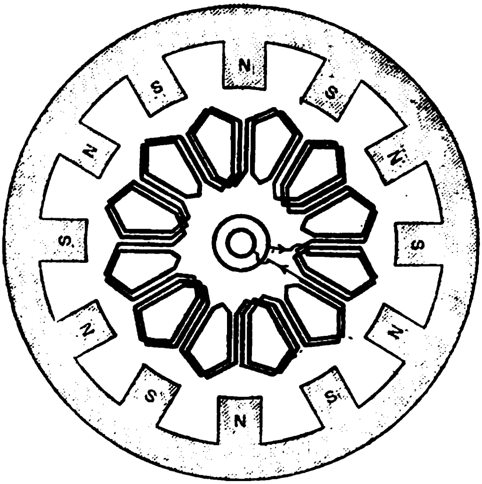


FIG. 290.—Lap-wound single-phase armature with divided coils.

§ 4. **Interlinked quarter-phase and four-phase systems.** —If the several phases of a polyphase alternator are kept entirely distinct and independent, each set of coils has its two collector rings or terminals, and requires two transmitting lines to its

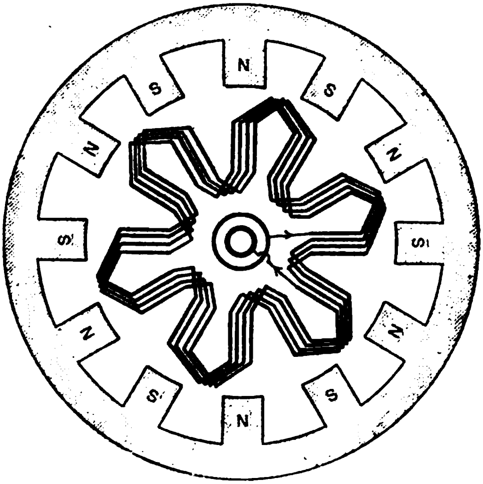


FIG. 291.—Wave-wound single-phase armature.

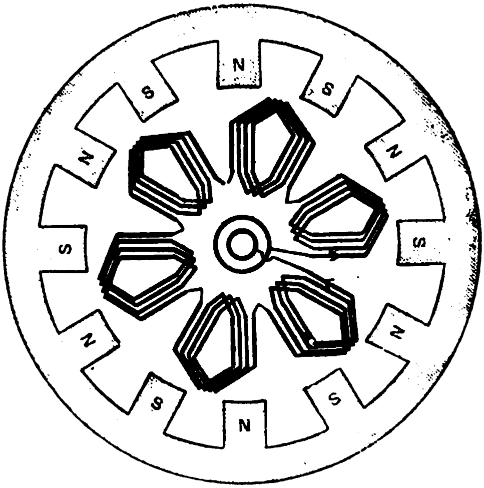


FIG. 292.—Lap-wound single-phase armature with undivided coils.

external circuit. It has, however, already been stated (Chap. VIII § 9) that it is possible to interlink the several phases, the advantage of so doing being that the number of transmitting lines to the external circuits and the weight of copper in them may be reduced for a given loss in the transmission. Before passing to the winding of polyphase alternators, the question of interlinking in general may first be considered. Obviously, if such interlinking is not to interfere with the proper flow of the currents, the algebraic value of the instantaneous currents reckoned as positive or negative according as they flow away from or towards any junction must be zero when in their original state before the one or more junctions are effected. To investigate the methods of interlinking, each circuit of the armature may be diagrammatically represented by a coil having a certain angular displacement relatively to the other circuits or coils. Thus fig. 294 shows two pairs of coils in quadrature with one another and giving E.M.F.'s which differ in phase by a quarter of a period. If the two wires *b* and *c* are replaced by the single wire *bc* (fig. 295), this third wire will serve as a common path for the currents of the two phases either away from or towards the armature, and we have a *quarter-*

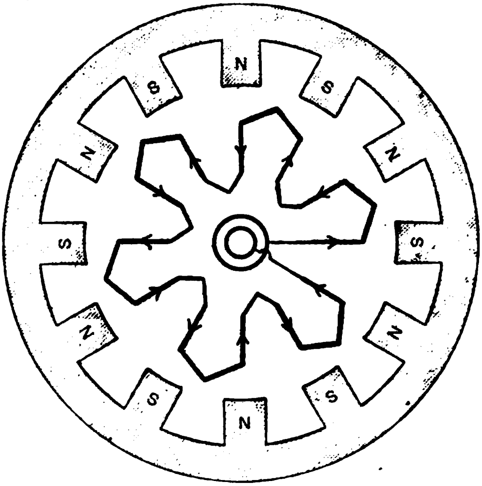


FIG. 293.—Single-bar drum winding.

phase interlinked system. For all calculations as to effective values the radii vectores representing the R.M.S. values as derived from the equivalent sine waves (p. 118) may be compounded together even if the actual periodic curves are not sinusoidal. Thus if the circuits both external and internal are alike in every respect, in resistance, inductance and capacity, so that the curves of their instantaneous E.M.F. and current and therefore their equivalent sine waves are identical, the effective value of the current in the common wire *bc* is the vector sum of the effective values of the currents in the other two transmitting wires; *i.e.*, it is $\sqrt{2}$ or 1·414 times the current in either of the two outer wires, and the effective value of the voltage between the outer pair of wires *a* and *d* is $\sqrt{2}$ or 1·414 times the effective voltage between either of the outer wires and the common wire *bc*. In the above method the two sets of coils are independent and are only interlinked at one of the external terminals or collecting rings. Although copper is thereby saved, the method is seldom used.

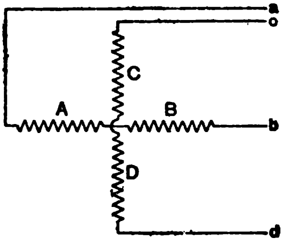


FIG. 294.—Two E.M.F.'s in quadrature.

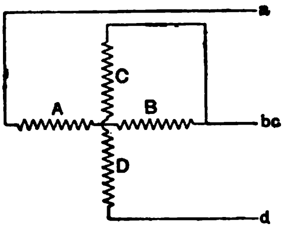


FIG. 295.—Quarter-phase interlinked system with three wires.

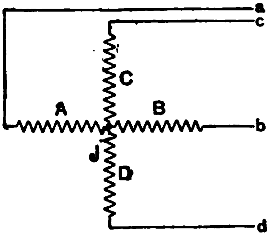


FIG. 296.—Star-connected four-phase system.

A second method unites the four coils at a common junction J, and four terminals and transmitting lines are again required (fig. 296). Such an interlinked arrangement is known as a *star-connected four-phase system*; although often called a two-phase system, it is best regarded as composed of four phases given respectively by the coils A, B, C, and D, just as a single-phase alternator can be regarded as composed of two phases, E and − E, the one being as it were the counterpart or return of the other. If e be the effective voltage of one coil A, the voltage between the two line wires c, d, is $2e$, and that between b, c, is $\sqrt{2}e$.

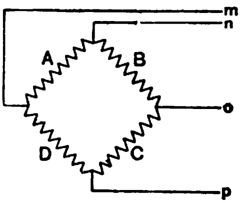


Fig. 297.—Mesh-connected four-phase system.

The four coils may also be connected up into a continuous helix just as in a continuous-current closed-coil machine, and four equidistant points are then connected to the collecting rings or external terminals (fig. 297). When so interlinked, the arrangement forms a *mesh-connected four-phase system* (also called 'ring-connected'). Again, since four lines are necessary, there is no saving of copper, but the currents obtained by it can be used directly to supply rotary converters, in which it is necessary that the winding of the armature should be a closed circuit. If e be the effective voltage of one phase A the voltage between any pair of adjacent lines in our diagram is e and between m and o or n and p is $\sqrt{2}e$. The current in any line is the vector sum of the currents in the two phases connected to it, and its effective value is $\sqrt{2}c$, where c is the effective value of the current in one phase.

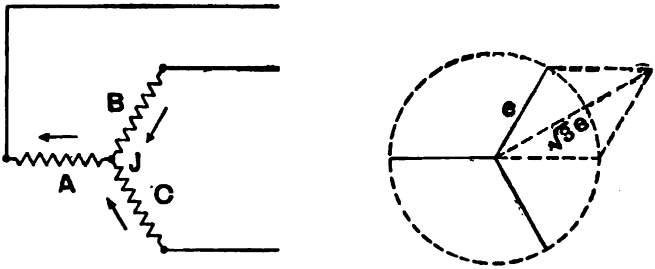


Fig. 298.—Y-connected three-phase system.

§ 5. **Interlinked three-phase systems.**—With three phases 120° apart, the same methods of star and mesh connections may be followed out. The three coils of fig. 298 can be united at their centre and three external leads are alone required. At any instant one wire will carry away from the centre the currents which are flowing towards the centre in the other two wires, or *vice versâ*. Such a star method of interlinking is also known as the *Y-connected three-phase system*. If e be the effective voltage of each phase or the voltage between any one of the three collecting rings and the common central junction, J, the volts between any pair of transmitting lines will be $\text{E}=\sqrt{3}e$, the effective current C in each of the three lines will be equal and the total output in watts will be $\text{W}=3\text{C}e=\frac{3\text{C E}}{\sqrt{3}}=\sqrt{3}\,.\,\text{E C}$, or 1·732 times the product of the effective voltage between the lines and the current in any single line.

On the other hand, if the three coils are closed upon themselves, and three transmitting lines are connected to the three junctions of the coils, we obtain the mesh or ring connection which in the three-phase case is also called the Δ or *delta-connected winding*. The voltage E between any pair of wires is that of one phase and the current in a line wire is the vector sum of the currents in two adjacent phases; *i.e.*, if c be the current in one phase the current in a line wire beyond a collecting ring is $\text{C}=\sqrt{3}.c$. The watts are

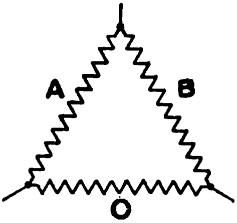


FIG. 299.—Δ-connected three-phase system.

$$\text{W}=3c\,\text{E}=\frac{3\text{C E}}{\sqrt{3}}=\sqrt{3}.\ \text{C E},$$

which gives exactly the same numerical value as before. Hence any three-phase winding can be changed over from the Y to the Δ connection, and will then give 1·732 times as much current but only $\frac{1}{1{\cdot}732}$ of the voltage, so that the output is unaffected although its two factors are altered.

For transmission purposes the three-phase system is now almost always adopted in practice in preference to the quarter-phase with common return or the four-phase with four wires. Further, except when rotary converters are to be fed, the star or Y connection is much more common than the mesh or Δ connection, since in the latter any ripples superposed on the main sinusoidal E.M.F. curve may cause internal parasitic currents round the closed triangle.

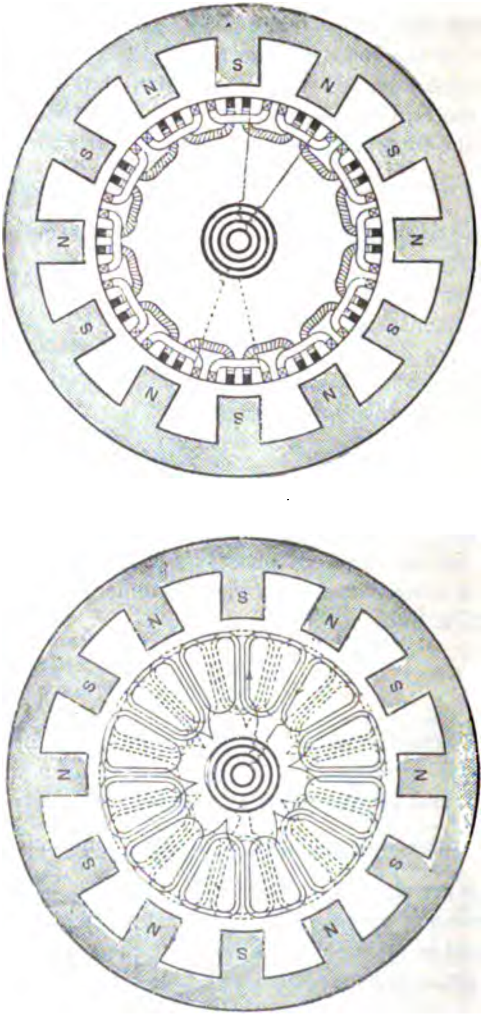


FIG. 300.—Quarter-phase winding with divided coils for rotating armature.

§ 6. **Polyphase armature windings.**—The conversion of the preceding winding diagrams into forms suitable for quarter- or four-phase generators is easily effected by superposition upon the original set of coils of a second set exactly similar but displaced through an angle corresponding to half the pole-pitch; while for three-phase machines, two new sets of coils must be added, displaced respectively through one-third and two-thirds of the pitch. Thus fig. 300 shows the quarter-phase equivalent of fig. 287 with as many coils in each phase as there are poles; the adjacent sides of the coils of one phase may be wound in the same slot, or may be divided between two slots (as in fig. 300), or between a still larger number. Fig. 301 shows the same arrangement in connection with an external stationary armature. In one set of coils (marked B) the end-connections after clearing the tunnels are bent up at right angles and lie against the sides of the armature; in the other set of coils (marked A) the end-connections after clearing the B set can then run practically straight across from tunnel to tunnel, care being taken that the length of the two sets of coils is alike. Fig. 302 is the quarter-phase equivalent of fig. 288, with one coil per pair of poles and per

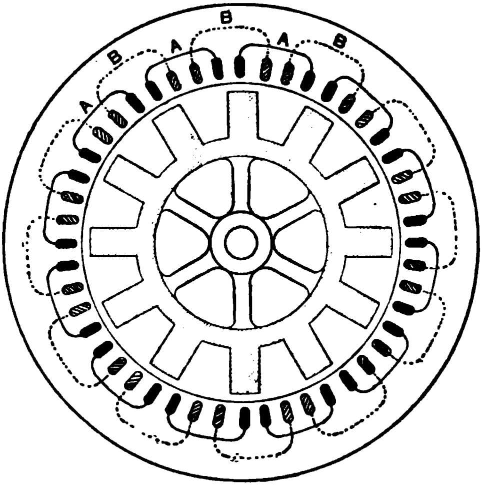


Fig. 301.—Quarter-phase winding with divided coils for stationary armature.

phase, and with the winding concentrated in one slot per pole and per phase. Or as in fig. 289, there may be one coil per pair of

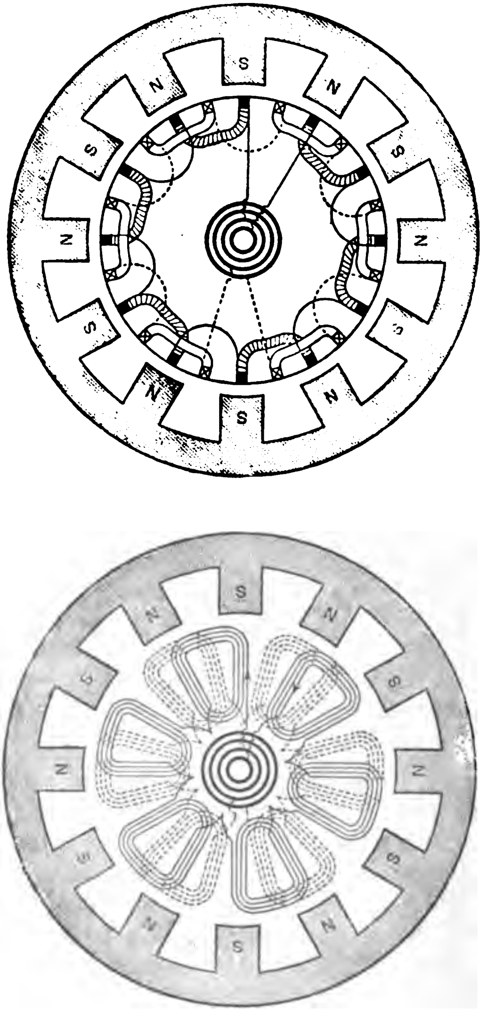


FIG. 302.—Quarter-phase winding with undivided coils.

poles, but with the winding grouped in two slots per pole and per phase. In all such cases where there is in effect only one large coil per pair of poles, the advantage is gained that the armature

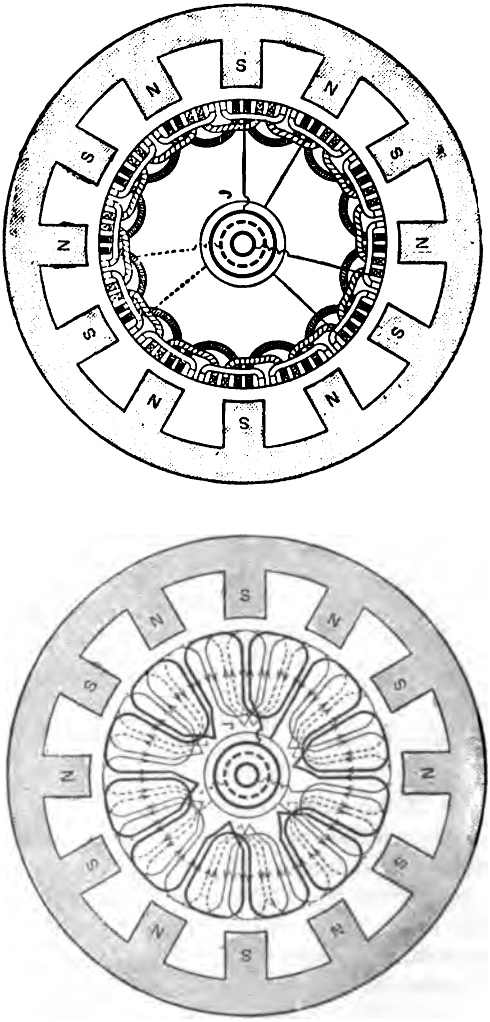


FIG. 303.—Three-phase winding with divided coils for rotating armature.

when external and stationary can be divided to enable the internal magnet to be removed; since at certain points on the circumference it will be seen that a radial section of the armature cuts no coils, and only the connecting wires between the coils need to be separated. Four collecting rings are in all cases shown, and the armature circuits may be either independent or interlinked as a star at their central points. The three-phase equivalent of fig. 287 with as many coils as there are poles is shown in fig. 303; it will be seen that with the stationary armature (fig. 304) the third or C set of coils would be most conveniently bent inwards in the opposite manner to the B set, but that they would then project in front of the magnet and prevent it being withdrawn sideways since the armature cannot be split; they must therefore in practice be bent up outwards. When, however, we pass to the three-phase equivalent of fig. 288 with one large coil per pair of poles, two cases are possible. Thus if each of the two additional windings has a relative displacement of ⅔rds of the pitch, although the armature cannot be split, only two kinds of coils are required, and the series of coils of any one phase consists alternately of straight and bent coils (fig. 305); the appearance of such a three-

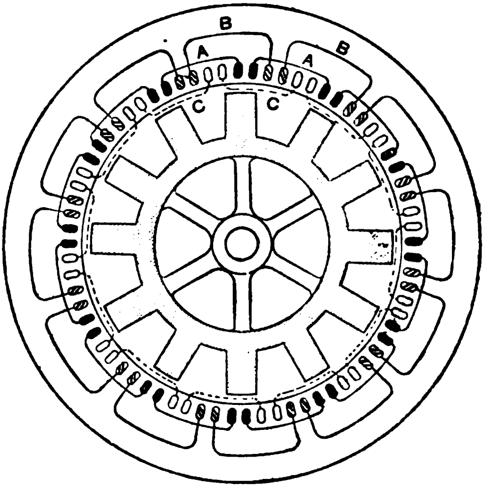


FIG. 304.—Three-phase winding with divided coils for stationary armature.

phase armature is therefore exactly the same as that of the quarter-phase armature of fig. 301, except in the number of sides of coils embraced within the pole-pitch. But if the three windings are displaced through ⅓rd of the pitch (figs. 306 and 307), although a section of the armature cuts through three sets of coils at certain points and therefore three kinds are required, one of these can be bent inwards since the armature can be split at certain points where no coils are cut. Fig. 306 is shown star- or Y-connected, as indicated by the common junction J, while fig. 307 is shown delta-connected as indicated by the two armature wires led to each of the three collecting rings. In Y-connected windings after completing one phase, it is immaterial whether the start of the second and third phases is made from a common junction of three points lying close beside each other on the armature or from a common ring joining points approximately 120° apart, but in order to reduce the difference of potential between adjacent coils to a minimum the latter is preferable; and for the same reason each ring of the Δ-connection should unite two points

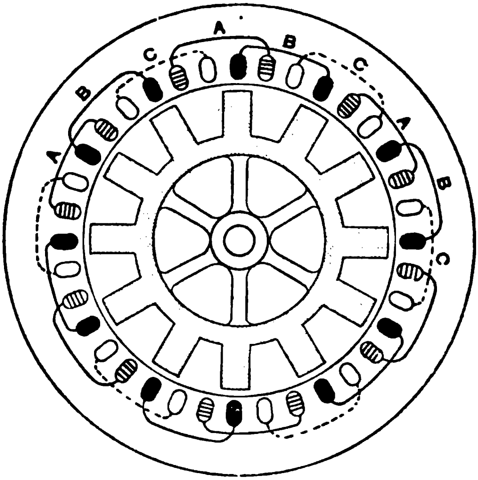


FIG. 305.—Three-phase winding for stationary armature with large coils of two kinds.

approximately 120° apart.[1] Bar windings are shown in figs. 308 and 309, the former being the lap-wound armature of fig. 292,

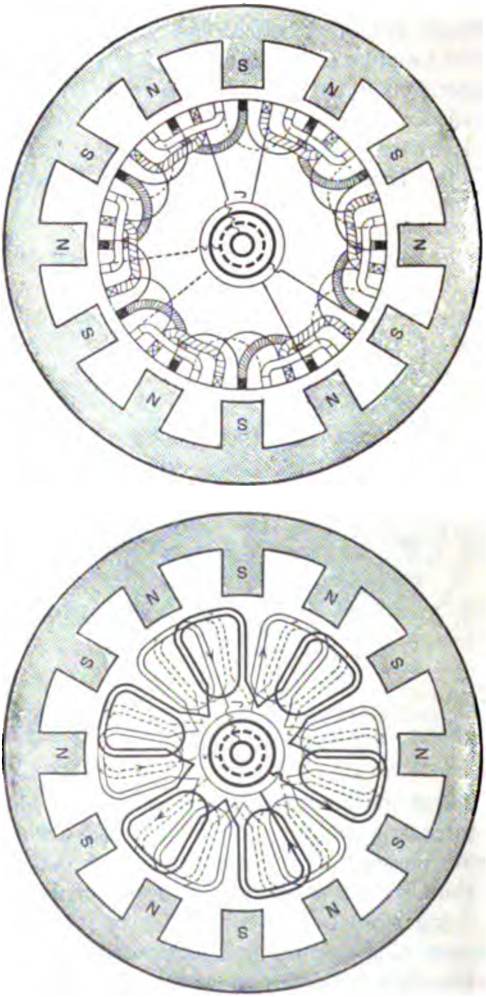


FIG. 306.—Three-phase winding with undivided coils of three kinds.

---

[1] If the number of poles is itself divisible by three, the junction points cannot be exactly 120° apart.

and the latter the wave-wound armature of fig. 291, both adapted to three phases and both Y-connected.

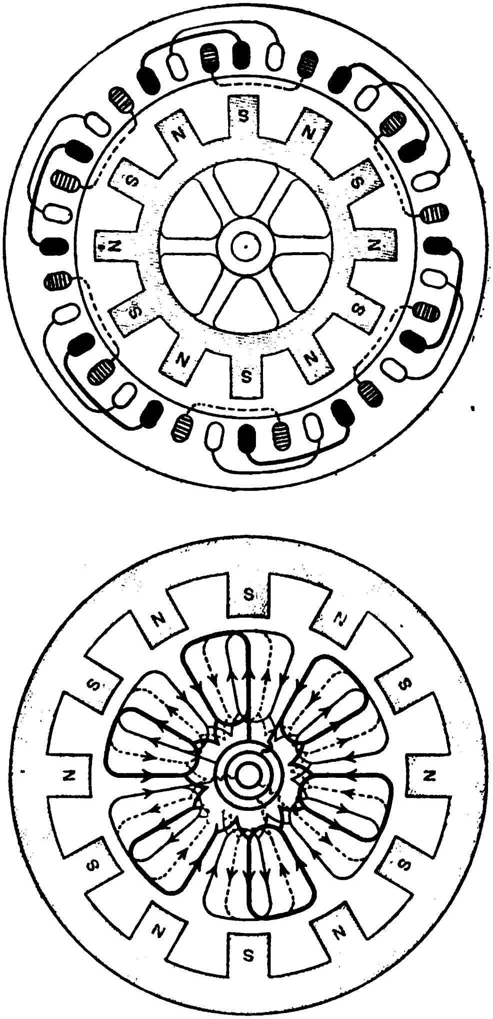


FIG. 307.—Three-phase winding for stationary armature with three kinds of coils.

All the above forms have been derived from the single-phase methods by duplication or triplication, and some are more convenient than others according to the nature of the output which

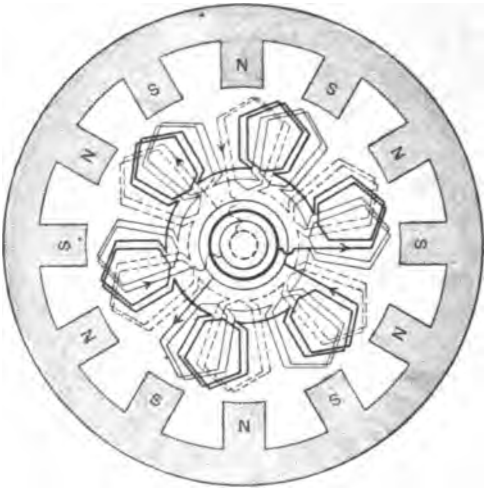


FIG. 308.—Three-phase lap-wound bar armature.

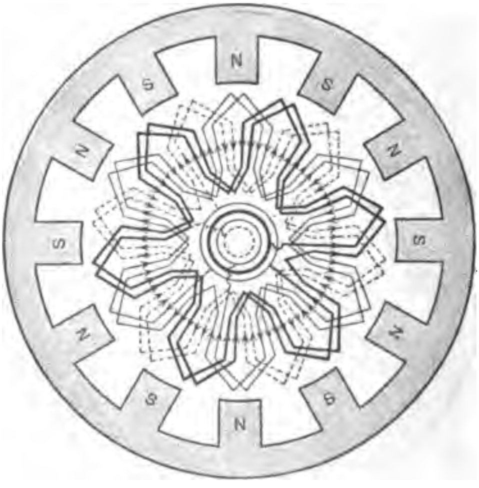


FIG. 309.—Three-phase wave-wound bar armature.

the generator is to give. There are, however, in addition other windings which find advantageous application in polyphase work, although not suitable for single-phase machines. In general it may be said that while the forms with one large coil in each phase per pair of poles lend themselves readily to quarter, and three-phase work, the forms with as many coils as there are poles, although utilising the end-space well and having a good cooling effect and small inductance, yet involve a considerable amount of overlapping of the coils. This disadvantage may be avoided by the employment of what is known as the *short-coil* or *non-overlapping method*, which may be obtained from a rearrangement of fig. 300 or 303. The coils are now narrowed until their over-all width instead of being equal to the pitch is no more than half the pitch in the quarter-phase machine, and two-thirds of the pitch in the three-phase machine. The poles may correspondingly be narrowed until their width is about equal to the width of the coils in order to avoid too great an amount of differential action, or if retained of normal width, all the lines of one field do not pass through a coil, and K has a low value, especially under full armature reaction. As will be seen from figs. 310 and 311, there is now no overlapping of the coils, and the ends of the armature are still well utilised for cooling, yet the reduction of the effective pole-area is so far disadvantageous that the method is seldom employed, any gain from the reduction in the mean length of an armature turn being of small importance.

There still remain to be considered certain forms special to the ring- or delta-connected polyphase machine, in which the winding must necessarily form a closed circuit. Any re-entrant drum or ring winding such as those employed in continuous-current dynamos can then be used; in the four-phase machine four approximately equidistant points in the winding are determined and connections brought from these points to the four collecting rings or terminals, while similarly in the three-phase machine three equidistant points must be taken and connection made to the three rings. Fig. 312 shows a re-entrant drum wave winding for a delta-connected six-pole armature, exactly analogous to a continuous-current series-connected drum armature. Strictly speaking, the number of inductors or of bars should be a multiple of three in such a three-phase winding, so that if n be the number of bars in one phase

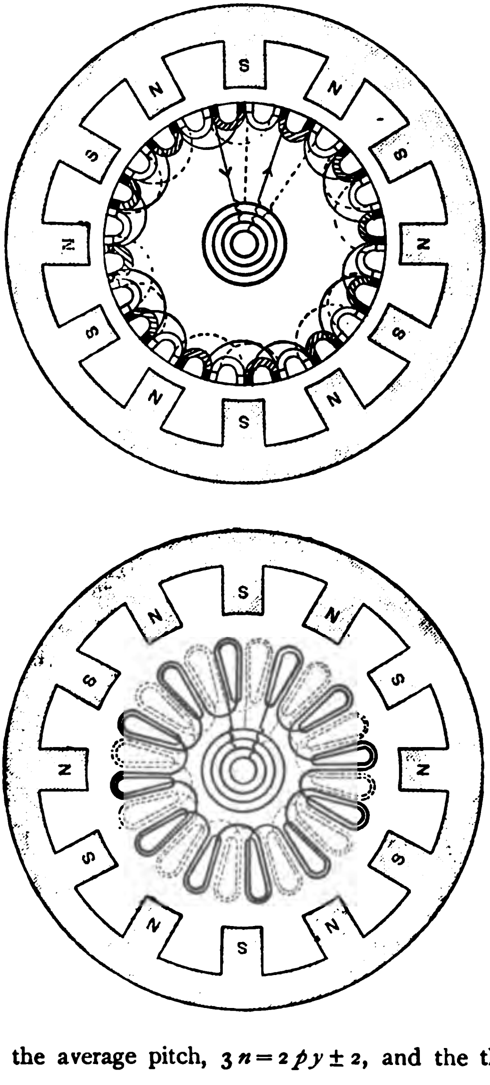


FIG. 310.—Quarter-phase non-overlapping winding.

and y is the average pitch, $3n = 2py \pm 2$, and the three points which are connected to the terminals are exactly equidistant, but as

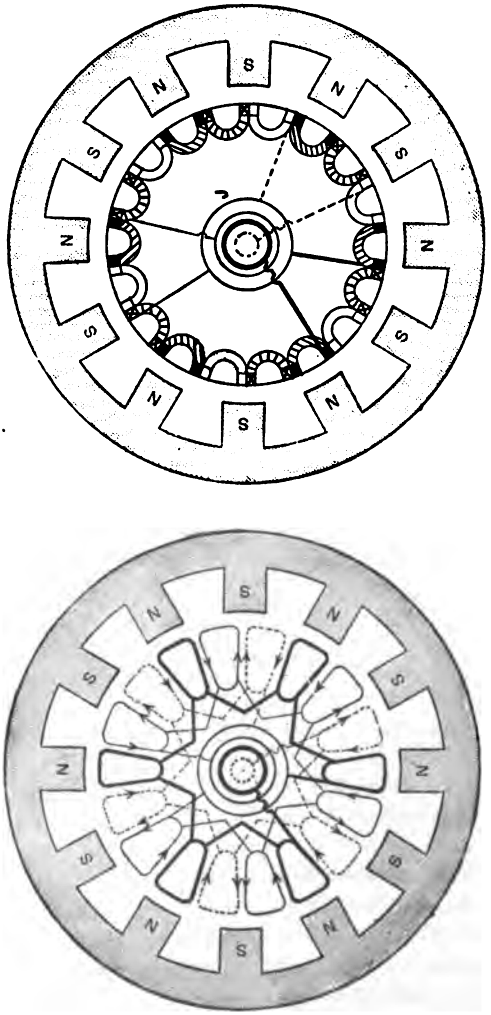


FIG. 311.—Three-phase non-overlapping winding.

this condition somewhat limits the possible number of inductors, it is permissible with a large number of bars to make one division larger than the other two to the extent of a pair of additional bars.

All the above windings although shown in connection with a field-system of alternate poles are equally applicable to homopolar machines with half as many poles of similar sign in each row. The only qualification that must be added is that in the case of the short-coil methods the pole-width must be equal to the pitch in order to avoid an unsymmetrical curve of E.M.F.

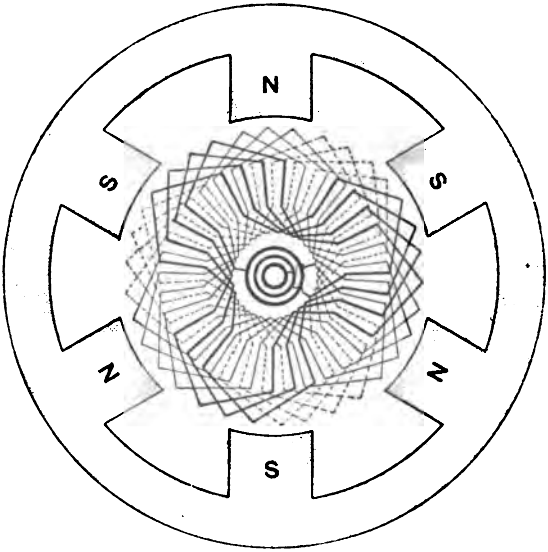


FIG. 312.– Three phase re-entrant drum wave-winding.

§ 7. **Rules for connecting up three-phase armatures.**— In order to connect the coils of a three-phase machine correctly, it is convenient to consider the instant when the inductors of one phase are situated centrally under the poles and to assume that they are then carrying the maximum current; if the machine is to be Y-connected, let one end of the phase in question be connected to the common junction, and let it be assumed that the current is flowing into the common junction. The direction of the current in any group of inductors of the first phase is then fixed, and the direction in the groups on either side in the two other phases must be the same under the same pole-piece. But the current in the two other phases must be flowing away from the common junction, so that their ends must be connected to the common junction in such a way that their E.M.F. agrees with the assumed direction of the current. If the machine is to be Δ-connected, and the ends of the first phase which is centrally under the poles are connected to two collecting rings, A and B, let it be assumed that the current flows from A to B. The current in the two other phases must also be flowing from A to B, passing through C on the way. The second phase must therefore be connected to A and C in such wise that the current flows from A to C, and the ends of the third phase must be connected to C and B in such wise that the current flows from C to B, the directions of their currents being the same under the same poles as that assumed in the first phase.

§ 8. **The value of K in the E.M.F. equation.**—That the value of the factor K in the electromotive force equation of the alternator on open circuit will vary according as the winding is concentrated, grouped, or uniformly distributed has been already shown in Chap. VIII. The effective E.M.F. induced in the armature per phase by eq. (11) on open circuit or at no load, is $E_0 = K \,.\, 2\, Z_a \times \frac{pN}{60} \times$ number of inductors in series in one phase $\times 10^{-8}$ where K is itself the product of the two factors, k' the form factor, and k'' the width coefficient. In order to examine the values of either the one or the other with various ratios of pole-width and coil-width to the pitch, some assumption must be made as to the shape of the curve of instantaneous E.M.F. of one inductor, as dependent on the shape of the curve of flux from one pole. If this were sinusoidal, the resultant E.M.F. of the several inductors which are more or less out of phase with each other could be easily determined by the process of adding together their E.M.F.'s vectorially; or, again, if the flux from each pole were strictly confined to the area of the pole-face with a constant length of air-gap, so that the flux would itself be uniform over this area, the curve of instantaneous E.M.F. given by one inductor in each half-period would be a rectangle, and the united E.M.F.'s of the several inductors or groups of inductors could be easily calculated by the superposition of a number of such rectangles displaced from each other by the correct interval. Since neither assumption is true, a closer approximation may be made by assuming a certain distribution of the useful fringe of lines which surrounds the pole, and by adding together the E.M.F.'s graphically. In a paper by the writer[1] the following tables of values of K have been thus calculated on the assumption that the length of the single air-gap is $\frac{1}{20}$th of the pitch,[2] and that the curve of induction in the air-gap shades off from its full value under the pole to zero on the interpolar line of symmetry after the law given by Mr. Carter in his note "On the Air-gap and Interpolar Induction" (*Journal Inst. Electr. Eng.*, vol. xxix. p. 925). As the ratio of the pole-width to the pitch is reduced, the effect of the fringe becomes more marked since it forms a greater percentage of the total number of useful lines entering the armature, but for ordinary values of the pole-ratio, the results may be taken without much error even if the exact distribution of the fringe is in practice somewhat different; any reasonable allowance for the effect of the fringe gives a much closer approximation to the truth than its entire neglect. The values given in the tables for large ratios of the pole-width to the pitch, say from unity to 0·75, may be applied to the case of homopolar or inductor alternators with the cautions already pointed out in Chap. IX § 4, viz., that the useful flux is the actual flux under a pole and within the arc of the pitch, *minus* the flux which enters the armature in the adjoining interpolar gap and within the arc of the pole-pitch; further, strictly speaking, the instantaneous curve of a single inductor is not precisely analogous to that of a machine with alternate poles of opposite sign, yet but little error is involved in the above identification, provided it be borne in mind that the useful flux is the difference between z_a' and z_a.

---

[1] *Electrical Review*, vol. xlvii. p. 655 ff.

[2] The values for other proportions of l_g to pitch may be interpolated between the maxima figures given below and those which are tabulated in the same paper for the hypothetical case of no fringe.

TABLE I.—*Values of* K *for uniformly distributed Winding and long Coils with Allowance for Fringe.*

Ratio of Coil-width to Pitch.	Ratio of Pole-width to Pitch.						
	Alternate-pole Machines.				Inductor Machines.		
	0·4	0·5	0·6	0·7	0·8	0·9	1
0	1·25	1·18	1·125	1·08	1·05	1·025	1
0·1	1·225	1·16	1·112	1·06	1·035	1	·968
0·2	1·19	1·135	1·09	1·05	1·008	·97	·932
0·3	1·162	1·11	1·075	1·025	·983	·94	·897
0·4	1·125	1·083	1·05	1	·955	·905	·857
0·5	1·09	1·046	1·015	·967	·92	·87	·82
0·6	1·03	1	·963	·92	·87	·816	·78
0·7	·976	·944	·91	·87	·82	·772	·731
0·8	·906	·88	·843	·803	764	·72	·682
0·9	·841	·812	·78	·745	·707	·67	·633
1·0	·77	·745	·716	·685	·654	·617	·578

TABLE II.—*Values of* K *for grouped Winding and long Coils with Allowance for Fringe.*

One slot per pole per phase	1·25	1·18	1·125	1·08	1·05	1·025	1
Two slots per pole per phase ⅙th of pitch apart . .	1·19	1·13	1·085	1·05	1·005	·966	·91
Two slots per pole per phase ¼th of pitch apart . .	1·15	1·102	1·058	1·02	·965	·915	·866

TABLE III.—*Ratio of Maximum to Effective E.M.F. for uniformly distributed Winding with Allowance for Fringe.*

0	1·54	1·385	1·265	1·18	1·11	1·045	1
0·1	1·57	1·405	1·28	1·2	1·121	1·07	1·034
0·2	1·61	1·435	1·31	1·22	1·155	1·104	1·075
0·3	1·65	1·468	1·33	1·245	1·182	1·14	1·116
0·4	1·68	1·505	1·36	1·278	1·218	1·188	1·168
0·5	1·6	1·525	1·406	1·32	1·265	1·23	1·225
0·6	1·49	1·499	1·49	1·39	1·34	1·31	1·29
0·7	1·41	1·43	1·45	1·45	1·42	1·39	1·366
0·8	1·365	1·395	1·44	1·49	1·51	1·49	1·462
0·9	1·325	1·36	1·418	1·476	1·535	1·59	1·58
1·0	1·295	1·34	1·405	1·47	1·54	1·63	1·73

TABLE IV.—*Ratio of Maximum to Effective E.M.F. for grouped Winding with Allowance for Fringe.*

Number of Slots.	Ratio of Pole-width to Pitch.						
	Alternate-pole Machines.				Inductor Machines.		
	0·4	0·5	0·6	0·7	0·8	0·9	1
One slot per pole per phase	1·54	1·385	1·265	1·18	1·11	1·045	1
Two slots per pole per phase $\frac{1}{6}$th of pitch apart . .	1·61	1·442	1·315	1·22	1·155	1·11	1·1
Two slots per pole per phase $\frac{1}{4}$th of pitch apart . .	1·64	1·48	1·35	1·25	1·205	1·174	1·155

It will be seen from the tables how the values of K for uniform and grouped distribution meet in the case of a concentrated winding, the hypothetical case of a single inductor of zero width giving exactly the same theoretical result as a single narrow slot in a toothed armature, while as the number of slots per pole per phase is increased, the value of K in the slotted armature becomes more and more nearly identical with the value for an equal width of coil in a smooth armature. Thus with three slots per pole and per phase $\frac{1}{9}$th of the pitch apart, the value of K is not far different from the value for a uniformly distributed winding having the equivalent width of $\frac{2}{9}$ths or 0·22 of the pitch; and the factor for a symmetrical bar winding with a large number of slots or tunnels becomes identical with that for an equivalent width of coil on a smooth armature, *e.g.*, with a three-phase closed circuit wave-wound armature, such as fig. 312, the winding is equivalent to a uniformly distributed coil of width = $\frac{2}{3}$rds or 0·66 of the pitch.

In all cases as the winding becomes more distributed the value of K decreases; but the reduction of the E.M.F. due thereto has its full force only on open circuit or no load. The reaction on the field when the full current is taken out of the armature is less in the distributed than in the concentrated winding, and as will be more apparent later the inductance of the armature wires is reduced, while incidentally a better exposure of the coils to the cooling action of the air is obtained. Hence it is frequently advantageous to employ a distributed winding even at the sacrifice of some of the possible E.M.F. at no load, and in recent alternators, especially when polyphase, the tendency has been to adopt two or three slots per pole and phase. The shape of the curve of instantaneous E.M.F. is thereby altered, and this possesses considerable importance from its bearing upon the performance of the alternator on loads of different kinds, such as lighting and motors, and also upon the ratio of the maximum to the effective E.M.F. The insulation of the inductors must be capable of withstanding the strain of the maximum E.M.F., but while with a concentrated winding the ratio of the maximum to the effective E.M.F. of the phase increases rapidly from unity as the pole-ratio is decreased, with a distributed winding or with a large number of slots the ratio decreases from a maximum of 1·73 corresponding to a triangular wave of E.M.F. and a pole-width ratio of unity. The two curves for concentrated and distributed winding respectively cross in fact at a value of about 1·33 for a pole-ratio in the neighbourhood of 0·5. Any number of intermediate cases are therefore possible, and in general it may be said that the maximum strain that the insulation of the coil has to withstand from the action of its own phase E.M.F. is about 1½ times its effective E.M.F. Thus in order not to lose too much space in the insulation of the slots, if the effective E.M.F. be high, the coil must needs be concentrated within one or a few slots. With a stationary armature, alternators of large size can be successfully wound directly for voltages up to 10,000; the insulation of their winding should then be subjected to a test at 20,000 alternating volts.

In ordinary cases the value of K varies within the limits of 1 and 1·25, which may be compared with the value for a sinusoidal E.M.F., viz., 1·11 and at the same time the shape of the curve usually more or less resembles a sine curve.

§ 9. **The form of the E.M.F. curve and its delineation.**—Various methods have been devised for the purpose of delineating the curves of instantaneous E.M.F. or current in alternators (see Fleming, *Alternate Current Transformer*, vol. i. Chap. vi §§ 2 and 3, and vol. ii. Chap. iv §§ 1 and 4), and a number of curves for different types of machines have been determined, among which especial reference may be made to those given in the *Electrician*, vol. xxxiv. p. 507 ('The Delineation of Alternating Current Curves when the Dynamo is not accessible,' by Dr. Fleming).[1]

Although for convenience the actual curves of E.M.F. and current are usually assumed to be replaced by their equivalent sine waves, it must be remembered that such an assumption is only valid for calculations of power, and cannot be used in estimating the maximum E.M.F. as affecting the insulation, save only as a rough approximation. Again, even if the curve of terminal E.M.F. on open circuit be a sine curve, its shape under full load may be very different, and further, will vary widely according to the nature of the external load; *e.g.*, if the alternator be working on a non-inductive load such as a water-resistance or fully loaded transformers, the shape may differ considerably from that for the same alternator working on an inductive load such as lightly loaded transformers or motors.

In general it may be said that a disc machine without any iron in the armature gives a curve of E.M.F. on open circuit closely corresponding to a sine curve, and so also, but to a less degree, do smooth-surface armatures, both ring and drum, although the presence of the iron in the armature of these latter may considerably distort the current-curve when under full load. In toothed armatures which may now be regarded as the most frequently used type, there are still greater deviations from a sine law even on open circuit, owing to the rhythmic passage of the teeth with their corresponding groups of inductors past the poles which causes pulsations of the magnetic reluctance. In a single-phase machine a concentrated winding exerts a great effect on the shape of the E.M.F. curve when carrying its full current owing to its high inductance to which corresponds a large armature reaction; when the winding is distributed among two or more slots per pole, the distortion of the field is reduced owing to the lesser inductance of the coils, and a smoother curve of E.M.F. is obtained. Analogously, in polyphase machines the distribution of the winding tends to reduce the peaked form of the E.M.F. curve which is in general given by the toothed armature.

---

[1] And in papers reprinted in the *Electrical Engineer*, vol. iv. p. 126, vol. vi. p. 509 (Tobey and Walbridge, on the Stanley Alternate Current Arc Dynamo), *Electrician*, vol. xxviii. p. 90, vol. xxxiii. p. 523.

The relative merits of various shapes of E.M.F. curve have been the subject of much discussion, and to a certain extent can only be decided after consideration of the nature of the load. The general conclusion that may be stated is that for a given effective E.M.F. a peaked wave requires less magnetic flux so that the hysteretic loss is less. The gain therefrom in the efficiency of a generator or in a synchronous motor driven by it is practically negligible, but in the case of a lightly loaded transformer the lesser iron loss from hysteresis becomes appreciable and the efficiency is improved. On the other hand, for transmission of energy over very long distances at very high potentials, a sinusoidal curve is better, as resonance, causing a rise of voltage at the far end of the line, is then less likely to be set up. In every case a peaked wave has the disadvantage that its maximum value is greatly above its effective value, and hence the insulation is more strained or must be thicker than in the case of a sine curve.

CHAPTER XXIII

ARMATURE REACTION IN ALTERNATORS

§ 1. **Regulation of alternators.**—In order to judge of the practical merits of an alternator, its behaviour under various loads with different amounts of inductance must be examined. The value of an alternator largely depends upon the degree to which it regulates for constant terminal voltage. It is therefore determined by its drop in volts, or the difference in the terminal voltages (with constant excitation and speed) at no load and under a certain definite load formed by an external current I_e with an angle of lag ϕ_e such that $E_e I_e \cos \phi_e =$ the full rated watts of output of the machine. If this fall of potential exceeds a certain amount suitable to the purpose and work for which the alternator is designed, the load has passed the limit set by armature reaction, even though the heating of the armature winding may be well within the permissible limit. Thus the output of the alternator is fixed by the two considerations of heating and armature reaction, just as is that of the continuous-current dynamo, the only difference between the two being that the limit set by armature reaction in the alternator does not arise from any question of sparking, but from the point of view of the *regulation*. Since in most cases the terminal voltage at which the external circuit is fed is required to remain practically constant for all loads within the capacity of the machine, it is evident that if the fall of potential with unaltered excitation is great, the alternator will require considerable attention to adjust the excitation to the amount required by varying loads; or, again, the fall may be so great that no reasonable increase in the field excitation can maintain the volts under full load, and the alternator becomes useless for the work in question.

§ 2. **Value of power-factor of external circuit.**—For lighting distribution with transformers fully loaded, ϕ_e is about 10° and the *power-factor* may be taken as nearly equal to 1. For power distribution with fully loaded asynchronous motors, ϕ_e on the average is about 35°, and cos ϕ_e is 0·85. In ordinary use, however, the motors will seldom all be fully loaded, and the average value of cos ϕ_e may for long periods not be more than 0·6 to 0·7. Next when a large motor is switched on, its starting current is high and lags behind the terminal voltage by a considerable angle; if the effect of this is to cause a large drop in the volts of the generator, it will seriously interfere with the steadiness of light in a mixed system of combined incandescent lamps and motors, or with motors only may cause these to stop, since their torque is proportional to the square of the terminal pressure. It is not, however, only in motor or combined light and power systems that it is necessary to consider the behaviour of the alternator under low power-factors. With a number of small transformers, each lightly loaded as may be the case during the daytime, the power-factor of a lighting installation may fall as low as 0·3 or even to 0·2. The full current of the alternator which is run to supply the network in the day-time may then be taken up in magnetising the transformers, or it may even be necessary to run two alternators in parallel in order to keep up the voltage although such a course is in itself uneconomical. An approximate calculation of the maximum angle of lag in the external circuit that may be expected in the day-time is then advisable.[1] Thus if the magnetising current of the transformers I_m, their hysteresis current I_h, and active or useful current I_n are calculated,

$$\cos \phi_e = \frac{\text{I}_n + \text{I}_h}{\sqrt{(\text{I}_n + \text{I}_h)^2 + \text{I}_m^2}}.$$

§ 3. **Fall of volts in armature of alternator.**—In order to deduce the effect of different loads on an alternator that is already built, it is first necessary to determine and plot the curve of its terminal or induced E.M.F. on open circuit as related to the exciting current or the ampère-turns of the field-winding. This is easily taken in the same way as the similar curve of a continuous current dynamo (cp. Chap. XV § 12), and is in fact the no-current flux-curve expressed in terms of the E.M.F. induced at a constant speed. Before proceeding to the next step, it will be necessary to consider more in detail the way in which the fall of potential arises by means of a clock or vector diagram.

---

[1] Fischer Hinnen, 'Alternator Design,' *Electr. Eng.*, vol. xx. p. 620.

At a given moment let the external current corresponding to one phase be zero, so that its vector has the direction of the horizontal axis OX (fig. 313), and upon this line let the length OI represent the maximum value of the active external E.M.F. per phase, this E.M.F. being itself the sum of the volts lost over the ohmic resistance of the external circuit and of any E.M.F. which may be absorbed in doing useful work against a back E.M.F., as, *e.g.*, in the primary coil of a transformer or the armature of an alternating-current motor. At the same moment the E.M.F. of self-induction of the external circuit is a maximum and has the direction OY and value OE'_s. This must be balanced by a component of the terminal voltage in the opposite direction, or OE_s, which is thus the voltage consumed by the external inductance. The maximum value of the terminal voltage per phase, being the resultant of OI and OE_s, is then given by the length of the line E_e, and in phase it is ϕ_e degrees in advance of the current, where ϕ_e may be any angle between the two limiting cases of $\phi_e = 0°$, and $\phi_e = 90°$. Now the fall of potential in the armature is due (1) to the loss of volts over the ohmic resistance of the winding, (2) to the weakening of the field through such portion of the eddies set up by the varying armature current as are in phase therewith, principally in the poles if these are not laminated, or in

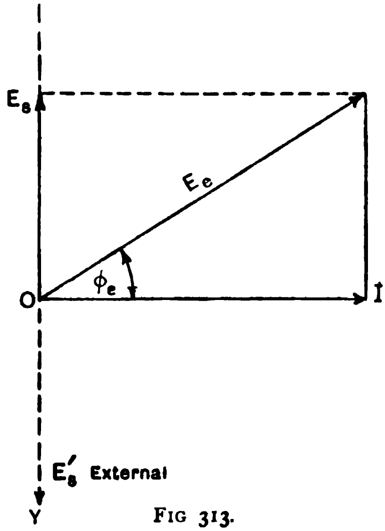


FIG 313.

the armature discs and their cast-iron case, and (3) to the reaction of the armature ampère-turns on the field, to which corresponds the inductance of the armature; this latter property may for the present be provisionally assumed to be located as it were in an imaginary inductive coil separate from the armature core and without resistance. Passing then to the armature, a second diagram may be drawn as in fig. 314; the volts consumed by the two first items coincide in phase with the current, and may therefore be represented at the same moment by the line $e_1 + e_2$; the internal self-induced E.M.F. will have a lag of 90° behind the current, and the voltage consumed in overcoming it must be exactly opposite as shown by e_s. The resultant of $(e_1 + e_2)$ and e_s is the total E M.F. e absorbed in overcoming all sources of counter E.M.F. in the armature together with its loss of volts, and its phase relatively to the current is given by the angle ϕ'_a. We now have to combine together the two separate E.M.F.'s E_e and e in their respective phases (fig. 315), and the vector sum of the two is E_0 with a total angle of lag ϕ_0. The two separate diagrams may also be conveniently combined into one as in fig. 316, and further the same diagrams are equally applicable to the determination of the values and phases of the effective E.M.F.'s;

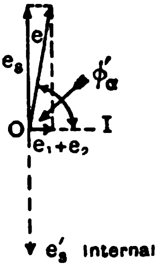


FIG. 314.

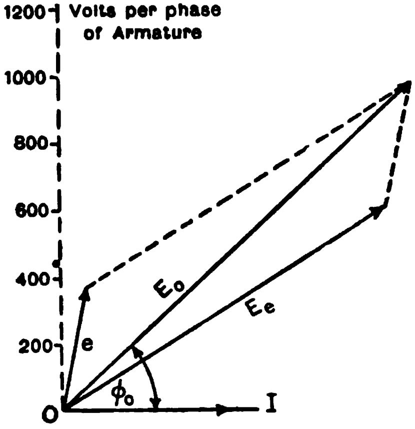


FIG. 315.

we have only to substitute in all cases for the maxima values of the components their effective values, and since sine waves are assumed, we may pass from the inductance to the reactance of the armature. Thus the fall of effective potential for a constant excitation and a given current and angle ϕ_e is $E_0 - E_e$. It must at once be clearly understood that while E_e is the effective terminal voltage due to one phase of the armature when the alternator is loaded, E_0 is here the terminal potential difference of one phase at no load with the same excitation, and this is identical with the E.M.F. induced per phase in the armature at no load; it is not the E.M.F. actually induced per phase in the armature when the machine is running under the load in question, and in fact never exists simultaneously with E_e, although its determination is of the greatest importance to the dynamo designer.

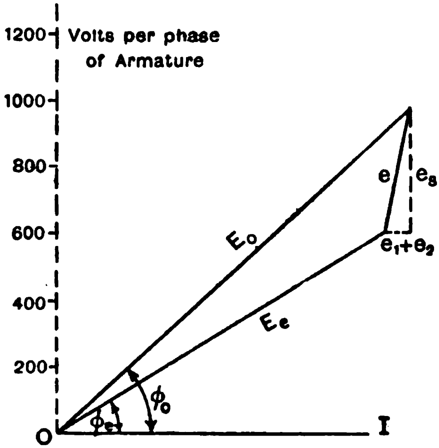


FIG. 316.

§ 4. **The short-circuit characteristic.**—In order to draw the diagram for any assumed load in the external circuit, the values of the two quantities e and ϕ_a' must be known. The determination of e must now be made by experiment, and since it cannot be directly measured when the machine is supplying current to an external circuit, such conditions must be chosen as will enable it to be indirectly deduced, without the necessity for actually trying the effect of different external loads. This may be done by determining *the short-circuit characteristic* of the alternator, when run with its terminals short-circuited; as may be traced from fig. 316, since E_e is now $= 0$, $E_0 = e$, and fig. 316 is reduced to a diagram analogous with fig. 314. An alternating-current ammeter is placed across the terminals of the armature in a single-phase machine, or in a polyphase machine all the phases are short-circuited and a low-resistance ammeter is inserted in one phase. A second characteristic is plotted with values of the exciting current or ampère-turns of the field as abscissæ and values of I_0 or the short-circuit current per phase as ordinates (fig. 317). The curve of I_0 as a rule is nearly a straight line passing through zero if there be no residual magnetism; with strong excitations it diverges slightly from a straight line.

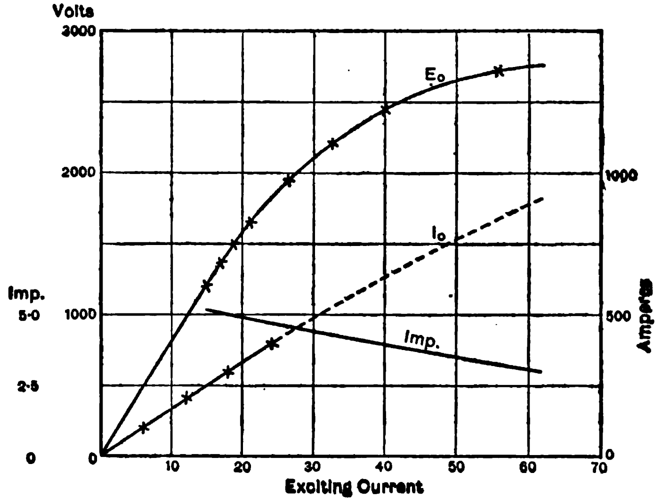


FIG. 317.

§ 5. **The synchronous impedance and reactance of the armature.**—When the values of $e_0 = E_0$ are divided by the corresponding values of I_0 the quotient $\frac{e_0}{I_0}$ being an alternating E.M.F.

divided by a current is of the nature of an impedance, and here represents what is called the *synchronous impedance of the armature*; this quantity may now be used to determine the value of $e = \text{I} \times$ impedance, for any value of I. The phase of e or the angle ϕ_a' at which it is inclined to the current line may be fixed by the following considerations. The component e_1 may be calculated from the ohmic resistance of the armature, being simply $e_1 = \text{I}_e\text{R}_a$, where R_a is the resistance of one phase of the winding. The component e_2 increases approximately as the square of the number of revolutions, and could be separated from e_1 in virtue of this difference; it is not, however, of large amount, and allowance may be made for it by assuming the resistance of the armature to be twice as great as it is in reality.[1] From the knowledge of the hypotenuse and of one side of the right-angled triangle, the angle ϕ_a' is at once obtainable. Experiment shows that with good alternators ϕ_a' is greater than 80° and cos ϕ_a' is then nearly $= 0$; in other words, even the sum of both the ohmic loss and that by eddy-currents is small as compared with the E.M.F. consumed by the synchronous reactance of the armature, and the synchronous impedance may practically be identified with the synchronous reactance, ϕ_a' being only little less than 90°.

§ 6. **Construction of full-load characteristic.**—Taking a given armature current, it is now possible to draw a curve connecting the terminal voltage as ordinate with different values of the exciting current or field ampère-turns as abscissæ for any particular value of ϕ_e that may be desired. In fig. 318 let OA be drawn at an angle ϕ_a' to OI which marks the phase of the current (cos ϕ_a' is in practice approximately $= 0{\cdot}15$) and of length $= e = \text{I} \times$ *impedance*. From A draw a line AC inclined to the horizontal at an angle ϕ_e. Taking any value for E_o, draw from

---

[1] Such an allowance is purely empirical, and is probably greatly in excess of the facts. The bulk of the eddy-currents must also be present on open circuit, and then weaken the field as much as at full load; they have no further share in causing the drop of volts between no load and full load. The additional eddy-currents due to the varying armature current have but a very small effect by way of increasing the apparent resistance of the armature, and their true effect will be more fully described in § 10. (Cp. Potier, *L'Éclairage Électrique*, vol. xxiv., p. 138)

O a line of such length as to represent it, cutting the line AC at the point C; then AC represents the value of the terminal voltage for the exciting current or ampère-turns corresponding to E_0 on the no-load voltage curve. The only difference between fig. 316 and fig. 318 is in fact that in the latter our starting-point is our knowledge of e and ϕ_e, so that the compounding of the several E.M.F.'s is carried out in reverse order. A number of points having been determined for, say, $I =$ full-load current and $\phi_e = 0$, a curve may be drawn which will fall below the no-load characteristic; below this again will fall a similarly drawn curve for $\phi_e = 90°$, and between these two limiting cases will lie

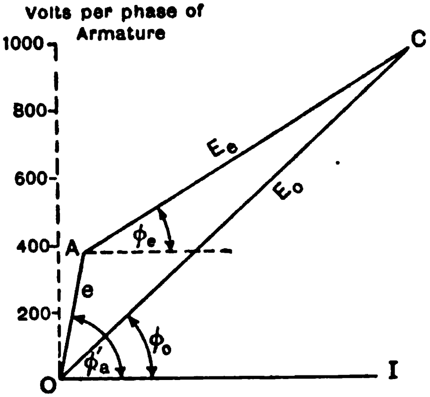


FIG. 318.

the terminal voltage for all values of ϕ_e with the given current I (fig. 319).

Since the armature contains iron, the reactance and the synchronous impedance of which it forms the chief component can not be really constant, as is indeed evident from fig. 317. The inductance has, however, been assumed to be located in an imaginary resistanceless coil separate from and in series with the armature. If such were in reality the case, the method would be strictly accurate, so long as the value of the reactance or of the impedance corresponding to the given current I was taken, as was done in the above example. The assumption that has been made as to the separate inductive coil and the physical meaning of the construction of fig. 316 or 318 requires, however, further examination, and it will be found that its legitimacy or otherwise turns on the nature of the inductance of the armature in an alternator.

§ 7. **The inductance of the armature and its two component elements.**—The composition of E.M.F.'s by the parallelogram law which has been the method so far followed is in fact only valid under one or other of two conditions. Either the various parts of the circuit in which the different induced E.M.F.'s are set up must be magnetically independent, or in default of this the flux-curve must be a straight line, the permeability of the iron being practically constant.[1] Now when current is flowing through the coils of the armature, a certain number of lines are linked therewith which do not pass through

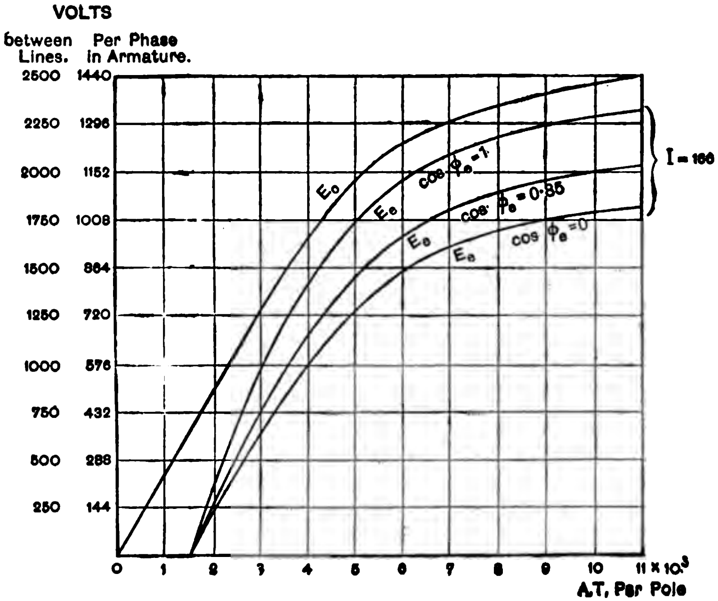


FIG. 319.—Load characteristics for different power-factors.

---

[1] Prof. Arnold, *E.T.Z.*, 1896, p. 774.

the main magnetic circuit, but are immediately closed round the inductors, passing through the iron teeth and across the tops of the slots within which the armature wires are wound.[1] There is, too, a further number of armature lines linked with the ends of the coils, which are similarly confined to local circuits outside the active length of the core. All these lines together form a certain amount of flux, or as it may be called from the analogy of the transformer, of *secondary leakage*, in local circuits which are, relatively speaking, magnetically independent of the main field circuit. To this leakage corresponds one portion of the total armature inductance which is practically independent of the position of the coil and is constant for any given current, but does not increase so fast as the current owing to the local paths in the armature iron becoming more or less saturated. The self-induced or reactance E.M.F., due to these leakage lines of the armature and lagging 90° behind the current forms a portion of the total synchronous reactance and must be counter-balanced by an equal E.M.F. from the main field; this E.M.F. preceding the phase of the current by 90° may be symbolised as *the leakage reactance voltage* $= e_{sa}$, and requires for its production a proportionate number of lines of the main flux.

Let t be the number of inductors in one group, *i.e.*, the number of inductors per pole and per phase; in the drum and disc machines they may form the sides of two adjacent coils or the single side of one undivided coil, but in either case they constitute in effect one large coil of t turns corresponding to a pair of poles. Let z_a' be the number of the secondary leakage lines, assumed to be all linked with each of the t turns of the coil. Then the average E.M.F. self-induced thereby in the coil is $4z_a' . f . t \times 10^{-8}$ volts, or since $t = \frac{\tau}{2mp}$, and there are p such large coils in one phase, the average E.M.F. of the whole phase is $\frac{1}{mq} . 2z_a' . f . \tau \times 10^{-8}$. The effective value of the voltage consumed owing to the leakage is therefore in one phase $\frac{K'}{mq} . 2z_a' . f . \tau \times 10^{-8}$

---

[1] Strictly speaking, this leakage does not exist separately in closed circles, but shows itself in a local distortion of the main flux.

volts, where K' is the ratio of the effective to the average E.M.F. For simplicity's sake, the equivalent sine wave is usually taken in place of the actual self-induced wave, and we pass to the effective reactance voltage with $K' = \frac{\pi}{2\sqrt{2}} = 1{\cdot}11$, and a corresponding value for the number of lines z_a'. At the same time if Z_a be the total number of lines from one pole due to the field winding and passing through the armature, the impressed E.M.F. caused thereby is by eq. (11)

$$E = \frac{K}{mq} \cdot 2Z_a \cdot f \cdot T \times 10^{-8} \text{ volts.}$$

If then K and K' have the same value (which is, however, by no means necessarily the case), it is evident that whatever number of leakage lines is linked with the entire armature coil, the same number must be supplied from each pole within the active length of the core by the main field in order to counterbalance them; or if K and K' are not alike, the field lines must be

$$z_a' \cdot \frac{K'}{K} = \frac{1{\cdot}11\, z_a}{K}.$$

The inductance of the armature is, however, not exhausted by the secondary leakage which has been above described. It further contains a second element, which is of radically different nature, and the armature reaction which is but another way of considering the inductance is similarly divisible into two parts which are different in kind. The first portion which has been described is practically unaffected by the position of the coil relatively to the poles. In virtue of the second portion, the inductance of an armature coil or set of coils in one phase, considered apart from any other phase, will be found to be dependent upon its position relatively to the poles, and even for the same current and state of saturation of the iron is not strictly a constant; it reaches a maximum when the inductors are immediately under the centre of a pole, and a minimum when they are midway between the poles in an interpolar gap.

§ 8. **The cross and direct ampère-turns of the armature.**—When an alternator is running under given conditions of load, each inductor or group of inductors has for each position which it occupies relatively to the poles a certain magneto-motive force. Further, if the paths of the magnetic circuit presented at any moment to the armature coils of the single phase are traced, they will be found to be different at different positions, and hence the reluctance presented to the armature magneto-motive force is not constant but varies. When the centre of a ring coil or of a group of wires forming the side of a drum coil is directly under the centre of a pole (fig. 320), all the ampère-turns act round a cross-circuit passing athwart the pole, and give a cross magneto-motive force displacing the resultant field towards one or other side. If the phase of the current neither lags nor leads, and its maximum value coincides with the centre of a pole, the density of the flux at the pole-corner ahead of the centre is strengthened, while that of the pole-corner behind the centre is weakened; or if we name the pole-corners in relation to the passage of a coil through an interpolar gap, as was previously done in the case of continuous-current dynamos, the leading pole-corner is weakened and the trailing pole-corner is strengthened. When the same inductor or group of inductors reaches a similar position relatively to the next pole, its current will be reversed, and so also will

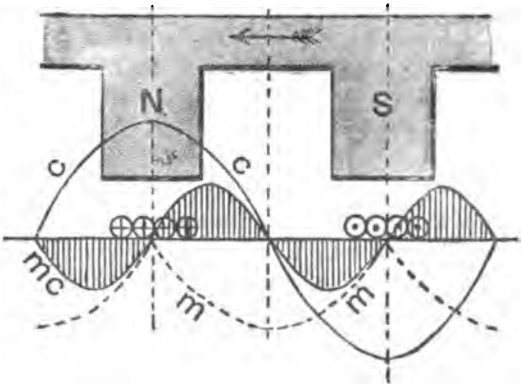


FIG. 320.—No angle of lag of current behind E.M.F. or centre of pole.

be the direction of the field, so that its magneto-motive force has precisely the same effect, viz., entirely cross-magnetising. But when the group of inductors moves away from the central position, two magnetic paths are presented to it, one of these being directly *through* a pair of poles round the main circuit of the field instead of *across* the pole. The ampère-turns are now divisible into the two groups of *cross and direct turns*, of which the latter directly affect the strength of the symmetrical field, while the former simply displace the weakened or strengthened symmetrical field. The proportion of the directly magnetising turns to the cross or distorting turns gradually increases until the group of inductors is exactly midway between the poles when there are no cross turns but all are directly magnetising. Later, as the coil passes under the next pole, the armature turns are again divisible, partly into direct and partly into cross magnetising turns. Thus in a single-phase alternator or in one phase of a polyphase machine considered by itself, the magneto-motive force of the armature for each point of the field is fixed in direction relatively to the poles, although as the inductors move relatively to the field, its amount varies synchronously with the period of the machine. The same is also true for either the cross or the direct magnetising effect when these are separated. In order therefore to estimate the effect of either the cross or the direct ampère-turns during a whole period, it is necessary to take simultaneously into account both the instantaneous value of the current for each position of the coil and the magnetising effect that a constant current in the same turns would have for the same position. Let the direct ampère-turns be considered in the first place, and let it be for the moment assumed that there is no lag or lead of the current relatively to the centre of a pole, so that the maximum value of the current coincides with a position of the inductors immediately under the centre of a pole. Let the dotted curve *mm* in fig. 320 be taken to represent the direct magnetising effect of the wires for each position with some constant value of current, say, one ampère. The current curve being that marked *cc*, the product of corresponding ordinates of the two curves may be plotted as a third curve *mc*, which will then show the varying M.M.F. due to the direct ampère-turns, back ampère-turns being plotted below and forward ampère-turns above the base line. The shaded areas show that in this case there is a periodic weakening and strengthening of the symmetrical field, but that the two balance during a complete period, and the average flux is unaffected.

The conditions assumed above would, however, seldom hold; when the circuit is closed in the first instance, the rise of the current causes a corresponding self-induced flux; the E.M.F. of self-inductance due thereto delays the growth of the current so that its instantaneous value is not in step with the instantaneous value of impressed E.M.F. although they gradually become more nearly alike in phase. The first few waves of current are therefore distorted, their exact shape depending on the moment in the period at which the circuit is closed, and it is only after some little time that the current curve reaches its final shape and phase. But the impressed E.M.F. is always changing in value; hence, unless the effect of self-inductance is in some way exactly balanced (as, *e.g.*, by capacity in the circuit), the current phase can never overtake that of the E.M.F. but must continue to lag behind it. Unless then the inductance of the armature and external circuit is precisely annulled, there must be a lag of the current by some angle ϕ_0 behind the impressed E.M.F., this latter being the E.M.F., E_0, of fig. 318, and being regarded as fixed in relation to the poles (its maximum coinciding with the centre of a pole). An examination of fig. 321 now shows that when there is such a lag of the maximum value of the current behind the centre of a pole, the weakening effect of the direct turns is longer and greater than the strengthening effect, so that on the whole there is a back effect from the armature ampère-turns. It remains to estimate its amount.

§ 9. **Calculation of the armature back ampère-turns in a single-phase alternator.**—Some idea must first be formed as to the way in which the direct effect of the turns with a constant current would vary with different positions of the wires,—in other words, as to the shape of the curve *mm*, and on this point there may be considerable variation in different cases. It depends, in fact, primarily upon the ratio of the pole-width to the pitch and upon the class of the alternator with its appropriate winding, either distributed, grouped, or concentrated in a single slot.

In an alternator with separately magnetised poles, the width of the pole-face being appreciably less than the pole-pitch, there is a comparatively large gap between the poles. If the group of inductors is wound in a single slot, then, as the slot moves away from the centre of a pole towards the edge, the direct effect of any current in the inductors rapidly increases. But when the slot has passed the edge of the pole, there is but little further increase in the direct effect for a given current. If, therefore, the angle of lag of the maximum value of the current behind the centre of a pole is increased, the demagnetising effect of the armature ampère-turns rapidly increases until the maximum value of the current coincides with a position of the inductors immediately at a pole edge, and for greater values of the lag ϕ_0 the increase is much smaller, until it reaches its maximum for a position of the inductors midway between the poles and a current whose maximum value lags 90° behind the centre of a pole. Such a case would be represented by fig. 322, where the armature is supposed to be stationary and the magnet to rotate; the direction of the current in the central group of wires shows that the resultant E.M.F. in them corresponds to passage under the field of a S. pole, and if the current is supposed to be at its maximum at the moment illus-

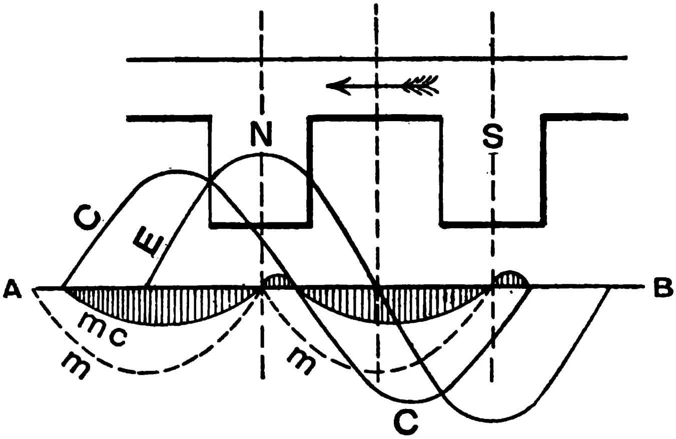


FIG. 321.—Maximum current lagging behind centre of pole.

trated, the self-induced flux which is indicated by the dotted lines is at its maximum and is seen to follow precisely the same paths as the main flux, but in exactly the opposite direction, so that it enters into a N. pole and issues from a S. pole.

The direct magnetising effect of the armature may also be established mathematically on the approximate assumption that the effect of a *constant* current in the inductors varies as a sine curve with their position reckoned from the centre of a pole, or as a cosine curve when reckoned from midway between the poles. In a heteropolar alternator or an inductor alternator with double armature, let time be reckoned from a position of the group of wires midway between the poles, when the impressed E.M.F. is zero, and when the back effect from any given current in the wires is a maximum.

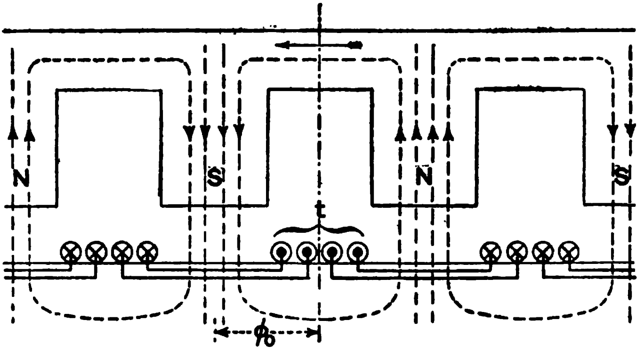


FIG. 322.—Maximum current lagging 90° behind centre of pole.

Such an initial position is shown in fig. 322, but for our present purpose of framing a general expression, instead of the maximum possible lag of 90° there shown, the angle of the current vector relatively to the no-load E.M.F. vector or to the centre of a pole must be allowed to take any value, such value being reckoned positive if the current lags and negative if the current leads. Let the instantaneous current i of the wires marked t in fig. 322 be reckoned as positive when it is in the direction of the E.M.F. induced as they come into the field of a N. pole, then for any movement of the poles corresponding to an angle a (expressed in terms of an electrical period) the instantaneous value is

$i = I \sin(a - \phi_0) = \sqrt{2} \cdot \text{I} \sin(a - \phi_0)$ where I is the effective value of the current. Then the instantaneous proportion of the turns which embrace one magnetic circuit and directly magnetise it will be for any angle of movement a away from our zero position $t \cos a$ if the wires are assumed as concentrated in a close group. The instantaneous value of the direct ampère-turns is thus $mc = \sqrt{2} \cdot t\,\text{I} \sin(a - \phi_0) . \cos a$. The maximum value of this can now be shown to occur when $a = \frac{1}{2}(90^\circ - \phi_0)$, and to be then $\text{X}_b = -\sqrt{2} \cdot t\,\text{I} \cdot \frac{\cos 2\phi_0 - \sin\phi_0}{2}$. It has however still to be asked how far the field is really affected by the maximum demagnetising influence. If the pole-pieces and magnet-cores are solid and the field varied rapidly in value, the amplitude of the variation would be speedily damped by the eddy-currents that would result. But even if the iron is laminated, the exciting coils with which the field is linked will act as a secondary and themselves limit the variation owing to their high inductance. The effect may therefore be considered from a different basis. Between the limits of a half period or $a = 0$ and $a = \pi$, the integral $\int_0^\pi \sin(a - \phi_0) \cos a \,.\, da$ in the expression for the instantaneous value of the direct ampère-turns is $= -\frac{\pi}{2} \cdot \sin\phi_0$, so that the algebraic sum of the shaded areas in fig. 321 corresponds to $-\sqrt{2} \cdot \frac{\pi}{2} \cdot t\,\text{I} \cdot \sin\phi_0$. The average ordinate of the curve, being $\frac{1}{\pi}$ of the last expression, is therefore $-\sqrt{2} \cdot \frac{t}{2} \cdot \text{I} \sin\phi_0$, and this value may also be established from the expression for the instantaneous value of mc. When expanded, this is equal to

$$\sqrt{2} \cdot t\,\text{I}\,(\sin a \cdot \cos a \cdot \cos\phi_0 - \cos^2 a \sin\phi_0);$$

the average value of $\sin a \cdot \cos a$ is zero, and the average value of $\cos^2 a$ is $\frac{1}{2}$; whence $\text{X}_b = -\sqrt{2} \cdot \frac{t}{2} \cdot \text{I} \sin\phi_0$.

It will thus be seen that from the two different stand-points accordingly as we consider the maximum value of the back ampère-turns or their average value in the single-phase alternator, two different expressions are obtained, viz.,

$$x_b = -\sqrt{2}\,.\,t\,\mathrm{I}\,\frac{\cos 2\phi_0 - \sin\phi_0}{2} \quad . \quad . \quad . \quad (101)$$

and

$$x_b = -\sqrt{2}\,.\,\frac{t\,\mathrm{I}}{2}\,.\,\sin\phi_0\,. \quad . \quad . \quad . \quad . \quad . \quad (102).$$

The divergence between the two is due to the fact that the armature M.M.F. pulsates between zero and the maximum corresponding to the former or highest value of x_b; but so far as any weakening of the field does actually take place, the latter expression is the more appropriate measure of its amount owing to the damping action of the exciting coils even if the field be laminated, and in practice the back-ampère turns are found to be nearly proportional to the sine of the angle of lag ϕ_0. In both cases if the coil be distributed, and the ratio of its width to the pole-pitch be *y*, the above values must be multiplied by a breadth-coefficient less than unity; if the magnetising curve of the concentrated winding be taken as sinusoidal, and the ratio of the width of the side of the coil or of one group of inductors to the pole-pitch be *y*, the value of the breadth-coefficient is

$$\frac{\sin\left(\frac{\pi}{2}\,.\,y\right)}{\frac{\pi}{2}y}.$$

While the above assumption that the curve of magnetising effect for a constant current is sinusoidal may be taken to hold for inductor alternators in which the width of pole is equal to the pitch, some allowance may require to be made in the case of heteropolar machines in which the width of the pole is considerably less. This has the effect of prolonging the time during which the wires well within the interpolar gap exercise their full demagnetising influence, yet the pole-face and air-gap areas are reduced. If the reluctance of a magnetic circuit R be identified with that of the double air-gap under the pole-faces, and time be reckoned from a position of the inductors (assumed to be closely concentrated) opposite the centre of a pole, the reluctance of the main circuit as presented to the coil falls gradually from ∞ to a value R as the coil edge approaches the edge of the pole; on reaching this, it remains constant at R until the coil-edge passes under the next pole. Thus the curve *mm* of the magnetising effect of a constant current instead of being sinusoidal would consist of horizontal portions joined by inclined straight lines. If x be the ratio of the pole-width to the pole-pitch, the reluctance for any angle of movement between the centre and the edge of a pole, *i.e.*, between $\alpha = 0$ and $\alpha = \alpha_1 = \frac{\pi}{2} \cdot x$ will be $R\frac{\frac{\pi}{2} \cdot x}{\alpha}$, and between α_1 and $\frac{\pi}{2}$ will be R. Meantime the current is itself varying, and if we assume a maximum value for ϕ_0, viz., $\frac{\pi}{2}$, its instantaneous value is $i = \sqrt{2} \cdot I \cdot \sin \alpha$. The instantaneous value of the back turns will vary inversely as the reluctance presented to them, so that as compared with the field turns acting on a constant reluctance R, their instantaneous value between $\alpha = 0$ and α_1 will be $\sqrt{2} \cdot I \cdot t \cdot \frac{\alpha}{\frac{\pi}{2} \cdot x} \cdot \sin \alpha$ and between α_1 and $\frac{\pi}{2}$ will be $\sqrt{2} \cdot I \cdot t \sin \alpha$.

Every quarter of a period will be alike, whence the average value is

$$X_b = \frac{2}{\pi}\left[\int_0^{\alpha_1} \sqrt{2} \cdot I \cdot t \cdot \frac{\alpha}{\frac{\pi}{2} \cdot x} \sin \alpha \cdot d\alpha + \int_{\alpha_1}^{\frac{\pi}{2}} \sqrt{2}\, I \cdot t \cdot \sin \alpha \cdot d\alpha\right]$$

$$= \frac{2 \cdot \sqrt{2} \cdot I \cdot t}{\pi}\left[\frac{2}{\pi x}\int_0^{\alpha_1} \alpha \sin \alpha \cdot d\alpha + \int_{\alpha_1}^{\frac{\pi}{2}} \sin \alpha \cdot d\alpha.\right]$$

The first integral is $= -\frac{\pi x}{2} \cdot \cos\left(\frac{\pi}{2} \cdot x\right) + \sin\left(\frac{\pi}{2} \cdot x\right)$, and the second is $= \cos\left(\frac{\pi}{2} x\right)$, so that the expression within the brackets reduces to $\frac{2}{\pi x} \cdot \sin\left(\frac{\pi}{2} \cdot x\right)$, and the whole becomes

$\frac{4\cdot\sqrt{2}}{\pi^2 x}\,.\,t\,.\,1\,.\sin\left(\frac{\pi}{2}\,.\,x\right)=\frac{0{\cdot}57}{x}\,.\,t\,.\,1\,.\sin\left(\frac{\pi}{2}\,.\,x\right)$. Assuming the effect from any different value of ϕ_o to follow a sine law as before, $x_b=\frac{0{\cdot}57}{x}\,.\,t\,1\,.\sin\left(\frac{\pi}{2}\,..x\right)\sin\phi_o$, where the value[1] 0·57 compares with 0·707 of eq. (102).

§ 10. **Calculation of back ampère-turns in polyphase alternators.**—When the same principles are applied to polyphase machines, they lead to an important result. Thus in a quarter-phase machine, a second curve of the direct magnetising action of the second set of coils may be drawn similarly to that of fig. 321, but displaced 90° relatively to the first (fig. 323). The instantaneous value of the back ampère-turns is then

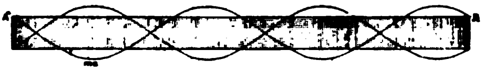


FIG. 323.—Composition of demagnetising effects in quarter-phase generator.

$$\sqrt{2}\,.\,t\,1\,\{\sin(\alpha-\phi_o)\cos\alpha+\sin(\alpha-\phi_o-90^\circ)\sin\alpha\}.$$

A simple trigonometrical calculation then shows that the instantaneous value of the two sets together is

$$-\sqrt{2}\,.\,t\,1\,.\sin\phi_o(\sin^2\alpha+\cos^2\alpha)=-\sqrt{2}\,.\,t\,1\,.\sin\phi_o\,;$$

hence it is independent of the angle α or is perfectly steady and for the same number of inductors per pole and per phase is twice as great as in the single-phase machine. Thus in fig. 323 the algebraic sum of the areas of the two *mc* curves gives the straight shaded band. Similarly in the three-phase machine, the instantaneous value of the direct turns is

$$\sqrt{2}\,.\,t\,1\,.\,\{\sin(\alpha-\phi_o)\,.\cos\alpha+\sin(\alpha-\phi_o-60^\circ)\,.\cos(\alpha-60^\circ)$$
$$+\sin(\alpha-\phi_o-120^\circ)\,.\cos(\alpha-120^\circ)\}\,,$$

and again simple calculation shows that the value of this is

$$-\sqrt{2}\,.\,t\,1\sin\phi_o\,.\frac{3}{2}\,(\sin^2\alpha+\cos^2\alpha)=-\sqrt{2}\,.\,\frac{3}{2}.\,t\,1\,.\sin\phi_o\,.$$

---

[1] Kapp, *Dynamos, Alternators and Transformers*, p. 380 (3rd ed.). See especially Guilbert, *Electr. World and Engineer*, vol. xl. p. 658 ff.

It is therefore again independent of the position of the coils and is perfectly constant at a value three times that of the single phase machine or one and a half times that of the quarter-phase machine.

Thus if any further effect of the width of the pole relatively to the pitch be neglected, and m be the number of phases, the final form of the equation for the polyphase generator with alternate poles is

$$X_b = -\sqrt{2}\,.\,\frac{m}{2}\,t\,\mathrm{I}\,.\,\sin\phi_0\,. \quad . \quad . \quad . \qquad (103)$$

The maximum value of this for an angle of lag of 90° behind the centre of a pole is then

$$X_a = -\sqrt{2}\,.\,\frac{m}{2}\,t\,\mathrm{I}\,. \quad . \quad . \quad . \quad . \qquad (104)$$

This maximum value may be called the ***magnetising armature turns***, and is in effect the ampère-turns of the armature as resulting from the composition of the currents in their several phases and as affecting one magnetic circuit. Since $mt = \frac{\tau}{2p}$,

$$X_1 = -\sqrt{2}\,.\,\frac{1}{2p}\,.\,\frac{\mathrm{I}\tau}{2}\,.$$

If the total effective ampère-turns on the armature, $\frac{\mathrm{I}\tau}{2}$, are symbolised by AT_a, $\frac{\mathrm{AT}_a}{2p} = \frac{\mathrm{I}\tau}{2 \times 2p}$, or the effective armature ampère turns per pole; we therefore also have for the magnetising turns of the armature the identical expressions,

$$\left.\begin{aligned} X_a &= -\sqrt{2}\,.\,\frac{\mathrm{AT}_a}{2p} = -\sqrt{2} \times \text{the effective ampère-turns per pole,} \\ \text{and } pX_a &= -0{\cdot}5\,\sqrt{2} \times \text{effective ampère-turns on the armature,} \\ &= -\sqrt{2} \times \text{effective ampère-wires on the armature.} \end{aligned}\right\} (105)$$

In all cases allowance may be made for a distributed winding by multiplying by a breadth-coefficient, as explained in § 9.

While the above formulæ are convenient in so far that they are in a form applicable to one, two or three phases, it must be borne in mind that the result which has been deduced in the case of two or three phases, viz., that the back effect is constant, is in many cases not strictly true. Especially is this likely to be so when the coils are of the non-overlapping kind, and on this account the back effect of a three-phase generator is sometimes regarded as varying periodically and as being proportional to a maximum number of back turns $= \frac{2}{3}\sqrt{2}\frac{\tau}{2p} . I$ instead of $\frac{1}{2} . \sqrt{2}\,\frac{\tau}{2p} I$, or to some proportion of $\frac{\tau}{2p} I$ lying between 0·707 and 0·94.[1] It may also again be remarked that in a single-phase machine x_b has different values with different writers according to the method by which it is estimated, and that if the above expressions are taken as applicable to the single-phase case, they give the average and not the maximum values.

Lastly, it must be added that the chief effect of any eddy-currents set up by the varying armature current is to render the actual value of the back ampère-turns lower than the theoretical value. Such eddy-currents are practically wattless, and the ampère-turns to which they give rise so far as they embrace the main magnetic circuit are almost exactly opposed to the ampère-turns of the armature; further, being themselves proportional to the armature current, they result in a diminution of the apparent number of armature magnetising turns or of the apparent inductance just as in a transformer or as in the amortisseur of Leblanc.[2]

§ 11. **Calculation of back ampère-turns in inductor generators.**—In the case of the homopolar or inductor alternator with single armature, a comparison of figs. 324 and 325 shows that there is a difference in the value of x_b according to the method of winding the coils. If these are not subdivided, and there are only as many as there are polar projections, the tendency of their magneto-motive force when the current lags as shown in fig. 324 is in the first place to cause a certain number of back lines to pass through each magnetic field, and of these the majority pass round through the main magnetic circuit by way of the second air-gap B at the other end of the armature. A smaller number pass sideways from the flanks of the poles through the inter-polar spaces, following the same path as the useless lines of the main field and having the same direction. The number of back ampère-turns acting on the air-gap under a polar projection is therefore exactly the same as in the case of the multipolar machine with alternate poles, or $X_b = -\sqrt{2}\,.\,m\frac{t}{2}\text{I}\sin\phi_0$; and owing to the greater fall of potential over the air-gap the number of useless lines passing into or out of the armature core through the interpolar spaces or s_a is increased. If, however, the armature coils are divided as shown in fig. 325, and as would usually be the case, the back ampère-turns through which the useful flux must be driven and with which it is linked, are only half the above, or $X_b = \sqrt{2}\,.\,m\frac{t}{4}\,.\,\text{I}\sin\phi_0$. The useless flux s_a is, however, also increased by the direct assistance of the remaining half of each pair of coils, so that the E.M.F. is thereby lowered more than in the first case. Next, when the inductor alternator has a double armature and divided coils are again used as in the last case, the same effects are repeated on the second half of the armature and in consequence the total number of back ampère-turns on the magnetic circuit as a whole is again

---

[1] Prof. Arnold, *E.T.Z.*, xvii. p. 731.

[2] Potier, *L'Éclairage Électrique*, vol. xxiv. p. 138.

$$X_b = \sqrt{2}\,.\,m\frac{t}{2}\,\text{I}\,.\,\sin\phi_0 \quad . \qquad (106)$$

It is evident how important is the number of the useless lines s_a in any calculation of the E.M.F. when there is lag of the current behind the centre of a pole.

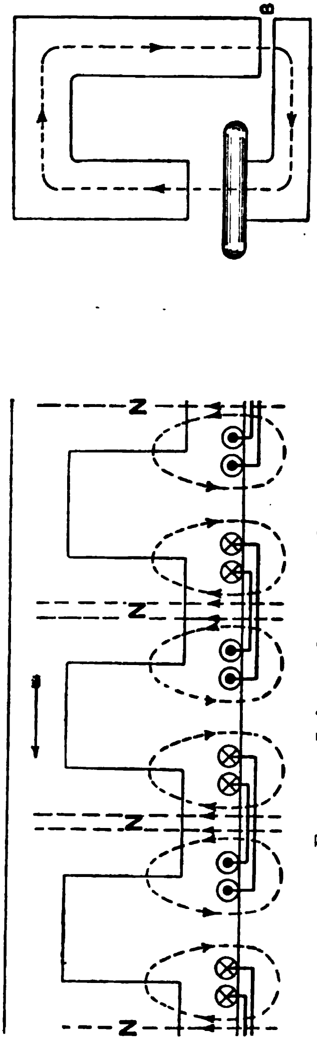


FIG. 324.—Inductor alternator with undivided coils and lagging current.

§ 12. The cross ampère-turns of the armature.

—Returning to the cross ampère-turns, these are to be treated on the same principles as were before applied to the direct ampère-turns. In a single phase considered by itself, if there be no lag or lead of the current relatively to the centre of a pole, the leading pole-corner is always weakened and the trailing pole-corner strengthened although to a varying degree according to the epoch of the period. But if the maximum value of the current lags behind the centre of a pole, each pole-corner is first weakened and then strengthened, the duration and amplitude of each fluctuation depending upon the angle of lag ϕ_0, until when this $= -90^\circ$, the strengthening is exactly balanced by the weakening and there is no net effect during a whole period, or the field is undistorted although its displacement may oscillate on either side of the centre line. If the current leads, the processes of weakening and strengthening are reversed in order of sequence at each pole-corner, and on the whole there is a strengthening of the leading pole-corner and a weakening of the trailing pole-corner, until when $\phi_0 = 90^\circ$, there is again no net effect on the field over a whole period. Reckoning time from the same moment as in § 9, *i.e.*, from the moment when the group of wires is midway between the poles and therefore has no cross magneto-motive force, the instantaneous proportion of cross ampère-turns is $t \sin a$ and the integral of the cross magnetising effect during a whole period is $\sqrt{2} \,.\, t \,.\, \mathrm{I} \int_0^\pi \sin(a - \phi_0) \sin a \,.\, da = \sqrt{2}\frac{\pi}{2} \,.\, t\,\mathrm{I} \cos \phi_0$,

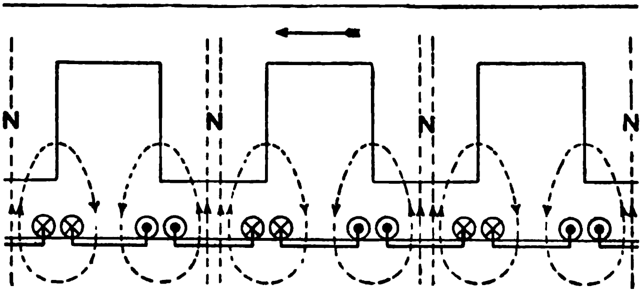


FIG. 325.—Inductor alternator with divided coils and lagging current.

and its average amount is $\frac{\sqrt{2}}{2} \,.\, t\,\mathrm{I} \cos \phi_0$.

Thus it follows the same law as that of the direct magnetising turns, save that it is proportional to the cosine of the angle of lag behind the centre of a pole instead of to the sine. If the angle of lag ϕ_0 is large and approaches 90°, the distortion disappears, the mean position of the centre of the field coinciding with the centre of a pole although there may be oscillation on either side of this line. In the case of the quarter-phase machine, the cross magnetising effect of the two sets of coils combined after the method already adopted for the direct turns is $\sqrt{2}\, t\, I \cos \phi_0$: it is therefore independent of the position of the coils or is entirely constant, so that the field is permanently displaced, and passage of an inductor through maximum field occurs later or sooner than its passage past the centre of a pole according as the current lags or leads relatively to the centre of a pole.

§ 13. **Armature reaction as explained on the assumption of a fictitious armature flux.**—The distinction between the cross and back ampère-turns of an alternator having now been established, it remains to consider how they affect the voltage of the machine. In the first place the magneto-motive force of the direct armature ampère-turns combines with that of the field-turns to determine the total number of lines of resultant flux through the armature. If the current lags behind the centre of a pole, this flux is smaller than would otherwise be due to the field ampère-turns. Such reduction of the main flux corresponds to the second portion of the inductance of the armature coils, and the term 'armature reaction' is often confined simply to this effect. If the problem be considered from the stand-point of two separate sets of lines, due respectively to the armature and to the field, and the one superposed on the other, the portion of the inductance now in question corresponds to the lines of the armature which follow the main magnetic circuit through the poles. Let Z'_a = the number of hypothetical lines corresponding to the magnetising armature turns, X_a, and so to the second portion of the inductance. Spacially these lines are at right angles to or in quadrature with the current. Like the leakage lines they are assumed to be all linked with every one of the t turns of the coil, and to give an effective self-induced E.M.F. $= \frac{K'}{mq} \cdot 2Z_a' f \cdot \tau \times 10^{-8}$. This must be balanced by an equal reactance voltage which may be distinguished as e_{sb}. If R_a and R_m be the reluctance of one entire circuit as presented to the armature and field-winding respectively $z'_a = \frac{1 \cdot 257 X_a}{R_a}$, and by the same reasoning as in § 7, the required number of counterbalancing field lines is $z_a = \frac{K'}{K} \cdot z'_a$. If K and K$'$ have the same value, and R_m and R_a may for a first approximation be identified, $z_a = z'_a$, and the ampère-turns on each magnetic circuit required to yield the annulling lines from the main field are X_a. In thus making use directly of X_a, the phase of the armature ampère-turns must be taken into account through a vector diagram; or if the back ampère-turns $X_b = X_a \cdot \sin \phi_o$ are substituted in the calculation, their value may simply be added algebraically to that of the field ampère-turns.

At the same time the cross ampère-turns of the armature distort the main flux, and this distortion corresponds to the inductance due to the armature lines which pass across the poles, but which neither strengthen nor weaken the field as a whole. In the single-phase machine the relative amount of the inductances corresponding to the back and cross ampère-turns varies, the one rising as the other falls, but in the polyphase machine the distortion remains practically constant and has therefore no effect except in the starting-up of the current wave; in any case its effect in reducing the flux due to the saturation of the pole-edges may be practically ignored. Thus the total E.M.F. of self-inductance or the voltage required to overcome it is divisible into the two parts, or $e_s = e_{sa} + e_{sb}$, and the importance of thus distinguishing them is that while the latter is a purely magnetic reaction the former is purely electric, and only indirectly affects the main flux by necessitating a corresponding increase in it and altering its phase.

Returning to the vector diagram of E.M.F.'s, let $2I_e \cdot R_a$ in fig. 326 represent the loss of volts over the ohmic resistance of one phase of the armature winding and by eddy-currents, the effect of the latter being empirically taken into account (as explained in § 5) by assuming an increased value for the resistance of the armature, say, twice its real value. Combining $2I_e \cdot R_a$ with the external E.M.F., E_e, preceding the current by an angle ϕ_e, we obtain vector E_1; if this be again combined with the voltage consumed by the leakage reactance or e_{sa}, a second vector $OB = E_2$ is obtained, which precedes the phase of the current by the angle ϕ_0'. While E_2 has been called the virtual induced E.M.F., E_1 has been called the real induced E.M.F.,[1] since it is undeniably present in the armature as due to the flux cut by the inductors within the limits of the armature core. Yet the E.M.F. E_2 is at least to some extent an equally real physical fact, so far as some of the self-induced lines are linked with the end connections of the coils outside the core length; the fluxes of secondary leakage and of counterbalancing field lines do not precisely overlap in their spacial distribution per unit length of core and do not immediately annul one another, so that, *e.g.*, the strain on the insulation is more than corresponds merely to E_1. Hence E_2 may be approximately identified with the actual E.M.F. induced in the armature or E_a. It is, however, more important to observe that proportional to the E.M.F. E_2 is the actual number of lines that cross the air-gap from one pole within the limits of the armature core, so that if an instantaneous photograph of the lines

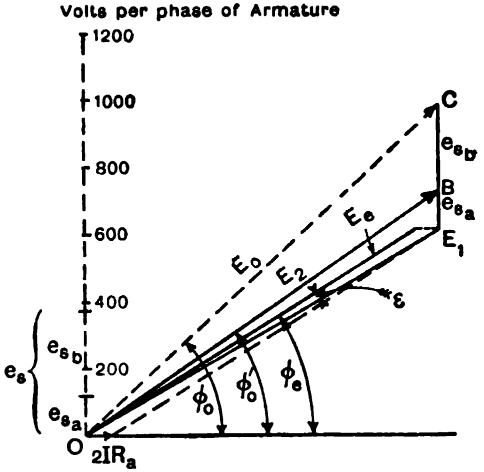


FIG. 326.—Composition of E.M.F.'s.

---

[1] Steinmetz, *Theoretical Elements of Electrical Engineering*, p. 128.

of the field could be taken, their number and position relatively to the armature coils would be found to correspond with the length and phase of the vector OB $= E_2$. The excitation over the armature iron and air-gaps is strictly that required by the flux corresponding to OB, and the phase of the current lags behind the maximum strength of the actual armature field by the angle ϕ'_a.

So far the process is perfectly legitimate since the self-induced E.M.F.'s corresponding to the inductance of the external circuit and to e_{sa} are to all intents and purposes magnetically independent. If we now produce e_{sa} by an amount e_{sb} corresponding to the fictitious number of lines Z_a', so that $e_{sa} + e_{sb} = e_s$, the total voltage consumed by the self-inductance of the armature as in figs. 314, 316, a diagram is obtained which exactly reproduces that of fig. 316. The resultant of E_a and e_{sb} or of OB and BC gives us an apparent impressed E.M.F. E_0, and proportional thereto an apparent total number of useful lines passing through the armature. From the line of apparent impressed E.M.F. is obtained the relative position of the line of maximum apparent field which is in reality to be identified with the centre of a pole; for it will be found that the vectorial combination of e_{sb} proportional to X_a and always at right angles to the phase of the current has the effect of taking into account the cross ampère-turns as displacing the field. As a matter of fact the field when distorted by the cross ampère-turns, even though the distortion remain constant as in a polyphase machine, cannot give a sine curve of E.M.F., so that no deductions can in practice be made from the relative positions of the lines E_0 and E_2 in fig. 326. Yet such a diagram does in fact give a clue to the actual state of the case. It shows that the maximum of the displaced field no longer coincides with the centre of a pole, and that the angle of lag of the current behind the maximum field is in general different from its lag when measured from the centre of a pole, except on short-circuit, when the lag is a maximum and the distortion is a minimum. Upon the lag of the current behind the actual distribution of the field depends the magnetic pull upon the armature inductors, and it is seen that any deductions as to its amount based on the angle of lag behind the centre of a pole or the line corresponding to the apparent impressed E.M.F.

E_0 will be quite erroneous. It is only in this connection that a steady distortion effect has any practical interest.

Evidently then the first of the two conditions under which the composition of E.M.F.'s is legitimate is not entirely fulfilled; the fictitious lines z_a' are not magnetically independent of the main flux-circuit of the armature. In reality they correspond to a certain number of back ampère-turns which are neutralised by the exciting turns, and which do not therefore affect the reluctance of the main magnetic circuit by directly increasing the flux-density therein. If the ampère-turns required for a given terminal voltage under current are deduced from the curve of E_0, and if ζ be the primary leakage lines which escape from the poles and do not pass through the armature, the lines of the actual field through the pole and yoke are not those given by $OC + \zeta$, but by $OB + \zeta$, and since the reluctance of the field-magnet is not a constant quantity, the loss of magnetic potential over the magnet is not so high as calculated. If the main magnetic circuit were of constant permeability, the method would still remain strictly correct, and this is the second of the two alternative conditions under which the application of the parallelogram law to E.M.F.'s remains completely valid. The loss of potential over the magnet would then be increased by the addition of the hypothetical flux z_a', but such increase would simply correspond to the back ampère-turns of the armature and would be proportional to the armature current. Thus in default of a constant permeability in the iron magnet, and so long as we deal with the undivided quantity of the synchronous reactance e_s which contains within itself not only the leakage reactance voltage, but also the armature reaction proper, the simple composition of E.M.F.'s makes the required amount of excitation greater than will really be necessary, or *vice versâ*, leads to a value for the terminal E.M.F. below that which will in reality obtain. As an alternative method, the composition of ampère-turns or of M.M.F.'s instead of E.M.F.'s must now be considered.[1]

§ 14. **Composition of M.M.F.'s instead of E.M.F.'s.**—Owing to the rotation of the armature or of the field, the M.M.F. of the field-magnet coils with respect to the armature may also be represented by a rotating vector, the relation of which to the rotating vector of the current or of the E.M.F. is one of space and not of time. In the usual case of a drum coil the vector of the E.M.F. is conventionally regarded as lagging 90° behind the flux or the M.M.F. which causes it, the spacial displacement of the centre line of the coil from the centre line of a field when the E.M.F. is a maximum being a quarter of a period, although the maximum E.M.F. coincides in point of time with the position of the coil when its inducing side is in the centre of a field. Thus,

---

[1] Cp. Rothert, *E.T.Z.*, 1895, *Electrician*, vol. xxxviii. p. 45, and Arnold, *E.T.Z.*, vol. xvii. p. 774.

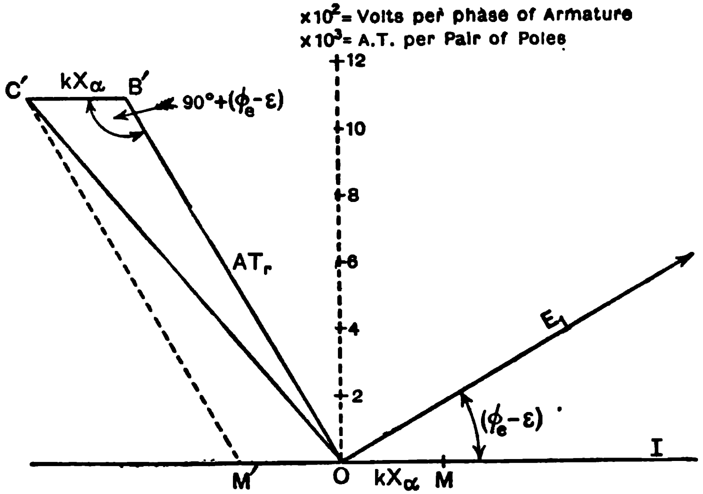

×10² = Volts per phase of Armature
×10³ = A.T. per Pair of Poles
+12
+10
+8
+6
+4
+2
C′
kXα
B′
90°+(φe−ε)
ATr
E1
(φe−ε)
I
M′
O
kXα
M

FIG. 327.—Composition of ampère-turns.

if the current be assumed as zero vector (fig. 327), and E_1 be the armature E.M.F., the resultant flux or the ampère-turns producing it must precede E_1 by 90°, as shown by the line AT_r. The armature ampère-turns coincide in phase with the current, and are negative since the current is assumed to lag behind the E.M.F. From the short-circuit characteristic the ampère-turns on one magnetic circuit which correspond to a value of the short-circuit current equal to the desired full-load current are read off. These turns being required to balance the combined effect of the armature reaction and armature secondary leakage may be expressed as a certain multiple of the armature magnetising ampère-turns X_a, *i.e.*, as kX_a, where k has some value greater than unity. If this amount be represented by OM in fig. 327, they must be balanced by an equal number of field-turns, OM′, and the composition of this with the resultant turns AT_r gives the total number of ampère-turns OC′ required on the field or X, which in its spacial position corresponds to the centre of a pole.

The triangle OB′C′ can now be applied to the construction of the characteristic curve for any given current and external angle of lag. The angle ϵ between E_e and E_1 (fig. 326) can be calculated from the equation

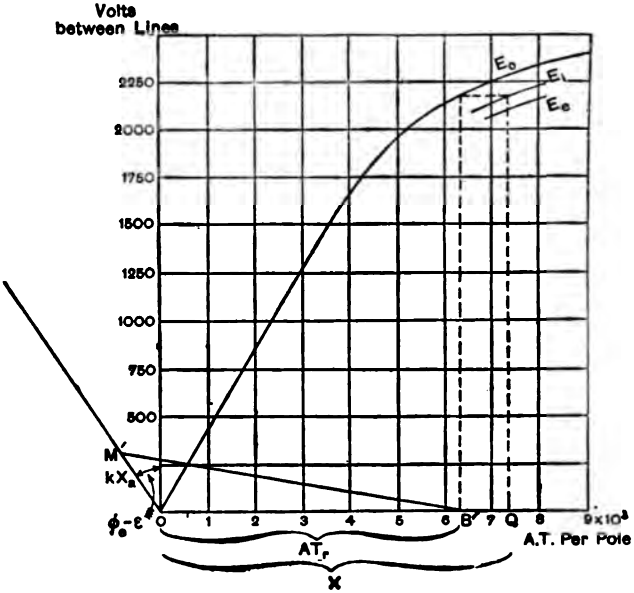


FIG. 328.

$$\sin \epsilon = \sin \phi_e \frac{2 I R_a}{E_1}$$

the ohmic resistance per phase being R_a, and E_1 being the vector sum of E_e and $2IR_a$. Draw OM′ $= kX_a$ making an angle $\phi_e - \epsilon$ with the vertical axis (fig. 328). Taking any number r ampère-turns OQ = X, describe a circle about the point M′ with radius = OQ, cutting the horizontal axis at B′. Then the no-load E.M.F. corresponding to the excitation OB′, when transferred to the excitation OQ, gives the armature E.M.F. E_1 for the given current and angle of lag, and is one point of an armature characteristic. Deducting from this the loss of volts by the resistance of the armature and by eddy-currents so far as such loss is in phase with the terminal E.M.F., *i.e.*, subtracting $2IR_a \cos \phi_\omega$ the terminal voltage and characteristic for full load is obtained. It will be seen that this is in effect the application of the triangle OB′C′ of fig. 327 in such a way as to deduct a certain number of ampère-turns off the total number of field-turns, so as to obtain the virtual number to which the resultant flux corresponds. The method is, however, still not correct. The actual flux through the armature is more than that corresponding to E_1 in virtue of the secondary leakage which must be balanced by the main field, and this leakage has only been taken into account by grouping it with the armature reaction proper. Further, the primary leakage is that corresponding not to OB′ but to the total number of exciting turns OQ, and with highly saturated poles the reluctance of the magnet will be higher than has been assumed. The method therefore yields too high an external characteristic.

§ 15. **Corrected combination of the two methods.**—A further correction is therefore required, and this can only be made if the component items e_{sa} and e_{sb} can be separated out, and the two methods be combined, E.M.F.'s being first dealt with and then M.M.F.'s. If e_{sa} is known, the first method of fig. 326 must be followed until the stage when the armature E.M.F. E_a due to the composition of $2I_eR_a$, E_ω and e_{sa} has been reached; after this the actual flux corresponding to E_a can be found, and the transition must be made to the composition of ampère-turns. The complete diagram is therefore to be drawn as follows.

The vector sum of E_e and $2IR_a$ by the usual trigonometrical formulæ for the solution of triangles becomes

$$E_1 = \sqrt{E_e^2 + (2\,I\,R_a)^2 + 2E_e(2\,I\,R_a)\cos\phi_\omega}$$

and the angle ϵ is calculated as before from the equation

$$\sin\epsilon = \sin\phi_e\,\frac{2\,I\,R_a}{E_1}.$$

The composition of E_1 with e_{sa} gives

$$E_a = \sqrt{E_1^2 + e_{sa}^2 + 2E_1 \cdot e_{sa} \cdot \sin(\phi_e - \epsilon)},$$

and $$\sin a = \cos(\phi_e - \epsilon)\,\frac{e_{sa}}{E_a}.$$

Thus the angle of lag of the current behind the centre of the field is $\phi_o' = \phi_e - \epsilon + a$. At right angles to E_a, let $X_{(g+a)}$ (fig. 329) be drawn representing the ampère-turns required per magnetic circuit over the air-gaps and iron of the armature to give the flux corresponding to E_a. Let X_a represent to the same scale the ampère-turns required to balance the armature magnetising turns. Then the interpolar excitation required per magnetic circuit upon the field is given by the line X_p, or the vector sum of $X_{(g+a)} + X_a$. It should be observed that here the value X_a must be used and not the greater value kX_a, since we are dealing simply with the armature reaction proper, and further that the lesser amount of X_b as compared with X_a when the angle of lag of the current behind the centre of the pole is less than 90° is automatically taken into

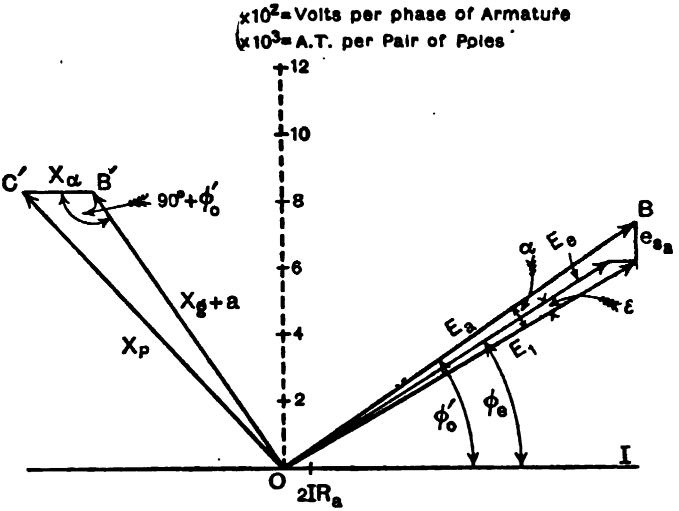

x10² = Volts per phase of Armature
x10³ = A.T. per Pair of Poles
+12
+10
+8
+6
+4
+2
C′
Xa
B′
90°+φ′o
Xg+a
Xp
B
α
Eе
esa
ε
Ea
E1
φ′o
φe
O
2IRa
I

FIG. 329.

account by the diagram, since the component entering into X is only $X_b = X_a \sin\phi_0$.

The effect of the primary leakage may now be taken into account graphically [1] or it may be as rapidly calculated by the following approximate method just as in a continuous-current dynamo.

The primary leakage is $1{\cdot}257 X_P \times P = \zeta$, where P is the leakage permeance of the air-paths reduced to a quantity that may be regarded as in parallel with the armature path. The true $Z_m = Z_a + \zeta$, and the density required for calculating the ampère-turns expended over the iron of the poles and yoke is $B_m = \frac{Z_a + \zeta}{AB}$, whence X_m can be calculated and by addition to X_P gives the total ampère-turns required on one magnetic circuit. Or if it be required to calculate the terminal voltage for a given excitation, a process of trial and error must be used, a certain terminal voltage being assumed and the excitation worked out until it is found to agree with the given value that will be actually used. It will be seen that this more elaborated method gets rid of the error involved in regarding the reluctance of the field-magnet as unaffected by the primary leakage which is itself determined in part by the armature ampère-turns. The method of fig. 327 is in fact only valid when the density in the field-magnet is so low that its reluctivity is practically constant. Thus on short-circuit the density in the field-magnet is almost entirely due to ζ, and the ampère-turns required over the field-magnet's reluctance are nearly negligible; the resultant turns AT_r may then be identified with X_g, and fig. 327 is reduced to the simple diagram of fig. 330.

§ 16. **The short-circuit diagram of ampère-turns.**—The latter diagram as representing the short-circuit conditions deserves further consideration from its importance to the theory and design of alternators. On short-circuit an alternator presents many points of similarity to a transformer, the primary being represented by the field-magnet winding, and the secondary by the armature winding. The maximum value of the short-circuit current occurs when the poles formed on the armature are exactly opposite to the poles of the magnet-system. The demagnetising force of the armature is then nearly equal to the magnetising force of the field-winding, or in other words, the secondary ampere-turns are nearly equal to the primary ampère-turns, and the difference between them merely corresponds to the flow of a small magnetising current and a small number of lines.

---

[1] As by the method of Dr. Niethammer, *E.T.Z.*, 1901, p. 255.

Let it be assumed that in an ideal case there is no loss of volts

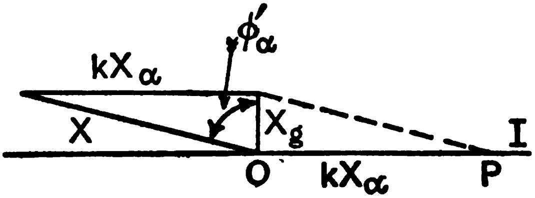


FIG. 330.

over the copper resistance of the armature and no eddy-currents therein; if z_a be the number of lines required to give E_0 on open circuit, then

$$z_a = \frac{1{\cdot}257\ X}{R_m}.$$

On short-circuit the fictitious demagnetising lines of the armature are

$$z'_a = \frac{1{\cdot}257\ X_a}{R_a},$$

where R_a and R_m as before are the reluctances of one entire magnetic circuit as presented to the armature winding and field winding respectively. The apparent synchronous reactance voltage e_0 is then exactly balanced by the impressed E.M.F. E_0, so that

$$\frac{e_0}{E_0} = \frac{K'}{K} \cdot \frac{X_a}{X} \cdot \frac{R}{R_a} = 1.$$

The angle of lag of the current behind the centre of a pole being in our imaginary case precisely 90°, the magnetising turns of the armature or X_a embrace practically the same magnetic circuit as the field ampère-turns, and $R_a = R_m$. If now K and K' have the same value, we have $X_a = X$. Such is the ideal case of a machine in which there is no secondary leakage and in which the impressed E.M.F. is assumed to follow the same sine law as the reactance voltage. The total apparent number of field lines is entirely annulled by the armature reaction; or in other words, the magnetising turns of the armature as resulting from the several phases or X_a are exactly balanced by the field excitation X, the assumption being also made that there are no eddy-currents to partially counterbalance the theoretical value of X_a.

There must, however, be some small surplus of ampère-turns on the field in order to provide for the loss of volts over the ohmic resistance and by eddy-currents, both of which have been so far treated as negligible. The lag of the current behind the centre of the pole is not quite 90°, but since from fig. 314 ϕ'_a exceeds 80°, $\sin \phi'_a$ is approximately $= 0{\cdot}989$

and $$X_a = X \sin \phi'_a = 0{\cdot}989X,$$

or $$X = 1{\cdot}01X_a$$

The effect is to reduce the maximum number of armature magnetising ampère-turns that can be actually balanced by a given number of field-turns, so that instead of the ideal ratio of 1, we find $X = kX_a$, where k is some constant slightly greater than one.

The same effect but to a much greater degree is produced when the secondary leakage which must be present round the end-connections of the coils and across the tops of the teeth is taken into account. The total number of leakage lines from the secondary as caused by the combined effect of the two or more phases is dependent upon the armature ampère-turns, and may therefore be expressed as the product of the armature magnetising ampère-turns multiplied by some constant, or $z'_a = c \, . \, X_a$. The number of field lines required to counterbalance these is $\frac{K'}{K} \, . \, z_a'$, and neglecting the reluctance of the armature by comparison with that of the double air-gap, the ampère-turns required to pass $\frac{K'}{K} \, . \, z_a'$ lines over the two air-gaps is $0{\cdot}8 \frac{K'}{K} \, c X_a \, . \, \frac{2l_g}{a_g}$. At the same time the ampère-turns required to pass the flux necessitated by the ohmic resistance and eddy-currents is $X_a \cot \phi'_a =$ say, $0{\cdot}15X_a$, and the phase of these latter is at right angles to the former. The true X_g being the hypotenuse of the right-angled triangle is therefore (on the assumption that the ratio $\frac{K'}{K}$ is approximately 1)

$$X_g = 0{\cdot}8\ X_a\, .\, c' \times \frac{2l_g}{a_g},\ \text{where}\ c' = \sqrt{c^2 + 0{\cdot}15^2},\ \text{and}$$

is very slightly greater than c. The back ampère-turns as affecting the main magnetic circuit are still present and the field excitation at the pole-tips must therefore be approximately

$$X_P = X_g + X_a = X_a\left(1 + 0{\cdot}8c'\, .\, \frac{2l_p}{a_g}\right),$$

or more strictly

$$= \sqrt{X_g^2 + X_a^2 + 2X_g\, .\, X_a\, .\, \sin\phi'_o}.$$

Thus $\quad Z_m = \zeta + c'\, .\, X_a$

where $\quad \zeta = 1{\cdot}257X_P \times P = 1{\cdot}257X_a\left(1 + 0{\cdot}8c'\, .\, \frac{2l_g}{a_g}\right) \times P.$

Thence $\quad X_m = f'\left(\frac{Z_m}{a_m}\right) . \, l_m$

and $\quad X = X_a + X_g + X_b + X_m$

$$= X_a\left(1 + 0{\cdot}8c'\frac{2l_g}{a_g}\right) + f(B_m)\, .\, l_m \quad . \quad . \quad (106)$$

The last term dealing with the loss of magnetic potential over the reluctance of the iron magnet-cores and yoke is in most cases quite negligible as compared with the first, since the total number of lines actually passing when the alternator is short-circuited is but small; yet in a machine with great leakage as in an inductor generator with overarching claws even the last term has some influence. Except, however, in such cases the resultant ampère-turns on the field may be identified with X_g, and we have a diagram similar to fig. 330, but in which the leakage lines of the armature are reckoned in calculating X_g, and X_a is plotted along the horizontal axis of current instead of kX_a. Even this may be simplified by taking into account the secondary leakage with the armature reaction under the general term kX_a, when X_g or AT_r itself becomes negligible, and we simply have two straight lines, X on one side balancing $X_a + X_a \times 0{\cdot}8c' \frac{2l_g}{a_g}$ on the other side. In fact the short-circuit characteristic measures the combined effect of the magnetising turns of the armature and of the secondary leakage expressed in ampère-turns.

§ 17. **Separation of armature reaction proper from secondary leakage by wattless-current method.**—It might now be thought that the separation of the effects of secondary leakage and of the true back ampère-turns could be easily made by deducting either X_a or the excitation corresponding to e_{sa} *i.e.*, $X_a \,.\, 0{\cdot}8c' \frac{2l_g}{a_g}$, whichever is the better known, from the X given by the short-circuit characteristic for the full normal number of ampères.[1] But although this may approximately be done and the results are useful in the work of design, the method is not free from objection, since in point of fact neither of the two component items is known with complete accuracy. In the first place, the calculated value of the magnetising armature turns which has been deduced in §§ 9 and 10, viz., $X_a = \sqrt{2} \,.\, \frac{m}{2} t I_o$ is itself based on a number of assumptions as to the shape of the magnetising curve for a constant current that may not in practice hold strictly true. Again, even on short-circuit in an actual machine R_a is never exactly the same as R_m; thus in the case of machines with alternate poles, the width of the centre of an armature coil is very often greater than the width of a pole face, while in the polyphase machine the reverse may be true, especially if the coils are of the non-overlapping kind. Again, the exact effect of any eddies due to the armature current in reducing X_a is not known. Still on the whole the theoretical value of X_a may better be used to determine e_{sa} than *vice versâ*.

It is, however, evident, that for the accurate predetermination of the terminal E.M.F. under different loads, especially if the field-magnet system is strongly saturated, it is very desirable to be able to separate the effect of the armature back ampère-turns from that due to the secondary leakage, and that this is only possible if some further datum be obtained other than the mere short-circuit characteristic. A method of separating the two has been given by M. Potier,[1] based on the previous work of M. Blondel, and the results deduced by it are found to be in close agreement with observed facts. The additional datum is derived from a curve of terminal E.M.F. taken for a considerable current I with such a low power-factor as to be practically wattless. Such a curve may be obtained by working the alternator on a highly inductive load such as induction motors running light, or in connection with a synchronous motor so excited as to shift the phase of the current relatively to the E.M.F.; or the machine may itself be operated as a synchronous motor. The field-exciting current A_s is progressively reduced while the armature current I is maintained as nearly as may be constant, until, if the machine is short-circuited, the final value of A_s is one reading of the field current in a short-circuit curve. The latter may, in fact, then be entirely dispensed with, the necessary data being all obtainable from a complete curve of the terminal voltage for different field-currents with a wattless current in the armature of constant value. Or, if only a few readings for the upper portion of the wattless-current curve are obtained, one supplementary reading giving the point where the curve cuts the axis of X may always be added from the short-circuit characteristic if this has been independently taken.

---

[1] Cp. Niethammer, *E.T.Z.*, 1901, p. 255.

M. Potier has shown that a wattless curve, such as the lowest of fig. 319, when cos ϕ_e is practically equal to 0, may be analysed as follows. In fig. 331 if any value of the field current $OA = A_s$ be taken and if all the magnet coils equal in number to the poles are in series with T_c turns per coil, the field excitation per magnetic circuit is $2A_sT_c$, from which must be deducted the back ampère-turns of the armature in order to arrive at the number which is really effective in causing a flux of lines through the armature. With the assumed low power-factor the back turns are practically equal to the magnetising turns of the armature or X_a. Neglecting then the difference in the primary leakage in the two cases, the flux under the load must be that corresponding to $(2A_sT_c - X_a)$ ampère-turns of the open-circuit curve of E.M.F., or if we mark off to the left from A, a distance AB along the axis of abscissæ equal to $\frac{X_a}{2T_e}$ or the ampères which must pass through the field coils in order to counterbalance X_a, the flux corresponds to an armature voltage BP. From this must be deducted an amount $PQ = e_{sa}$, so that the actual terminal voltage $E_e = BQ$. Since this value occurs with the field current $A_x = OA$, it must be transferred horizontally across as shown at AP′. Thus P′ is one point on the curve of terminal E.M.F. for a wattless current of value I, and is seen to be virtually obtained by shifting the point P on the curve of E_0 along the line PP′ inclined to the horizontal axis at the angle θ. Any other point may be similarly treated as shown by the triangle KLS, where the special value of the field ampère-turns or current is that which gives the value I for the armature current on the short-circuit curve. It follows that the curve SP′ may be at once derived by shifting the open-circuit curve of E.M.F. horizontally

---

[1] *L'Éclairage Électrique*, vol. xxiv. p. 133 ff.

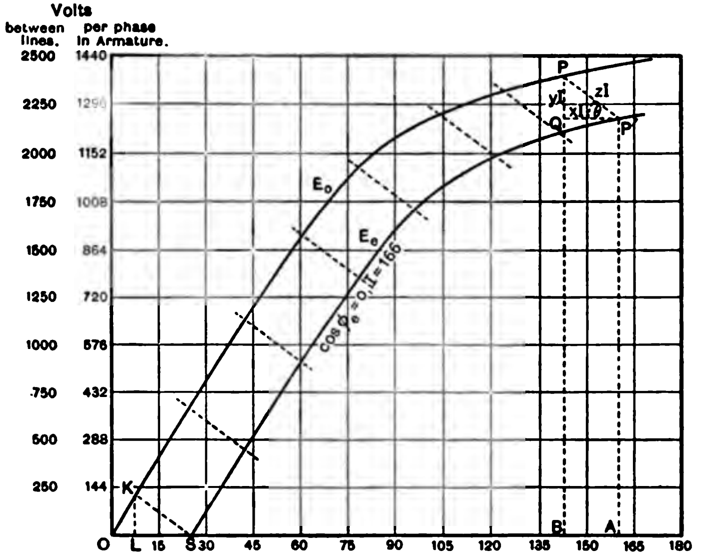


FIG. 331.—Analysis of terminal E.M.F. curve for wattless current.

along by a distance QP′, at the same time that it is dropped down by the distance PQ; in other words, by shifting it aslant through a distance PP′, as shown by the inclined dotted lines. When running as a synchronous motor, the curve is shifted along the same lines but in the opposite direction, so that it lies above the open-circuit curve E_o. If then such a curve as that marked SP′ is obtained experimentally and be compared with OKP, the open-circuit curve, the latter, with the marked point P, being drawn on tracing paper and shifted parallel to itself in a slanting direction, such a position may be found that it exactly fits over the lower curve. The line joining the old and new positions of P gives the hypotenuse of the right-angled triangle, and the two sides PQ and QP′ are at once determined. The vertical PQ is $e_{sa} = y\,\text{I}$, where y is a factor converting armature current into volts of self-inductance and is assumed to be a constant. The abscissæ being in ampères, the horizontal distance $\text{QP}' = x\text{I}$, where x is some constant by which the back turns of the effective armature current are expressed in terms of the equivalent current through the 2T_c turns on each magnetic circuit, its theoretical value being $x = \dfrac{\sqrt{2} \cdot \frac{m}{2} \cdot t}{2\text{T}_c}$. The actual angle θ depends upon the scale to which the volts and ampères are respectively plotted.

The expression for the terminal E.M.F. under the wattless load is thus $E_e = f(\text{A}_x - x\,\text{I}) - y\,\text{I}$. This also holds on short-circuit, when $E_e = 0$, and $y\,\text{I} = f\,(\text{A}_0 - x\,\text{I})$ where $\text{A}_0 = \text{OS}$ is the field current on short-circuit corresponding to the armature current I. Since on short-circuit the point K falls on the straight initial portion of the open-circuit characteristic, let a_x be some small value of the field-current which gives an open-circuit E.M.F. E_0; then ampère-turns are converted into volts if multiplied by $\frac{E_0}{a_x}$, or, in other words, the function f is then given by the constant multiplier $\frac{E_0}{a_x}$. Hence, if A_0 is the field-current corresponding to the short-circuit current I_0,

$$v\text{I}_0 = \frac{E_0}{a_x}(\text{A}_0 - x\text{I}_0)$$

or

$$A_0 = I_0 \left\{ x + y\left(\frac{a_e}{E_0}\right) \right\} = I_0\left(x + y''\right)$$

from which it is seen how the field-current is balanced by the short-circuit current with the two components into which it is divisible, the one due to the back ampère-turns and the other due to the secondary leakage.

If a single reading E_e on the wattless-current curve is taken experimentally, and the short-circuit characteristic is known, the armature reaction proper may still be separated from the leakage reactance by the following construction due to M. Blondel.[1] From the open-circuit characteristic deduct the fixed amount E_e, the derived curve being that shown by $O'aM'$ in fig. 332. Let the field-current corresponding to the wattless current I and terminal voltage E_e be OA, and let $I = aI_0$, where I_0 is the short-circuit current corresponding to the same field excitation OA. Drawing

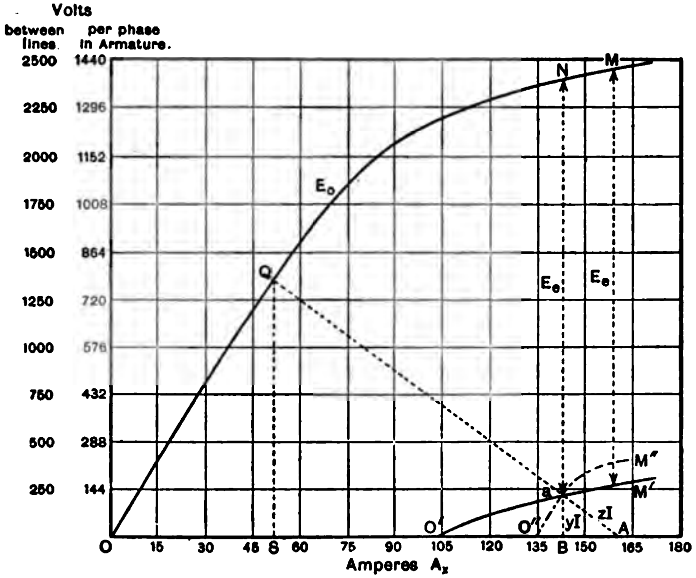


FIG. 332.

---

[1] *E.T.Z.*, vol. xxii. p. 474.

radii from A to the open-circuit curve E_0, reduce each in the proportion $\frac{I}{I_0} = \alpha$, $\left(e.g., \ \frac{Aa}{AQ} = \frac{I}{I_0}\right)$ the reduced values when plotted giving the curve O″aM″, which is similar to OQM about the centre A. Then the leakage reactance yI of fig. 331 must be given by some ordinate to the curve O′aM′, while the hypotenuse to the right-angled triangle of fig. 331 or zI must be given by some radius to the curve O″aM″. The intersection, therefore, of the two curves at a reproduces the right-angled triangle of fig. 331,

$$\text{and } x = \frac{AB}{I} \text{ or } \frac{AS}{I_0}, \text{ while } y = \frac{aB}{I} \text{ or } \frac{QS}{I_0}.$$

§ 18. **Approximate estimate of secondary leakage and of e_{sa} from experimental results.**—Turning next to the question of predetermining the value of e_{sa}, measurements have been made of the real inductance of armature coils under different conditions and in different positions relatively to the poles, but in any deductions from such experiments care must be taken to consider how far the measured inductance includes not only that due to the secondary leakage, but also that which corresponds to the cross magnetising effect which is unimportant and to the demagnetising effect which is to be estimated by other methods. The apparent inductance with which we are now concerned is best reproduced when the coil on which the experiments are made is embedded in its slots on the iron core, but is otherwise free, the poles being absent. From the data given in Mr. Hobart's valuable paper[1] to which frequent reference has been made in Chap. XVIII § 20 when discussing the inductance of continuous-current armature coils, the flux of lines per cm. length of an embedded coil and per C.G.S. current-turn is found to range from about 25 with a shallow wide slot (1·5 cm. deep × 2 cm. wide) to 50 with a narrow deep slot (5 cm. deep × 1 cm. wide), the top of the slot being in both cases open. If the turns of one coil-side are distributed among two or three slots per pole, there is a considerable reduction in the value on the assumption that the flux is all linked with every turn; yet this reduction becomes proportionately much less if the number of slots is increased above three, so that the values would become approximately as follows:—

---

[1] 'Modern Commutating Dynamo Machinery,' *Journal Inst. Electr. Eng.*, vol. xxxi. pp. 192 ff.

	Deep.	Medium.	Shallow.
One slot,	50	35	25
Two slots	32·5	22·8	16·3
Three slots	28·2	19·8	14·1
Four slots	25	17·5	12·5

While the above values might be used to estimate the total reactance voltage e_s, they are still too high for the purpose of calculating e_{sa}, since they include a considerable number of back lines; it will therefore be necessary to take a lower value of, say, on the average some 17 lines per cm., and even then this may further be modified to say 20 for small currents and 12 for heavy currents to allow for the increased saturation of the iron. The inductance of the free end-connections of the coils may be calculated on the same basis as in Chap. XVIII § 20, viz., on the assumption of a flux of 8 lines per cm. length and per C.G.S. current-turn. If then l be the length of the armature core in cm., and t be the number of inductors in a group per pole and per phase, and there are as many coils as there are poles, the flux linked with the two sides is partly self-induced and partly mutually induced from the neighbouring coils; it is therefore due to t wires and is $t\ (17\ l) \times 2$. If l' be the length of the end-connections on one side or of the two half-connections corresponding to a half loop, the flux linked with both ends is $\frac{t}{2}\ (8l') \times 2 = t\,.\,8\,l'$. The hypothesis is that the whole of this flux is linked with all the $\frac{t}{2}$ turns of the coil, so that the total number of linkages in one coil for each

C.G.S. unit of current is $t\ (17l+4l')$, or the self and mutual inductance of the coil is $t^2(17l+4l') \times 10^{-9}$ henrys. The total inductance of the $2p$ coils of one phase is therefore

$$L = 2pt^2(17l+4l') \times 10^{-9} \text{ henrys,}$$

and on the assumption that the inductance is constant or that the current-change is sinusoidal, the reactance of the phase is

$$2\pi f L = 2\pi f \,.\, 2pt^2(17l+4l') \times 10^{-9} \quad \text{ohms.}$$

The voltage consumed by the reactance is equal to the product of the current and the reactance, so that if I be the effective value of the current in ampères, the effective value of the voltage consumed by the reactance which alone we require is

$$e_{sa} = 2\pi f L \,.\, I = 2\pi f \,.\, 2pt^2 I(17l+4l') \times 10^{-9} \text{ volts.}$$

This is more conveniently expressed if the flux be reckoned per ampère-turn instead of per C.G.S. current-turn; the numerical constants per cm. length are then only $\frac{1}{10}$th as large, so that

$$e_{sa} = 2\pi f \,.\, 2p \,.\, I \,.\, t^2\ (1{\cdot}7l+0{\cdot}4l') \times 10^{-8} \text{ volts.} \qquad (107)$$

The corresponding lines that must be furnished in each magnetic circuit or from each pole in order to yield this voltage are $z_a = s'_a \frac{K'}{K}$, or by equating e_{sa} to $K \,.\, 2Z_a \,.\, f \,.\, \frac{T}{m} \times 10^{-8}$ (eq. 11), the counterbalancing $z_a = \frac{\pi}{K} \,.\, I \,.\, t\,(1{\cdot}7l+0{\cdot}4l')$.

If $K = \frac{\pi}{2\sqrt{2}}$ as with a sine wave,

$$\begin{aligned} z_a &= 2\sqrt{2}\ I\ t\,(1{\cdot}7l+0{\cdot}4l') \\ &= 2{\cdot}83\ I\ t\,(1{\cdot}7l+0{\cdot}4l') \quad . \quad . \quad . \quad . \quad . \quad . \qquad (108) \end{aligned}$$

If the t inductors are divided between two or more slots the first constant 1·7 would be lower, while if each side of the small coil is divided between two or more slots, the second constant 0·4 would be reduced, since the paths of the wires at the ends would not then be so nearly coincident. While the above is the more usual case, there may alternatively be only one large coil per pair of poles. In this case, while the flux linked with the inductors remains the same, the flux linked with the end-connections is four times as great; there are therefore twice as many linkages with the inductors and four times as many linkages with the ends, but as there are now only p coils, it is only necessary in calculating the reactance voltage to double the second constant, or the counterbalancing

$$z_a = 2{\cdot}83 \ \mathrm{I} \ t \, (1{\cdot}7l + 0{\cdot}8l') \quad . \quad . \quad . \quad (109)$$

In a polyphase generator, the mutual inductance of the other phases increases the apparent inductance, and for this an approximate allowance may be made in the case of a three-phase generator by multiplying by $\frac{m}{2} = \frac{3}{2}$; in other words, by taking the flux as one and a half times as great, so that with $2\,p$ coils

$$\mathrm{L} + \mathrm{M} = 1{\cdot}5 \times 2pt^2(1{\cdot}7l + 0{\cdot}4l')$$

$$e_{sa} = 2\pi f \, . \, 3p \, . \, t^2\mathrm{I}(1{\cdot}7l + 0{\cdot}4l') \times 10^{-8} \text{ volts,} \quad . \quad (110a)$$

and the counterbalancing $z_a = \frac{1{\cdot}5\pi}{\mathrm{K}} \ \mathrm{I} \, . \, t \, (1{\cdot}7l + 0{\cdot}4l') \quad . \quad (110b)$

$$\text{or with a sine wave} = 4{\cdot}25 \ \mathrm{I} \ t \, (1{\cdot}7l + 0{\cdot}4l')$$

§ 19. **Approximate calculation of secondary leakage.**— The shape of the slot and the degree of saturation in the teeth have in reality great effect in modifying the secondary leakage, and in default of experimental data, the flux must be estimated on the general principles of Chap. XVIII § 21 after the method of Dr. Niethammer.[1] Thus in the case of a half-closed slot of shape such as that of fig. 333 the inductance may be divided into the several portions corresponding to the flux within the slot, across the bridge or overhanging edges of the opening, and within the air-gap for a short distance on either side of the opening.

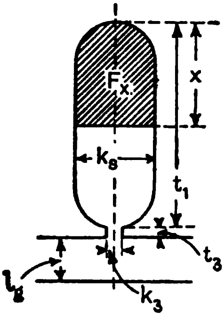


FIG. 333.—Calculation of secondary leakage.

Within the slot the permeance of an elementary strip of width dx is $\frac{dx}{k_s} \, . \, l$. If q_1 = the number of slots per pole and per phase, the total wires in the slot are $\frac{t}{q_1} = \frac{\tau}{2mpq_1}$; if F be the total area of cross-section of the slot, and F_x be the area up to the height x, the M.M.F. acting across the elementary strip with one C.G.S. unit of current flowing in each wire is $4\pi \frac{t}{q_1} \cdot \frac{F_x}{F}$. The lines in the strip are therefore $4\pi \frac{t}{q_1} \cdot \frac{F_x}{F} \cdot \frac{dx}{k_s} \cdot l$ linked with $\frac{t}{q_1} \cdot \frac{F_x}{F}$ wires. Hence

---

[1] *E.T.Z.*, 1900, p. 550, and 1901, p. 255.

$$dL_1 = 4\pi \left(\frac{t}{q_1}\right)^2 \left(\frac{F_x}{F}\right)^2 \frac{dx}{k_s} \cdot l$$

and

$$L_1 = 4\pi \left(\frac{t}{q_1}\right)^2 \int_0^{t_1} \left(\frac{F_x}{F}\right)^2 \frac{dx}{k_s} l.$$

With straight sides to the slot and square corners, $\int_0^{t_1} \left(\frac{F_x}{F}\right)^2 dx = \frac{t_1}{3}$ and $L_1 = 4\pi \left(\frac{t}{q_1}\right)^2 l \cdot \frac{t_1}{3k_s} \times 10^{-9}$ henrys.

The flux across the opening is $4\pi \cdot \frac{t}{q_1} \cdot l \frac{t_3}{k_3}$, and is linked with all the $\frac{t}{q_1}$ wires, and similarly the flux within the air-gap may be roughly estimated from an assumed permeance $\frac{l_g \cdot l}{k_3 + 1{\cdot}6 l_g}$.

In a complete phase there are $2pq_1$ slots, so that

$$L_1 + L_2 + L_3 = 2p \cdot t^2 \cdot \frac{4\pi}{10} \cdot \frac{l}{q_1}\left(\frac{t_1}{3k_s} + \frac{t_3}{k_3} + \frac{l_g}{k_3 + 1{\cdot}6 l_g}\right) \times 10^{-8} \text{ henrys.}$$

To this must be added the inductance of the end-connections, and if there are $2p$ coils then as before explained,

$$L_4 = 2p \cdot t^2 \cdot 0{\cdot}4 l' \times 10^{-8},$$

or less if $q_1 =$ four or more, and the total inductance is

$$L = 2p \cdot t^2 \left[\frac{1{\cdot}257 l}{q_1}\left(\frac{t_1}{3k_s} + \frac{t_3}{k_3} + \frac{l_g}{k_3 + 1{\cdot}6 l_g}\right) + 0{\cdot}4 l'\right] \times 10^{-8} \text{ henrys.}$$

By the same steps as before, and equating $e_{sa} = 2\pi f \mathrm{L} \,.\, \mathrm{I}$ to $\mathrm{K} \,.\, 2Z_a \,.\, f \,.\, \frac{\tau}{m} \times 10^{-8}$ volts, we find the counterbalancing flux

$$Z_a = \frac{\pi}{\mathrm{K}} \mathrm{I} \quad t \left[\frac{1{\cdot}257 l}{q_1} \left(\frac{t_1}{3k_s} + \frac{t_3}{k_3} + \frac{l_g}{k_3 + 1{\cdot}6 l_g} \right) + 0{\cdot}4 l' \right]$$

or if $\mathrm{K} = \frac{\pi}{2\sqrt{2}}$,

$$Z_a = 2\sqrt{2}\, \mathrm{I}\, t \left[\frac{1{\cdot}257 l}{q_1} \left(\frac{t_1}{3k_s} + \frac{t_3}{k_3} + \frac{l_g}{k_3 + 1{\cdot}6 l_g} \right) + 0{\cdot}4 l' \right]$$

$$= 2{\cdot}83\, \mathrm{I}\, t \left[\frac{1{\cdot}257 l}{q_1} \left(\frac{t_1}{3k_s} + \frac{t_3}{k_3} + \frac{l_g}{k_3 + 1{\cdot}6 l_g} \right) + 0{\cdot}4 l' \right] \quad (111)$$

Similarly in the three-phase machine

$$Z_a = \frac{1{\cdot}5\pi}{\mathrm{K}} \mathrm{I}\, t \left[\frac{1{\cdot}257 l}{q_1} \left(\frac{t_1}{3k_s} + \frac{t_3}{k_3} + \frac{g}{k_3 + 1{\cdot}6 l_g} \right) + 0{\cdot}4 l' \right] \quad (112a)$$

and with a sine-wave of E.M.F.

$$= 4{\cdot}25\, \mathrm{I}\, t \left[\frac{1{\cdot}257 l}{q_1} \left(\frac{t_1}{3k_s} + \frac{t_3}{k_3} + \frac{l_g}{k_3 + 1{\cdot}6 l_g} \right) + 0{\cdot}4 l' \right] \quad (112b)$$

§ 20. **Use of the short-circuit characteristic in designing.** —In default of complete knowledge of either X_a or e_{sa}, the designer must fall back upon the results obtained from the short-circuit characteristic as experimentally observed. In every case the actual ratio of X to X_a on short-circuit is found to differ considerably from unity. The value of k in the expression $\mathrm{X} = k\mathrm{X}_a$ as tabulated by several observers[1] from actual experiment varies with different types of machines and arrangements of winding; and it is instructive to compare it with the approximate expression $\left(1 + 0{\cdot}8c' \frac{2l_g}{a_g}\right)$. In a given machine the value of k increases with increasing values of the excitation and of the short-circuit current I_0; in other words, the short-circuit characteristic is not a straight line but tends to become slightly convex; the field-leakage increases with the increasing armature reaction, and so also does the resultant flux, both combining to raise the saturation of the field-magnet. Yet if the teeth of the armature are highly saturated by the leakage across the slots, this secondary leakage may increase so slightly with large currents that the tendency of the curve to bend downwards is more than counterbalanced, and it becomes slightly concave. Indeed, by varying the proportions of the magnetic circuit, much may be done to decide the shape of the curve, and its upward or downward tendency will have an important bearing upon the regulation of the machine. In some cases for small values of the exciting current or of I_o, k is less than unity and as low as 0·9; this result which from what has been said might appear to be impossible is due to residual magnetism in the iron cores, which in effect makes the virtual ampère-turns of the field greater than those given by the product of the actual exciting current into the number of turns on the magnet.

---

[1] Arnold, *E.T.Z.*, 1896, p. 731; Fischer-Hinnen, *Electr. Eng.*, vol. xx. p. 597.

In the expression $X = k \cdot \sqrt{2} \cdot \frac{m}{2} t I_o$, since $t = \frac{\tau}{2mp}$ in a heteropolar machine with each pole separately magnetised, we have

$$pX = k \cdot \frac{1}{\sqrt{2}} \cdot \frac{\tau}{2} I_o;$$

but pX = the total number of ampère-turns on the field, and $\frac{\tau I_o}{2} = AT_a$ = the total number of effective-ampère-turns on the armature. Thus on short-circuit, the total AT on the field $= k \times 0{\cdot}707$ × effective AT on the armature, or $\frac{AT_f}{AT_a} = k \times 0{\cdot}707$. If there is but a single exciting coil on the field, its ampère-turns must be multiplied by p in order to obtain $pX = AT_f$ with which to compare the armature ampère-turns.

In the inductor alternator with single armature and divided coils, p being used with the signification that has been given in Chap. IX as appropriate to the type, t is again $= \frac{\tau}{2mp}$, but X_a has half its previous value; hence $pX = k \times 0{\cdot}35 \frac{\tau}{2} I_o = k \times 0{\cdot}35 \times$ effective-ampère-turns on the armature. But if the armature is of the double kind, and τ be reckoned for one armature, X_a has the same value as in the heteropolar machine, and thus as before $pX = k \times 0{\cdot}707 \frac{\tau}{2} . I_0 = k \times 0{\cdot}707 \times$ effective-ampère-turns on one armature. Since there is but a single exciting coil in the inductor type, its ampère-turns must as above be multiplied by p for the purpose of comparison with the armature ampère-turns.

It is found in practice that the value of k ranges from 1·2 to 1·8 in three-phase heteropolar machines, and on the average may be taken as = 1·4. In single-phase alternators, it varies according to the method on which X_b or X_a is calculated (§ 9); if the maximum value be taken, it may be from 1 to 1·5, or if the average value be taken, from 2 to 3, and in general on the latter method may be reckoned as about 2·25.

As the exact estimation of the magnetising ampère-turns of the armature is open to some uncertainty, especially in the case of single-phase alternators, for such an expression as $X = k\sqrt{2}\,\frac{m}{2}\,t\,I_0$ may be substituted the simpler form $X = \lambda\, t\, I_0$,[1] and it is then sufficient for the designer to tabulate the value which λ is found to have in a given type of machine and with certain proportions. Thence

$$\frac{\lambda}{m} = \frac{pX}{\frac{\tau}{2} I_0} = \frac{AT_f}{AT_a} \quad . \quad . \quad (113)$$

or the ratio of the ampère-turns of the field to the effective ampère-turns of the armature on short-circuit,—a ratio which will be found to be one of the chief data in the design of alternators. Knowing the value of λ, we can calculate the short-circuit current $I_0 = \frac{X}{\lambda t}$ for any value of the field ampère-turns, and thence the synchronous reactance voltage. The smaller the value of λ, the better the regulation of the machine, and it is evident that it is dependent not only upon the actual value of the demagnetising effect of the armature ampère-turns, but also upon the secondary leakage.

§ 21. **Use of synchronous reactance method in practice.** —While the composition of E.M.F.'s as first carried out in § 6 yields too low a terminal E.M.F. for a given number of ampère-turns of excitation, the simple composition of ampère-turns without allowance for primary leakage yields too high an E.M.F. The curves of figs. 319 and 328 are in fact based on the same data derived from the alternator of which the full design is given in Chap. XXV, and as the case chosen is one in which the poles are strongly saturated, it will be seen that there is a marked divergence between the the two results. The actual terminal E.M.F. for any current and power-factor must lie between the two curves, but when we are limited to one or other of the two alternative methods, the simple composition of the total synchronous reactance voltage e_s (as determined from λ) with E_1 has the advantage of safety, since it yields a value lower than the actual value by perhaps some 5 or 10 per cent. Experience soon shows when slightly better results or a slightly fewer number of ampère-turns may be expected.

---

[1] Behrend, *Electr. Eng.*, vol. xxvi. p. 727.

From fig. 316 certain approximations[1] can be made which are convenient in the work of designing, and which are therefore shortly given below.

From the triangle of fig. 316, $E_0^2 = E_e^2 + e^2 + 2E_e . e\cos(\phi'_a - \phi_e)$ or $E_e^2 + 2E_e . e\cos(\phi'_a - \phi_e) = E_0^2 - e^2$. Adding $e^2\cos^2(\phi'_a - \phi_e)$ to both sides and solving the quadratic equation,

$$\begin{aligned} E_e + e . \cos(\phi'_a - \phi_e) &= \sqrt{E_0^2 + e^2\{\cos^2(\phi'_a - \phi_e) - 1\}} \\ &= \sqrt{E_0^2 - e^2\sin^2(\phi'_a - \phi_e)} \end{aligned}$$

In order to complete the square, let $\frac{1}{4} . \frac{e^4\sin^4(\phi'_a - \phi_e)}{E_0^2}$ be added to the term under the square root, which is permissible since e is small as compared with E_0; thence

$$E_e = E_0 - e\cos(\phi'_a - \phi_e) - \frac{1}{2}\frac{e^2\sin^2(\phi'_a - \phi_e)}{E_0}$$

Since ϕ'_a is approximately = 90°, $\sin\phi_e = \frac{E_0^2 - E_e^2 - e^2}{2E_0 . e}$

and $$E_e = E_0 - e\sin\phi_e - \frac{e^2\cos^2\phi_e}{2E_0}.$$

---

[1] Fischer Hinnen, *Electr. Eng.*, vol. xx. p. 596. and 620.

Thus by neglecting the effect of the ohmic resistance and of eddy-currents as is permissible with good machines, for the limiting cases of a non-inductive and a purely inductive load, we obtain respectively

$$\phi_e = 0 \qquad \cos\phi_e = 1 \qquad E_e^2 = E_o^2 - e^2 \text{ or } E_e = \sqrt{E_o^2 - e^2} \quad . \quad (114)$$

and

$$\phi_e = 90^\circ \qquad \cos\phi_e = 0 \qquad E_e = E_0 - e \quad . \quad . \quad . \quad . \quad . \quad (115)$$

while for intermediate cases we have (fig 334)

$$E_e = \sqrt{E_o^2 - e^2 \,.\, \cos^2\phi_e} - e\sin\phi_e \quad . \quad . \quad (116)$$

Since the apparent total impedance is not strictly constant even for the same current in the armature, it has been shown that there is some little error in using the value of e derived by experiment from the short-circuit current with a large angle of lag for the same current if the angle of lag be much smaller as on a non-inductive load.

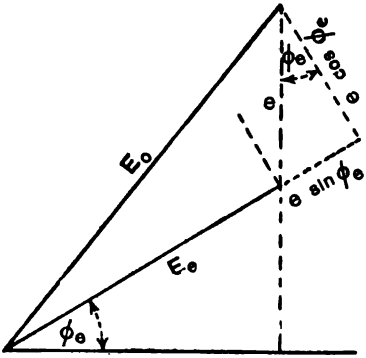


FIG. 334.

In determining the short-circuit characteristic, readings of the short-circuit current are to be taken for different values of the field excitation as it is increased in steps from zero, and during the short time necessary for each observation, the current may safely be allowed to rise to two or three times the full-load normal current. Further, during the readings exact constancy of the speed is not imperative; as the speed is increased, the reactance of the armature rises equally, and the only effect is to alter the relative proportion of the eddy-currents and of $e_1 + e_2$ to e_s. As this is in any case small, the current is almost entirely determined by the reactance and remains practically constant for a given excitation even if the speed does vary somewhat. With any power-factor in the external circuit less than 0·6, the drop of volts is practically the same as that due to a truly wattless current; hence if an inductive load can be applied, a wattless curve can be added to the short-circuit curve.

§ 22. **Example of the synchronous reactance method.**—An actual case of a three-phase 500 horsepower inductor generator giving 2000 volts between any pair of terminals may now be cited[1] as an example of the application of the synchronous reactance method when the magnet system is not very highly saturated. When run at a speed of 406 revs. per min. with a frequency of 54, the open-circuit curve of E.M.F. for different values of the

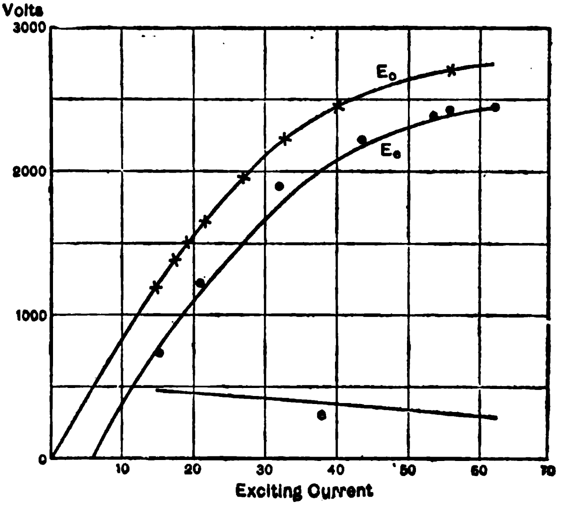


FIG. 335.

field-excitation was that marked E_0 (fig. 335), giving a value of 2-60 volts for an exciting current of 62 ampères. When short-circuited, the armature ampères for low values of the field current are given by the full line I_0 (fig. 317), which for currents above some three or four times the normal may be prolonged as shown by the dotted extension. Dividing E_0 by I_0, we obtain the corresponding values of the apparent impedance of the armature. Thus with a field current of 62 ampères the value of the short-circuit current is 920 ampères, and the apparent impedance is $\frac{2760}{920} = 3$. The value of the impedance gradually rises when the field excitation is reduced as shown in the fourth column below.

---

[1] Taken by permission of Mr. B. A. Behrend from his paper on 'The Factors which determine the Design of Monophase and Polyphase Generators" (*Electr. World and Eng.*, republished in *Electr Eng.*, vol. xxvi. p. 670.

A_x	E_0	I_0	Imp.	$e_0 =$ I × Imp.	$E_0 - e_0$	E_e observed.
62	2760	920	3	288	2472	2470
56	2710	840	3·22	309	2401	2410
54	2690	815	3·3	317	2373	2390
44	2550	680	3·75	360	2190	2230
32	2220	510	4·3	420	1800	1900
21	1650	340	4·85	465	1185	1230
15·5	1270	249	5·1	490	780	747

It is now desired to calculate the fall of potential with a highly inductive load of 96 ampères. The values of the product of the fixed current into the varying impedance is given in the fifth column, and since the load is highly inductive, it may be assumed that approximately $\phi_e = 90°$. It may further be assumed that ϕ_a' is $= 90°$; under these circumstances the counter E.M.F. e may be simply deducted from the value of E_0. This has been done in the sixth column, and by its side are added the observed terminal volts, from which it will be seen that the agreement is very good except in the case of the fifth observation where there is probably some error in the reading of either speed or voltage. Neither ϕ_a' nor ϕ_e can strictly speaking be 90°, but in such a case as the present they may at least be taken as equal, and then $E_e = E_0 - e_0$ becomes very closely true, as shown by fig. 335.

§ 23. **Values of inherent regulation.**—For the purpose of estimating the degree to which different alternators tend to maintain a constant terminal voltage under varying loads, their *inherent regulation* may be compared. This latter quality may be defined in several slightly different ways; *e.g.*, as the *fall* in volts which occurs if the machine is excited to give its normal terminal voltage at no load and full load is then switched on while the excitation and speed remain constant. The fall, being expressed as a percentage of the no-load terminal volts, then gives some indication as to the amount by which the excitation must be altered by the attendant in order to maintain the terminal volts constant. Or it may be defined as the *rise* in volts which occurs when the full load is removed, the speed and excitation being maintained constant at the value corresponding to the full rated output ; the rise expressed as a percentage of the terminal volts at full load then rather indicates how far the alternator under varying loads may be left without attention. Owing to the greater degree of saturation of the machine in the latter case, the second definition gives slightly better values than the first. Thus the inherent regulation varies according to the nature of the full external load, and the power-factor of this must be stated ; or in general, the inherent regulation may be given for the two limiting values, viz., cos $\phi_e = 1$ with a non-inductive load, and cos $\phi_e = 0$ with a completely inductive load. In a machine with a large amount of armature reaction and correspondingly poor inherent regulation the short-circuit current may only exceed the full-load current by some 50 per cent. even with the normal excitation for full load. It is then impossible for the machine to be very seriously overloaded or for its armature to be immediately burnt out through an accidental short-circuit. This advantage is, however, bought at the expense of the regulation, and so important is this latter quality that in modern designs it is usual to specify as low a rise of voltage as can be obtained with reasonable cost of copper and manufacture. For a few minutes a current two or three times the normal will do no serious harm, and beyond this reliance must be placed on fuses or automatic circuit-breakers to protect the machine in the case of a prolonged short-circuit. Hence on the whole the larger the short-circuit current of the alternator, the better is the machine. It will, however, be seen later in Chaps. XXIV and XXV that a small percentage rise or fall of volts if obtained by the employment of a very high degree of saturation in the poles may involve some loss of flexibility in meeting emergency loads, unless the alternator is designed at the start for a low power-factor ; it may be then unduly large for its normal load and power-factor, or uneconomical in exciting energy. Thus economy of exciting watts as affecting the efficiency must be given its due weight, and in any case when the alternator is employed to supply induction motors, a considerable overload capacity is even more important than good inherent regulation. For electric lighting, the drop of volts is always more than can be permitted at the consumers' terminals, so that the excitation must necessarily be adjusted, and the amount of the adjustment within reasonable limits becomes more or less a matter of indifference.

A good polyphase generator with alternate poles and a distributed winding will regulate within about 6 per cent. on a non-inductive load with $\cos \phi_e = 1$, and within 15 to 18 per cent. on an inductive load with $\cos \phi_e = 0{\cdot}8$. With concentrated windings the results obtained are not quite so good, and in the single-phase machine an equally good inherent regulation can only be obtained at greater cost of iron and copper in the field-magnet. Inductor alternators with poles of similar sign in each crown do not show such good regulation especially on loads with low power-factor, and it is on this account that for motor work and transmission of energy they have been practically displaced by the generator with alternate poles of opposite sign. With $\cos \phi_e = 0{\cdot}8$ the highest figure that can be allowed in practice for the inherent regulation may be set at 15 to 20 per cent.

§ 24. **Constant-current and constant-power alternators.**—Only in certain special cases is a high inherent regulation actually desirable, and of these two instances may be mentioned. The first is the so-called *constant-current alternator*. Historically, some of the earliest alternators such as those used for series lighting by Jablochkoff candles belonged to this class. When the alternator is applied to series arc-lighting, a large amount of armature reaction is sought for, just as in the series-wound continuous-current dynamo, and the machine is worked on the drooping portion of its external characteristic in order to obtain a nearly constant current with widely varying values of the voltage. Even when entirely short-circuited, the current then hardly exceeds the normal value required by the arc lamps, so that some of their number may be cut out without affecting the working of the remainder.

The second case is that of the alternator when used for electric smelting or electro-metallurgical work for which only the heating and not the chemical properties of the current are required, as for instance in the production of calcium carbide. For such use an approximately constant power is needed in order to maintain the temperature constant; as fusion proceeds in the electric furnace, the resistance of the circuit rapidly falls; a greatly increased current with violent fluctuations would result, but is held in check by the drop in volts which proceeds *pari passu* with the increase of the current, so that the alternator delivers an approximately constant watt output.

CHAPTER XXIV

CONSTRUCTION OF ALTERNATOR ARMATURES AND FIELD-MAGNETS

§ 1. **The iron armature core.**—The construction of the laminated iron core of the ring or drum alternator armature is similar to that of the analogous continuous-current dynamo. The chief difference is that the radial depth of iron is given more lavish proportions in the alternator, in order to reduce the flux-density therein and so to keep within reasonable limits the loss from hysteresis, which would otherwise be high owing to the greater frequency. Usual values for the induction in the armature iron below the teeth, or B_a are from 8000–10,000 lines per sq. cm. for 25 cycles to 6000 for 40 cycles, 5000 for 50 cycles and 4000 for high frequencies of 100 or more. In the homopolar drum alternator, owing to the fact that the direction of the field is never reversed, the hysteresis loss is lower (Chap. XII § 11) and the flux-density may be increased to $B_a = 12,000$ for 40 cycles and 10,000 for 60 cycles. In all cases the discs require to be very tightly compressed by cross bolts and securely held, in order to prevent the humming noise due to the rapid variations of the magnetic field from becoming disagreeably loud. Ventilating ducts are arranged every few inches along the length of the core, and it is even more important in the alternator than in the continuous-current dynamo that the separation of the laminæ should be effective so as to minimise eddy-currents.

§ 2. **Drum-winding.**—In the *drum armature*, the two classes of wire- and bar-winding,—each with its subdivisions according as it is hand- or former-wound,—may again be distinguished, but are not so sharply divided from each other as in the continuous-current dynamo.

If the armature be smooth, rectangular copper strip may be wound in the lathe round a former giving the required shape to the inside turn; with an armature of considerable diameter and a number of poles, the former-wound coils are nearly flat and need to be but slightly curved to fit the rounded surface of the core. When completed, the central shaper is removed, and the coil is fastened with the edges of the strip facing the poles to the inside of the stationary or the outside of the rotating armature. In the latter case the rounded ends of the coils are bent down over the ends of the iron core; the central openings are filled in with wooden driving-pieces screwed to the core, and the whole is in addition well served with bands of binding wire. Such a method of construction was formerly employed even for peripheral speeds up to 6500 ft. per min., but in order to better withstand the centrifugal force, it became more usual to adopt a toothed armature with open slots having straight sides, into which the coils could be pressed and afterwards drawn tight and fastened by wooden keys entering into notches on the sides, as in fig. 336 iv. The separate armature coils could then still be wound on a former and be heavily taped prior to their being assembled on the core.

The rotating drum, whether toothed or smooth, has, however, now been practically discarded except for small machines in favour of the stationary armature and rotating field-magnet. The latter can be given greater mechanical strength to withstand centrifugal force, and greater moment of inertia to secure a uniform angular velocity, while even if the exciting coil or coils revolve with the field-magnet, the rubbing contacts of the collecting rings have only to transmit an exciting current of which the voltage can be made low. Further, the coils of the high-tension circuit of the armature are relieved from any strain on their coverings due to rotation. Thus with the exception of single-phase disc alternators for high frequency, all modern machines for outputs above, say, 100–150 kilowatts have stationary ironclad drum armatures.

For high voltages and moderate outputs, round wire is usually employed, and with open slots the advantages of former-wound coils in that they are easy to wind, are more regular and stiffer, can be thoroughly insulated before being placed in the slot, and are quickly replaced, are all obtained. So far as loss of space by insulation is concerned, a single large slot per pole and per phase is the least wasteful, and with the open slot the pole-faces must in any case be laminated. Rectangular or oval-shaped tubes of micanite with a long scarfed opening down one side have lately been introduced, which enable the straight sides of the coils to be inserted into their insulating envelope before they are placed within the slots. If the winding is embedded in closed tunnels stamped out round the edge of the armature discs (fig. 336 i.), the turns must be threaded through by hand,—an expensive process and somewhat wasteful of space since plenty of room must then be allowed to the winder; the holes or tunnels are first completely lined with seamless tubes of micanite or paper projecting well out beyond the ends of the armature core, and the wires must be well insulated but need only to be just sufficiently tight in the holes to prevent chafing, since, as explained in Chap. XIII § 21, the greater part of the mechanical stress is taken up by the iron core. The closed slot avoids the necessity for lamination of the pole-shoes and slightly reduces the reluctance of the air-gap, but it has the objection that, however thin the iron bridge across the mouth of the slot, it considerably increases the inductance of the coil and its secondary leakage. This objection is largely obviated by employing a half-closed slot such as fig. 336 ii. and iii.; or the narrow iron bridges left in the stamping may be sawn through subsequently, and one or other of these methods is to be preferred in almost every case to the completely closed slot. The

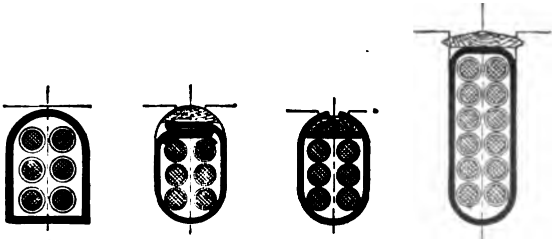

i. ii. iii. iv.

FIG. 336.

half-closed slot is easier to wind than the tunnel, but still involves more labour than the open slot with former-wound coils, so that its advantage or otherwise as compared with the open slot largely turns upon the price of labour. Yet, as a matter of fact, it is often possible to employ former-wound coils even with half-closed slots if the opening be not very small, and with a normal length of air-gap solid poles may still be retained. If the required section of copper in the inductors becomes too great for a single round wire, a number of wires in parallel must be adopted, or a stranded cable. Rectangular wire or band copper can also be wound on shapers and used in connection with open slots. With armatures of narrow width and with well-exposed ends to the coils, the current-density in the armature wires is usually about 2000 to 2500 ampères per square inch.[1]

For low voltages or very large outputs at high voltages, the transition is gradually made to the simple hand-wound bar armature with solid bars of rectangular or circular section, alternately long and short, threaded through insulated tunnels and soldered to end-connectors of flat copper strip bent to a double evolute or forked shape (fig. 337). Two or more slots per pole and per phase can then be easily obtained, and the wave of E.M.F. becomes more nearly sinusoidal.

§ 3. **Number and shape of slots.**—The number of slots per pole and per phase varies; in a three-phase machine with a single slot per pole and per phase, the curve of E.M.F. differs considerably from a sine form and shows a marked triple harmonic. To avoid this, two or three slots per pole and per phase are to be preferred, but beyond this latter number the loss of space in insulation becomes too pronounced, at least with high tensions of, say, 10,000 volts. With a given number of inductors, the effect of the number of slots on the secondary leakage is dependent upon a variety of complex conditions, chief among which is the width of the opening and the ratio of the width of the slot to its depth. Various shapes are in use, but in general they approximate

---

[1] For measurement of the temperature of the stationary armature coils in an alternator when running, an alcohol thermometer is to be preferred if there is much stray field, as a mercury thermometer may give false readings if eddy currents are set up in it.

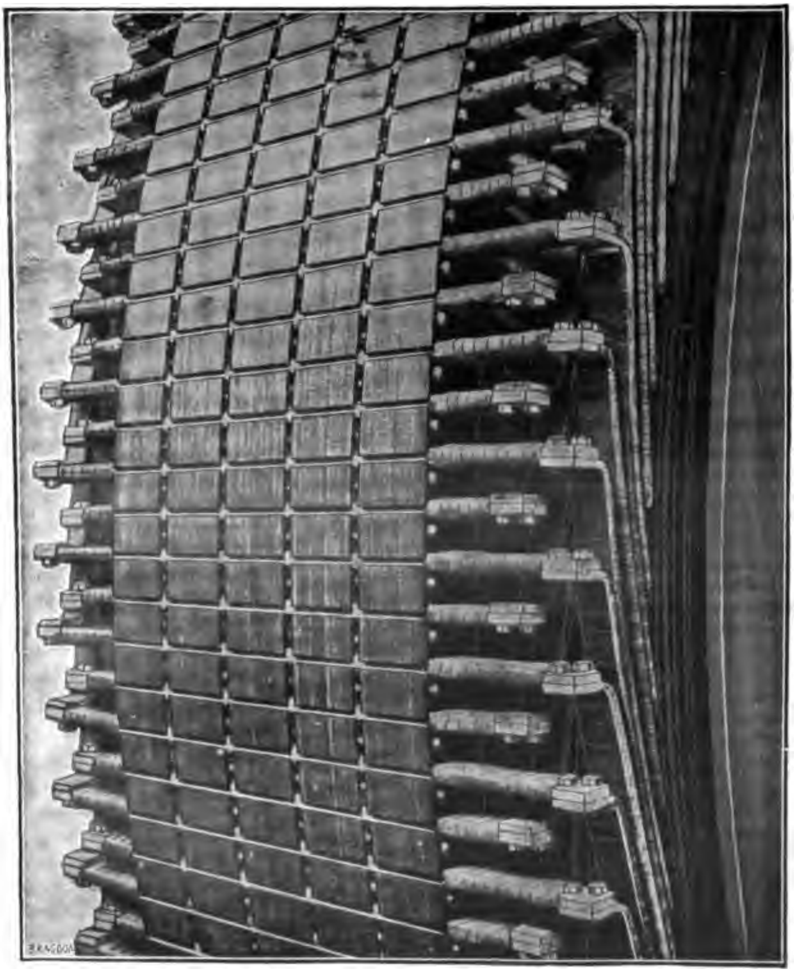


FIG. 337.—Portion of bar-wound rotating armature of Westinghouse alternator.

[*To face p.* 814.

to rectangles with a depth three times their width, and in many cases the corners are rounded off (fig. 336).

The apparent density in the teeth may be taken nearly as high as in continuous-current machines, *i.e.*, from 16,000 to 20,000. A fairly high density has the advantage that it lessens the depth of the slot, and reduces the weight of iron and with it the total loss by hysteresis and eddy-currents, although the loss per unit volume of the iron is increased.[1]

§ 4. **The air-gap.**—A small air-gap is advantageous on the score of economy in copper on the magnet, but from the mechanical point of view it necessitates very accurate workmanship and alignment of the bearings in order to avoid unequal magnetic pull. The mechanical advantages of a large air-gap are further reinforced by the electrical consideration that it may render lamination of the pole-shoes unnecessary. Hence l_g seldom is less than 0·25″, except in quite small machines. Means for equalising the air-gap in case of wear of the bearings are usually provided. The densities employed in the air-gap are generally comparable with those of continuous-current dynamos, and vary from 5500 to 9000; and like the induction in the armature core, they are higher in the homopolar than in the heteropolar type.

With the large number of poles usual in alternators, the expression of Chap. XIV § 15 viz., $\frac{B_g^2 A \cdot P}{11,183,000} \times \frac{d}{c \cdot l_g}$ may be used to calculate approximately the magnetic pull due to a displacement d of the magnet wheel from its true position concentric with the armature. If $\beta =$ the ratio of pole-width to pole-pitch, $AP = \pi DL\beta$, and if $\beta = 1$ as in an inductor generator the formula reduces to the expression for two nearly co-axial cylinders as given by Mr. Behrend (*American Inst. Electr. Eng.*, vol xvii. p. 606) save for the introduction in the denominator of the factor c. From the assumptions made in its construction, the formula probably errs in giving too small a value for the pull, so that, to be on the safe side, c might be neglected.

§ 5. **The external stationary armature.**—Evidently an

---

[1] Rothert, *L'Éclairage Électrique*, vol. xxix. p. 307 ff, which in addition to a complete analysis of the alternators of the Paris Exhibition of 1900 contains much useful information bearing on the design of alternators.

external stationary armature ring although free from centrifugal strains yet requires to be of great mechanical strength to resist collapse. If the cast-iron case be round when bored out in a horizontal position, it naturally suffers a certain deformation when raised to a vertical position and supported by brackets on either side; and this deformation then becomes further reinforced by very powerful stresses due to any unbalanced magnetic pull when its internal surface is not perfectly concentric with the magnet wheel, or by reason of any inequality in the strength of the poles. Up to diameters of 30 ft. the requirements of mechanical strength may be met by the employment of a deep cast-iron casing of inverted U-shape; round the inside of this the segmental laminations, assembled so as to break joint (cp. figs. 142 and 366), are in the process of building threaded over numerous transverse bolts and are clamped tightly together by plates engaging in a turned rim on the armature case (cp. fig. 406). The back or sides of the armature casing are usually pierced with a number of large openings to allow of the exit of the air driven through the ventilating ducts of the core. Fig. 338 of a 106 H.P. alternator built by Brown, Boveri & Co. for the St. Petersburg Polytechnic, and giving 230 volts at 50 cycles and 215 revs. per min., illustrates the standard type adopted by that firm for sizes up to about 150 H.P., while fig. 339 shows their standard type for larger sizes up to about 500 H.P. In order to render the inside of an armature with its numerous coils readily accessible for cleaning, it may in small sizes be arranged to slide horizontally along an extension of the baseplate by means of rackwork or screws worked by hand (fig. 340).

With very large diameters the prevention of the mechanical deformation of the armature ring becomes more difficult, and several supporting feet (fig. 341)[1] may be required. Fig. 342 shows the lower half of a 3000 kilowatt alternator built by the

---

[1] From the brochure, 'Der Aufbau und die planmässige Herstellung der Drehstrom-dynamomachinen' by Herr O. Lasche of the Allgemeine Elektricitäts Gesellschaft, to whom we are indebted for permission to reproduce many of the accompanying figures and data illustrative of the construction of alternators. An English translation appeared in *Engineering*, vol. lxxii, 1901, p. 173 ff, to which the reader is especially referred.

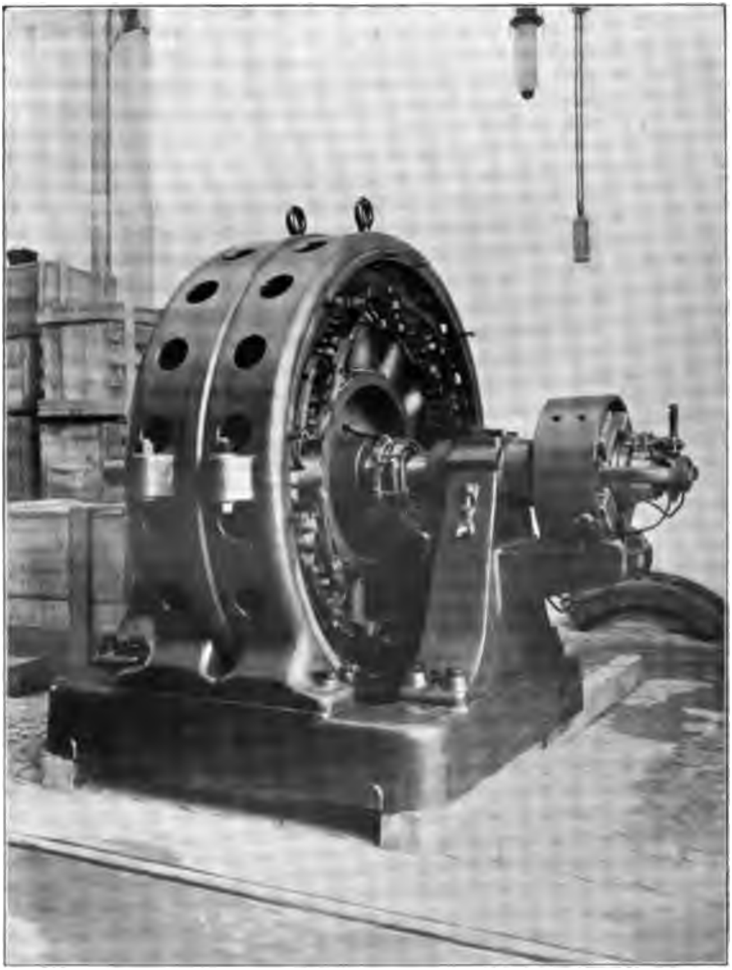


FIG. 338.—Alternator of Brown, Boveri & Company.

[*To face p.* 816.

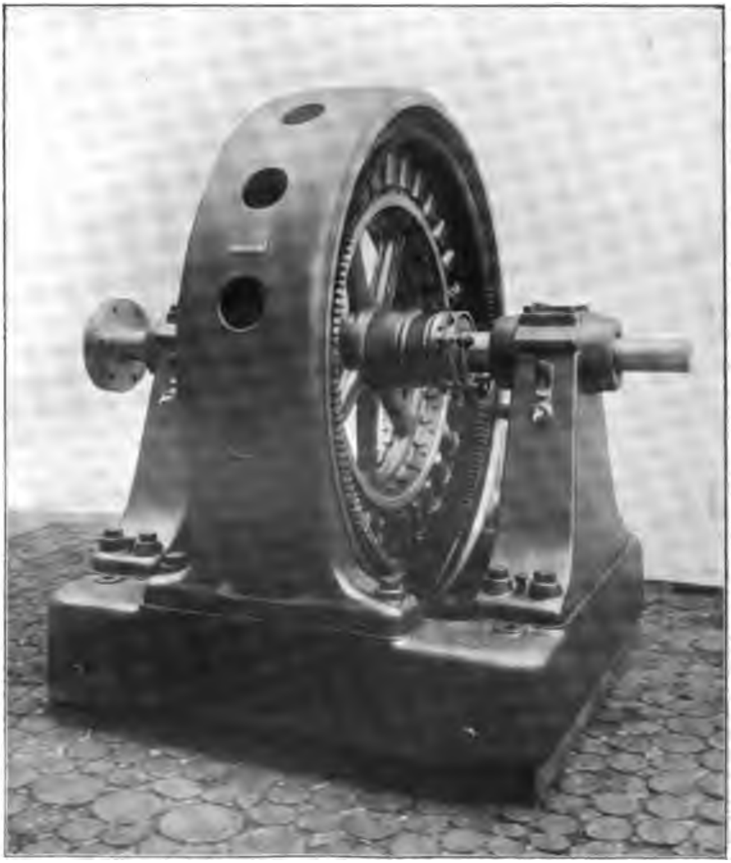


FIG. 339.—Alternator of Brown, Boveri & Company.

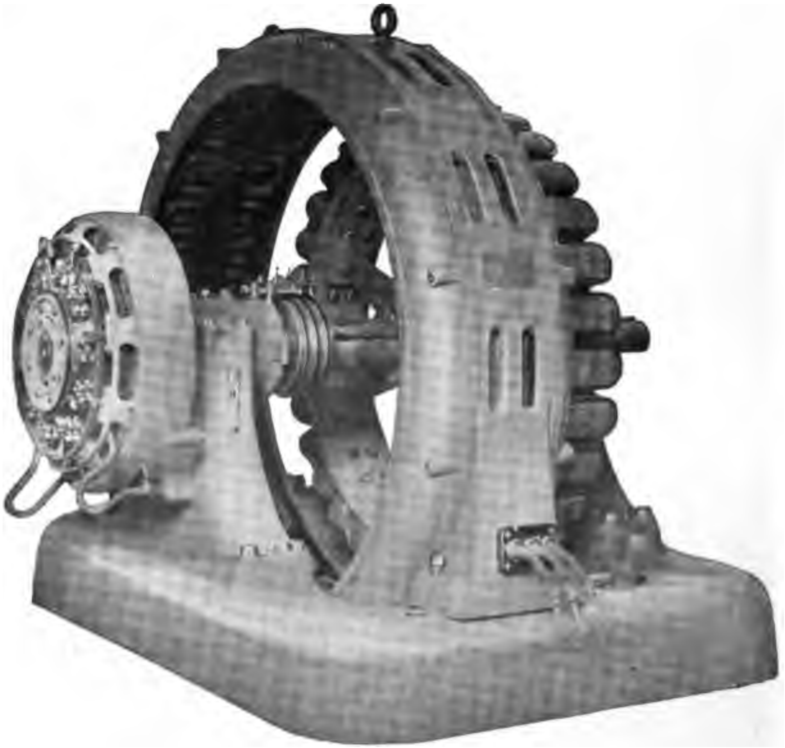


FIG. 340.—120 K.W. generator of British Thomson-Houston Company with direct-connected exciter, showing method of sliding armature for field or armature inspection.

[*To face p.* 816.

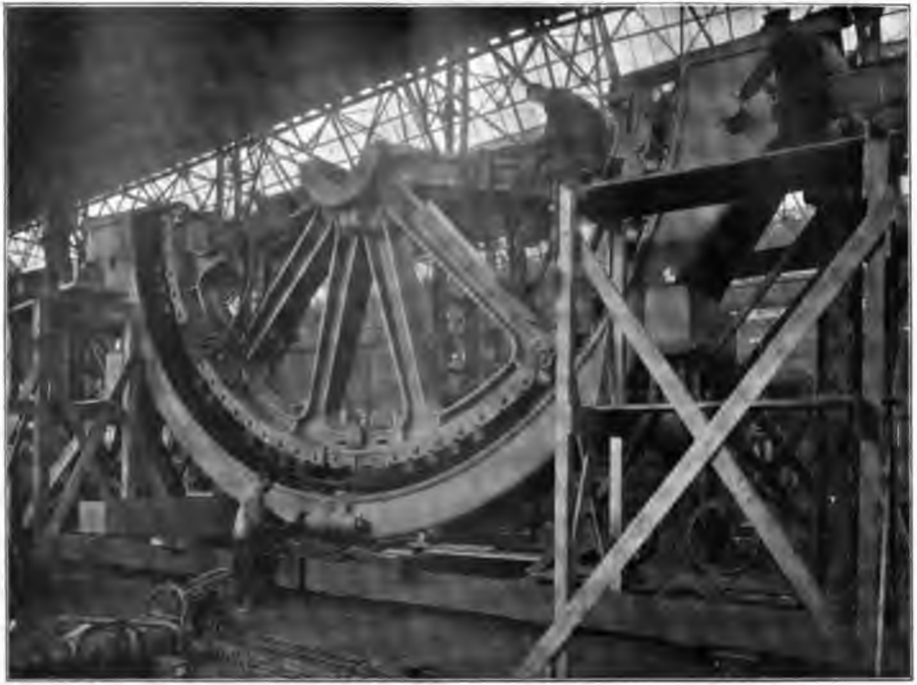


FIG. 342.—Lower half of 3000 K W. alternator of Allgemeine Elektricitäts Gesellschaft.

[*To face p.* 816.

Allgemeine Elektricitäts Gesellschaft of Berlin, and is more fully described in Chap. XXVI; it gives an idea of the size to which the armature casing may attain, and it will be seen that there are three pairs of feet, the chief part of the weight being taken by the middle pair halfway up the lower semicircle while the upper pair are given considerable proportions to satisfy the eye in the matter of appearance. In some cases greater stiffness is imparted by radial tie-rods terminating in a ring, concentric with but free from the shaft (fig. 343). A convenient and advantageous method is that devised by Mr. C. E. L. Brown and employed by Brown, Boveri & Co.; the armature case is mounted on a pair of star frames, one on either side, with massive arms which terminate in rings embracing trunnion journals on the plummer blocks that carry the revolving magnet (fig. 344). The armature is thus maintained in accurate concentricity with the magnet wheel, without increase of its external diameter or of the dimensions of the foundations, while the further advantage is gained that when the bolts which fix the armature to the concrete foundation and prevent it from turning are withdrawn, the whole armature can be barred round on the trunnion bearings so as to bring any coil into a convenient position for inspection or cleaning.

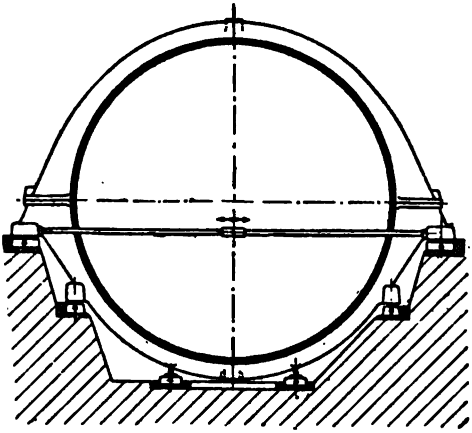


FIG. 341.

Fig. 345 shows the large armature wheel of one of the Frankfort single-phase 1500 H.P. alternators during construction; the core is divided into similar segments which are seen in process of being fixed between the two side star frames. The outside diameter is 26 ft., and the weight of the complete armature is 53 tons, that of the whole machine which runs at 86 revs. per min. being 110 tons. A similar method of stiffening the armature is employed in the 15,000 volt three-phase alternators of the Paderno-Milan transmission, and also in the 3000-volt 650-H.P. three-phase generator of fig. 346, which shows clearly the side frame and holding-bolts. The latter machine is one of two directly-coupled to blast-furnace gas-engines at the Düdelingen Iron Works, and it is of interest to mention that no difficulty has been experienced in running them in parallel.

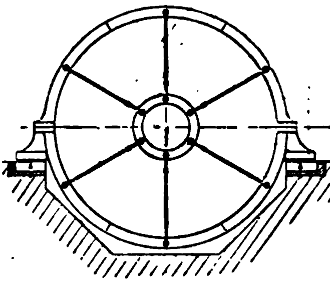


FIG. 343.

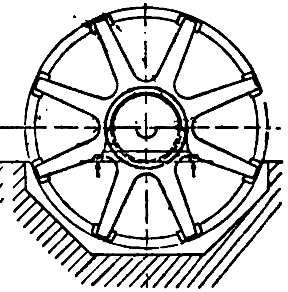


FIG. 344.

Fig. 405 shows the method adopted by the Westinghouse Co. for stiffening the large armature casing of the 5000 kilowatt generators supplied to the Manhattan Railway Co.; as will be more fully described in Chap. XXVI, the outside diameter is about 42 ft., and the weight of the stationary part of the machine is 234 tons, although the laminated iron core itself only weighs some 40 tons.

The mechanical problem has been met in a drastic and novel manner in the type of alternator recently brought out by the Allgemeine Elektricitäts Gesellschaft. The cast-iron casing is here entirely abandoned, a triangulated system of tie-rods fastened to the side-cheeks of the armature (fig. 347) being used in its place. Fig. 348 shows such an armature as built up in the

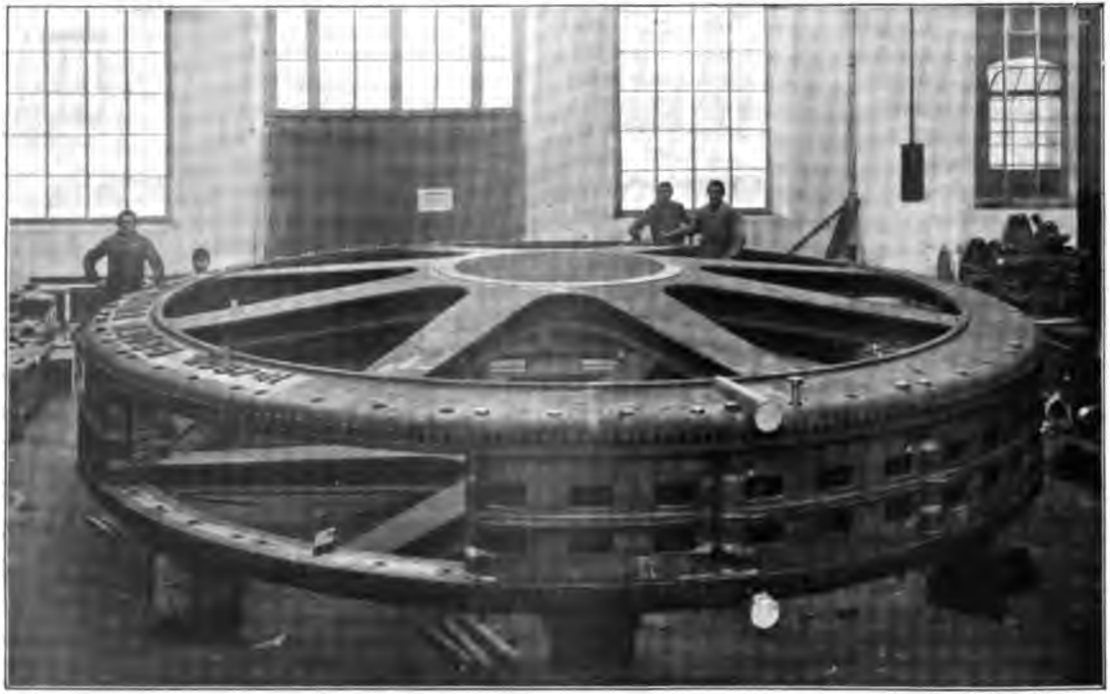


FIG. 345.—Armature wheel of 1500 H.P. single-phase alternator (Brown, Boveri & Company).

[*To face p.* 818.

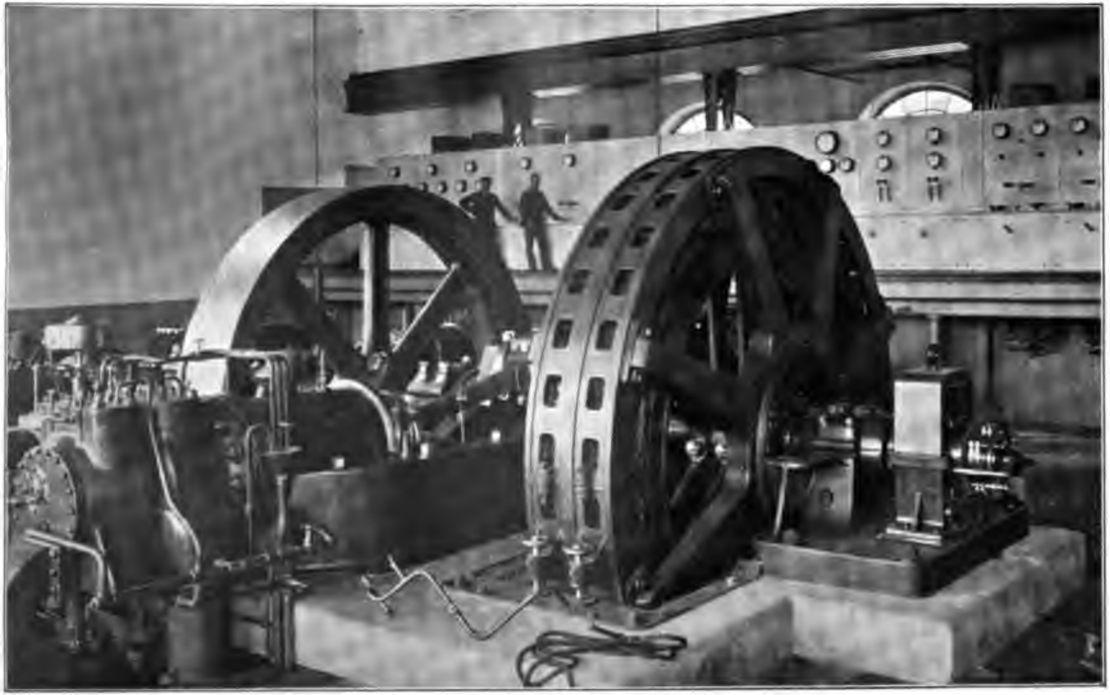


FIG. 346.—650 H.P. three-phase alternator (Brown, Boveri & Company).

[*To face p.* 818.

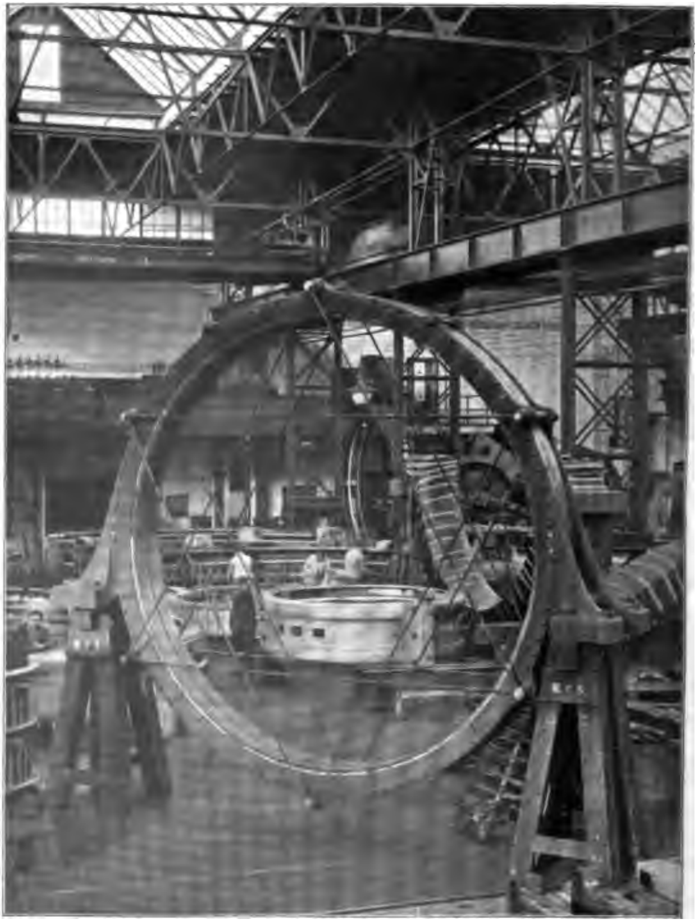


FIG. 348.—Stiffened armature of the Allgemeine Elektricitäts Gesellschaft.

[To face p. 819.

factory, while in the three-phase 1200-kilowatt alternator of fig. 349 is seen an earlier form in which the tie-rods are arranged on

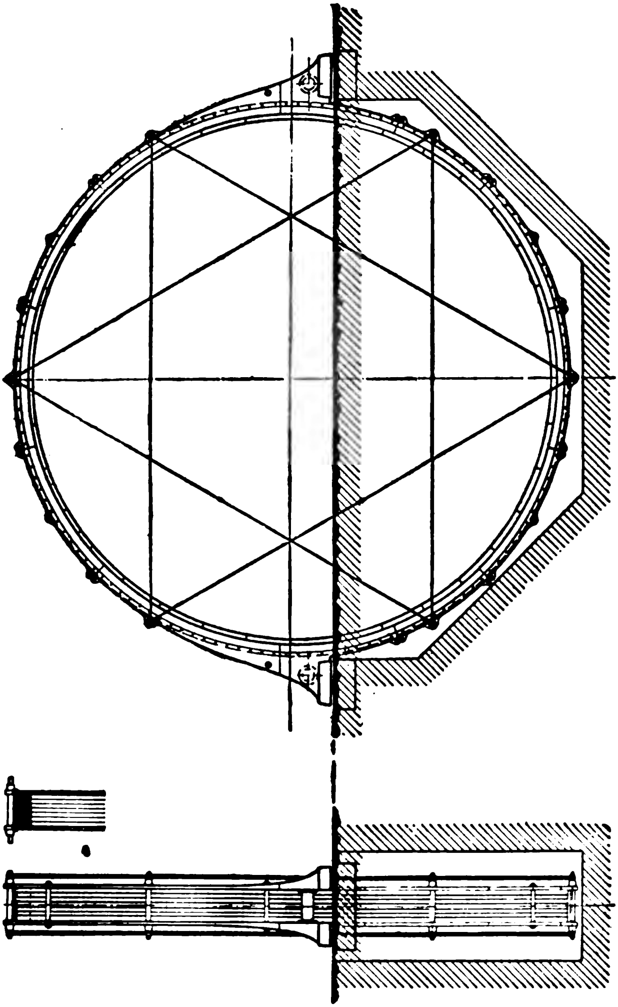


FIG. 347.—Stiffened armature of the Allgemeine Elektricitäts Gesellschaft.

the back of the armature. The great advantage that is gained by the method is that the stresses on the ring are brought under complete control, and that in bringing the rim to a true circular shape, no undue demands are made on the mechanical skill of the erector; the rods may be given ample proportions at very little cost, and the concentricity of the complete ring is easily secured and checked during erection by measuring the clearance between the inside of the laminated ring and the magnet wheel. At the same time a noteworthy saving of weight is made, since in very large machines with the usual cast-iron case as much as four-fifths of the total weight of the stationary armature is employed in giving constructional stiffness, and only the remaining fifth is magnetically active. The necessary depth and width of the foundations are also reduced, to which must further be added the incidental advantages that the armature core is much better exposed to the air, and has in consequence greater cooling power.

§ 6. **Discoidal ring winding.**—In the *discoidal ring alternator* the armature is usually the rotating portion. The grip of the turns on the elongated section of the core is good, and indeed they serve more or less as binding-wires, so that the armature may be run at a high peripheral speed without danger from the effects of centrifugal force. Speeds as high as 8500 feet per minute are found in Kapp alternators of this type. Since the turns are usually numerous, the length of wire required to complete one coil is considerable; hence in the process of manufacture, it is convenient to wind it on to two shuttles, and the coil is then begun at its centre. The middle part of the wire is laid on the core and thence the winding proceeds to one end of the first layer and back to the centre, the shuttle being passed backwards and forwards through the interior of the ring. This is repeated until one half of the coil is wound from each shuttle. Both ends of the coil are thus brought to the outside layer, at its centre or at the ends, according as the number of layers is even or odd. The cooling effect of the rotating armature is good, and the output is limited not so much by the heating as by the armature reaction. Yet the rise of temperature of the armature wires must be kept small, in order to avoid any danger from their becoming slack on the radial sides of the core when they expand under heating.

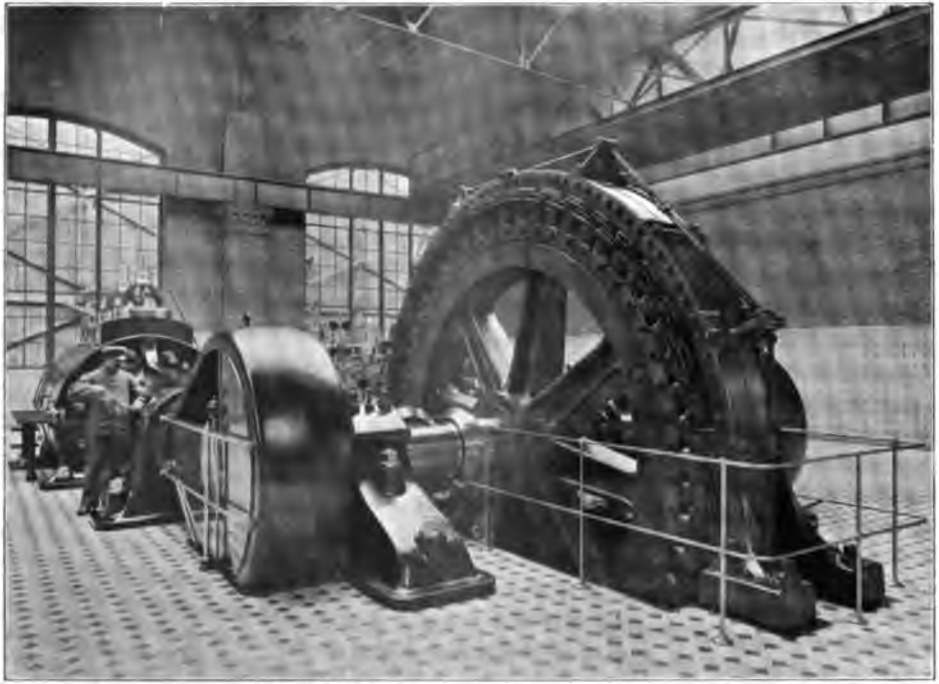


FIG. 349.—1200 K W. three-phase alternator, 107 revs. per min. (Allgemeine Elektricitäts Gesellschaft.)

[*To face p.* 820.

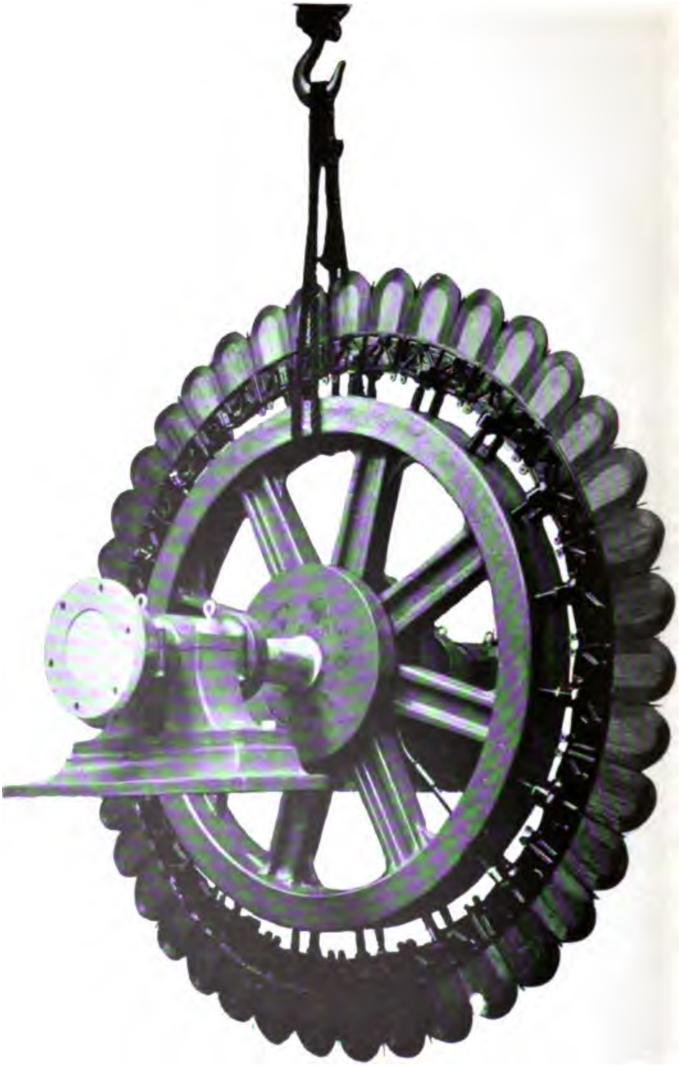


FIG. 350.—Flywheel armature of Ferranti 500 K W. disc alternator.

§ 7. **The disc armature.**—From its mechanical construction the armature is best adapted to be the revolving element of the disc alternator, since a high peripheral speed exceeding 5000 feet per min., which may be found in such machines, assists in keeping the built-up structure of the armature rigid in the plane of rotation. The separate bobbins are wound on cores of some substance other than iron, such as slate (Mordey-Victoria) or wood (Siemens), so that there is no loss by hysteresis in the armature. Owing to the low inductance of its coils, the disc armature without iron is not so suitable for parallel running with slow-speed engines,[1] and as its construction does not lend itself to polyphase windings, its use is practically confined to lighting work on circuits of high power-factor.

The construction of the Ferranti disc alternator is in many ways unique, and is illustrated by fig. 350, which shows the fly-wheel armature of a 500 kilowatt single-phase machine running at 270 revs. per min., and giving 2100 volts with a periodicity of 90. The coils, of which there are as many as there are pairs of opposing poles, are wound on laminated brass cores, the direction of lamination being parallel, and not, as in other types, at right angles, to the inductors. To form the core, a number of pieces of brass strip, each with a central corrugation running up it, are taken and separated from one another by slips of asbestos of tapering thickness: the brass and asbestos layers are put into an hydraulic press, and when under great pressure a brass eye-piece is burnt on at one end, the molten metal being run on to and fusing with the corrugated strip. Thus the core is split into a large number of comb-like teeth, and is egg-shaped in outline, owing to the asbestos strips gradually becoming thicker towards the outer ends of the teeth. When placed in position on the armature, with its small end towards the shaft, the radial teeth of the core have E.M.F.'s induced in them up or down their length; but owing to their being insulated from one another save at their inner ends, no circuit is formed by which a current may pass from one tooth into the next and back again; eddy-currents cannot, therefore, flow save within the limits of the several teeth, and these being narrow, the loss of power and the heating from eddies become unimportant.

---

[1] Eborall, *Journal Soc. Arts*, vol. xlix. p. 751.

On the laminated core is wound a narrow bare copper strip, a central corrugation being impressed upon it in the process of winding, to prevent one layer from slipping over another; in fact, the whole bobbin is locked sideways by the corrugation, so that it cannot become distorted out of its flat shape. Each layer of copper is separated from the next by a thin band of vulcanised fibre, which is wound on at the same time. The inside end of the copper strip is brazed on to the solid end of the brass core. The completed bobbins are mounted in pairs between two gun-metal plate castings, riveted together to form a holder, and are secured in place by means of screws which pass through the eye-holes cast in the solid ends of each core: thus the holder serves to connect the two inside ends of a pair of coils. The holder is itself lined with fibre, and between the two coils in the same holder, where the maximum difference of potential due to two coils occurs, is interposed an insulating strip of ebonite. The outer ends of two coils where they approach one another in adjacent holders, are connected together by bolted lugs (cp. fig. 79 and p. 168). In the rim of the flywheel are a number of cored holes of elliptical section, each of which contains an elliptical cast-iron nut tapped to receive a steel bolt which thus protrudes from the periphery of the wheel. Between the nut and the cavity in the driving ring in which it is placed there is a clearance space all round of about ¾", and into this is run a composition of melted sulphur, which on hardening completely insulates the nut and its steel bolt from the rim, while at the same time holding it mechanically firm. By means of lugs cast on it, the bobbin-holder is threaded over and screwed to a pair of the insulated bolts. The removal of a coil for examination or repair is then simply effected by detaching a bobbin-holder from its supports. In a large machine of the same type supplied by Messrs. Ferranti, Ltd. to the Deptford electric light station, and giving 10,000 volts and 150 ampères at a speed of 156 revs. per min., the external diameter of the armature is 22 feet, which gives the peripheral speed of 10,800 feet per min., the weight of the whole flywheel armature being 35 tons. The diameter of the flywheel rim is 16 feet, and the cast-iron wheel is made in two pieces bound together by steel rings shrunk on; it is further strengthened by having large tie-bolts, each 6" in diameter

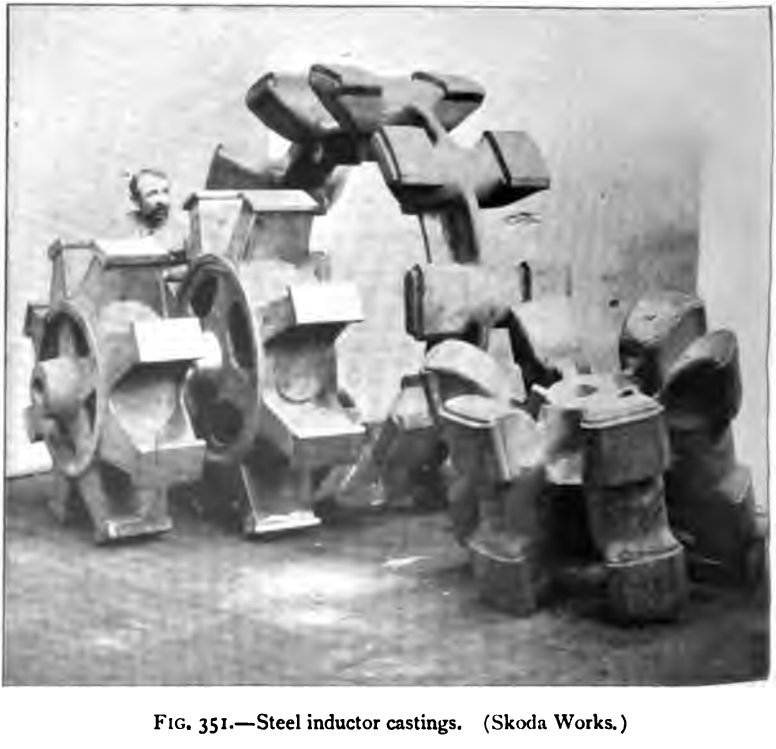


FIG. 351.—Steel inductor castings. (Skoda Works.)

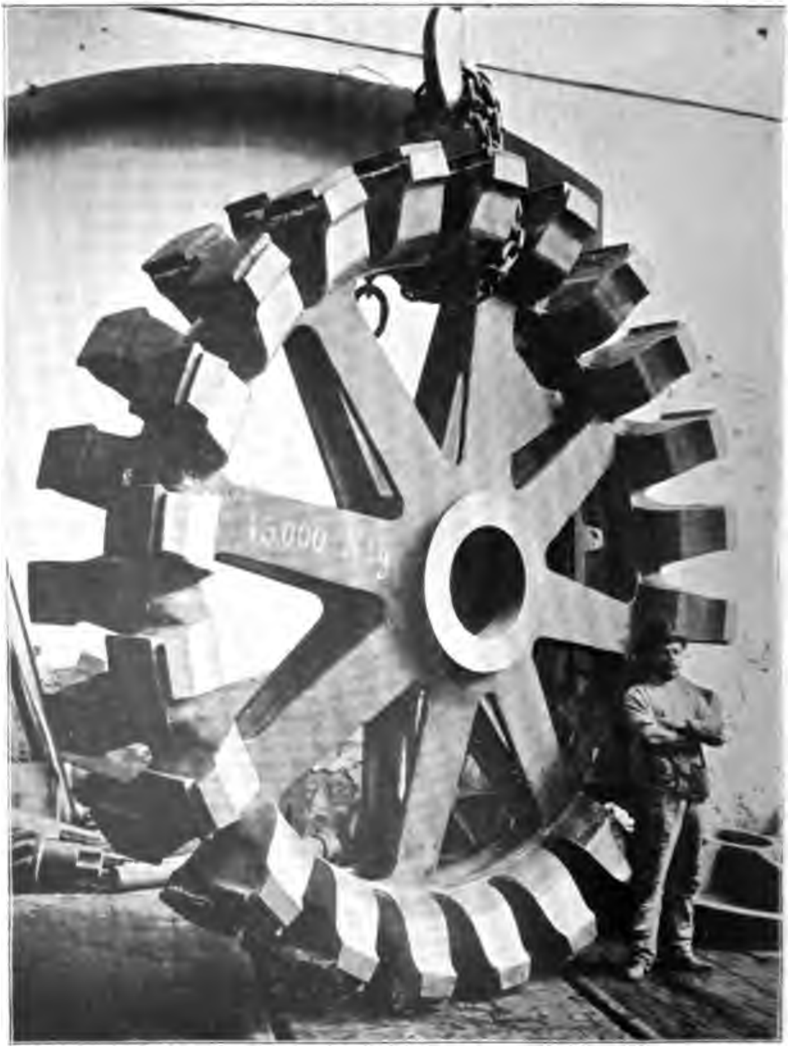


FIG 352. Cast-steel inductor wheel, 11½ feet diameter. (Skoda Works.)

[*To face p.* 823.

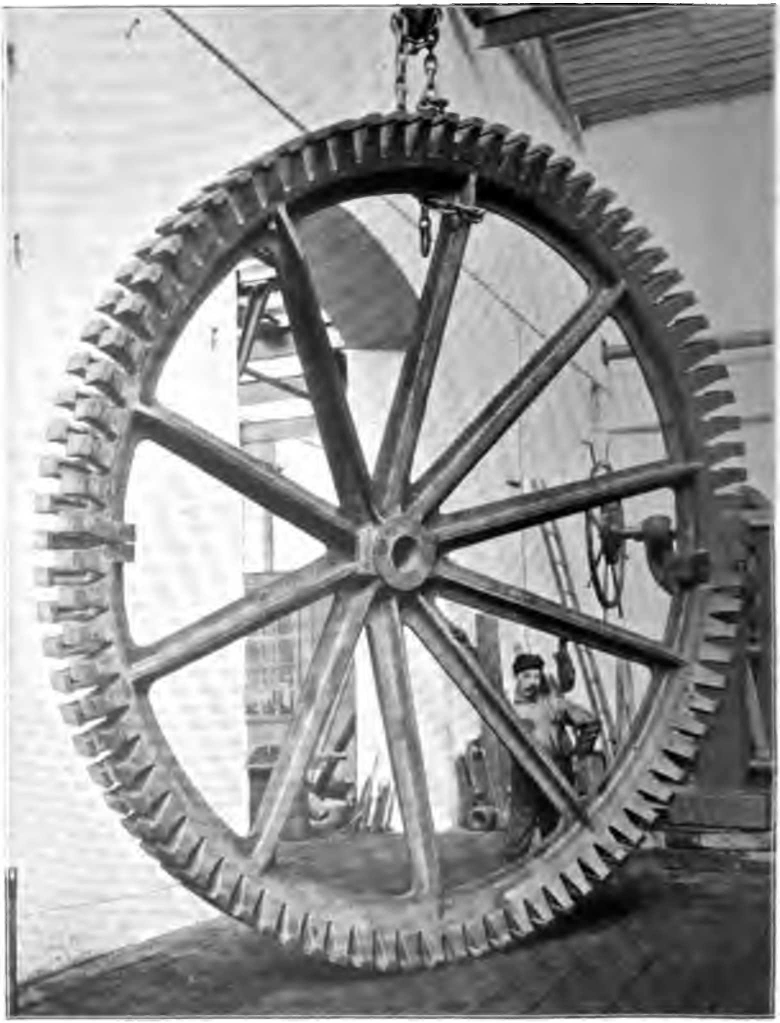


FIG. 353.—Cast-steel wheel of inductor generator for Olten-Aarburg electricity works.

[*To face p.* 823.

passing through the rim, screwed into a steel block shrunk on to the shaft.[1]

§ 8. **The magnet system of inductor generators.**—The general nature of the magnetic systems of alternators of various types have been diagrammatically illustrated in Chaps. VIII and IX. They may be divided into the two classes, viz., those of heteropolar alternators in which poles of alternate sign occur in succession in one ring round the armature, and those of inductor or homopolar generators in which in any one ring drawn round the armature there are only poles of one sign, those of the opposite sign forming a second ring spacially displaced.

The latter class is illustrated by fig. 351,[2] which shows a number of inductor wheels of different forms in their unmachined state as cast in steel by the Skoda Works. In the foreground is seen the inductor of a double-armature alternator corresponding to that shown diagrammatically in fig. 85, and in the back ground is a larger ring of the same kind. In figs. 352 and 353 the poles are staggered as in fig. 86, but the finished machines are designed to give two phases. Owing to the unfavourable nature of the strains on the bolts carrying the overhung inductors of fig. 87, the method of construction there shown has now been abandoned.

The use of the polyphase generator for transmitting energy obtained from waterfalls has led to the development of alternators with a vertical axis of rotation specially suitable for direct coupling to the vertical shaft of slow-speed turbines on low falls, and in the design of such 'umbrella' types Mr. C. E. L. Brown was the pioneer from 1891 onwards. The six two-phase generators, each of 300 H.P., installed by Brown, Boveri & Co. in the Olten-Aarburg central station are directly coupled to the vertical shafts of their turbines and are probably unique from the extremely slow speed at which they run, viz., 28·5 revs. per min. Fig. 353 shows the inductor wheel of one of these generators; its diameter is 15 feet, its weight nearly 8 tons, and the 84 poles forming one crown give a periodicity of 40. Fig. 354 shows in section half of a double-armature three-phase inductor generator with vertical shaft built by the Allgemeine Elektricitäts Gesellschaft for the large water-power installation at Rheinfelden; the five-armed star has bolted to it a ring carrying 55 pairs of laminated polar projections arranged in two tiers, the outside diameter of the inductors being 18 ft. 9 in. The stationary exciting coil of T section has a mean diameter of nearly 20 ft., wound with 270 turns of 0·315″ wire, and is placed within the two halves into which the armature casing is divided on the horizontal plane; the section of iron in the magnetic circuit is shown black. The single air-gap is only 5 mm. = 0·197.″ The armature is wound with 110 coils of 8 turns each, and gives 6800 volts between the leads and 61 ampères per phase, *i.e.*, 575 K W with cos ϕ = 0·8, at 55 revs. per min. and a periodicity of 50.

---

[1] *Engineer*, April 21, 1899.

[2] Reproduced together with figs. 352 and 353 by permission of the Skoda Works of Pilsen, Bohemia.

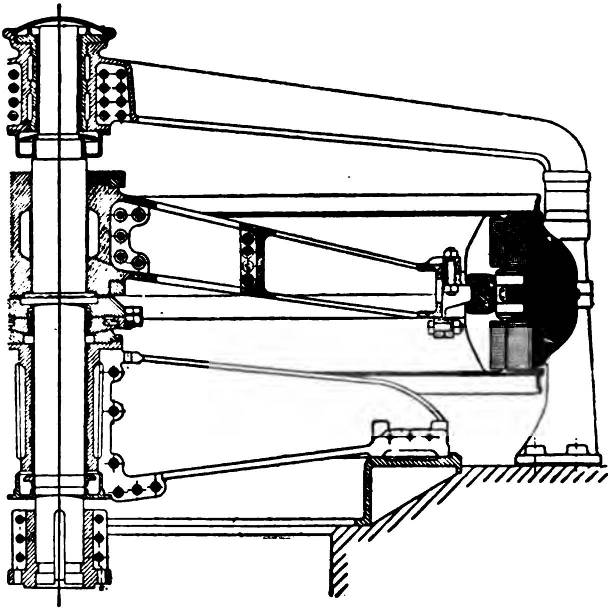


FIG. 354.—Inductor generator of Allgemeine Elektricitäts Gesellschaft at Rheinfelden.

The inductor generator has certain defects inherent to the type, chief among which is the flux of lines which must necessarily pass into the armature core between each pair of adjacent poles of the same sign. Although not to be called leakage in the proper sense of the word, yet any such flux is doubly disadvantageous. Not only does it assist in saturating the field-magnet and so call for more ampère-turns of excitation, but it directly causes a back E.M.F. in the armature, so that, as explained in Chap. IX § 4, the useful flux is the difference between that which enters the armature under a pole (z_a') and that which enters within the pole-pitch between a pair of poles (z_a). If the flux under the pole-face and between the poles be measured separately and the latter be deducted from the former, $z_a' - z_a$ as so determined agrees closely with the value of the useful flux as deduced from the open-circuit characteristic curve, and in a machine so tested the proportion of the harmful flux z_a to the useful flux ($z_a' - z_a$) averages about 15 per cent., but varies with the ratio of the air-gap to the width of the pole, and, further, is dependent upon the absolute value of the pole-pitch.[1] Thus if an increase in frequency is required with the same speed, and it is not permissible to increase the diameter of the armature so as to increase the pole-pitch proportionately, the absolute value of the latter is reduced, and the machine for the higher frequency gives an open-circuit characteristic appreciably below that for the lower frequency. This effect is illustrated by the two curves of fig. 355 taken from the above-quoted paper; the higher is the open-circuit curve of a three-phase inductor generator at 500 revs per min. and 50 periods per second, the number of polar projections being six on each side of the exciting coil, while the lower curve shows the voltage of the same armature at 250 revs. per min., and with 12 poles on each side so as to give the same periodicity at the lower speed. In each case the width of the poles is equal to the pitch, and the latter is 10·85 and 5·42 inches in the two cases.

Next, in the inductor generator there is considerable leakage from the flanks of the inductor and from the shaft and its pedestals into the armature case. If the harmful flux through the armature be added to the flux which does not enter into the armature and is therefore leakage in the strict sense of the term, and the ratio of their sum to the useful flux be compared with the leakage factor of the machine with poles of alternate sign, the latter will be found to have the advantage. Thus in an inductor machine with an air-gap of $\frac{1}{16}''$ and a pole-pitch of 9″, the leakage and harmful fluxes together amount to 20 per cent. of the useful flux, but with a pole-pitch of 5″ and an air-gap of $\frac{1}{8}''$ the ratio rises to the high figure of 45 per cent. The leakage coefficient in the case of the inductor generator giving the lower curve of fig. 355 with a pole-pitch of 5·42″ and an air-gap of 0·118″ is similarly found to be about $\nu = 1{\cdot}45$; yet in a generator with alternate poles of nearly the same pole-pitch, viz., 5·5″ and an air-gap slightly longer or 0·138″, it is only 1·13 for the same number of exciting ampère-turns. It is

---

[1] Behrend, 'The Factors which determine the design of Monophase and Polyphase Generators,' *Electr. Eng.*, vol. xxvi. p. 598.

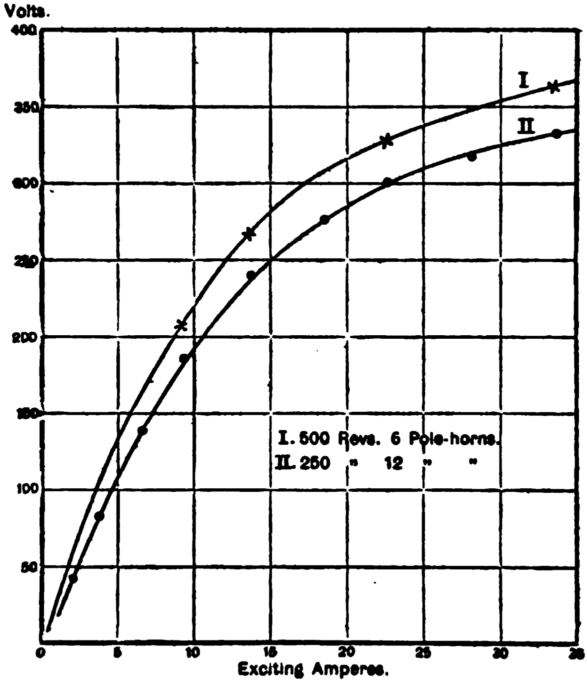


FIG. 355

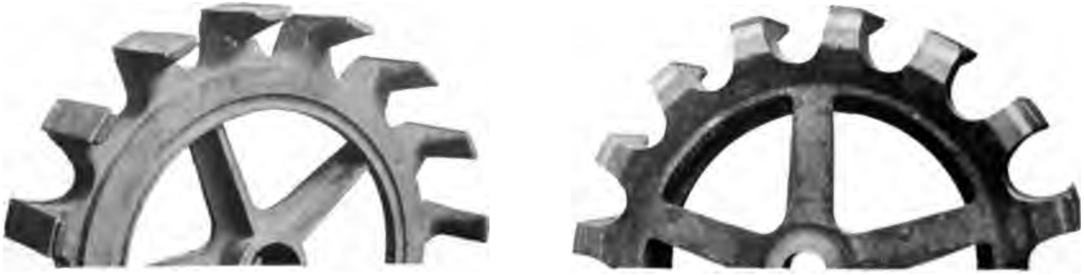


FIG. 356 --Portions of cast-steel claw magnet, showing front and back view of overarching poles.

[*To face p.* 827.

therefore very important to keep the percentage of the useless flux in the inductor generator low, and as this involves working with very small air-gaps, any eccentricity of the revolving part is liable to throw great stresses on the bearings and shaft from the unequal magnetic pull; due precautions must then be taken to meet such stresses by employing an exceptionally large diameter of shaft, and by frequent taking up of any wear in the brasses of the bearings.

Finally, the secondary leakage of lines immediately round the armature coils across the tops of the slots is proportionately greater; in the homopolar generator not only the wires which at any moment are useful inductors, but those also which at the moment are merely connectors, are embedded in iron and possess considerable leakage inductance, while in the alternate-pole machine the end-connectors are comparatively far removed from any iron. Thus although the single exciting coil is in itself an efficient means of obtaining the necessary numerous fields and the absence of any rotating copper is an attractive recommendation, yet experience has shown that the inductor type is only suitable for high speeds and low frequencies which allow of the pole-pitch being made large. Especially is it unsuitable for motor work, since it is not adapted to meet the problem of large lagging currents owing to its poor regulating properties. Yet it may again come into favour when the use of steam-turbines running at very high speeds becomes more widely extended.

§ 9. **The magnet-system of heteropolar generators.**—Turning to heteropolar alternators, the advantages of the single exciting coil may still be retained by adopting the type of field used by Mr. C. E. L. Brown in the design of the Lauffen generators, in which overarching claws of alternate sign are made to spring from a single central magnet core (cp. fig. 202). The upper halves of two side portions of such a field as cast in steel by the Skoda Works are shown by fig. 356, the diameter of one complete half being 8 ft. 9 in. In this type the primary leakage reluctance is but small owing to the necessarily large amount of the adjacent surfaces of opposite polarity, so that again it is ill adapted for inductive loads. Further, the single circular coil being concentrated at the base of the overarching poles is so situated that its entire magneto-motive force is in the best position for causing a large leakage flux. Lastly, if the coil revolves, it is subjected to strains due to the heating and cooling of the wire which are prejudicial to the life of the insulation, while if stationary all advantage of the peripheral speed in cooling it is lost. The type has therefore passed out of favour, and of late there has been a return to the simple form of revolving field with internal yoke-ring and radial poles of alternate sign, each separately magnetised (cp. figs. 77, 338 and 339). In exceptional cases where the machine is small and the number of poles is high, every other pole may be left unwound; but this arrangement is seldom advantageous, as the mean length of a turn is increased, the leakage is greater, and the two poles, the one wound and the other unwound, are not of equal strength, so that the two half-waves of a period are unsymmetrical.[1] Hence the form with half the poles unwound has now been practically abandoned.

With six or more half-closed slots per pole, *i.e.*, two per phase in a three-phase machine, solid poles may be employed if the width of opening of the slots is not more than 1·5 l_g, but with wider openings it is advisable to laminate at least the pole-shoes in order to avoid the eddy-currents arising from the variation of the flux as it sweeps over the pole-faces. Although the eddies do not penetrate far into the mass of the pole, it is frequently the practice to laminate the entire pole. This construction is employed partly from its cheapness in manufacture, since it is not easy to dovetail or fasten the laminated pole-shoes into solid poles, and partly also because the connection of the two usually involves a considerable radial depth of pole-shoe, so that the primary leakage between the pole tips is increased and the exciting coils do not approach close to the ends of the poles. An objection to the laminated pole-shoe as also to the laminated pole is that it renders parallel working more difficult; if there are periodic fluctuations in the flux or speed, the solid pole-face itself plays the part more or less of an amortisseur winding (Chap. XXV. § 16) owing to the eddy-currents set up in it. Laminated poles are bolted up to or are cast into the yoke-ring, and in some cases are dovetailed into the rim of the yoke-wheel, a method which has the advantage that the pole may then be drawn out sideways for examination or removal of the bobbin without shifting the armature or magnet as a whole. The section of the pole if laminated must be rectangular, but with solid cast steel poles, although it is seldom possible to employ the most economical section of a circle owing to the necessity of arranging a large number of poles round a wheel of moderate diameter, yet at least an approximation to the best form may be made by giving the poles an oval shape. A considerable saving of copper is thus obtained and the bobbins are themselves easier to wind. When once the secondary leakage of an armature has been reduced to the minimum that the circumstances of the design allow, the rise of volts when the full current is removed can be brought to a small value only in one way; the primary leakage must be kept as small as possible and a high saturation must be employed in the poles—as high in fact as is compatible with reasonable certainty of obtaining material of the estimated permeability. A small coefficient of leakage is given by the use of very short poles, but on the other hand this must not be pushed to such an extreme that the thickness of the copper winding unduly increases the mean length of a turn and the consequent weight of copper, while the cooling surface is reduced. The use of laminated poles enables a very high density to be employed in the magnet since the sheet-steel employed can be carefully tested in small packets and the material itself is very constant in permeability. Ingot iron or steel is more uniform in permeability than cast-steel, gives greater exactness of section and is free from the danger of blow-holes; yet with cast-steel poles if carefully tested, the average flux-density may be carried as high as $B_m = 18{,}000$ when a small rise of volts is specially required, or say B_m at the outer end of the bobbin $= 16{,}000$ and at the base $= 18{,}500$. With such high densities the yoke-ring must itself also be of cast-steel, since with cast-iron the lines would be throttled at the point of junction between pole and yoke, or the pole must be dovetailed into the ring; and in any case it is safer to allow a certain small number of ampère-turns for the reluctance of the joint. Thus the advisability or otherwise of a high degree of saturation in the magnet turns upon the conditions of the design and the nature of the service for which the alternator is intended. In the highly saturated machine the value of the armature reaction must be accurately known or very carefully estimated at the outset, and unless it be designed for a low power-factor, an overload or even the normal load with a smaller power-factor may not be able to be carried without an impossible increase in the field excitation. In generators of low frequency for the supply of rotary converters a moderate degree only of saturation is to be recommended.

---

[1] Eborall, *Journal Soc. of Arts*, vol. xlix. p. 754, and Rothert, *L'Éclairage Électrique*, vol. xxix. p. 319.

The yoke may be given considerable thickness and be worked at a lower density, since lavish proportions are here not disadvantageous in so far as they increase the flywheel effect and the steadiness of the running.

The leakage coefficient of the field-magnet with alternate poles varies with the ratio of the pole-width to the pitch, with the absolute value of the pitch as determined by the frequency, and with the length of the poles, and is also affected by the load. It averages from 1·2 to 1·4, but as in the case of continuous-current dynamos, unless it has been experimentally ascertained from machines of similar proportions and similarly saturated, even an approximate calculation of the leakage permeance, P, is to be preferred, and such calculation should be made with the greatest possible accuracy.

The use of the field-magnet as the sole fly-wheel in the case of alternators directly coupled to their steam-engines is very frequent, yet is only to be recommended when the most favourable diameter of armature and of flywheel naturally approximate to each other, and this is usually the case only with machines of large size. In other cases if the flywheel is given the dimensions which would best suit the alternator, an unduly large weight of cast-iron in the yoke-ring may be necessitated and a larger shaft to carry it, while if the diameter of the alternator be increased to suit the flywheel requirements, the armature frame must be of greater depth and size to give it sufficient rigidity.[1]

---

[1] Rothert, *L'Éclairage Électrique*, vol. xxix. p. 313. See especially the brochure above-cited. 'Der Aufbau und die planmässige Herstellung der Drehstrom-Dynamomaschinen,' where various arrangements of magnet and flywheel are shown diagrammatically and discussed.

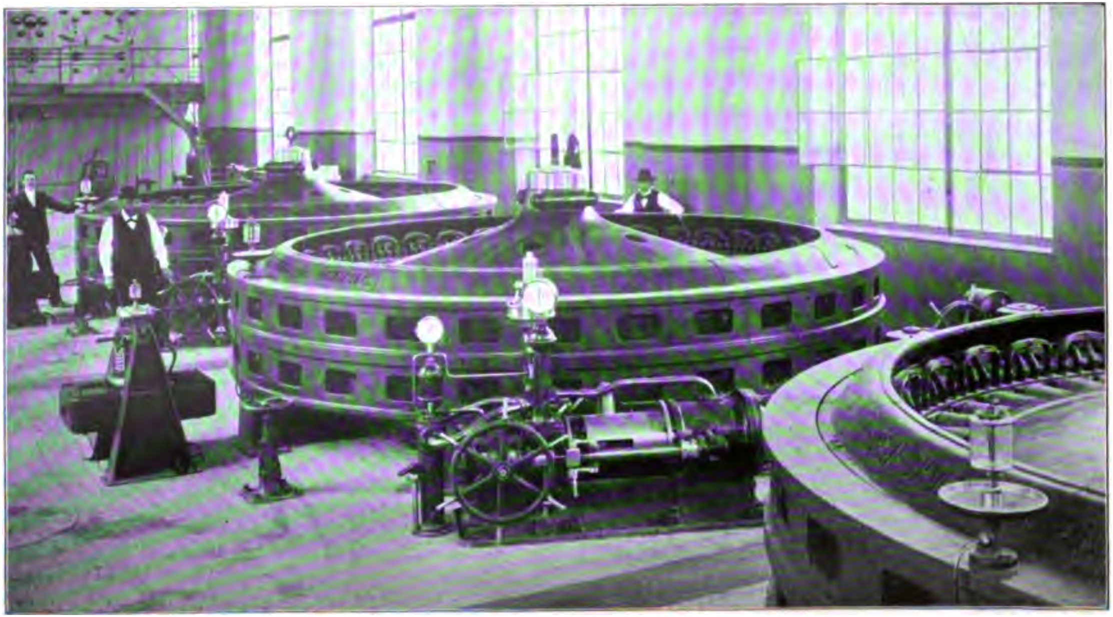


Fig. 358.—Hagneck Electricity Works. (Brown, Boveri & Company.)

[*To face p.* 831.

The revolving-field heteropolar type when adapted to the vertical shaft of a low-speed turbine is illustrated in fig. 357, which shows in section half of one of the later three-phase generators built by the Allgemeine Elektricitäts Gesellschaft for the Rheinfelden installation; it should be contrasted with the homopolar form of fig. 354. The weight of the rotating portion is carried chiefly by a step-bearing fed with oil under a pressure of 450 lbs. per square inch.

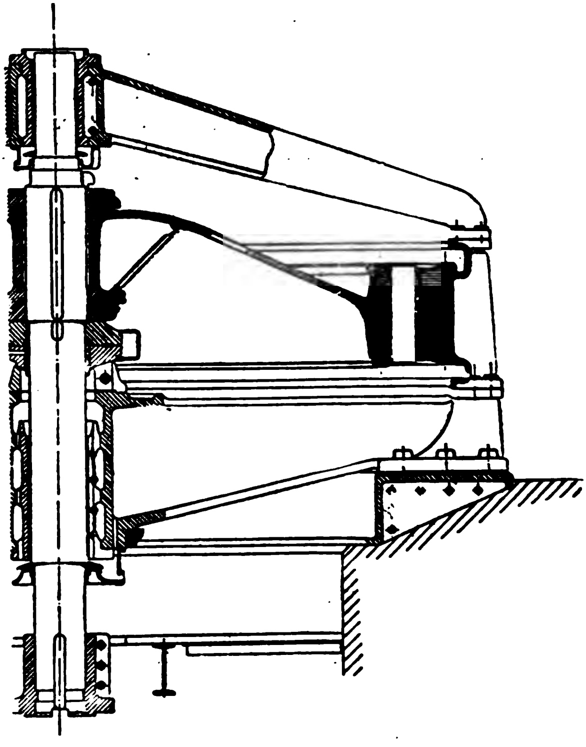


FIG. 357.—Three-phase heteropolar generator at Rheinfelden. (Allgemeine Elektricitäts Gesellschaft.)

Fig. 358 gives a general view of the three-phase generators of 'umbrella' type by Brown, Boveri & Co. in the central station of Hagneck, near Biel. Each of the four machines is directly coupled to a vertical turbine of 1300 H.P. and gives at 100 revs. per min. a voltage of 8000 and a periodicity of 40, the number of poles being 48. The armature casing rests on a foundation having a circular opening of 8 ft. 6 in. diameter, through which when the magnet-wheel has been removed the parts of the turbine can be raised from below.

While a peripheral speed of 6000 ft. per min. is permissible in the case of a plain cast-iron flywheel with solid rim and arms well rounded into the rim, such a limiting speed must be considerably reduced to, say, 4000 ft. per min., if the poles are bolted to the wheel and the further centrifugal force due to their mass and to the exciting coils which they carry is added to that of the rim itself. Especially must this be the case when the rim of cast-iron is divided and has to be locked together. The employment of a cast-steel wheel is then necessary for greater safety at speeds up to 5000 ft. per min., but finally for still higher speeds a different construction must be adopted. In the latest type of machine constructed by the Allgemeine Elektricitäts Gesellschaft, as shown in figs. 359 and 361, this takes the form of an interlocked ring of laminated sheet-steel plates wedged on to the arms of the cast-iron centre-wheel. Each packet of segmental plates is held by several

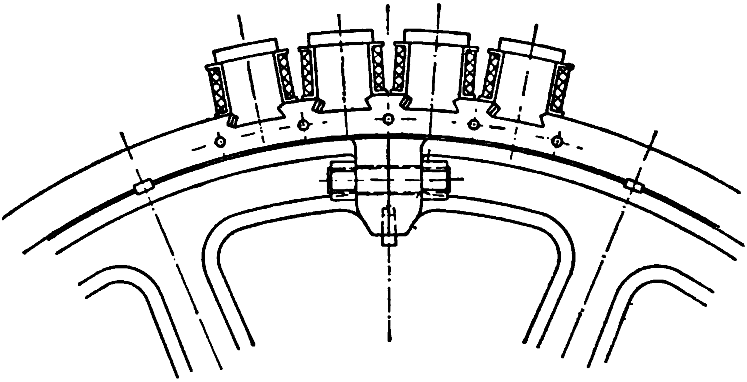


FIG. 359.—Magnet-wheel construction of Allgemeine Elektricitäts Gesellschaft.

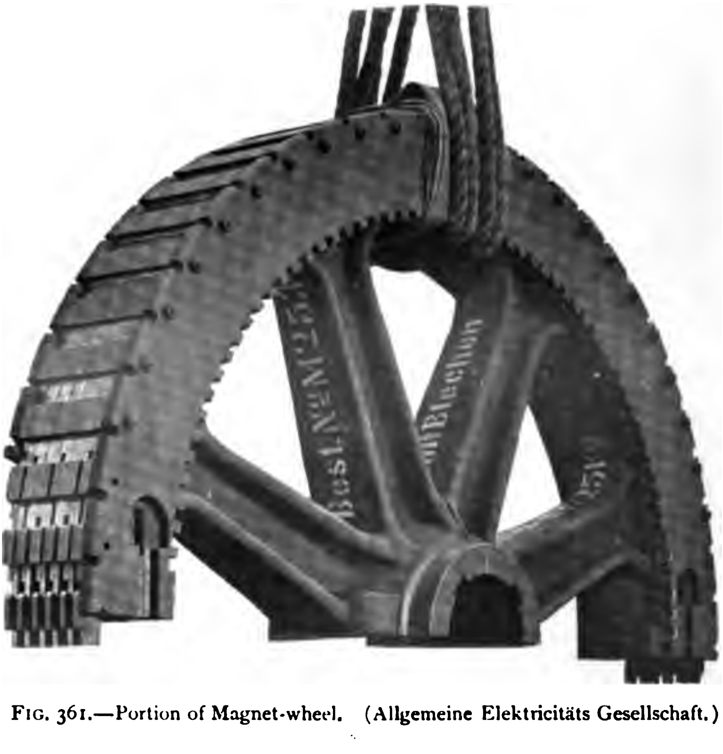


FIG. 361.—Portion of Magnet-wheel. (Allgemeine Elektricitäts Gesellschaft.)

[*To face p.* 832.

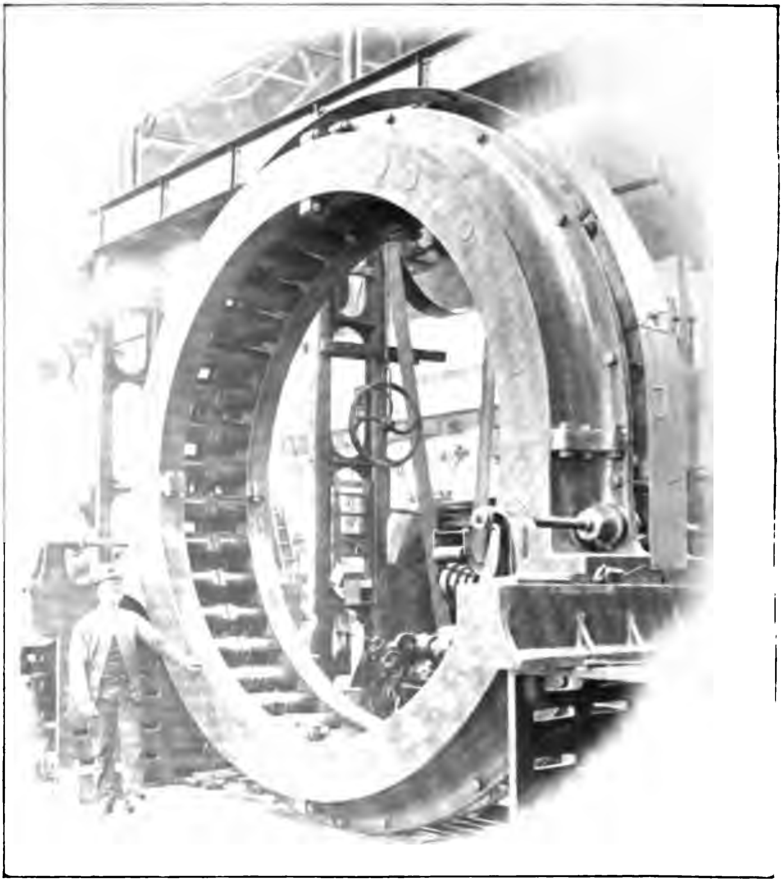


FIG. 362.—Magnet-frame of 650 K W. Ferranti disc alternator.

[*To face p.* 833.

bolts (fig. 360) and the whole ring is free to expand under the action of centrifugal force without undergoing deformation. The further advantage is gained that the tensile quality of the materials of the plates and the shearing quality of the bolts can be experimentally ascertained, while the stresses in each case admit of close calculation. The pole-pieces are also made of sheet-steel laminations so as to secure a material thoroughly reliable both mechanically and magnetically; they are dovetailed into the yoke ring and are fastened by double wedges, these latter being themselves locked by a wire passing through them and bent over in opposite directions at the two ends.

The type of field appropriate to the disc alternator has been diagrammatically shown in fig. 79, and is further illustrated by fig. 362. The magnet system of the 650 KW. Ferranti alternator here shown consists of 40 pairs of circular magnet-cores driven into machined holes, and secured by grub screws in the supporting framework of the two massive cast-iron yoke-rings. Each of these is divided into four quarters, bolted together to form a complete crown. A curved flange bends backwards over the magnet-cores, so that when the two crowns are brought together and fixed with suitable distance-pieces, the space between the opposing pole-faces only exceeds the width of the armature coils by a sufficient amount to give the necessary mechanical clearance on either side of the rotating armature. The exciting coils are of copper strip wound on edge, and are held in place by the pole-shoes which are fixed to the cores by countersunk screws. Thus 20 pairs of magnetic fields of alternate sign are formed, giving at the speed of 250 revs. per min. a periodicity of 83. The same periodicity is obtained in the 1500 kilowatt machine (alluded to above in § 7) by 64 poles on each side of the armature; the magnet-cores are of cast steel 5″ in diameter, arranged on

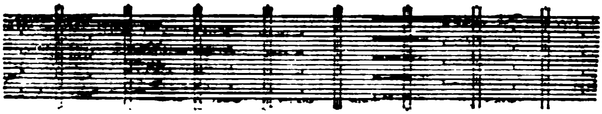


FIG. 360.—Portion of laminated rim of magnet-wheel.

a pitch-circle of 20 ft., and in order to prevent sparking across from the armature to the pole-faces under the high tension of 10,000 volts, micanite caps $\frac{5}{32}''$ thick are fitted over the rectangular pole-shoes. Each one of these pole-caps is separately tested to a pressure of 21,000 volts with an alternating current. The width of the armature strip is ¾", and the clearance between armature and micanite $\frac{11}{32}''$ on each side, making a total air-gap from pole to pole of 1¾ in. The field-winding is again of bare copper strip wound on edge with insulation between the turns, and is excited at a low voltage.

§ 10. **The exciting coils.**—In the case of revolving field-magnets owing to their high peripheral speed (4000–6000 ft. per min.), the exciting coils are best formed of rectangular copper strip wound on edge with insulation coiled round between the turns (fig. 363); the effect of the centrifugal force is then only to compress the whole spiral towards the periphery and the insulation does not suffer by chafing. Copper band wound on the flat is not so economical; since a number of turns of thin sheet nearly equal in width to the length of the poles implies a considerable amount of space lost in insulation. Round wire should never be employed on rotating magnets owing to the danger of the wire slipping on itself and damaging the insulation. Coils of rectangular or square wire necessitate insulating bobbin-cases, and are not so self-supporting as spirals of copper strip. The coils or their cases are held in place by the overhanging edges of the pole-shoes, or by special gun-metal bridges which also serve in some measure to steady the machine for parallel working by their damping action (cp. fig. 403). The current density in the field wire usually ranges from 1300 to 1900 ampères per square inch, or higher than in continuous-current dynamos owing to the greater cooling action of the rotating magnet. The voltage of the exciting current varies according to circumstances from 50 to 100 volts, yet good insulation of the entire circuit of coils and exciter is necessary, since owing to the great self-inductance of the circle of bobbins there is a considerable strain on the insulation when the circuit is broken; indeed on this account

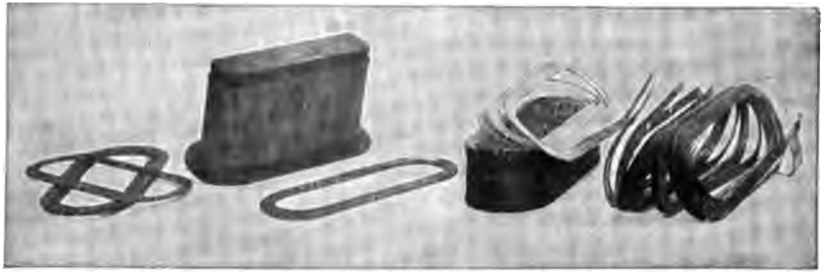


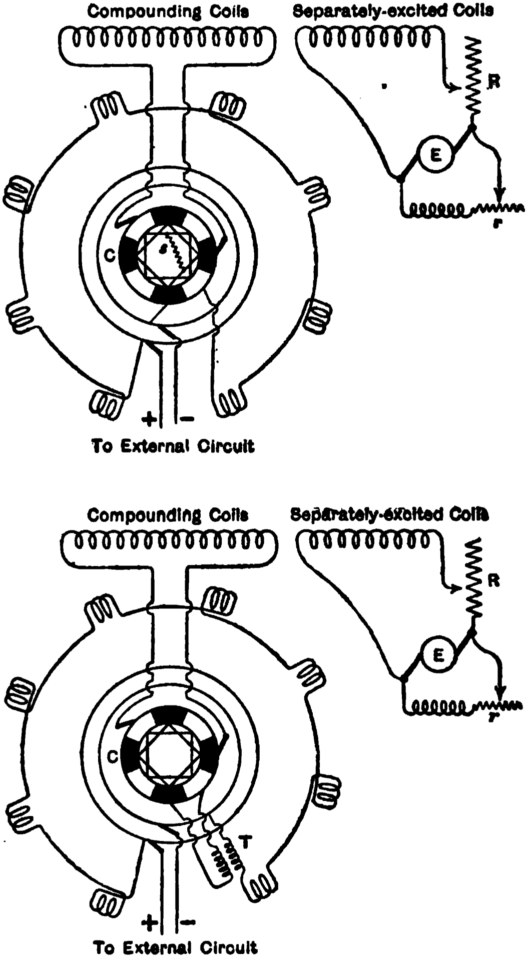
FIG. 363.—Strip-winding of field coil of British Thomson-Houston Co.'s alternator.

[*To face p.* 834.

the opening of the circuit is best effected through a non-inductive shunt and carbon-break switch.

§ 11. **Methods of field excitation.**—The field-magnet system of the alternator must necessarily be excited by a continuous current. Separate excitation by an auxiliary continuous-current dynamo or '*exciter* E (fig. 364) is therefore almost universally employed, its great recommendation being that the exciting current can readily be altered by means of a rheostat R in series with the field-winding so as to maintain the alternating voltage at the required value when the load varies. If a separate small exciter is used for each machine, its armature may be driven directly from the alternator shaft (figs. 338, 340, and 346), or through rope or belt gearing. The method then, however, has the disadvantage that any change of speed of the prime mover also causes a change of speed and of voltage in the exciter. Hence it is better that the exciter should be driven by a separate steam-engine or turbine, in which case one machine will serve to excite several alternators in combination, each having its own field-regulating resistance. The alternating current supplied by the alternator itself can also be used to excite the field if *rectified*, or converted into a direct form; the machine can then be made self-exciting, or if given an initial excitation from a separate source exercises a compounding effect which increases with the load just as in the compound-wound continuous-current dynamo. Two sets of magnet bobbins are then required, the one separately excited and the other self-excited by the armature current (fig. 364). The rectifying commutator C has as many sectors as there are poles, and alternate sectors are connected together to form two separate sets into and out of which the alternating current is led. A pair of brushes is then arranged to press upon the commutator at two points corresponding to zero value of the current. Thus the circuit between the two sets of sectors is completed through the brushes and the series or compounding coils; the connections of these latter with the sectors are reversed at each reversal of the alternating current, so that the current through them is unidirected. It is not, however, necessary to rectify the whole of the armature current; a portion only as derived from a branch circuit may be used. Thus in fig. 364

if the sets of sectors are connected across by a resistance *s* as shown, only a part of the whole current is shunted through the



FIGS. 364 and 365.—Compounding of alternator.

brushes. Further, the whole of the voltage of the alternator need not be impressed upon the rectifying commutator, but only that of one or two coils. Even then the voltage may be too high to be convenient, so that the alternating current may be passed through a transformer (fig. 365), and the secondary current thus obtained be rectified. The commutator must be attached to the rotating portion of the alternator, but the transformer T may be independent of the machine if the field-magnet rotates and the armature is stationary. If the width of the brushes is less than the width of the insulation between the sectors, there must be some sparking owing to the discontinuity of the circuit and the inductance of the field-winding. On the other hand, if the width of the brushes be greater than the width of the insulation between the sectors, both the supply circuit and the field-circuit are short-circuited at the moment when the brushes bridge over the insulation. Various devices have been tried to obviate these difficulties, but on the whole have not found much favour in practice.

Automatic compounding can also be applied to polyphase machines. Thus in a three-phase alternator each of the three currents may be taken through a transformer, the secondary of which feeds the rectifier; the compounding effect is thus dependent upon the combined currents of all the three phases. Or the common junction of the star winding may be replaced by a small delta, and the three terminals of the delta are then connected to the sectors of the rectifier which are united into three sets. The result is that some small portion of the current is shunted through the rectifier, and as the phases are each represented separately, if the currents in the three lines are not equal or the circuits are not balanced, the regulation is but little affected.

§ 12. **Compounding of alternators.**—Although the self-exciting principle has been used with some success for compounding purposes in the case of small machines, the problem cannot be said to have been satisfactorily solved. While the series turns may be adjusted to suit the normal load and normal load-factor so as to maintain the voltage fairly accurately, there remains the difficulty that the compounding effect is really required to vary not merely with the value of the current, but also with the angle of lag of the current according as the external load is inductive or non-inductive. Hence an alternator which is adjusted for a load of lamps is not so well adjusted for a load of motors, and will require an alteration of its separate excitation.

Various attempts have been made to meet this difficulty, among which may be mentioned the 'compensated exciter'[1] in which the alternating armature current is caused to react upon the field of the exciter to an extent depending not only upon its strength, but also upon its phase relatively to the E.M.F. The exciter armature is built upon the same shaft as the rotating field of the alternator, and has the same number of poles so as to give the same frequency. In addition to the collecting rings by which the continuous current is supplied to the field-winding of the alternator, the exciter armature winding is joined to three collecting rings through which the three-phase alternating current from a transformer in series with the main circuit is passed. The reaction of the latter current upon the field of the exciter causes its voltage to rise not only with increase in the main current but also with increase of its angle of lag.

Different principles of compounding[2] are again to be found in the exciter due to M. Leblanc and exhibited in connection with the Grammont alternator at the Paris Exhibition of 1900, and in the Boucherot alternator exhibited by MM. Breguet et Cie., the the latter being of special interest from its being designed for operation as an asynchronous generator.

§ 13. **The choice of frequency.**—The frequencies for which alternators are commercially designed range from 100 to 25, and even lower values are beginning to be considered in special cases. Indeed it may be said that, while in the early days of electric lighting frequencies of 125 or 133 were not uncommon, these are no longer now in use, and the general tendency is towards moderate frequencies by which may be understood any value below 60. When the alternators are driven by belt at a considerable speed, a high frequency can be obtained without the necessity for a large number of poles; hence the machines are economical in first cost, and so also are the transformers for use in connection with them on lighting circuits. On the other hand, the drop of voltage over the transmitting lines is increased owing to their greater reactance with the higher frequencies, and when the generators are of large size and are directly connected to the shaft of the steam-engine or turbine wheel, the large number of poles that is required makes the machines somewhat expensive in labour and in copper or exciting energy; further they do not regulate so well for constant potential. An additional disadvantage in the case of engine-driven sets is that at high frequencies they do not work so well in parallel; the permissible deviation from a perfectly uniform speed of rotation varies inversely as the number of poles, and if the machines for high frequencies are to be run in parallel the engines must be better balanced as regards their moving parts or must have larger and more expensive flywheels. If the alternators are to be used for the supply of energy to asynchronous motors, still another reason is found for the adoption of a low frequency, since with high frequencies the power-factor and efficiency of the motors are less, at least if their speed is to be moderate. Lastly, if rotary converters form a large part of the load, uniformity of the angular velocity becomes of special importance in order to prevent hunting.

---

[1] Fully described in *Electrical World*, vol. xxxvii. p. 676, by Mr E. J. Berg.

[2] Both forms are described by M. Guilbert in the *Electrical World*, vol. xxxvii. pp. 302 and 352.

In the other direction there are, however, again certain limits. Owing to the periodic heating and cooling of the carbons in the alternate-current arc, the eye can detect and is fatigued by the variation in the light if the frequency is less than 40, and even in the case of incandescent lamps the same effect becomes noticeable with frequencies below 30. Thus for systems combining the supply of both light and of energy to motors, and in which the former is an appreciable part of the whole, a frequency of 60 is often adopted. Except on very long lines the reactance is then moderate, and the regulation of voltage is good, while asynchronous motors can be built to run well and at reasonable speeds with such a standard. Rotary converters of moderate size can also be operated successfully at 60 cycles unless the alternators are directly coupled to very low-speed engines. Both motors and converters are, however, better suited by a lower frequency, so that, if they form the principal part of the load and complete steadiness of light is not of first importance as in factory lighting, a frequency of 50 or 40 affords a better compromise and the former is in general use on the Continent. Finally, if the whole of the energy is to be supplied to motors of several hundred horse-powers and rotary converters of large size, or if the transmitting lines are so long as to have considerable capacity, a frequency of 25 is to be recommended. Such a value was adopted by the engineers for the transmission of energy at Niagara, where rotary converters of large size are extensively used, and the results have been such as to amply justify the choice of what was then considered an abnormally low frequency.[1]

---

[1] Cp. Steinmetz, *Electrical Review* of New York, April 20, 1901, vol. xxxviii.

CHAPTER XXV

ALTERNATOR DESIGN

§ 1. **Preliminary calculation of dimensions of alternator.**—The starting-points in the design of an alternator for a given output are usually our knowledge of the desired frequency and of the speed of the driving engine or turbine, whence the number of pairs of poles is at once fixed from the equation $f=\frac{pN}{60}$. The peripheral speed in the case of a rotating magnet may vary between 4000 and 6000 ft. per min. (20 to 30 metres per sec.) and even when no portion of the copper winding revolves, seldom exceeds 8000 ft. per min. (40 metres per sec.). If then v be the peripheral speed in ft. per min. which is assumed as permissible, the diameter in ft. is given by the equation $D'=\frac{V}{\pi N}$, and the pole-pitch can be thence approximately determined. The power-factor of the circuit on which the alternator is to work is either given or is known from previous experience, and the permissible drop or rise in volts is usually specified, so that $E_e=\eta E_0$, where η may be, say, 0·9 or 0·8.

From the approximate equation (116) applicable to the intermediate case of a partially inductive load, or

$$\eta E_0=\sqrt{E_0^2-e^2\cos\ \varphi_e}-e\sin\phi_e$$

we have

$$e\sin\ \phi_e+\eta E_0=\sqrt{E_0^2-e^2\cos^2\ \phi^e}.$$

Squaring and solving the quadratic equation,

$$\frac{e}{E_0}+\eta\sqrt{1-\cos^2\phi_e}=1-\eta^2\cos^2\phi_e$$

or

$$\frac{e}{E_0}=\sqrt{1-\eta^2\cos^2\phi_e}-\eta\sqrt{1-\cos^2\phi_e}=Q$$

The numerical ratio Q can be worked out and tabulated once for all for various values of η and $\cos\phi_e$. (See Fischer Hinnen, *Electr. Eng.*, vol. xx. p. 620.)

Since $I_0 = \dfrac{pX}{\dfrac{\tau}{2} \cdot \dfrac{\lambda}{m}}$, and the impedance $= \dfrac{E_0}{I_0}$, the reactance voltage e for the full-load effective current I per phase is

$$e = I \times E_0 \cdot \frac{\dfrac{\tau}{2} \cdot \dfrac{\lambda}{m}}{pX}$$

and

$$\frac{e}{E_0} = Q = \frac{I \cdot \dfrac{\tau}{2} \cdot \dfrac{\lambda}{m}}{p \cdot X}.$$

Hence $\dfrac{I\dfrac{\tau}{2}}{pX}$ or the ratio $\dfrac{\text{effective A.T . on armature}}{\text{A.T . on field}}$ must be $Q \cdot \dfrac{m}{\lambda}$, or the *ampère-wires per pole* must be

$$\frac{I\tau}{2p} = X \cdot \frac{Q \cdot m}{\lambda} \quad . \quad . \quad . \quad . \quad . \quad (117)$$

As a general rule $\dfrac{Q \cdot m}{\lambda}$ or the ratio $\dfrac{AT_a}{AT_f}$ averages about 0·25, and under favourable conditions with good design may be taken as high as 0·3.

A somewhat different ratio is employed by Rothert[1] as one of the fundamental constants of alternator design, and is termed by him the *constant of reaction.* For the effective ampère-turns of the armature are substituted the magnetising armature-turns or X_a, and for X are taken the ampère-turns required on each magnetic circuit to give the full terminal voltage under load with both secondary and primary leakage but without any allowance for the actual back ampère-turns themselves. X thus becomes a theoretical quantity which can only be calculated, and is a little more than the ampère-turns required at no load, say 10 to 15 per cent. greater. The ratio $\dfrac{X_a}{X}$ averages about 0·3 but may be taken as high as 0·4 without introducing too much drop for the proper working of asynchronous motors.

---

[1] *L'Éclairage Électrique*, vol. xxix. p. 324.

§ 2. **The value of λ.**—Reverting to the considerations of § 20, Chap. XXIII, we have, independent of any question as to the method of reckoning the armature back-turns, the value $\lambda = \frac{X}{n_0}$ as determined experimentally from the short-circuit characteristic. This is found in practice in three-phase heteropolar alternators to range from 2·55 to 3·8, and on the average may be taken as about 2·95, while in the single-phase alternator it varies from 1·4 to 2·1, and may in general be reckoned as 1·6. Machines with short non-overlapping coils show a higher value for λ than those with long coils owing to the fact that the former do not embrace the magnetic circuit so completely, or R_a is different from R_w, and local flux is then set up which has to be counterbalanced by an equal main flux. Bar-wound armatures with closed slots also show large values for λ. In general the value of λ is influenced by the number of slots per pole and per phase, and is less the greater the number of such slots, since the secondary leakage is then less and the back ampère-turns are not so concentrated on the main magnetic circuit. Thus in a single-phase alternator[1] with one slot per pole, λ' was 1·56; when the same type and size of machine was wound as a three-phase generator with three slots per pole, the subdivision of the winding between the three slots partly compensated for the increased back ampère-turns which would otherwise have resulted, so that the new value of $\lambda''' = 2·9$ was considerably less than three times the λ' of the single-phase case.

But still more important is the influence upon λ of the absolute value of the pole-pitch and so indirectly of the frequency. The secondary leakage is practically a constant whatever the pole-pitch, and therefore its ratio to the main flux varies inversely as the pole-pitch. Further, the value of λ depends not only upon the actual number of the demagnetising turns of the armature, but also upon their effect in increasing the primary leakage. The greater the permeance P of the leakage paths, the higher the density of the resultant flux in the field-magnet; its reluctivity rises and $f(B_m)$ in the second term of the right-hand side of eq. (106) increases. But with reduced value of the pole-pitch, P rather tends to increase, and in consequence with an increase of frequency unless the pole-pitch can be maintained constant, X_m increases and the value of λ is higher. Fig. 366 taken from

---

[1] Behrend, 'Factors which determine the Design of Monophase and Polyphase generators,' *Electr. Eng*, vol. xxvi. p. 727.

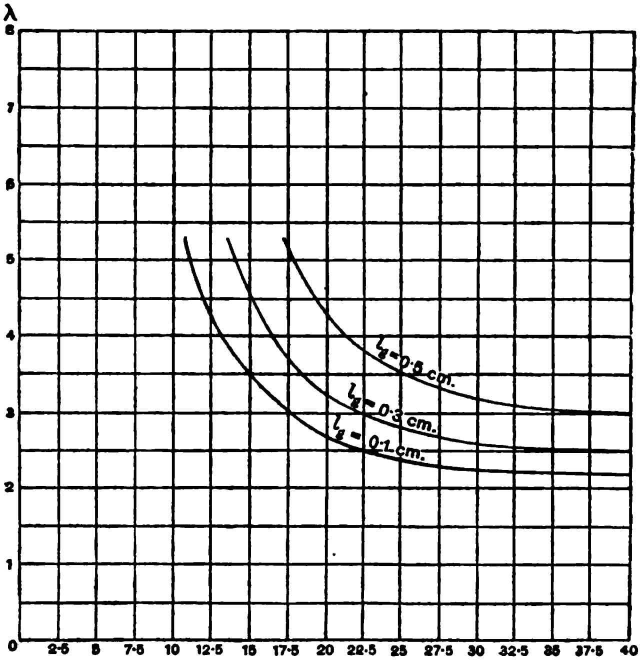


FIG. 366.

Mr. Behrend's paper above-cited shows how the value of λ varies not only with the air-gap, but also with the absolute value of the pole-pitch in the case of three-phase inductor generators of the type shown in fig. 85 with one slot per pole and per phase and widths of armature-coil and of pole equal to the pitch. For large values of the pole-pitch the curves tend to converge towards the theoretical value of $\sqrt{2} \cdot \frac{3}{2} = 2{\cdot}12$, which it would have if both primary and secondary leakage were absent. The two upper curves may also be used for heteropolar machines if their pole-pitches exceed 20 cm., when the value of λ becomes more nearly independent of the air-gap.

If it is desired to find whether a machine can be advantageously wound for a higher frequency, let f', I_o' and λ' be the new values for the higher frequency; if the total number of inductors remains the same, the number per pole and per phase for a given voltage and the same magnetic frame will vary inversely as the frequency. Therefore

$$I_o' = \frac{X}{\lambda' t'} \propto \frac{f' X}{\lambda'}$$

$$I_o = \frac{X}{\lambda t} \propto \frac{f X}{\lambda}$$

$$\text{whence } I_o' = I_o \times \frac{f'}{f} \times \frac{\lambda}{\lambda'}.$$

The larger I_o, the better; so that if the higher frequency is to be advantageous to the dynamo design, the quotient $\frac{f'}{f}$ must be greater than the quotient $\frac{\lambda}{\lambda'}$. When thus examined, it is found that with a fixed diameter, the output reaches a maximum for a certain frequency, and if this favourable frequency is widely departed from, a new diameter should be chosen, so far as considerations of peripheral speed or of manufacturing economy permit.

§ 3. **The armature ampère-wires per pole.**—The ampère-turns on one magnetic circuit may be expressed as a multiple of the ampère-turns required over the double air-gap, or $X = 0{\cdot}8B_g . 2\, l_g \times c$; the value of the multiplier c which expresses the ratio of the total ampère-turns of the circuit to those of the air-gaps varies from 1·2 to 2, but may approximately be regarded as constant for the same type and proportions of machine, say, at 1·4. The consideration of mechanical clearance demands a certain minimum air-gap, and any increase beyond this increases X_g and the ratio of the primary leakage. The value of l_g is therefore in a given type confined practically within small limits, especially in the case of machines with a single exciting coil or in homopolar machines. There is further a certain maximum value of B_g which it is not advisable to exceed, having due regard to economy in the excitation, to hysteresis and eddy-current losses, and to the magnetic pull on the armature if there is any want of symmetry in the several fields. Thus if $X = 0{\cdot}8\ B_g \cdot 2l_g \times c$ be regarded as constant, we are led to a more or less constant value under given conditions of regulation for the ampère-wires per pole, or

$$\frac{I\tau}{2p} = X \cdot \frac{Qm}{\lambda} = \text{a constant.}$$

Although the ratio of the armature to the field ampère-turns, or to use the new form of expression given above, the number of armature ampère-turns per pole is a most important factor in the design of alternators, it is evident from the wide variations of λ with different types of machines and different arrangements of the winding that the reactance voltage is not definitely determined by the armature ampère-turns per pole. In different cases, therefore, much may be done to vary the one or the other so as to suit the exact nature of the load. Thus if the load factor varies through a wide range, it is especially important that the armature ampère-wires per pole should be low, although the total armature reactance voltage may not be comparatively high; such a case is met by the employment of a very small number of turns per coil, entirely embedded in the iron of the armature in closed slots or tunnels. On the other hand, if the power factor is high and does not vary very much, and especially on non-inductive loads it would be better to employ a larger number of armature ampère-wires per pole, arranged in shallow open slots, so that the total reactance voltage is not so high; in other words, it becomes then of more importance to minimise the electric part of the reaction due to the secondary leakage rather than the magnetic part due to the back ampère-turns.

§ 4. **The armature ampère-wires per unit length of circumference.**—But as in the case of continuous-current dynamos, a knowledge of the permissible number of ampère-wires per pole as dependent upon the drop of volts between no load and full load is but little else than a warning to the designer, and must be further extended to give some indication of the dimensions of the armature. In other words, the normal relation of the ampère-wires to unit length of the circumference as given by practice must be introduced, in order to make a preliminary calculation as to the diameter of the armature. Experience shows that in three-phase generators, the effective ampère-wires per unit length of circumference are $a_w = \frac{I \cdot \tau}{\pi D}$ = approximately 300 per inch, or 120 per cm. up to 500 kilowatts, and above that rising to 375 per inch or 150 per cm. of circumference, if cos ϕ_e lies between 1 and 0·85. With lower power-factors, a_w must be reduced in proportion to cos ϕ_e if the drop of volts is to be maintained at the same percentage value, but since within the limits of practice a greater drop of volts is usually expected if the power-factor be low, a_w remains practically constant at the values given above. From this approximate result the preliminary forecast of the diameter must now be checked.

If the drop on inductive loads is to be the same in a single-phase alternator as in a three-phase generator, the value of a_w would have to be reduced to 200–250 per inch, or 80–100 per cm. of circumference (or even less if the maximum back ampère-turns be taken into account with laminated poles), but in practice as the average value of the back ampère-turns better corresponds to actual results, a_w may reach as high as 300 per inch or 120 per cm. of circumference, especially when it is remembered that the single-phase alternator is mostly used on lighting circuits with high power-factors. Indeed, in such cases there is but little difference between the output of a given machine when wound for a single-phase circuit of high power-factor and when wound for three-phase circuits of only moderate power-factor.

§ 5. **The dimensions as dependent upon the watts per rev. per min.** If W be the total watts of output of the machine, and m be the number of phases, while E_e and I are the effective values of the terminal voltage per phase of the armature and of the current per phase, then $\frac{W}{m} = E_e \cdot I \cos\phi_e$ and I which is also $= \frac{a_w \cdot \pi D}{\tau}$ is at once determined.

The E.M.F. induced per phase at no load is

$$E_0 = \frac{K}{m} \cdot 2Z_a \cdot f \cdot \tau \times 10^{-8} \text{ volts},$$

and the terminal voltage is to be ηE_0. Therefore the watts per rev. per min. are

$$\frac{W}{N} = \frac{mE_e \cdot I\cos\phi_e}{N} = \cancel{m} \cdot \eta \cdot \frac{K}{\cancel{m}} \cdot 2Z_a \cdot \frac{p}{60} \times \cancel{f} \times 10^{-8} \times \frac{a_w \cdot \pi D}{\cancel{f}} \cdot \cos\phi_e$$

The width of the pole-face is approximately

$0{\cdot}66$ to $0{\cdot}75\ \frac{\pi D}{2p}$ in heteropolar machines

or $0{\cdot}45$ to $0{\cdot}5\ \frac{\pi D}{p}$ in homopolar machines $= 0{\cdot}9$ to $1\ \frac{\pi D}{2p}$

so that it is in general $= \beta \cdot \frac{\pi D}{2p}$.

Hence for Z_a may be substituted $\beta \cdot \frac{\pi D_{\prime\prime}}{2p} \cdot L_{\prime\prime} \times B_g \times 6{\cdot}45$, and

$$\frac{W}{N} = \cancel{2}K \cdot \beta \cdot \frac{\pi D_{\prime\prime}}{\cancel{2}\cancel{p}} \cdot L_{\prime\prime} \times B_g \cdot \frac{\cancel{p}}{60} \times \pi D_{\prime\prime} \cdot a_w \cdot \eta \cdot \cos\phi_e \times 10^{-8} \times 6{\cdot}45\ .$$

$$= \frac{\pi^2 D_{\prime\prime}^{\ 2} \cdot L_{\prime\prime} \cdot a_w \cdot \beta \cdot B_g \cdot K \cdot \eta \cdot \cos\phi_e \times 6{\cdot}45}{60 \times 10^8} \quad . \quad . \quad . \quad (118)$$

which is exactly analogous to eq. (89) for the continuous-current dynamo. The output is here expressed in terms of ηE_0, and if η be very low, or a large drop of volts is allowed, E_0 is correspondingly high. But since E_0 does not in reality exist at full load, there would be no risk in giving B_g or the density of the fictitious lines corresponding to E_0 an equally high value. Or again if η be low, a_w may itself be given a somewhat higher value provided that the heating limit is not exceeded. It thus results that a_w, B_g, and η may be grouped together as forming a more or less constant quantity. For a given type of machine, β and K are constant, and on the average β may be taken as $= \frac{2}{3}$rds., which gives an approximately sinusoidal curve of E.M.F., so that finally we have watts per rev. per min. $\propto D^2L \cos \phi_e$. From this it will be seen how important a part is played by the requirements of regulation on an inductive load in determining the size of machine, especially when it is remembered that for the same percentage drop a_w must be reduced if $\cos \phi_e$ is low. With such values for a_w, β, B_g, K and η as 300, 0·66, 7400, 1·11 and 0·85,

$$\frac{W}{N} = D_{''}^2 L_{''} \cos \phi_e \times 0{\cdot}0147$$

$$\text{or } D_{''}^2 L_{''} = \frac{W}{N} \times \frac{68}{\cos \phi_e} \text{ up to 500 KW} \qquad . \quad (119)$$

or with $a_w = 375$

$$D_{''}^2 L_{''} = \frac{W}{N} \times \frac{55}{\cos \phi_e} \text{ up to 1000 KW} . \qquad . \quad (120)$$

Any such approximate formulæ admit of wide deviations according to the circumstances of the design, and in them it is assumed that the periodicity is 50; for lower frequencies, the constants may be reduced. They could also be thrown into the same form as eq. (90) for the determination of the diameter, but since this has already been decided by other considerations, they may rather be used to determine L, and it must then be seen whether the ratio of the pole-width to the length of the core is such as to give an economical section to the field-magnet cores from the point of view of exciting energy or copper. In most cases a large diameter and narrow width are no disadvantage since they give a large flywheel effect for a given mass of copper and iron, and further lend themselves to placing the alternator between the cylinders of the steam-engine. With short magnet-cores the cost of the magnetically and electrically effective portions of the alternator is also more independent of the diameter than is the case with the continuous-current machine.

§ 6. **Design of three-phase generator.**—Let it be required to design a three-phase generator with Y or star-connected armature to give 2000 volts of interlinked pressure between the lines and 166 ampères per phase with a frequency of 50 when running at about 95 revs. per min. The power-factor is given as $\cos \phi_e = 0{\cdot}85$, so that while the kilovolt-ampères or kilowatts of output with unity power-factor would be $\sqrt{3} \times 2000$

× 166 = 575, the actual output on the partially inductive external load is only 575 × 0·85 = 488 kilowatts.

In order to give the frequency required, $f = \frac{p\text{N}}{60} = \frac{32 \times 94}{60} = 50$, so that the speed must be 94 revs. per min., and the number of poles 2 × 32 = 64. The armature is to be of the stationary external type, and the magnet system is to have poles of alternate sign arranged radially on the rim of a centre-wheel. For a cast-steel yoke-ring a suitable peripheral speed will be given if we assume a speed of 5000 ft. per min. at the face of the poles. The external diameter of the rotating portion will therefore be

$$\text{D}' = \frac{\text{V}}{\pi\text{N}} = \frac{5000}{3{\cdot}14 \times 94} = 17 \text{ ft.}$$

A rise of 15 per cent. of the terminal volts is to be permitted when the full load is switched off, so that the open-circuit E.M.F. with constant speed and the full-load excitation may be 2300; hence $\eta = 0{\cdot}85$, and by reference to the calculated table of $\sqrt{1 - \eta^2 \cos^2 \phi_e} - \eta \sqrt{1 - \cos^2 \phi_\omega}$ the corresponding value of Q is found to be 0·245. The pole-pitch at the face of the pole-shoes is $\frac{204'' \times 3{\cdot}14}{64} = 10''$, an average figure for which λ may be taken as = 3. The ratio of the effective ampère-turns on the armature to those on the field, or $\frac{\text{AT}_a}{\text{AT}_f} = \frac{\text{Q}m}{\lambda}$ by eq. (117) $= \frac{0{\cdot}245 \times 3}{3} = 0{\cdot}245$. Assuming the single air-gap to be $l_g = 0{\cdot}375'' = 0{\cdot}95$ cm., and the flux density therein at full load to be slightly less than the B_g corresponding to E_ω, $\text{X}_g = 0{\cdot}8 \times 7000 \times 1{\cdot}9 = 10{,}600$, and $\text{X} = c \,.\, \text{B}_g = 14{,}800$, if $c = 1{\cdot}4$ as in § 3. While the value of X has here been approximately estimated, it is usually more or less known from previous experience with machines of the same type, similarly proportioned, so that the designer thence arrives at the number of ampère-wires per pole, or $\frac{\text{I} \,.\, \tau}{2p} = \text{X} \,.\, \frac{\text{Q}m}{\lambda} = 14{,}800 \times 0{\cdot}245 = 3620$,

and $\text{I} \,.\, \tau = 3620 \times 64 = 232{,}000.$

An average value for the ampère-wires per inch length of the circumference being $a_w = 300$, we obtain for the present diameter of a little over 17 ft., $1\tau = 640 \times 300 = 192{,}000$, a value which falls below the limit above given and which may therefore be safely taken. The number of inductors will then be $\tau = \frac{192{,}000}{166} = 1156$, but as this must be divisible by the number of phases and by the number of slots, either one or two per pole and per phase, we are limited to either 64 slots per phase with 6 wires in each slot, or 128 slots per phase with 3 wires per slot. In both cases $\tau = 1152$, and in order to minimise the leakage reactance and produce an approximately sinusoidal wave of E.M.F., two slots per pole and per phase will be chosen. The ratio of pole-width to pole-pitch will be taken at $\beta = 0{\cdot}66$.

The value of K for such an arrangement will be less than 1·11, say 1·07 by Table II, Chap. XXII. Inserting then the values of 300, 0·66, 7400, 1·07 and 0·85 for a_w, β, B_g, K and η, we have by formula (118)

$$D_{''}^2 L_{''} = \frac{W}{N} \cdot \frac{10^8}{1{\cdot}06 \times a_w \times \beta \times B_g \,.\, K \,.\, \eta \,.\, \cos\phi_e}$$
$$= \frac{W}{N} \cdot \frac{71}{\cos\phi_e} \,.$$

The watts per rev. per min. are $\frac{488{,}000}{94} = 5200$, whence $D_{''}^2 L_{''} = 435{,}000$ and $L_{''} = 10\tfrac{1}{2}''$. A small increase in B_g will enable the round dimension of 10″ to be employed for the length of the armature core, and this change will also have the advantage of slightly improving the section of the magnet-core.

Allowing a loss of volts over the armature resistance of about $2\tfrac{1}{2}$ per cent, or of 50 volts on 2000, the loss over one branch of the armature winding will be $\frac{50}{\sqrt{3}} = 29$, or, say, 30 volts. The resistance of the one branch of the armature must therefore be $\frac{30}{166} = 0{\cdot}18$ ohm. If $2\tfrac{1}{2}''$ are allowed on each side of the core for the projection of the micanite trough past the core and its end-plates, the length of the straight side of each coil will be 15″, while, as the pole-pitch is 10″, the length of one curved end of a coil will be, say, $10'' \times 1{\cdot}66 = 16{\cdot}6''$. The length corresponding to one inductor will therefore be $31{\cdot}6'' = 2{\cdot}63$ ft., and the total length of one phase of 384 inductors = $2{\cdot}63 \times 384 = 1010$ ft. If the heating coefficient reckoned from a temperature of 60° F. be 1·17, the resistance per 1000 ft. of the wire must be $\omega' = \dfrac{0{\cdot}18}{1{\cdot}17 \times 1{\cdot}01} = 0{\cdot}152$ ohm and the required area is $\dfrac{0{\cdot}00813}{0{\cdot}152} = 0{\cdot}0535$ sq. inches. As the round wire corresponding to this area will be somewhat stiff to bend, and when arranged with the three turns one above the other will necessitate a deep slot, two wires each 0·185″ in diameter may be wound in parallel, giving 6 wires per slot disposed in three layers of two side by side. The slot can then be made 1·25″ deep over all × 0·75″ wide, with an opening of 0·275″ at the top, closed by a hard-wood strip (fig. 336 ii.).

The number and arrangement of the slots having now been settled (fig. 367), an approximate estimate of the leakage reactance voltage must be made. From the results of Chap. XXIII § 18, with two slots of medium depth per pole and per phase, the flux per ampère-turn and per cm. length of the active embedded side of the coil (= 25·4 cm.) may be taken as 1·7, while the length of wire in one end is $l = 21{\cdot}6'' \times 2{\cdot}54 = 55$ cm. The flux per ampère-turn linked with the coil is therefore $1{\cdot}7 \times 25{\cdot}4 + 0{\cdot}4 \times 55 = 43{\cdot}2 + 22 = 65{\cdot}2$, and, since $t = 6$, the leakage reactance voltage is by eq. (110*a*)

$$e_{sa} = 2\pi \times 50 \times 3 \times 32 \times 36 \times 166 \times 65{\cdot}2 \times 10^{-8} = 1{\cdot}8 \times 65{\cdot}2 = 117$$

volts per phase of the armature winding. Or by § 19, Chap. XXIII, from the dimensions of the slot we have in eq. (112*a*),

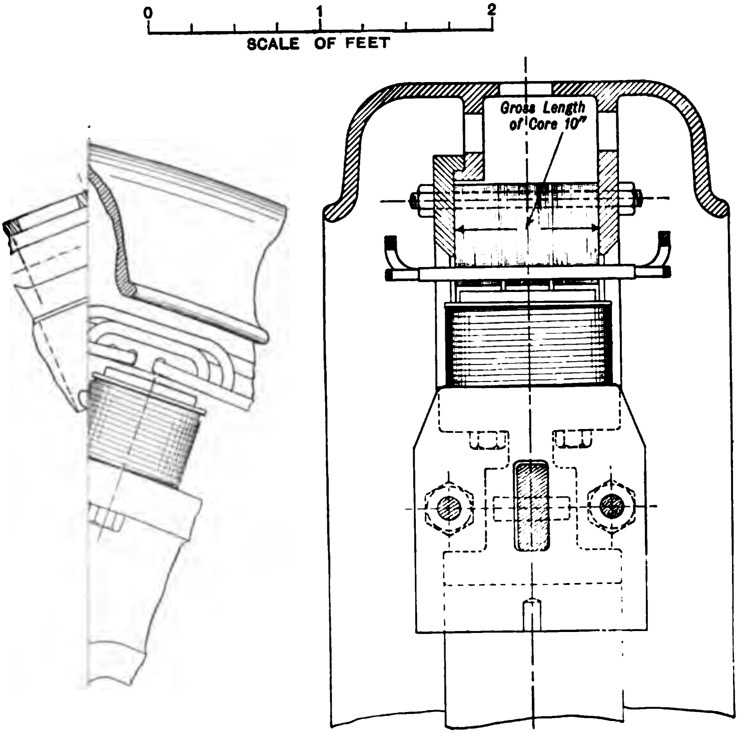
$$\frac{1{\cdot}257 \times 25{\cdot}4}{2}\left(\frac{0{\cdot}75}{3 \times 0{\cdot}75} + \frac{0{\cdot}5}{0{\cdot}275} + \frac{0{\cdot}375}{0{\cdot}275 + 1{\cdot}6 \times 0{\cdot}375}\right) + 0{\cdot}4 \times 55$$
$$= 16 \times (0{\cdot}333 + 1{\cdot}82 + 0{\cdot}43) + 22 = 63{\cdot}5\,,$$

and $$e_{sa} = 1{\cdot}8 \times 63{\cdot}5 = 114 \text{ volts.}$$

A value of $e_{sa} = 1{\cdot}8 \times 66 = 119$ may therefore safely be taken. At the same time the magnetising ampère-turns of the armature are by eq. (104)

$$X_a = 1{\cdot}414 \times 1{\cdot}5 \times 6 \times 166 = 2{\cdot}12\, tI = 2110.$$

The combined effect of the secondary leakage and of the calculated magnetising armature-turns may now be compared with



[*To face p.* 852.

the value previously assumed for λ from experimentally obtained curves. On short-circuit when the armature current has its full-load value, the flux per pole required to give the voltage consumed by the leakage reactance is by eq. (110*b*)

$$\frac{1{\cdot}5 \times 3{\cdot}14}{1{\cdot}07} \times \frac{X_a}{2{\cdot}12} \times 66 = 137\, X_a,$$

and the ampère-turns required to pass this flux across the air-gaps are $0{\cdot}8 \times 137\, X_a \times \frac{2l_g}{a_g}$. The flux necessary to give the volts which are lost over the resistance of the armature is practically negligible, since it is in quadrature with the preceding flux and increases the total amount but little.

The width of the pole may be made $\frac{1}{8}''$ less than that of the core, so that $\frac{a}{l_g} = \frac{0{\cdot}25}{0{\cdot}375} = 0{\cdot}666$, and $K_1 =$ say $0{\cdot}85$ by Table I. of Chap. XV. The interpolar gap being $3{\cdot}4''$, $\frac{c}{l_g} = \frac{1{\cdot}7}{0{\cdot}375} = 4{\cdot}55$, and $K_2 = 2{\cdot}05$ by Table II. of Chap XV. The effective polar area is therefore by eq. (30)

$$\begin{aligned} a_g &= (9{\cdot}5 + 0{\cdot}85 \times 0{\cdot}375)(6{\cdot}66 + 2{\cdot}05 \times 0{\cdot}375) \\ &= 72{\cdot}2 \text{ sq. in., or } 466 \text{ sq. cm.} \end{aligned}$$

On short-circuit then and neglecting any loss of potential over the field-magnet as very small, eq. (106) gives

$$X_a\left(1 + 0{\cdot}8 \times 137 \times \frac{1{\cdot}9}{466}\right) = X$$

$$X_a(1{\cdot}446) = 2110 \times 1{\cdot}446 = 3050 = X.$$

Hence $\lambda = \frac{3050}{1t} = \frac{3050}{996} = 3{\cdot}06$, which is sufficiently close to the assumed value of 3, especially as the armature will in working be more saturated. A calculation of the open-circuit voltage for $X = 3050$ (cp. p. 858) gives 650 volts between the lines, or 376 volts per phase of the armature, whence the synchronous impedance per phase is $\frac{376}{166} = 2{\cdot}26$ ohms. This value as deduced from λ might at once have been used to calculate the full-load characteristic by the synchronous reactance method, but in the present case the more accurate division of the leakage reactance from the magnetising armature turns is followed for the sake of illustration.

Combining $E_e = \frac{2000}{1 \cdot 73} = 1150$ volts per phase with $2IR_a = 60$ as in § 15, Chap. XXIII, we have $E_1 = 1200$, and combining this again with $e_{sa} = 119$, the required armature volts at full load are found to be $E_a = 1268$, (cp. figs. 326 and 329 which are drawn to scale to illustrate the present machine). By eq. (11)

$$E_a = 1268 = 1 \cdot 07 \times 2Z_a \times 50 \times 384 \times 10^{-8}$$

the flux corresponding to E_a is found to be

$$Z_a = 3 \cdot 085 \times 10^6 .$$

With a flux-density in the armature of $B_a = 5000$, $2ab = 617$ sq. cm. or the cross-sectional area of iron in the core below the slots must be half this amount, *i.e.* = 47·8 sq. inches. If there are two air-ducts in the core each 0·4″ wide and a deduction of 10 per cent. is made for the insulation between the laminations, the net width of iron in the 10″ gross width of core is $9 \cdot 2 \times 0 \cdot 9 = 8 \cdot 28''$. The radial depth must therefore be 5·77″, say 5¾ inches, which with a slot 1½″ deep makes the external diameter of the discs 218·75″. The number of teeth under a pole varies between three and four, or on an average and with allowance for the fringe say 4. The width of a tooth is 0·94″, whence the area of iron in the teeth corresponding to one field is $0 \cdot 94'' \times 8 \cdot 28'' \times 6 \cdot 45 \times 4 = 200$ sq. cm., and the maximum apparent density is $\frac{3{,}085{,}000}{200} = 15{,}425$, a comparatively low figure.

In order to make a preliminary determination of the requisite size of magnet-core, a leakage coefficient of, say, $\nu = 1 \cdot 2$ must be assumed, whence $Z_m = 1 \cdot 2 \times 3 \cdot 085 \times 10^6 = 3 \cdot 7 \times 10^6$. With poles of cast-steel carefully tested, the flux-density may be carried as high as $B_m = 18{,}000$, so that in our case the necessary area is $\frac{3 \cdot 7 \times 10^6}{18{,}000} = 206$ sq. cm. or 32 sq. inches. An overhanging edge is required for the pole-shoe to hold the exciting coil in position, and 7½″ length ×

$4\frac{1}{2}''$ width with well-rounded corners may in the first place be chosen for the two dimensions of the magnet core. An approximate sketch of one magnetic circuit such as fig. 368 (which will subsequently have to be corrected if not found suitable) must now be made in order to give the length of path in the pole and yoke, and more especially to enable the designer to calculate as closely

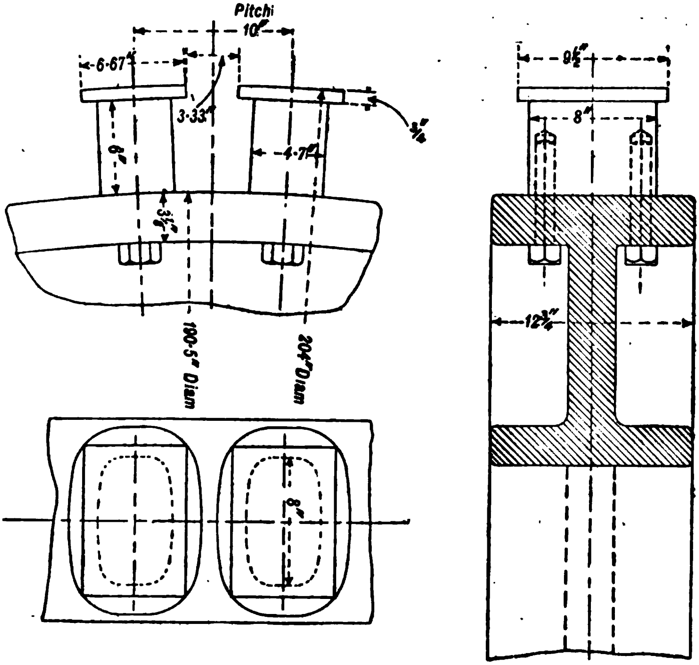


FIG. 368.—Yoke and poles of 488 KW. alternator.

as possible the value of P, the field leakage permeance. In order to minimise the leakage, the length of the pole will be as short as is compatible with the requirement of room for the exciting coils and a small loss of watts therein, say 6" between the overhanging edge of the pole-shoe and the yoke-ring. The radial depth of the pole-shoe will also be small, say $\frac{3}{4}''$, in order to keep the exciting coil as near to the top of the pole as possible. The yoke may be given ample proportions so as to increase the flywheel effect, say $12\frac{3}{4}''$ wide × $3\frac{1}{8}''$ deep, making a total cross-section, $2a_p$ of 510 sq. cm.

The permeance between the pole-tips on either side of a pole is by eq. (31)

$$\frac{0{\cdot}75 \times 9{\cdot}5}{3{\cdot}33} \times 2{\cdot}54 \times 2 = 10{\cdot}85\,,$$

and that between the flanks of the pole-shoes in either direction and on both sides by eq. (33)

$$1{\cdot}86 \times 0{\cdot}75 \times \log \frac{\pi \times 3{\cdot}33 + 3{\cdot}33}{3{\cdot}33} \times 4 = 3{\cdot}44,$$

both the above being acted on by the full excitation between the poles, or X_P. The permeance between the sides of the poles is approximately

$$\frac{6 \times 9}{5{\cdot}2} \times 2{\cdot}54 \times 2 = 52{\cdot}6$$

and from the flanks in either direction and on both sides of the core is

$$1{\cdot}86 \times 6 \times \log \frac{\pi \times 2{\cdot}25 + 5{\cdot}2}{5{\cdot}2} \times 4 = 16{\cdot}6.$$

The two latter being acted on by $\frac{X_P}{2}$ must be halved if regarded as in parallel with the armature path, whence

$$P = 10{\cdot}85 + 3{\cdot}44 + 26{\cdot}3 + 8{\cdot}3 = \text{say } 50.$$

The full-load excitation can now be calculated as follows, and is conveniently reckoned out for a half circuit or per pole. For greater accuracy the equivalent length of the air-gap may be calculated from fig. 216, and m is found to be about 1·02. Thence by eq. (38)

$$\frac{X_g}{2} = 0{\cdot}8\, Z_g \,.\, \frac{1{\cdot}02 \times 0{\cdot}95}{472} = 0{\cdot}00164\, Z_g\,.$$

For $Z_a = 3{\cdot}085 \times 10^6$, we thus find $\frac{X_g}{2} = 5060$, the real average density in the air being $B_g = 6620$.

$B_a = 5000$, and the length of path corresponding to an arc between the pole-tips and at a radius below the slots gives $\frac{l_a}{2} = 4{\cdot}3$

cm.; $f'(B_a) = 2{\cdot}4$ so that the loss of potential over the armature core only demands some 10 AT, and is practically negligible. As the density in the teeth is also low, the loss of magnetic potential over them may roughly be taken into account by doubling the loss over the armature core. Combining $\frac{X_g + X_a + X_t}{2} = 5080$ with $\frac{X_a}{2} = 1055$, vectorially, as shown graphically in fig. 329, the angle ϕ_o' being $34{\cdot}85^\circ$, $\frac{X_P}{2} = \sqrt{5080^2 + 1055^2 + 2 \times 5080 \times 1055 \sin \phi_o'}$

$= 5750$ ampère-turns.

The leakage flux is then $1{\cdot}257\ X_P \,.\, P = 724{,}000$, and $Z_m = Z_a + \zeta = (3{\cdot}085 + 0{\cdot}724) \times 10^6 = 3{\cdot}809 \times 10^6$, whence $\nu = 1{\cdot}235$, which agrees nearly with the assumed value when determining the size of the magnet core. Since, however, it is slightly higher, and the core is more highly saturated than was anticipated, it will be advisable to make the core rather more oval with an extreme length of 8″ and an extreme width of 4·7″ which with allowance for the rounded corners will give an area of 33·4 sq. in., or 215 sq. cm. The leakage permeance will be hardly affected by this change. The length of path in the magnet-core $= 6{\cdot}75'' \times 2{\cdot}54 = 17{\cdot}1$ cm., and in the yoke up to the dividing line between the poles is, say, $5'' = 12{\cdot}7$ cm. In order to allow for the magnetic joint between the core and the yoke at the point of maximum flux, it will be safer to assume an equivalent air-gap rising to about $\frac{1}{4}$ mm. with the full-load density within it of $B_m = 17{,}700$. Hence

$$B_m = 17{,}700 \qquad f'(B_m) \times \frac{l_m}{2} = 86 \times 17{\cdot}1 = 1470$$

$$\text{Allowance for joint} = 0{\cdot}8 \times B_m \times 0{\cdot}025 \qquad = 354$$

$$B_y = 7{,}490 \qquad f'(B_y) \times \frac{l_y}{2} = 4{\cdot}1 \times 12{\cdot}7 = 52$$

$$\overline{1876}$$

Adding $\frac{X_m + X_y}{2} = 1876$ to $\frac{X_P}{2} = 5750$, we find the total ampère-turns to be provided on each pole to be $\frac{X}{2} = 7626 = AT_c$.

The open-circuit or no-load characteristic is quickly calculated in tabular form as below, and is plotted in figs. 328, 331 and 332.

z_a in 10^6 C.G.S. lines	1		1·5		2		2·5		2·75		3		3·25		3·5	
$\frac{X_g}{2} = 1640 z_a$		1640		2160		3280		4100		4510		4920		5330		5740
$\frac{X_a}{2} =$		4		5		6		8		9		10		12		15
$\frac{X}{2}$		4		5		6		8		9		10		12		15
$\frac{X_P}{2}$		1648		2470		3292		4116		4528		4940		5354		5770
$\zeta = 1·257 \times X_p \times P$	0·207		0·31		0·414		0·516		0·57		0·62		0·675		0·726	
$z_m = z_a + \zeta$	1·207		1·81		2·414		3·016		3·32		3·62		3·925		4·226	
$B_m = \frac{z_m}{·215}$	5620		8420		11,200		14 000		15,400		16.800		18,250		19,650	
$f(B_m)$	1·6		3·1		4·8		12		20		48		116		250	
$\frac{X_m}{2} = f(B_m) \times 17·1$		27		53		82		205		342		820		1980		4272
Allowance for joint		40		75		140		210		280		300		365		415
$\frac{X_y}{2}$		5		8		15		24		35		48		75		115
$\frac{X}{2}$		1720		2606		3529		4555		5185		6108		7774		10,572
E.M.F. per arm. phase	410		615		820		1025		1130		1230		1335		1438	
E.M.F. between lines	710		1065		1420		1775		1960		2130		2310		2490	

It is evident that on open-circuit the full-load excitation will give 2300 volts, or the rise will be 300 volts = 15 per cent. of the terminal volts as required. It is interesting to note that on a completely inductive load fig. 331 shows but little increase in the excitation required for the full terminal voltage, viz., 7800 A T per pole as against 7626. This result is due partly to the fact that the loss of volts over the armature resistance is then entirely out of phase with the terminal voltage and in our instance $2IR_a$ bears a somewhat exaggerated proportion to e_{sa}; but still more is it due to the fact that the method of fig. 331 is not entirely accurate for a highly saturated magnet, and when the excitation for 2000 volts with $\cos\phi_e = 0$ is strictly calculated, the ampère-turns per pole are found to be more nearly 8175.

After making allowance for a thick end-plate under the pole-shoe to hold the coil in place and for insulation, the net winding length is, say 5·6″. The periphery of the pole when insulated is 23·6″, and if the radial depth of the turns is about 1·6″, the mean length of the turn is $l_s = 29{\cdot}25'' = 2{\cdot}44$ ft. Thence by eq. (46) if the exciting voltage is 100,

$$\omega' = \frac{100 \times 1000}{7626 \times 64 \times 2{\cdot}44 \times 1{\cdot}15} = 0{\cdot}073 \text{ ohm}$$

and the necessary area of the wire is 0·1115 sq. inches. A rectangular section wound edgewise gives a thin strip difficult to wind, and too great a loss of space in insulation, so that a rectangular wire 0·385″ × 0·29″ double-cotton-covered to 0·41 × 0·315 and wound flatwise will be chosen. Deducting one turn from the quotient $\frac{5{\cdot}6}{0{\cdot}41}$, we obtain 12½ turns per layer and five layers, making 62½ turns per pole or 4000 turns in all, of length 9760 ft. Thus $R_s = 0{\cdot}073 \times 9{\cdot}76 = 0{\cdot}712$, or when hot, 0·82, and $A_s = \frac{100}{0{\cdot}82} = 122$ ampères. The total excitation loss is therefore 12,200 watts, or 2·52 per cent. of the net output. The current-density is 1100 ampères per sq. inch, and the cooling surface of 180 sq. inches per pole or 1150 sq. inches in all with its ratio of 1·06 watts per sq. inch will lead to a small rise of temperature in agreement with the low figure assumed for the mean heating coefficient of the coil throughout its entire depth.

§ 7. **Parallel working of alternators.**—The series operation of alternators is not possible, unless they are connected together by a rigid mechanical coupling. If only connected together electrically, they exert no mutual control on one another; if for any reason one machine falls slightly behind another in phase, the lagging machine has the greater part of the load thrown upon it, which causes it to lag still more, until finally the two settle down into exact opposition of phase, their E.M.F.'s neutralise each other, and no current flows.

Alternators can, however, be run in parallel, the mutual control which they exert electrically upon one another being in most cases sufficient to keep them 'in step,' as it is termed. If two alternators be running at the same frequency, and further with equal E.M.F.'s rising and falling together, that is to say, if they *synchronise* in phase, their armature circuits may be coupled together, and they will continue to run in parallel, dividing the load between them in proportion to the power supplied to each. If a generator is to be thrown into parallel with another, or is to be joined to a network on which other generators are already working, then at any instant there should be no difference of potential between any pair of corresponding terminals which are to be united together, so that no cross current will flow between the machines due to such difference of potential. Thus the necessary conditions for parallel operation are (1) *equality of frequency*, (2) *synchronism of phase*, and (3) *equality of voltage*. The general principle of the connections for working two low-tension three-phase alternators in parallel is shown in fig. 369. Machine A is at work, its triple-pole switch S being closed on the three leads, *a*, *b*, *c* of the distributing network. The current in one phase is read on the ammeter I, while the wattmeter W measures the power of one phase, its main terminals being in series with one main lead and its shunt coil being connected to the ends of one arm of the Y winding, *i.e.*, between *a*' and the centre of the Y winding. The shunt-wound exciter E supplies the exciting current to the field-magnet M with a regulating rheostat R in series. The machine B is now run up to its full speed, and its excitation regulated by means of the rheostat R′ in series with its field-winding until the voltmeter V′ gives the same reading as the voltmeter V. The effective voltages of the two machines are now alike, and assuming their E.M.F. curves to be similar, so also are the maxima values of the E.M.F.'s. A further and finer regulation of the speed and phase must now be made by means of the phase lamps PP belonging to machine B. It will be seen that in virtue of their connection they are in series between two arms *a′* and *c′* of the armature winding on the one side and two wires *a* and *c* of the network on the other side, so long as the main switch of B is open; they therefore indirectly measure by their light the difference of potential between these points, and when this is zero, their light goes out, and the triple-pole switch of B may be closed.

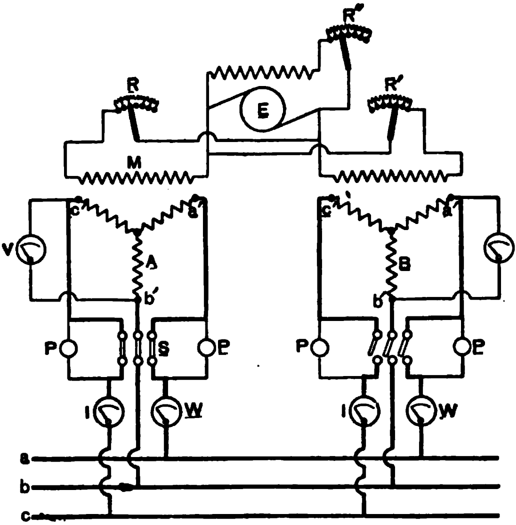


FIG. 369.—Connections for two three-phase machines to be worked in parallel.

Let us now suppose that the periodicity of machine B is not quite the same as that of the network or of machine A. Starting from a point of time when the instantaneous voltage of both machines is zero, let the dotted curve E_1 (fig. 370) represent the instantaneous E.M.F. between the wires *a* and *c*, while the thin curve E_2 represents the instantaneous E.M.F. between the terminals *a′c′* of machine B. The algebraic sum of the two shown by the heavy full line *e* gives the difference of potential on the lamps, proportional to which is the current through them. From the nature of the resultant curve it will be seen that the lamps are traversed by an alternating current of which the amplitude waxes and wanes in accordance with the effective value of the current as determined by its amplitude at the time; in other words, periods of brightness and darkness alternate. The number of complete waves of the alternating current through the lamps itself coincides with the number of periods of the more rapid of the two periodic voltages, and one 'beat' or pulsation of the light from complete darkness to maximum brightness and back to darkness occurs in the time in which the number of complete periods passed through by the voltage E_2 and the voltage E_1 differs by 1 (in our figure 8 and 7 respectively), *e.g.*, if the periodicity of the machine A be $f = 50$, and that of machine B is only 49, at the end of one second the difference of their numbers of periods will be 1, and the lamps will have passed through one pulsation. One beat per second of the light will therefore indicate a difference in the periodicities of $\frac{1}{50} = 2$ per cent. But by regulation of the driving power each pulsation of the light may easily be extended to, say 5 seconds,

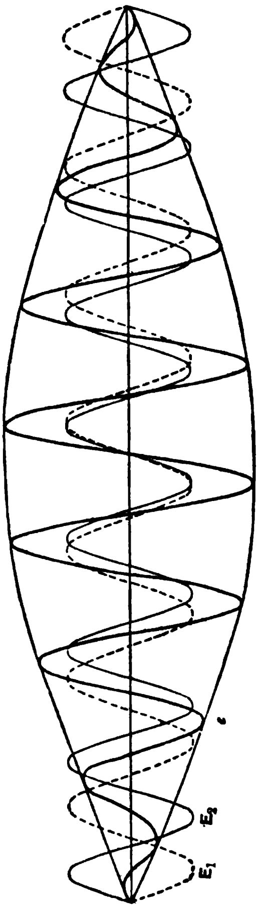


FIG. 370.—Pulsating voltage on phase lamp.

when the difference of the periodicities will be only $\frac{2}{5}$ per cent. Complete synchronism may not be obtained, yet if the triple-pole switch is now closed when the lamps are dark, but little cross current will flow, and the two machines will fall into step.

The two instruments of fig. 369 enable a machine to be conveniently taken out of parallel as follows. The steam stop-valve or inlet gate of the turbine is closed until the wattmeter reads zero. If the ammeter still shows some cross current passing, the field rheostat of the machine is altered until it is reduced to zero, when the main switch can be opened without disturbance to the machines still remaining at work and without spark.

§ 8. **Phase-indicator.**—The arrangement of phase lamps shown in fig. 369 is the simplest form of *synchronizer* or *phase indicator*, to insure that the difference of phase between the two machines may not be too great at the moment when they are thrown into parallel. It is, however, only suitable for low-tension machines, and many other devices are employed based on the same or similar principles. Thus the method may be immediately adapted to high-tension machines, if instead of the phase lamps being directly connected between the corresponding terminals the primaries of two small transformers be inserted, to the secondaries of which the lamps are applied as before. A still more common variation is to employ a transformer with two primaries wound on it; one of these p_1 is connected to the bus bars, and the other p_2 to the terminals of the alternator which is to be thrown into parallel as in fig. 371, which shows the arrangement for single-phase machines. A common secondary *s* has its terminals connected to an incandescent lamp, and the advantage is gained that the phase is indicated without any direct connection of the machine to the network, so that when the phases are in synchronism, both poles of the incoming alternator B may be simultaneously closed on the bus bars by means of the double-pole switch S. The primaries may be arranged to oppose one another when the machines are in step so that the lamp is dark at the proper moment for closing the switch; or they may be arranged so that when the E.M.F.'s synchronise the lamp burns brightly. The first is perhaps the best, since the period of total darkness is better distinguished than the period of maximum brightness, but it has the disadvantage that rupture of the filament of the lamp might happen to exactly coincide with the coming of the machines into step. A similar arrangement may also be used with a paralleling voltmeter in place of the lamps.

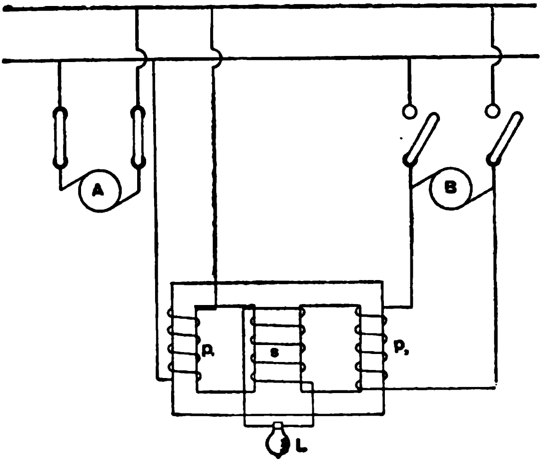


FIG. 371.—Phase-indicator for parallel working.

§ 9. **Effect of want of synchronism of phase.**—In the case of two similar machines with the same E.M.F. and frequency, but not quite in phase when coupled together, let their E.M.F.'s be represented by the dotted lines E_1 and E_2 of fig. 372. These E.M.F.'s may also be shown as nearly in direct opposition with respect to the cross circuit through their armatures just as in fig. 370, in which case the synchronising E.M.F. causing the cross current is the algebraic sum of the two; for our present purpose, however, they are more suitably shown as nearly alike in phase with reference to the external circuit, but the one leading the resultant E.M.F. of the bus bars E by the angle

$\frac{1}{2}\beta$, and the other lagging behind the resultant E.M.F. by the same angle. The E.M.F. which causes the cross current is then the algebraic difference of E_1 and E_2, and, as shown by the thin full line e, this is nearly in quadrature with either of the machine E.M.F.'s. To the left of the figure is given the corresponding clock diagram, the moment chosen for the translation into a system of rotating vectors being that at which the cross E.M.F.(e) is a maximum. To this E.M.F. corresponds a certain cross current, $I_c = \frac{e}{\text{impedance of cross-circuit}}$ lagging behind the E.M.F. by an angle ϕ_a such that $\tan \phi_a = \frac{\text{reactance}}{\text{resistance}}$. This current I_c may be resolved into its two components,—I_h or the energy component

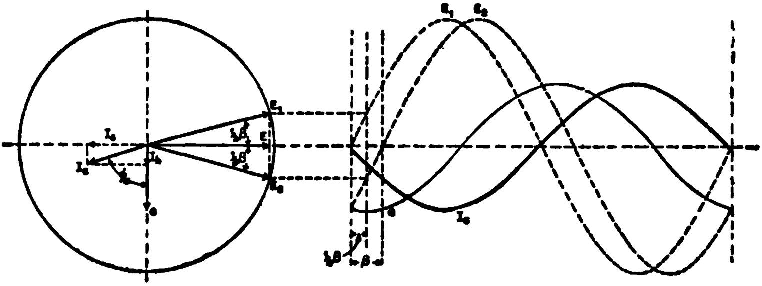


FIG. 372.

which is in phase with the cross E.M.F. and simply heats the armatures of both machines, and I_s which is the synchronising current proper. The latter is wattless in relation to the cross E.M.F., being at right angles to e, but on the same account it is nearly in phase with the E.M.F. of one machine, and in opposition to that of the other. The former machine therefore acts as a generator and drives the latter as a motor, so that it is only in virtue of this second component of the current that the machines are drawn into step. The value of the cross current through each armature is

$$\frac{\frac{1}{2}e}{\text{impedance of one armature}} = \frac{E_1 \cdot \sin \frac{1}{2}\beta}{\sqrt{R_a^2 + 4\pi^2 f^2 L_a^2}},$$

$$\text{and } I_s = I_c \cdot \frac{\text{reactance}}{\text{impedance}} = E_1 \cdot \sin\frac{1}{2}\beta \cdot \frac{2\pi f L_a}{R_a^2 + 4\pi^2 f^2 L_a^2} \cdot$$

A certain amount of reactance is therefore a necessity if there is to be any transfer of energy from one machine to the other; further, with a given R_a and phase difference, the maximum synchronising current would be reached when $\frac{2\pi f L_a}{R_a^2 + 4\pi^2 f^2 L_a^2}$ is a maximum, *i.e.*, if the reactance were equal to the resistance.[1] Such a low value of the reactance would, however, subject the machines to too great a mechanical strain, so that the higher ratios of the reactance to the resistance which alone occur in practice are the more suitable even from the point of view of easy parallel working.

§ 10. **Effect of inequality of frequency due to incorrect speed.**—The action in our first case of § 7 when the machines had not quite the same frequency is analogous, but is not so easily represented diagrammatically. If the E.M.F.'s of the two machines at the moment when they are switched into parallel are not in direct opposition with regard to the cross circuit or as soon as a resultant difference of potential arises between them, a cross current grows up, which is no longer determined almost entirely by the resistance of the lamps, but by the synchronous impedance of the armatures in series. The phase of the cross current is thereby shifted more nearly into agreement with the E.M.F. of the machine which is at the moment leading and into opposition to the E.M.F. of the lagging machine. In other words, the load is not shared equally between the two machines, and the falling into step comes about through the action of the mechanical governors of the prime movers. The greater load thrown on to the leading machine lowers its speed slightly as compared with that of the lagging machine which rises. In order therefore that the two may come into step easily, a fairly long range of speed from no load to full load is desirable, so that any inequality of the loads on the two machines may cause a definite change of speed in each case and bring them into synchronism. Further, in order that the loads may be equally divided after the machines have been thrown into parallel, the curve connecting speed and load should be similar for the two machines, and a very close mechanical regulation is not to be recommended, since with it a slight inaccuracy of speed means a large variation in the power delivered by the engine.

---

[1] $I_s = I_c \cdot \sin\phi_a$, and $I_c = \frac{E_1 \sin\frac{1}{2}\beta \cdot \cos\phi_a}{R_a}$, hence

$I_s = \frac{E_1 \sin\frac{1}{2}\beta}{R_a} \cdot \sin\phi_a \cdot \cos\phi_a$ is a maximum when $\sin\phi_a \cdot \cos\phi_a$ is a maximum, and this occurs when $\phi_a = 45^\circ$.

§ 11. **Effect of inequality of voltage.**—If the frequencies of the two machines are identical and their phases synchronous, but the E.M.F. of one E_1 is greater than that of the other E_2, the difference again causes a cross current to flow between the two. But in this case since the difference of voltage which causes the current is in phase with the E.M.F.'s of the machines, and the reactance is under ordinary conditions many times higher than the resistance, the cross current is itself nearly in quadrature with the induced E.M.F. It is therefore almost wattless and no energy is transferred by it from one machine to the other. Yet since it lags in the machine of greater E.M.F., it partially demagnetises it, while in the machine of weaker E.M.F. it leads and so produces forward ampère-turns. Thus its effect is purely magnetic, and it rises to such an amount that the terminal E.M.F.'s of the two machines are equalised. The lower the armature reaction, or in other words the greater the short-circuit current for a given excitation, the greater the cross current which must flow in order to counterbalance a given difference in the field strengths of the two machines. Hence at least a moderate amount of armature reaction is on this score to be desired, so as to prevent the cross current becoming unduly high. It is, however, evident that equality of voltage is not so supremely necessary for parallel operation as equality of frequency and synchronism of phase, since the cross current due to inequality of voltage is practically wattless.

When once thrown into parallel, equal division of the load between two alternators cannot be secured as in continuous-current dynamos simply by altering the excitation of one. Any such change only alters the phase angle of the current relatively to the E.M.F., or in other words, causes a cross current magnetising one machine and demagnetising the other, so that the voltage of the entire system is affected. Hence it is desirable that all generators which are worked in parallel should have similar load characteristics, in which case when in parallel their separate field rheostats may be mechanically coupled together, or the exciting voltage of all may be simultaneously altered for any change of load by altering the rheostat in the shunt circuit of the exciter.

§ 12. **Effect of inequality of frequency due to want of uniformity of speed.**—Inequality of frequency is, however, not necessarily confined to the moment when two alternators are thrown into parallel. When running together, their average frequencies, as determined by the mean speed of the prime movers to which they are coupled, may be the same, yet their instantaneous frequencies may differ if the speeds of the prime movers vary periodically during one or more revolutions. Owing to their perfectly uniform speed of rotation, the waterwheel and turbine afford an ideal means of driving alternators which are to be run in parallel. But in the case of an alternator connected directly to a gas- or steam-engine, the want of complete uniformity in the angular speed of the prime mover has important consequences. With a single-cylinder steam-engine there are two points of maximum speed and two minimum points in each revolution; and so also in a compound engine if the cranks are set at 180°, while if the cranks are set at 90°, there are four maximum and four minimum points in each revolution. Unless therefore the moment of paralleling when the electrical phases are identical happens or is arranged to coincide with synchronism of the mechanical phase, there will result a cyclical displacement of the relative phases of the two machines although their mean frequencies are the same. It is then always possible that the maximum speed of one machine may coincide in time with the minimum speed of the other machine, and if the one runs ahead of its mean position by an electrical angle $\frac{1}{2}\beta$, the other may lag behind the mean position by the same angle, so that the total phase displacement is β. The maximum cross current $I_c = \frac{E_1 \cdot \sin\frac{1}{2}\beta}{\text{impedance of one armature}}$ may also be expressed in terms of the short-circuit current, $I_0 = \frac{E_1}{\text{impedance}}$; *i.e.*, $I_c = \sin \frac{1}{2} \beta . I_0$; thus in fig. 372, $\frac{1}{2} \beta = 15°$, and $I_c = 0{\cdot}26\ I_0$. Now it has already been shown that the better the regulation of the alternator, the larger the short-circuit current as compared with the full-load current, but the corollary of a large short-circuit current is that the cross current due to any inequality in the instantaneous speeds of the prime movers is much increased. With a large transfer of energy from one machine to the other or interchange of the load, rhythmic hunting may be set up, and if reinforced by any approach to the natural period of oscillation of the governor or other revolving masses, may increase to such a degree as to throw the machines out of step, or at least to cause a periodic rise and fall of the voltage of the system. Hence from the point of view of parallel operation when coupled to steam-engines, a certain amount of armature reaction is necessary to avoid too large cross currents due to fluctuation of their speeds; yet on the other hand it must not be so high as to prevent the machines pulling one another into step at the moment of switching into parallel.

§ 13. **Permissible displacement of electrical phase.**—After every precaution has been taken to balance the reciprocating parts and to distribute the load as equally as possible between the several cylinders of the engine, the want of uniformity in the speed which still remains must be brought within practical limits by the addition of sufficient flywheel effect in the machine as a whole. The heavy revolving mass of the magnet system and even a comparatively light armature of the revolving disc type will assist considerably, but may require to be supplemented by the flywheel proper, and in order to reduce the size and cost of this as much as possible, it becomes of importance to determine the maximum deviation from a perfectly uniform speed which is compatible with ease of parallel working. As has been already stated (Chap. XXIV § 13) the greater the number of the poles, the more uniform should be the speed, and any general formula for the permissible want of uniformity should be expressed in terms of degrees of the electrical phase.

A practical limit for the degree of irregularity of speed is given by the rule that the maximum displacement from the mean position corresponding to uniform rotation must not exceed $2\frac{1}{2}^\circ$ in the electrical phase. Rotary converters form a very sensitive load, since owing to their great inertia they tend to maintain a constant speed and so run ahead of or fall behind the alternator, should the speed of the latter vary; yet if the deviation of the electrical phase on either side of the correct value for perfect uniformity of speed does not exceed $\theta = 2\frac{1}{2}^\circ$, even the most delicate load can be run with freedom from hunting.[1] Since the passage of a coil past a pair of poles yields one complete wave equivalent to 360 electrical degrees, the mechanical degrees of displacement from the true mean position must not exceed $\frac{2\frac{1}{2}^\circ}{p}$, where p is the number of pairs of poles in the heteropolar alternator. In other words, if a point on the revolving portion of the alternator or on the flywheel were marked, and its position relatively to that which it would have with perfectly uniform rotation could be followed by the eye, its maximum displacement from this latter position on the circumference of the circle which it traces must not be found to exceed $\frac{\theta}{p} = \frac{2\frac{1}{2}^\circ}{p}$. A standard speed for an engine-driven alternator for 1500 or 2000 kilowatts would be about 75 revs. per min., giving 40 poles or $p = 20$ in a generator for 25 cycles. The maximum angular displacement would then be $\frac{2\frac{1}{2}^\circ}{p} = 0{\cdot}125$ of a mechanical degree. If the same output had to be obtained at a frequency of 60, the number of poles must be increased to 96, and for the same want of uniformity of engine speed the phase displacement would be $2\frac{1}{2}^\circ \times \frac{96}{40} = 6$ electrical degrees, which might render parallel working impracticable. Hence in the latter case for the same phase-displacement of $2\frac{1}{2}^\circ$ to be attained, a much heavier flywheel must be employed, if a more even turning moment cannot be secured by a different distribution of the steam in the cylinders. An excessively heavy flywheel is, however, to be avoided for other reasons, since the governors of the engines running in parallel are then more liable to hunt. It may thus become preferable to adopt a low frequency which will secure freedom from hunting, having regard to the necessary want of complete uniformity of speed in the driving engines.

---

[1] If I_0 is $2\frac{1}{2}$ times the full-load current, the maximum cross current is then, by the reasoning of § 12, 10·9 per cent. of the full-load current.

§ 14. **The permissible variation of speed.**—The requirement that the displacement in space of the alternator should not exceed $\frac{2\frac{1}{2}^\circ}{p}$ still needs to be expressed in terms of velocity and time for the purpose of designing the steam-engine and its flywheel, since the amount by which the rotor forges ahead of or lags behind its proper position entirely depends upon the time during which the increased or lessened velocity acts. From the indicator diagrams[1] combined with a consideration of the inertia of the reciprocating parts must be plotted a torque curve in relation to time, thence an acceleration-time diagram, and finally a velocity-time diagram. The integral of this last curve plotted in relation to the mean velocity as a base line will give at any moment the space which has been traversed away from the mean position. When thus considered, it is evident that the maximum permissible variation of velocity depends upon the time during which the excess speed acts, and that it may be higher in an engine with two cranks than in one with a single crank, since in the former case the increased velocity only lasts half the time of the latter case. If the approximate assumption is made that the actual acceleration curve is replaced by a uniform acceleration of the same mean value, and if $\delta = \frac{v_{max} - v_{min}}{v}$, or the fraction which the maximum difference of speed is of the mean speed v, the maximum variation from the mean will be $\frac{\delta v}{2}$, and the excess of speed averaged over the time during which it lasts will be half this amount again or $= \frac{\delta v}{4}$. The mechanical angle passed through at the mean velocity v during the same time depends upon the time during which the excess speed lasts as determined by the number of impulses in a revolution; that is, if n be the number of maximum points of speed in a revolution, the mechanical angle passed through at the mean velocity is $\frac{360^\circ}{2n}$. The ratio therefore of the average excess speed to the mean speed will give the ratio of the mechanical displacement to $\frac{360^\circ}{2n}$, or

---

[1] See especially Kruesi, 'Speed Variations of Engines Direct-connected to Alternators,' *Electrical World*, vol. xxxvii. p. 591 ff.

$$\frac{\frac{\delta v}{4}}{\frac{v}{1}} = \frac{\text{mechanical degrees}}{\frac{360^\circ}{2n}} = \frac{\frac{\theta^\circ}{p}}{\frac{360^\circ}{2n}}, \text{ or } \delta = \frac{\theta^\circ \times 2n \times 4}{p \times 360^\circ}. \quad . \quad (121)$$

where θ° is the electrical phase-displacement. Thus in a single-cylinder engine, $n = 2$ or the average excess lasts during a quarter of a revolution, and $\delta = \frac{\theta^\circ}{p} \times \frac{16}{360^\circ}$. If θ is limited to $2\frac{1}{2}^\circ$, we then have $\delta = \frac{2{\cdot}5^\circ}{p} \times \frac{16}{360} = \frac{0{\cdot}11}{p}$ or $\frac{11\%}{p}$.

With two cranks set at right angles, the excess speed only lasts for one-eighth of a revolution, or $n = 4$, so that $\delta = \frac{\theta^\circ}{p} \times \frac{32}{360^\circ}$; the permissible variation of speed for the same limiting value θ° of the phase-displacement is thus twice that of the single-cylinder engine, or if $\theta = 2\frac{1}{2}^\circ$, $\delta = \frac{22\%}{p}$. A three-crank engine is only slightly better than a two-crank engine when the distribution of the load between the several cylinders and the inertia of the reciprocating parts are taken into account and indeed in some cases may even be inferior.

It is now evident that just as the permissible mechanical angle of displacement varies inversely with the number of poles, so also does the permissible variation of the speed, and if the mechanical displacement from the mean is not to exceed $\frac{2\frac{1}{2}^\circ}{p}$, the total fluctuation of the speed expressed as a percentage of the mean speed, or $\frac{V_{max} - V_{min}}{V} \times 100$, in a compound steam-engine with 2 cranks at 90° may be assumed to be of the order of $\frac{20\%}{p}$. In the case of the 60-cycle alternator instanced above, this would give a total fluctuation of speed less than one-half of one per cent., or strictly $\delta = \frac{1}{240}$. In many cases, however, the permissible δ is stated without reference to the number of poles, although in this form the expression is not so scientifically correct. Thus with a normal number of poles and under average conditions it is sometimes laid down that the total fluctuation in large machines which are to be run in parallel must not exceed $\frac{1}{250} = 0{\cdot}4$ of 1 per cent., or for smaller machines $\frac{1}{180} = 0{\cdot}55$ of 1 per cent., while above these limits damper winding becomes indispensable.

It may again be mentioned in conclusion that the possible amount by which two alternators running in parallel may be out of phase or the angle β in § 12, is twice the angle θ, since one machine may be ahead of its correct position by as much as the other is behind it.

§ 15. **Calculation of necessary flywheel effect.**—The flywheel effect required to keep the variation of speed within any assigned limits is calculated as follows. Let a complete cycle of speeds be performed in y revolutions; usually y is some fraction of a revolution, but in the case, *e.g.*, of a single-cylinder gas-engine after one explosion two revolutions may be required to complete a cycle of operations before the second explosion occurs. The mean power is H.P × 33,000 ft.-lbs. per minute, and the mean energy developed in the y revs. is $E = y \times \frac{33{,}000 \text{ HP}}{N}$ ft.-lbs. or $= y \times \frac{33{,}000 \text{ HP}}{2240 \text{ N}}$ ft.-tons. If W be the weight of the flywheel, and V be the mean velocity in ft. per sec. of its rim at the radius of gyration, the energy stored in the flywheel at its maximum work which it does in retarding or accelerating the rotating mass has the effect of actually increasing the possible angular displacement due to the unequal turning moment of the prime-mover. A marked distinction must in fact be drawn between the action of the synchronising current in the case of a transient disturbance as in switching into parallel and its action when rhythmically recurring impulses which never cease so long as the engine is running cause periodic oscillations of speed. In this latter case when the speed has its mean value, the displacement is a maximum, the machines are out of phase, and the synchronising torque which is proportional to the displacement also reaches its maximum; the excess or deficiency of the driving torque is then in either case augmented by the synchronising torque, causing the speed to diverge from the normal still more widely than it otherwise would, as may be seen from the following considerations.[1] The curve of the driving torque relatively to time may be regarded as made up of a variable portion, alternately positive or accelerating and negative or retarding, superposed on a constant torque as a base line; the periodicity of the variable portion is $\frac{60}{N \times n}$, where n is the number of impulses per revolution. The speed being proportional to the integral of the driving torque, the curve of the variable portion of the speed plotted above and below the mean speed as a base line has the same periodicity but lags a quarter of a period (*i.e.*, in a single-cylinder double-acting engine $\frac{1}{8}$th of a revolution) behind the curve of the variable portion of the driving torque. The space traversed being the integral of the speed, the variable part of the space traversed or the angular displacement again lags a quarter of a period behind the variable part of the speed or half a period behind the variable part of the driving torque. The synchronising torque is proportional to the displacement, but as it always tends to annul the displacement its phase agrees with that of the varying torque, or its curve is that of the angular displacement when inverted. Take the moment when the crank-pin is passing the dead centre; the rotor is then in advance of the position corresponding to uniform speed owing to the previous accelerating action of the excess driving torque, hence the synchronising torque is holding back the rotor; at the same moment the driving torque is a minimum, or in other words the variable part of the driving torque is negative and also retarding the rotor. When the driving torque is normal, the speed is a maximum or a minimum, but there is no displacement and no synchronising torque to check it. Thus the synchronising torque acts in harmony with the variable part of the driving torque, and the amplitude of the oscillations is increased. M. Boucherot has shown that if the effect of any electrical damping be neglected, the mechanical angle of displacement is $\propto \frac{T_0}{n^2 I \omega^2 - T_s}$ where T_0 is the maximum value of the excess driving torque; T_s = the synchronising torque, and its appearance in the denominator with a negative sign shows that it increases the angular displacement and acts against the effect from the moment of inertia (I) of the rotor. If $T_s = n^2 I \omega^2$, the mechanical displacement should be infinity, and this is in fact the case when the periodic time of oscillation of the rotor as determined by its mass is equal to the periodic time of the variable part of the driving torque. If this critical condition holds or is too nearly approached, the phenomenon of resonance magnifies the pendulum action, and the oscillations once started by the small amount of irregularity which must persist even in an exceptionally good engine will increase to such an extent that the machine is eventually thrown out of step entirely. If $T_s < n^2 I \omega^2$ but is too nearly equal to it, the remedy must be to increase the flywheel effect, but in the reverse case of $T_s > n^2 I \omega^2$, a decrease in the flywheel will cause a greater divergence between the two quantities, and in certain exceptional cases in practice a reduction of the rotating masses has been found to be attended with good results.[1]

---

[1] Boucherot, *Bulletin de la Société Intern. des Électr.* 2nd series, vol. i. pp. 529–553 (abstracted in *L'Eclairage Electrique*, vol. xxx. p. 287; Longwell, 'Paralleling of Alternators,' *Electr. World and Engineer*, vol. xxxix. p. 958; Oscanyan, *Electr. World and Engineer*, vol. xl. p. 416.

---

[1] For fuller treatment of the subject the reader may be referred to P. Boucherot, *La Lumière Électrique*, vol. xlv. pp. 201 and 260; A. Blondel, *Bull. Soc. Inter.*, 1893, p. 147; Kapp, *E.T.Z.*, vol. xx. p. 134, 1899; L. Wilson, *Journal Inst. Electr. Eng.*, vol. xxviii. p. 395, 1899; G. Benischke, *E.T.Z.*,

§ 16. **Damper winding.**—For the purpose of rendering polyphase machines more stable when run in parallel and more independent of any fluctuation of the prime movers' speed, M. Leblanc introduced the device of an additional *damper* or '*amortisseur*' winding. This consists in principle of a number of copper bars embedded in holes close to the edges of the pole-pieces parallel to the armature wires or slots, and riveted at their projecting ends to two solid rings of copper or bronze.[1] The damper system is thus closely analogous to the squirrel-cage armature of an asynchronous induction motor, and its action is to be explained on precisely the same principles.[2] The eddy-currents which would flow in a solid pole-piece are canalised or confined to paths at right angles to the flux-direction and are rendered fully effective. The progressive rise and fall of the current in the armature windings of a polyphase generator sets up a rotary field similar to that caused by the primary coils of an induction motor; when only one generator is at work, this field rotates in exact synchronism with the field-magnet, so that the bars of the damper winding are in no way cut by it, and the same is true if the speed agrees precisely with the alternations of the current in the case of two machines running in parallel. But if there is phase-swinging between the two machines due to irregularity of speed of the prime movers, the rotating field-magnet regarded as a secondary lags or leads at regular intervals, and so has a certain slip relatively to the armature regarded as a primary. The secondary currents thereby set up in the damper winding have the effect of assisting the machine to recover exact synchronism or to rotate in unison with the mean speed of the two machines which are running in parallel. On the analogy of a squirrel-cage armature, the primary pressure impressed on the lagging machine through the armature coils is that of the cross current, or, as in § 12, $= \text{E}_1 \sin \frac{1}{2} \beta$, and neglecting

---

vol. xx. p. 870, 1899; H. Görges, *E.T.Z.*, vol. xxi. p. 188, 1900; *Proc. Amer. Inst. E.E.*, Oct. 25, 1901; E. Rosenberg, *E.T.Z.*, vol. xxiii. p. 450, 1902.

[1] Cp. *Electr. Eng.*, Nov. 20, 1896, vol. xviii. p. 580; *Electrician*, Sept. 28, 1900, vol. xlv. p. 844.

[2] Cp. Fischer-Hinnen, *Electr. World and Eng.*, vol. xxxviii. p. 1058.

the copper losses, the no-load current of the motor is determined by the inductance alone, or is $\frac{E_1 \sin \frac{1}{2}\beta}{2\pi f L_a}$, *i.e.*, is identical with the synchronising current proper, as above considered. But since owing to the fall in speed of the driving engine a load is thrown on the secondary, the actual motor-current in the primary is greater than the no-load current in order to counterbalance the secondary current in the squirrel-cage winding; it may therefore be expressed as some multiple of the no-load current, and the pull tending to bring the machine up to speed is equally increased. Taking the most unfavourable moment in a single-cylinder engine, and making the same assumption as to the acceleration as in § 14 the electrical degrees $\frac{1}{2}\beta$ are $= \frac{\delta \times 2\pi \times p}{16}$, or $\beta = \frac{\delta \times \pi \times p}{4}$, whence the primary pressure can be calculated. The angular velocity of the rotary field of the primary is $\omega_1 = 2\pi f$, and the relative velocity of the secondary is $\omega = \omega_1 \times \text{slip} = \omega_1 \times \frac{\delta}{2}$. An approximate estimate can then be made on this basis as to the pull on the field-magnet due to the currents in the damper winding, and the work done by this pull in the time of one quarter of a revolution may further be compared with the energy given out by the flywheel in the same time. Since in the time considered the speed of the flywheel is supposed to have fallen from its mean value to v_{min}, the energy which it has given out is half of that in § 15, or $= \frac{W}{g} . v^2 . \frac{\delta}{2}$. At the same time the speed of the second machine has been similarly retarded, so that the whole combination is held in step against irregularities of speed with much greater power than would be the case without the damper winding.

It is not necessary that the rings short-circuiting the damper bars should be continuous, provided that there is no interference in the paths round which the secondary currents flow, and this will be ensured if there are only small gaps occurring midway between the poles. Further, the bars of copper may in the interpolar gaps be replaced by solid brass pieces. Fig. 373 shows a form of damper winding employed by the Allgemeine Elektricitäts Gesellschaft in which each pole is self-contained; the pole-faces are encircled by brass castings into which a few round copper bars passing through the laminations of the pole are riveted. The casting at the same time serves to hold in place the winding of the field-coils.

In the single-phase alternator the action of the damper winding is quite different. The secondary currents set up in it have the effect of partially neutralising the armature ampère-turns, and so of reducing the armature reaction or of increasing the output for the same drop of volts. The field of the armature does not rotate, yet although stationary, it pulsates in strength; it may therefore be replaced in imagination by two rotary fields, each of constant strength equal to half the maximum of the varying field. The one component field rotates synchronously with the magnet or the damper, and has therefore no effect on it. The other rotates at the same speed of synchronism but in the opposite direction, so that its velocity relatively to the damper is $2\omega_1$, and currents are set up in the damper of twice the periodicity of the machine. These currents are nearly opposed to the ampère-turns causing the field to which they are themselves due, so that in the ideal case the inductance of the armature should be halved. In practice so large a reduction is not quite realised owing to the leakage of lines in the relatively large air-gap between the armature coils and the damper, but there is a

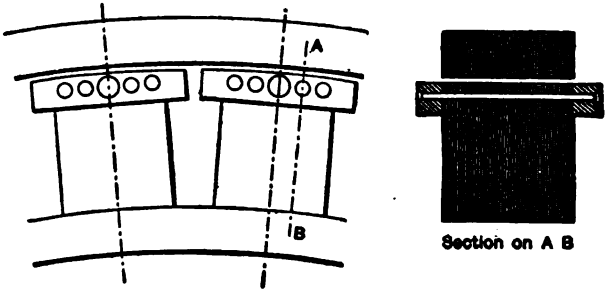


FIG. 373.—Damper winding.

marked reduction of the value of λ as compared with the same machine without the damper winding. Since the damper bars demand a considerable amount of copper (in many cases more than half of that of the exciting coils) and add to the expense of manufacture, it is an open question whether the same results in the single-phase case could not be more cheaply obtained by increasing the field strength and by adding part only of the extra weight of copper to the magnet-coils.[1]

§ 17. **The testing of alternators.**—If a pair of similar alternators are available, the best method of determining their efficiency is by a modification of the Hopkinson test which renders it suitable for alternating currents. The two machines are coupled rigidly together, but with a slight difference in the positions of their armature coils relatively to the poles in the two cases, so that a difference of phase of the induced E.M.F.'s results. When the combination is now driven through a transmission dynamometer or through a continuous-current motor of which the efficiency and losses at different speeds are accurately known, the one alternator acts as a motor and the other as a generator; by altering the difference of phase of the two machines, the input of the one and output of the other are regulated until they reach the required amounts. Then as in Chap. XX § 10, if W^1 be the output of the generator as measured by a wattmeter, and L be the total power supplied from the external source less the loss of energy over the two armature resistances, *i.e.*, $L = W - I^2(R_{aD} + R_{aM})$, the efficiency of the generator is similarly to eq. (99)

$$\eta = \frac{W^1}{W^1 + I^2 R_{aD} + \frac{L}{2} + A_s^2 R_s}.$$

A very complete account of such a test with its valuable results is to be found in *Proc. C. E.*, vol. cxxx. p. 247, 'Alternating-Current Dynamo Tests' (H. F. Parshall).

An ingenious extension of the Hopkinson test to the case of a single alternator has been given by Mr. Mordey, 'On Testing and Working Alternators,' *Journal Inst. Electr. Eng.* (vol. xxii. p. 116).

---

[1] Fischer-Hinnen, *loc. cit.*; and *Electr. Eng.*, vol. xx. pp. 597 and 654.

The retardation method of testing the various separate losses under different conditions is especially suitable to the case of alternators owing to the large moment of inertia of the rotor which allows of the readings being taken very accurately. The results of such a test on a 150 kilowatt alternator are given by M. Routin in *L'Éclairage Électrique* (vol. ix. p. 170), and an electrical method measuring the change of speed as the alternator slows down has been suggested by Dr. Sumpner (*Journal Inst. Electr. Eng.*, vol. xxxi. p. 634).

The most convenient form of non-inductive resistance for absorbing the output of an alternator on a full-load test is a water-bath; cos ϕ_e may then be taken as = 1, and the product of the volts and ampères will give the power developed. With wire coils or transformers lightly loaded, a wattmeter must be used to determine the true power.[1]

§ 18. **Efficiency of alternators.**—The efficiency of polyphase alternators varies but little from that of continuous-current dynamos of the same output, so that it is equally well represented by the curves of fig. 282 and 283, the losses in the exciter itself not being included (cp. figs. 409 and 413). The exciting energy varies from about 3 per cent. of the output in a 30 kilowatt alternator to 2 per cent. in a 500 kilowatt machine and 1 per cent. in machines of 800 kilowatts or over; but great differences exist between different types, and even in the same machine the amount varies greatly with the nature of the load if the regulation is much affected by the presence or absence of external inductance. Indeed the increased field excitation required for a power-factor of 0·8 as compared with unity will as a general rule lower the efficiency of the whole machine some 1 or 1½ per cent. With inductor generators for a power-factor = 1, the loss in excitation is less than in heteropolar machines, and may be brought as low as ½ per cent., but this advantage becomes less marked as the power-factor is decreased. The copper loss in the armature varies in machines of 500 kilowatts and upwards from 1 to 2·5 per cent. of the output at full load, and the loss by eddy-currents and hysteresis varies from 3 to 4 per cent., so that in general the latter is the greater of the two. The one may in fact be reduced at the expense of the other, and if a high efficiency is required at light loads, the copper loss in the armature should be given a higher ratio to the constant loss in the armature iron.

---

[1] For suggestions as to the testing of alternators in the factory, see Goldschmidt, 'Artificial Loading of Alternating-Current Machinery,' translated from the *E.T.Z.* in *Electr. Eng.*, vol. xxviii. p. 848.

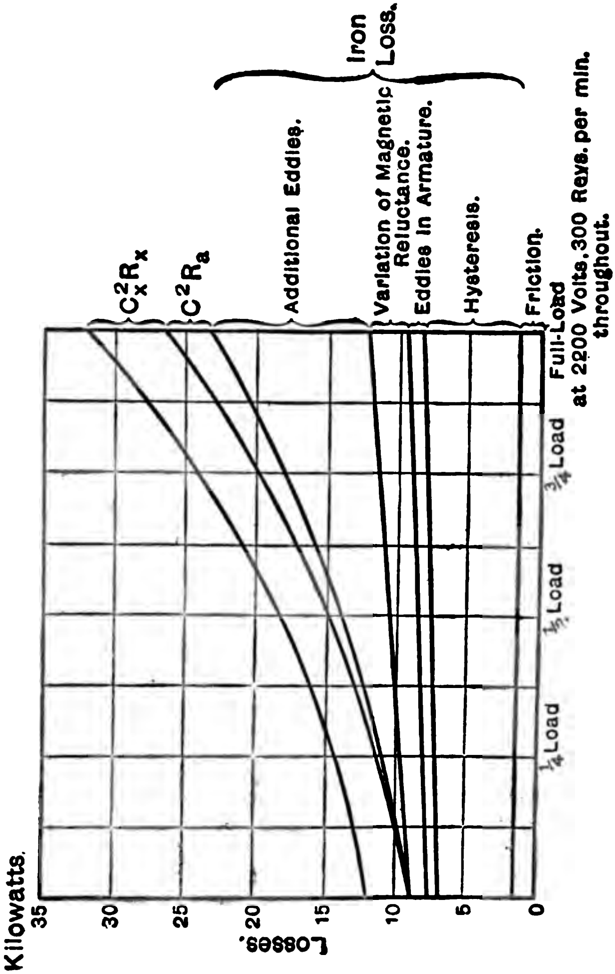


FIG. 374.—Losses in 385 K W. single-phase alternator.

The estimate of the loss by hysteresis and eddy-currents is a matter of some uncertainty, but its amount is greater in the single-phase alternator than in the polyphase machine, and on this account the efficiency of the former is appreciably lower than that of the latter. The reason for this is to be found in the fact that a current in the armature of frequency *f* sets up eddy-currents in the field-magnet of twice that frequency as described in § 16. Thus in the case of the 385 kilowatt machines of frequency 100 when running at 300 revs. per min. of which the full tests are given in the above-cited paper of Mr. Parshall, the eddies in the armatures of the two machines amounted to about 1400 watts on no load, but when the armature current was given values corresponding to about half, three-quarters, and full load, the additional eddy-loss in the field-magnets of the two machines was as follows :—

Current in System.	Additional Eddy-loss in Generator and Motor.
84·2 ampères	5,820
126·4 ,,	10,560
177·8 ,,	21,000

It thus increased approximately as the square of the current, and the total iron loss is shown (fig. 374) to be by no means a constant quantity in the single-phase alternator. Owing to the completeness of the tests the following figures may be added as an accurate analysis of the various losses in a single-phase alternator of high frequency for electric lighting, when working on a non-inductive load.

		No Load.	¼ Load.	½ Load.	¾ Load.	Full Load.
External power in kilowatts,		0	96·25	192·5	288·75	385
Current in ampères,		0	43·75	87·5	131·25	175
Terminal volts,		2200	2200	2200	2200	2200
Friction,	per cent. of input	12·5	1·36	0·71	0·48	0·36
Hysteresis in armature,	,, ,,	44	4·98	2·8	2·015	1·62
Eddies in armature,	,, ,,	5·85	0·905	0·544	0·415	0·288
Variation of magnetic reluctance,	,, ,,	11·75	1·45	0·946	0·767	0·696
Additional eddies in magnet due to armature current,	,, ,,	0	1·175	1·66	2·11	2·645
Copper loss in armature,	,, ,,	0	0·226	0·426	0·608	0·816
Copper loss in field-winding,	,, ,,	25·9	2·98	1·66	1·375	1·272
Total loss as percentage of input,		100	13·076	8·746	7·71	7·697
Percentage efficiency,		0	86·93	91·25	92·29	92·31
Total input in kilowatts,		12	110·5	211	313	417

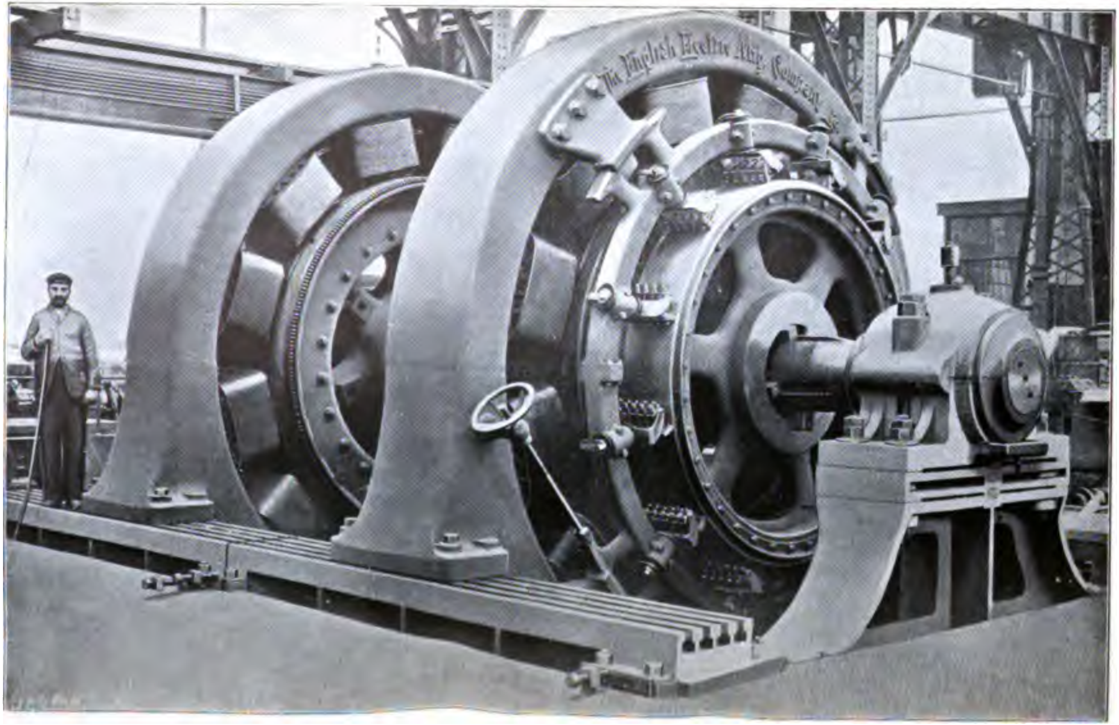


FIG. 375.—Traction generators of English Electric Mfg. Co., Ltd., arranged for Hopkinson efficiency test.

[To face p. 885.

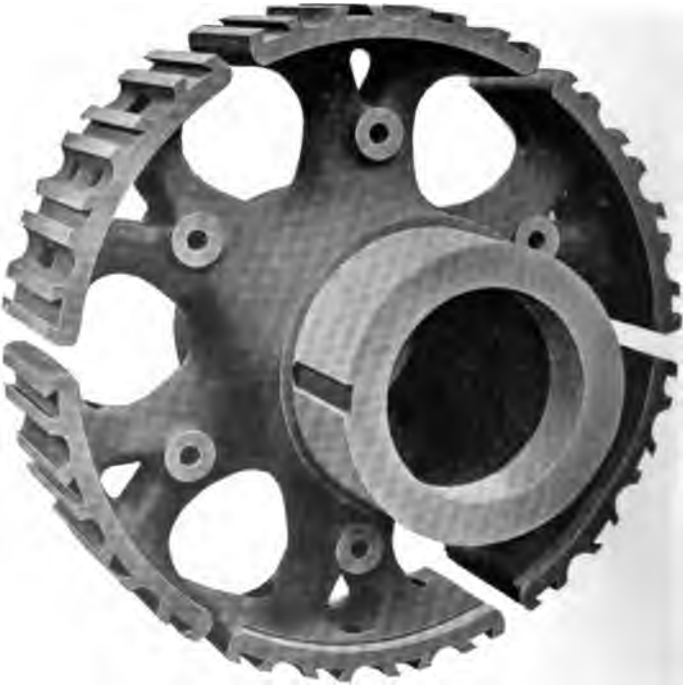


FIG. 376.—Armature hub.
(English Electric Mfg. Company, Ltd.)

CHAPTER XXVI

DESCRIPTIONS OF TYPICAL DYNAMOS

So many and so varied are the dynamos now manufactured that no attempt can here be made to describe all of even the leading types. It is only proposed in the present chapter to describe a few typical continuous- and alternating-current dynamos by way of familiarising the student with the construction and general appearance of complete machines of modern design, and to illustrate the manner in which the principles that have been discussed in preceding chapters are carried out in practice.

§ 1. **Multipolar traction generator of English Electric Manufacturing Company.**—The standard generator of the English Electric Manufacturing Company, Limited, of Preston, designed by the late Prof. S. H. Short, is a representative example of a modern multipolar continuous-current dynamo with slotted armature for direct coupling to the crankshaft of the driving engine. Two such machines, each of 1100-kilowatts output, are shown in fig. 375, as coupled together for a Hopkinson test under full load on the testing bed. For the purpose of carrying out the test the armature hubs are mounted temporarily on a common shaft by means of wedges driven into large keyways cut in the shaft.

The general description of the machine is as follows. The rotating armature is carried on a massive cast-iron hub, bored out so that it may be forced directly on to the engine shaft, where it is further secured by two keys. In order to avoid shrinkage strains when cooling, the several arms of the hub, each carrying a segmental portion of the rim, are when cast unconnected except through the nave of the hub (fig. 376). In the forked end of each arm a pair of holes is cast and so arranged as to drive air through radial ducts in the core when the armature is rotated. The armature core is formed of thin annealed sheet-steel laminations, of segmental shape, each thoroughly varnished on both sides and carrying on the inner edge at least two wedge-shaped keys fitting into dovetailed grooves on the rim of the hub. During the process of assembling the laminations, there are introduced at intervals of about three or four inches special spacing discs which have projecting bosses punched on them, and in which the teeth are twisted at right angles to the flat surface of the plate. By this means the plates are here kept about ⅛″ apart, and an air-duct is formed in the core. The complete core is clamped together between two end-rings by bolts passing through the interior below the discs. A flange on the end-plate at the engine side, perforated to give good ventilation, serves to support the ends of the barrel-wound armature coils, while the end-plate next to the commutator has a number of clamps fastened to facings on its outer surface by which equalising rings are held securely in place. These rings seen in fig. 377 are electrically connected at intervals with the commutator sectors, and equalise the voltage and currents of the several parallel paths of the winding (Chap. XI § 17).

The hub of the commutator is of cast iron, and is keyed to an extension of the armature hub, so that no movement can take place between the two. The hard-drawn copper sectors are insulated from one another by plates of soft amber mica 1/32″ thick, and are clamped firmly between steel end-rings from which they are insulated by moulded rings of mica, ⅛″ thick and projecting beyond the edge of the copper an inch or more. The numerous bolts which pass from end to end through the commutator under the sectors are threaded through mica insulating tubes. The wearing depth of the copper is not less than one inch.

When the commutator is in place on the hub, the armature core is ready to receive the coils. These are of copper bent on edge at the centre so as to form two layers, and shaped on a former into a hexagon, so that there are no joints except at the end where they are soldered into the commutator lugs. The latter are of laminated copper riveted and soldered into slots sawn in the commutator sectors. The armature coils are covered with mica, red rope paper and Japanese paper, and painted with varnish; two

and sometimes three, according to the nature of the design, are taped over with oiled linen, dipped and baked to form a composite coil, which is then placed in the slots on the surface of the core. Bands of binding-wire at intervals along the length of the armature complete the winding.

The magnet frame (fig. 378) consists of a circular yoke-ring of cast iron, divided on a horizontal plane, and bolted together by four bolts let into pockets on the inside of the circle. The radial poles project inwards and are composed of rectangular sheet-steel laminations, about $\frac{1}{8}''$ thick, cast into the yoke-ring. A number of the laminæ, punched to the correct shape, are assembled and clamped between two thicker steel plates to which they are riveted. The required number of poles thus formed are placed in position in the mould prepared for the casting, and have their outer ends coated with a flux to assist the union of the molten metal and the steel; the outer ends are left rough and are further provided with a longitudinal wedge-shaped groove on either side, to ensure their being held firm in the casting. After the poles have been heated up to prevent chilling of the metal at the first contact, the cast-iron is run into the mould, and the whole is left untouched in the sand for several days, in order to thoroughly anneal it, and secure a soft casting easy to machine and of high permeability. When the two half-rings have been bolted together, the pole-faces are turned to the correct bore, and at the same time there is turned in four cast-iron brackets bolted to the face of the yoke, the groove which is subsequently to receive the rocker ring carrying the brushes. The inner end of each pole has a dovetailed groove on either side parallel to the axis of the armature, and on this is clamped a detachable cast-iron frame[1] in two halves, which forms a polar extension and at the same time holds the magnet coil in place. The magnet coils are wound on a rectangular sheet-steel shell, just large enough to slip easily over the pole, light open-work brass flanges pierced with holes being riveted to each end of the shell. The spool is insulated with mica and paper, and the shunt coil which occupies about two-thirds of the winding space is kept apart from the series coil which occupies the remaining third by $\frac{1}{4}''$ of insulation. Double-cotton-covered wire is used for the shunt, while the series coil is composed of a few turns of wide copper ribbon wound on the flat. As a further mechanical protection the entire coil is covered with braided rope, filled with waterproofing, and thoroughly dried by baking; it is then tested for insulation by an alternating difference of potential of 2500 volts. When placed on the poles, the several sets of shunt and series coils are respectively connected in series.

---

[1] One such is seen in fig. 378 resting against the plummer block.

On the lower half of the magnet frame are cast two feet, which when planed rest on a pair of horizontal foundation girders and carry the whole magnet. Adjustment of the clearance between the poles and the armature is obtained horizontally by screws, and vertically by thin packing-pieces inserted under the feet. In order to expose the armature or to withdraw a magnet coil, the whole magnet frame can be moved lengthwise along the girders.

The brush rocker is a cast-iron ring in halves bolted together, of channel section with projections fitting into the groove on the four brackets on the magnet frame, and provided with a screw and hand wheel for moving it concentrically with the commutator to the best position for sparkless commutation of the current. Concealed in the inside of the channel are two cross-connecting rings, tapped at intervals with leads to the brushes of the same sign. The frame carrying each set of brushholders terminates in a spindle passing through an insulated hole in the rocker ring, so that the frame can be turned round through 180° to obtain the right set of the brushes for either direction of rotation. The carbon brushes are in holders attached to laminated copper strips, and as many sets of brushes are employed as there are poles, the armature winding being parallel-connected.

The output of the machines shown in fig. 375 is 2000 ampères, 550 volts at 100 revs. per min., the field being over-compounded so that the voltage rises from 500 volts at no load to 550 at full load. After a prolonged run at full load, the highest rise of temperature of the armature was 47° F., and of the field was 58 F°. The total weight of the machine giving 11,000 watts per rev. per min. was 56 tons, divided into 21 tons for the armature and 35 tons for the magnet. The efficiency was 95 per cent., and by calculation from the tests the following approximate division of the losses in watts was made:—

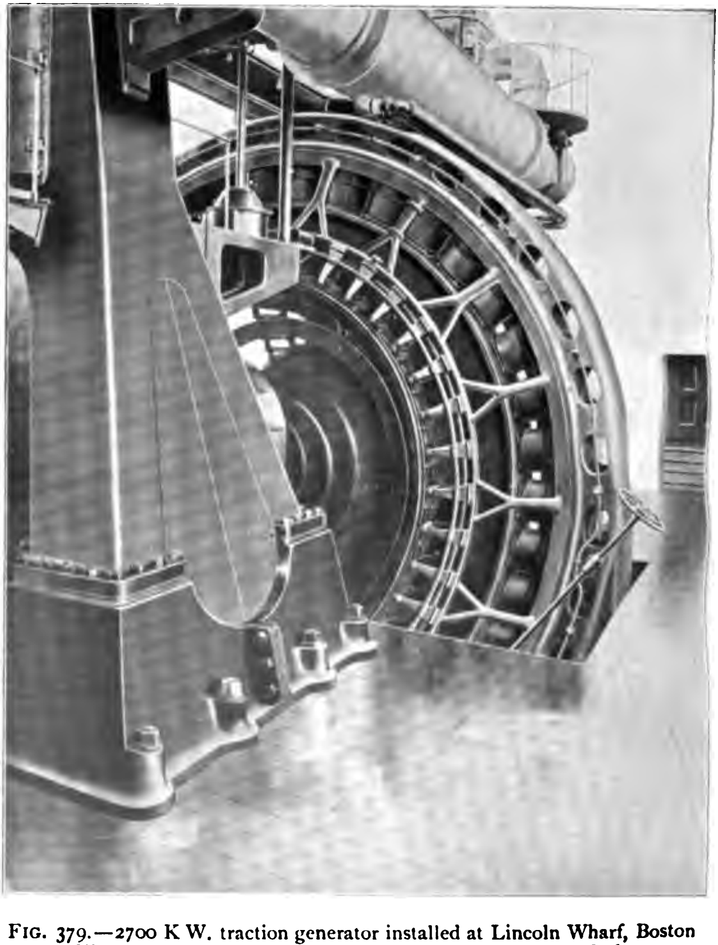


FIG. 379.—2700 K W. traction generator installed at Lincoln Wharf, Boston Elevated Railway, by the General Electric Co., of New York.

[*To face p.* 889.

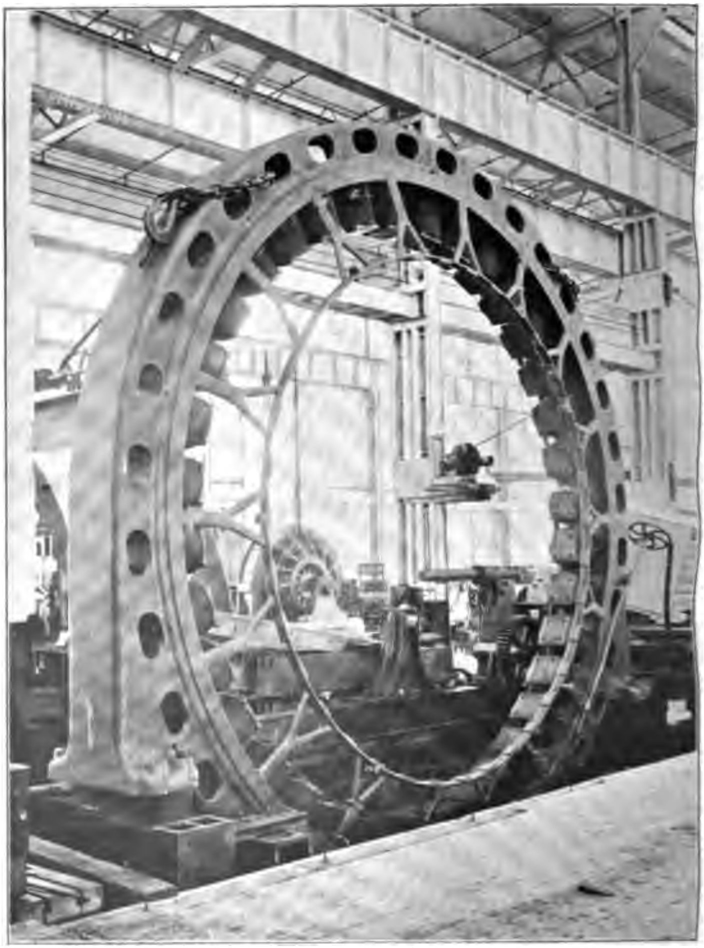


FIG. 380.—Field frame of 2700 K W. generator.
(General Electric Company of New York.

[*To face p.* 889.

	¼ Load.	½ Load.	¾ Load.	Full Load.
Hysteresis and eddy-currents, .	15,000	16,680	18,370	20,000
Armature resistance and brushes,	1,310	5,250	11,800	21,000
Shunt coils, ...	8,000	8,820	9,530	10,000
Series coils, ...	310	1,250	2,800	5,000
Total losses apart from friction, .	24,600	32,000	42,500	56,000
Output, ...	257,000	526,000	800,000	1,100,000
Input, ...	281,600	558,000	842,500	1,156,000
Efficiency percentage, . . .	87·5	94·5	95	95

§ 2. **2700 KW. generator of General Electric Company of New York.**—One of the largest continuous-current generators for traction work is a 2700 kilowatt machine built by the General Electric Company of New York, and installed at Lincoln Wharf, Boston Elevated Railway. For the details of this fine machine and for the accompanying photographs which illustrate it when completed and at various stages in the process of erection, we are indebted to the courtesy of the British Thomson-Houston Company.

The machine is a 36-pole generator with slotted armature, and has an output at 75 revs. per min. of 575 volts, 4700 ampères, or 36,000 watts per rev. per min. The diameter of the armature is 21′ 8″, and of the outside of the magnet frame is 31′ 0″. The weight of the armature complete is 71½ tons, or nearly half of the total weight of the entire machine, which is 138 tons. Fig. 379 from a photograph taken while the generator is running shows the completed machine mounted on the crankshaft between the standards of its vertical engine.

The magnet frame (fig. 380) is of cast iron of deep section with lightening holes, and is divided horizontally so that the upper half can be lifted for inspection or repair to either armature or magnet coils. On account of the large size of the frame the lower and upper halves were each subdivided into three sections in order to facilitate transportation, and the junctions of the sections are registered by feathers sunk into the adjacent surfaces of contact. Adjusting screws are provided for the purpose of centering the field with the armature. The magnet-cores, 36 in number, are of cast steel with laminated pole-pieces securely fastened to them by screws; they are bolted to the yoke in such wise that any pole-piece with its winding can be readily withdrawn after removing the fastening bolts. The laminations of the pole-faces are of two lengths, arranged alternately, so as to produce the graduated field effect given by a chamfered pole-edge. The field coils are wound on spools, having a double sheet iron body and malleable iron flanges; the shunt and series windings are kept separate from one another in each bobbin.

The armature spider (fig. 381) is of cast iron, with arms divided at the periphery so as to avoid shrinkage strains; its two halves are bolted together and the whole keyed to the engine crankshaft. The iron core is built up of laminations japanned prior to being assembled; these are secured to the spider by dovetail projections, and are held in place by end-flanges which also serve to support the end-windings (figs. 382 and 383). Spacing bolts are inserted at equal intervals to provide five ventilating ducts between the laminations. The armature spider is so arranged that it produces a fan effect, forcing air through the ventilating spaces. The barrel winding is that of a parallel-connected lap-wound drum, and has equaliser rings connected to points of equal potential. These rings are secured by insulating pieces attached to the arms of the spider at the back of the armature, similarly to the armature shown in fig. 384. The armature coils are made in halves, the back connection being made by soldering two ends together, and metal bands bind the conductors firmly to the end-flanges.

The commutator is 15′ 7″ in diameter with sectors of hard-drawn copper made with a dovetail on the lower edge to hold them in place. The mica between the sectors is of a soft quality, and as thin as the requirements for insulation will permit so as to ensure even wear of mica and copper. The whole is carried on a cylinder with internal arms bolted to the spokes of the armature hub (fig. 385).

The large circular frame carrying the 36 sets of brushes is moved in a groove on the magnet frame by means of a handwheel at one side of the yoke-ring (figs. 380 and 386). The carbon

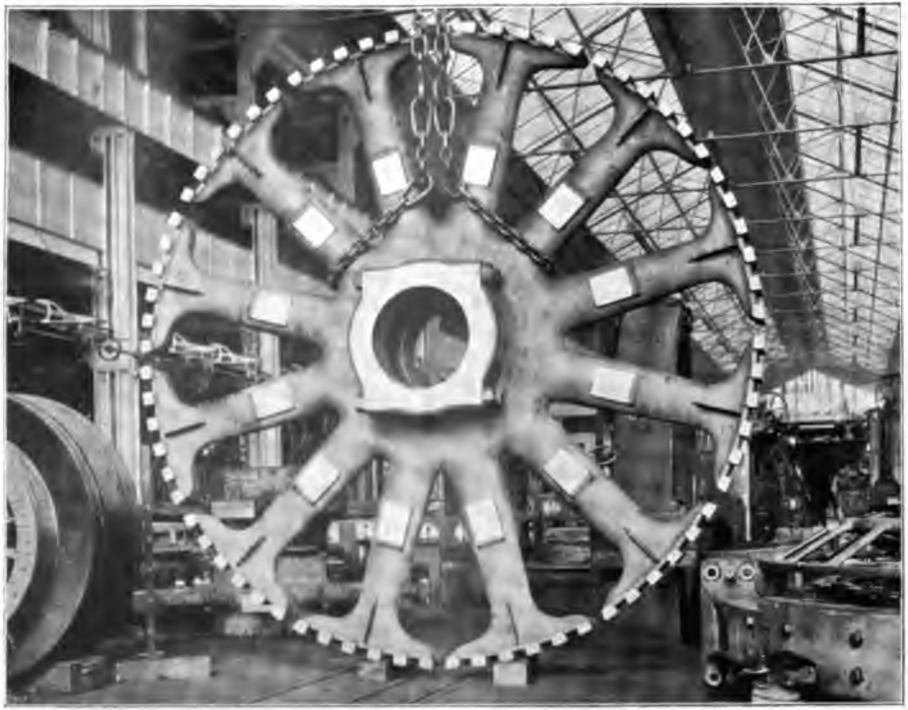


FIG. 381 —Armature spider casting for 2700 K.W. generator.
(General Electric Company of New York.)

[*To face p.* 890.

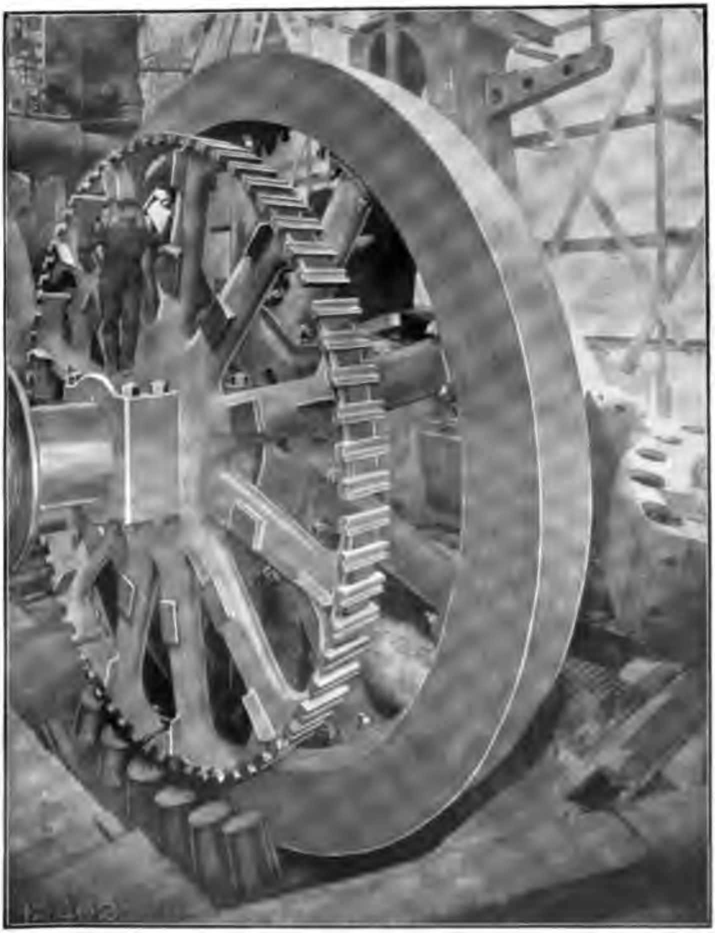


FIG. 382.—Armature spider of 2700 K W. generator during erection.
(General Electric Company of New York.)

[*To face p.* 890.

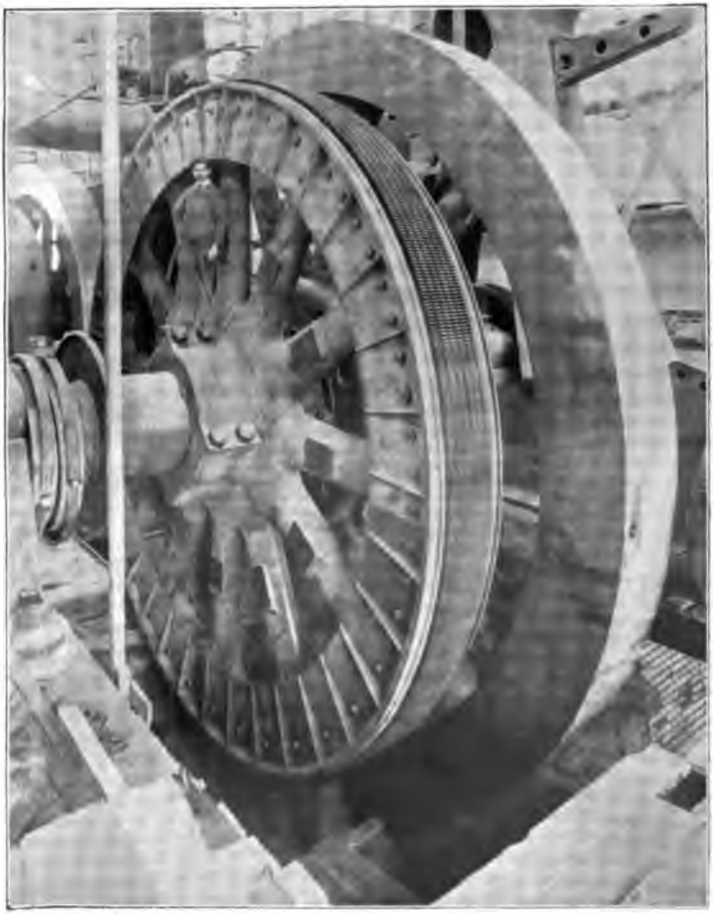


FIG. 383.—Armature core of 2700 K W. generator during erection.
(General Electric Company of New York.)

[*To face p.* 890.

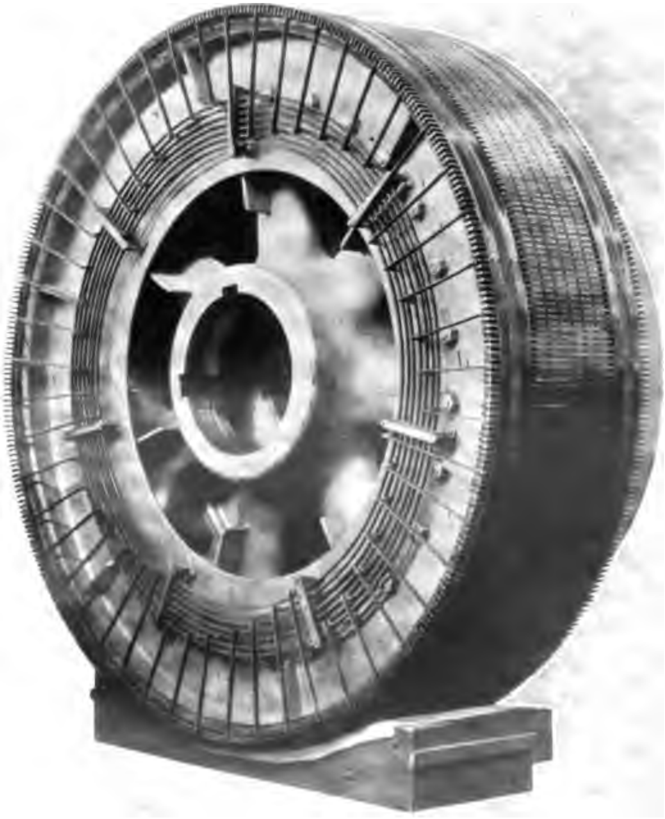


FIG. 384.—Rear view of armature showing equaliser rings.
(General Electric Company of New York.)

[*To face p.* 890.

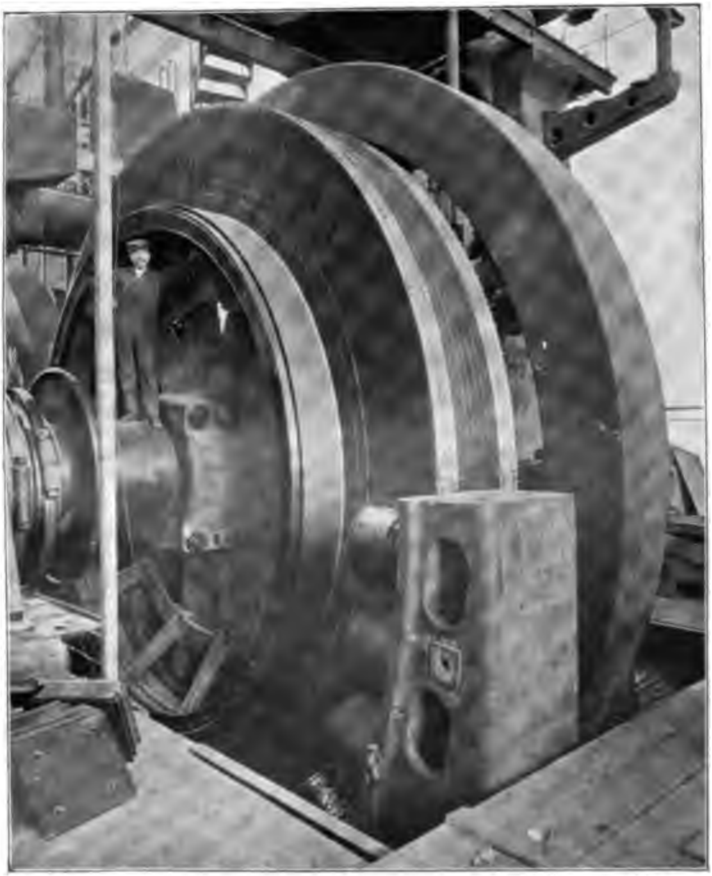


FIG. 385.—Complete Armature of 2700 K W. generator during erection.
(General Electric Company of New York.)

[*To face p.* 890.

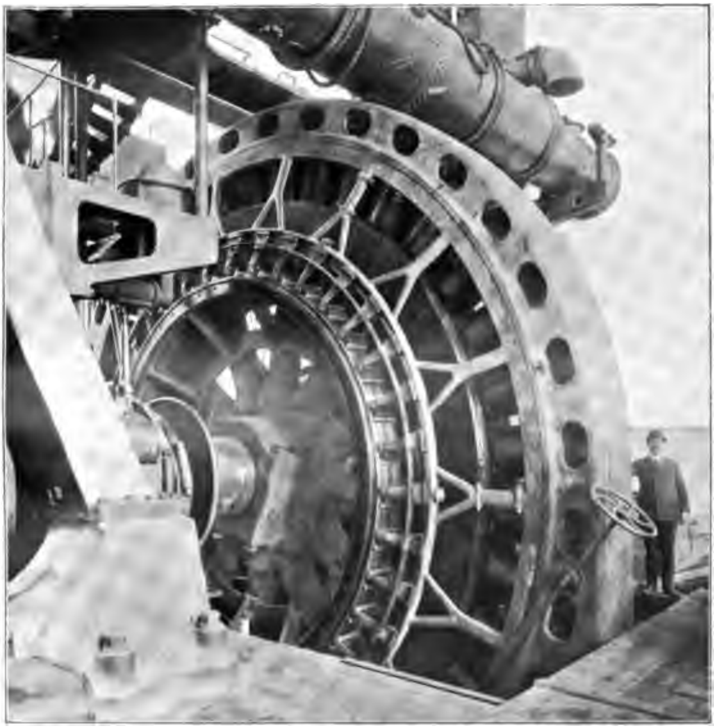


FIG. 386.—2700 K W. generator erected.
(General Electric Company of New York.)

[*To face p.* 890.

brushes are inclined slightly forward of a radial position and are each furnished with a flexible copper conductor connecting it to the brush-holder.

The insulation between the field coils and magnet frame and also between armature winding and armature core is tested with an alternating difference of potential of 4000 volts for 10 seconds or with 2000 volts for 60 seconds. The rise of temperature of the generator is guaranteed not to be more than 63° F. after a run of 24 hours at full load, and an overload of 50 per cent. for two hours following a run of 24 hours at full load will not cause the rise to exceed 99° F. The generator is guaranteed to carry an overload of 50 per cent. at the rated voltage for two hours, and of 100 per cent. momentarily without movement of the brushes and without injurious sparking. The compounding is arranged to give 525 volts at no load and 575 volts at full load. The efficiency rises from 94 per cent. at half load to as much as 95½ per cent. at full load.

§ 3. **Schuckert 900 KW. dynamo.**—A different type of armature construction is illustrated by the 900 kilowatt dynamo of Schuckert & Co., shown at the Paris Exhibition of 1900 (fig. 387). At a speed of 100 revs. per min. the output is 1500 ampères and 600 volts. The magnet has 14 radial poles external to the armature; the yoke-ring is of cast steel in two halves, and of a polygonal shape with 14 sides. The poles are of nearly square cross-section giving an area of 248 sq. inches, and are cast in one with the yoke, with a recess on the exterior opposite to each pole, so as to lighten the weight by removing all metal which is not used magnetically. The lower half of the magnet frame is carried by two feet resting on the concrete foundation. Fastened by screws to the interior faces of the poles are steel pole-shoes which also serve to keep the magnet bobbins in place. The external diameter of the magnet is 13 ft., and its axial width 16⅛ inches. The bore of the poles is 10 ft., and the axial width of both the pole-faces and the armature core is the same as that of the yoke. The bobbins are shunt-wound and are all in series.

The armature core is built up of soft sheet iron discs, 5 mm. thick, insulated by paper, and traversed from end to end by insulated steel bolts. The cast-iron armature wheel has six double arms of T form, and is made in two parts which are connected together by screws and also by rings shrunk on. Radial screw bolts fasten the core to the arms of the hub. The surface of the armature core is smooth, and the drum-winding is composed of bars united by evolute end-connectors of forked shape (fig. 388). The bars number in all 1072, in one layer, and are of rectangular cable 60 sq. mm. in section; they are insulated from the core by mica, and are driven by means of insulated driving-strips sunk at frequent intervals into the core and extending the whole length of the iron. Bands of steel binding wire backed by mica prevent any movement of the winding under centrifugal force; the finished diameter of the armature being 118 inches, the peripheral speed is 3090 ft. per min. Owing to the spacing out of the end-connectors, they form at either end a kind of grid through which air can pass, and a very effective cooling action is obtained. The centre of each forked end-connector is fastened at the engine end by a hooked wire caught

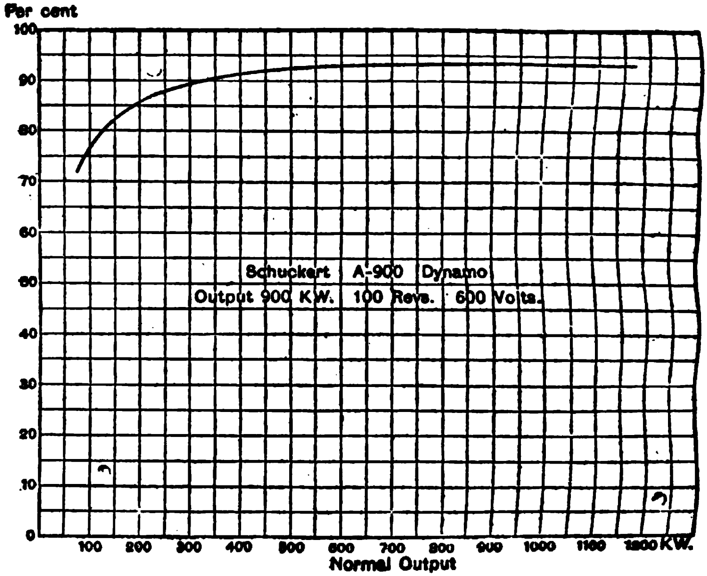


FIG. 389.—Efficiency Curve of Schuckert dynamo.

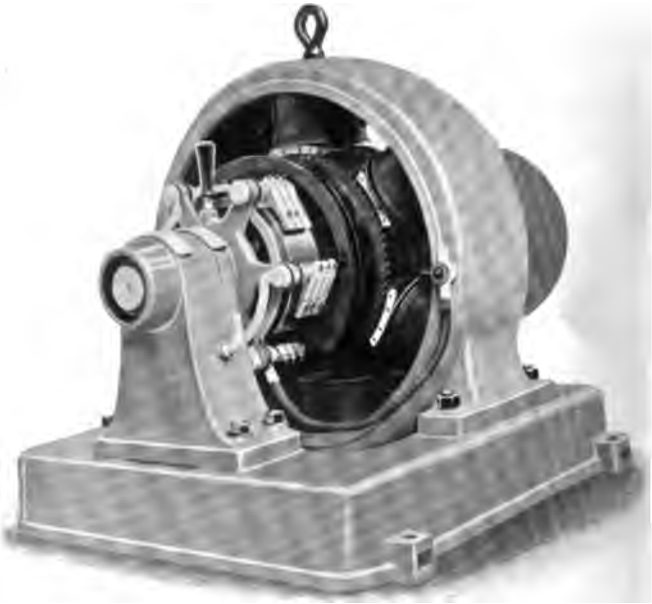


FIG. 390.—75 K W. belt-driven dynamo of International Electrical Engineering Company.

[*To face p.* 893.

under a wood ring, and at the front end the junction is effected between each end-connector and the commutator by means of a round wire passing into a hole drilled in the copper sector and fastened by three grub screws. The commutator has 536 parts; its diameter is 71″ and its breadth 10¼″. The over-all length of the wound armature is about 23⅝″. The winding is parallel-connected, and on each of the 14 brush arms are 4 carbon brushes to collect 215 ampères. The brush arms are carried by a star-shaped rocker, which is movable round a ring cast on the outer plummer-block by means of two hand-wheels and a worm actuating a spur wheel which engages with the rocker.

The weight of the complete armature is about 15¾ tons, and of the whole machine 44 tons. The losses at full load are approximately

in hysteresis and eddy-currents	26·3	kilowatts
over the copper resistance of the armature	14·7	„
and in excitation	7·8	„
total	48·8	„

giving an efficiency of nearly 94¾ per cent. at full load and of 92 per cent. at half load (fig. 389).

Mounted on the magnet frame is seen in fig. 387 an automatic maximum current switch with magnetic blow-out.

§ 4. **Belt-driven 75 KW. dynamo of International Electrical Engineering Company.**—Fig. 390 illustrates a small multipolar direct-current generator of 75 kilowatts output for belt-driving, as manufactured by the Compagnie Internationale d'Électricité, of Liége. The six circular poles, each 7½″ in diameter, are cast in one with the yoke ring, and each is furnished with a pole-shoe of laminated strips fastened by two screws with countersunk heads to the solid pole. The magnet coils of 1500 turns are wound on detachable spools, and the edges of the pole-shoes are slightly chamfered off to give a graded magnetic field at the poletips where commutation is effected. The air-gap is 0·236″. The slotted armature 22·6″ in diam. × 8¼″ long is barrel-wound with 304 inductors, and the commutator has 152 sectors. The six sets of brushes are carried by a rocker-ring journaled on a seating turned in the plummer-block, and provided with an insulated handle for the purpose of shifting it round the commutator. The bearings have slightly tapered cylindrical brasses each lubricated by means of two oiling rings dipping into the oil reservoir cast in the hollow interior. The brush arms of the same sign are connected together by means of flat copper strips. When run at a speed of 560 revs. per min., the machine gives 120 volts and 625 ampères, or 134 watts per rev. per min.; its constant of size thus works out to $\frac{134}{D_{a}^{2}L_{a}} = \frac{134}{4200} = 0{\cdot}032$.

§ 5. Electroplating dynamo of W. H. Allen, Son and Co.—Electroplating and electrodepositing dynamos form a class by themselves, and to the designer present certain difficulties owing to the requirements of a very low voltage in combination with enormous currents. Figs. 391 and 392 show the general arrangement and armature of a belt-driven electroplating machine manufactured by W. H. Allen, Son & Co., Ltd. for an output of 3 volts, 3000 ampères at a speed of 600 revs. per min. The drum armature is furnished with a commutator at each end, and the two windings are joined in parallel at the main terminals of the machine; connected to each commutator is a singly re-entrant triplex winding composed of 74 inductors or 37 lap-wound loops in a four-pole field, and when the four sets of brushes are in position on the commutator this is divided into $q = 2np = 2 \times 3 \times 2 = 12$ parallel paths. Thus by the adoption of two armatures on a common core, each with its triplex winding, the total number of inductors is raised to 148, and the total number of parallel branches on both armatures to 24, while conversely the current to be commuted in any one section is reduced to $\frac{3000}{24} = 125$ ampères. The flux of each of the four poles is $Z_a = 1{\cdot}46 \times 10^6$, and by eq. (14) the induced E.M.F. is

$$\frac{2}{q} \times 74 \times \frac{2 \times 600}{60} \times 1{\cdot}46 \times 10^{-2} = 3{\cdot}6 \text{ volts.}$$

The armature core has a diameter of 18″ and a length of 4½″, and is built up of discs with an internal diameter of 11½″ measured over the arms of the cast-iron hub on which they are

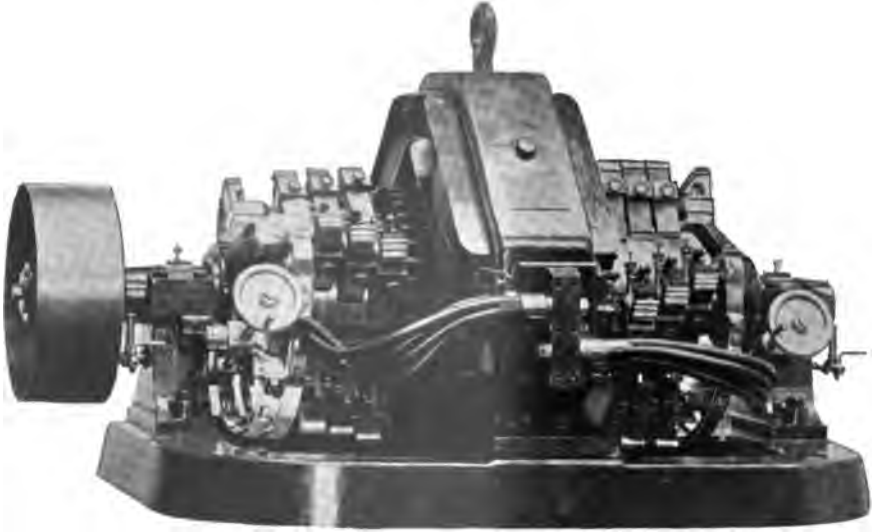


FIG. 391.—Electroplating dynamo with double commutator.
(W. H. Allen, Son & Company, Ltd.)

To face p. 894.

supported. The discs are insulated with varnish and are clamped between two wrought-iron end-plates by bolts which pass lengthwise through the whole and are themselves insulated by being threaded through fibre tubes. The armature surface is smooth, and the bars are driven through the magnetic field by means of 16 hardwood strips let into slots milled in the armature core. The bars are of rectangular section, 0·3″ deep × 0·32″ wide, of stranded copper double-cotton-covered, and are soldered to forked end-connectors of copper strip, ¾″ wide × 0·080″ thick × 2 in parallel; the centre of the forks is held in a slotted ring surrounding the shaft, and the space between adjacent connectors is sufficient to allow the air to pass freely between them so as to ensure ample ventilation. The resistance of the two armatures in parallel is 0·00005 ohm when hot, giving a loss of 0·15 volt, and a joint loss of 450 watts at full load. The commutators are 12″ in diameter and of large size; indeed in such a dynamo the commutators are the chief source of heating, and the commutator lugs and armature end-connectors must be relied upon to assist in dissipating the heat necessarily caused by the collection of 3000 ampères. Each of the 37 sectors in one commutator being a little over one inch wide in a circumferential direction, the effective width of each of the four sets of brushes must be at least 2½ or preferably 3 inches, in order that they may span some 2½ to 3 sectors. There are three sets of brushes on each arm, and the necessary great width of brush is obtained by setting them in a slanting line across the length of the commutator, one in advance of the other, so that the toe of the leading brush is 3″ ahead of the heel of the trailing brush. Each brush is 2″ wide in an axial direction, and in each brush-holder are arranged three gauze brushes, each ¼″ thick, one above the other so as to form a composite brush. There are thus in all 24 of these composite brushes on the whole machine, the total contact area of one arm being 2″ × 0·306″ × 9 = 5½ sq. inches, to carry 250 ampères. On the assumption of a specific contact-resistance of 0·003 ohm per sq. inch, the contact-resistance of one set of brushes is $\frac{0{\cdot}003}{5{\cdot}5} =$ 0·000545 ohm, giving a loss of 250 × 0·000545 = 0·136 volts, or 0·272 over the two sets of brushes of opposite sign, and a total loss of $0{\cdot}272 \times 1500 = 410$ watts over each commutator. The friction loss is even greater, and is by eq. (86)

$$\frac{0{\cdot}3 \times (1{\cdot}5 \times 5{\cdot}5 \times 4) \times (\pi \times 600 \times 1) \times 746}{33{,}000} = 420 \text{ watts,}$$

and in consequence of the relatively large proportion of these losses to the total output, the efficiency of the electroplating dynamo is necessarily low.

The magnet is of cast steel, with four poles each $7\frac{1}{2}'' \times 3''$ in area, bolted to an octagonal yoke-ring. The polar arc is only 55°, since owing to the nature of the winding the span at the commutator end of each winding differs greatly from that at the further end, the latter being 90° and the former only 61°. The density of the flux in the air-gap is thus $B_g = 5650$, and the bore of the pole-pieces is 19·2″. The machine is separately excited at 120 volts to give 13,000 ampère-turns per pair of poles. The winding consists of 36 layers of 45 turns of 0·098″ round wire on each pole with a total resistance for the four coils of 24·5 ohms when hot, a shunt current of 4 ampères, and a total loss of 480 watts over the field coils and the regulating resistance in series with the shunt.

§ 6. **1500 KW. three-phase generator of Brown, Boveri and Co.**—Figs. 393 and 394 show in section and elevation the three-phase 1500 kilowatt generator exhibited by Brown, Boveri and Co. at the Paris Exhibition of 1900 in connection with a 2000-2500 H.P. triple-expansion Sulzer steam-engine. The generator is designed for a terminal E.M.F. of 6000 volts interlinked pressure, or 3460 volts per armature phase and 170 ampères in each phase, or 1760 kilovolt-ampères; the real output with the designed power-factor of 0·85 is therefore 1500 kilowatts. The poles, 72 in number, giving a periodicity of 50 at 83·3 revs. per min., are arranged round the periphery of a cast-iron wheel with a hollow rim of box section, nearly 21 ft. in diameter, the whole serving as flywheel to the engine. The wheel is cast in two halves, which are united along a pair of arms by means of four bolts, the rim being also locked at each point of junction by a bolt and nuts let into a recess in the casting; there are thus 10 arms in all, and the nave

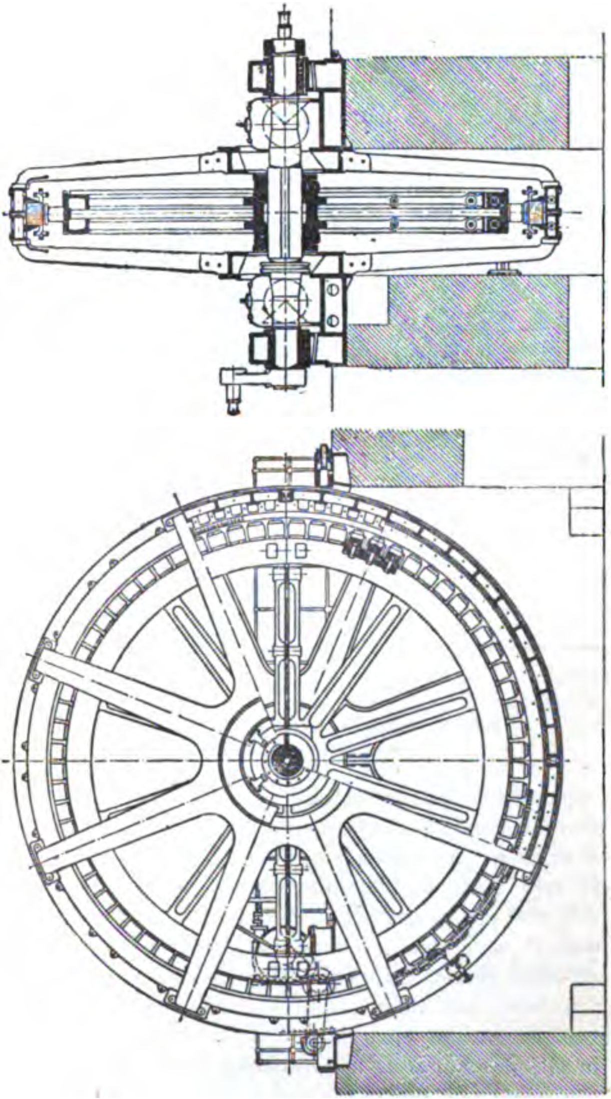


FIG. 393.—1500 K W. three-phase generator of Brown, Boveri & Company.

is itself fastened to the shaft by four bolts and two rings shrunk on, reinforced by a single driving key in the shaft. On the rim are 72 shallow recesses bored to receive the circular poles, each of which is fastened by a bolt and small steady pin. The poles are of iron, 8¼″ in diameter, with a rectangular pole-shoe at the outer end, 12·6″ long axially and 7½″ in width or 0·63 of the pole-pitch. The outer diameter of the pole-faces is nearly 22′ 7,′ or exactly 690 cm., while the bore of the external stationary armature is 692 cm., so that the single air-gap is 1 cm. = 0·394″. Each pole carries 45 turns of bare copper strip 1″ deep and 0·177″ thick, wound on edge; the several coils are united in series, and have a total resistance of 0·33 ohm at 15° C.

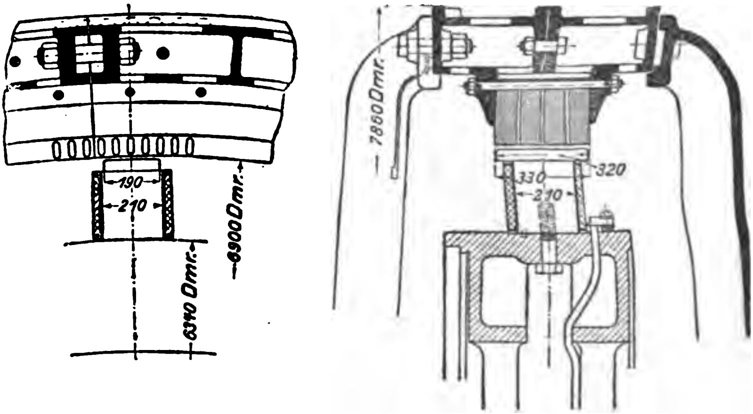


FIG. 394.—Detail of 1500 K W. generator.
(Brown, Boveri & Company.)

The armature laminations are about 0·020″ thick, and are pierced very close to their inner edge with 432 tunnels, or 2 per pole and per phase. Each tunnel is of oval shape, 2·36″ deep × 0·79″ wide, and is closed at the top. There are 36 coils in each phase, each coil being divided into two portions giving 6 wires per tunnel, and the coils of any one phase are arranged at the ends in one tier, so that there are three tiers in all. The tunnels are insulated with tubes of micanite, and the windings of the three phases are star-connected.

The armature core is 13″ long, divided by three ventilating air-ducts, and the total depth of the laminations is 10″. To carry the armature a massive star with eight radial arms is bolted to each side of the casing as described in Chap. XXIV § 5, the general appearance of the machine being thus very similar to that of fig. 346. The over-all diameter of the armature casing is 25 ft. 10 in., and it is made in two halves between which the laminations are clamped, the halves being bolted together by two rows of bolts, one of which also traverses the laminations and the cheeks of the casing. The armature is prevented from turning round when at work by two radial screws which abut on the outside of the casing on the floor level. When these are slacked back, it can be revolved on its trunnion bearings cast in one with the engine bearings by means of the same barring gear which also turns the flywheel; this operates by chain gear a pinion which can be made at will to engage in the teeth of a rack surrounding the edge of the armature casing. Inside the ring forming the trunnion bearing on one side are the two collector rings through which is passed the exciting current under a pressure of 110 volts.

The results obtained from tests of the performance of the machine are given in fig. 395. The excitation required to give 6000 volts on open circuit is 176 ampères or 7900 ampère-turns per pole, on full load of 1760 kilowatts with $\cos\phi_e = 1$ is 193 ampères or 8700 ampère-turns per pole, and on full load with $\cos\phi_e = 0{\cdot}85$ is 231 ampères, or an increase of 30 per cent. on the no-load excitation. The rise of volts when the circuit is opened is in the second case 5 per cent., and in the third case 13 per cent. The saturation of the magnets is carried to a medium value, so that the machine has an overload capacity of some 20 per cent. or 1800 kilowatts with a power-factor of 0·8. On the normal full load of the machine, the loss in excitation works out to 1·18 per cent. of the real output.

The watts per rev. per min. being 18,000, the constant of the machine as in eq. (120) is $\dfrac{45{\cdot}5}{\cos\phi_e}$.

§ 7. **350 KW. three-phase generator of Brown, Boveri and Co.**—A second three-phase generator exhibited by Brown

Boveri & Co. at Paris was of a somewhat different type, inasmuch as the rotating poles were external to the armature and projected radially inwards from an overhanging rim on the flywheel. This construction was first employed by Brown, Boveri & Co. in 1894 for the Cairo alternators, and is specially suitable for machines of moderate output at low speeds; its advantage is that a very large moment of inertia is thereby gained in the flywheel without the necessity for adopting dimensions for the armature which are unfavourable from an electrical point of view. At the same time there is no strain due to centrifugal force on the bolts which hold the poles, and indeed the magnetic pull tends to counterbalance the centrifugal force on the poles and rim. The output of the machine (figs. 396 and 397) was 200 volts at the terminals, and 1200 ampères per phase or 415 kilovolt-ampères with any power-factor between the limits of unity and 0·85; *i.e.*, 350 kilowatts. The speed was 92·5 revs. per min., giving with 52 poles a periodicity

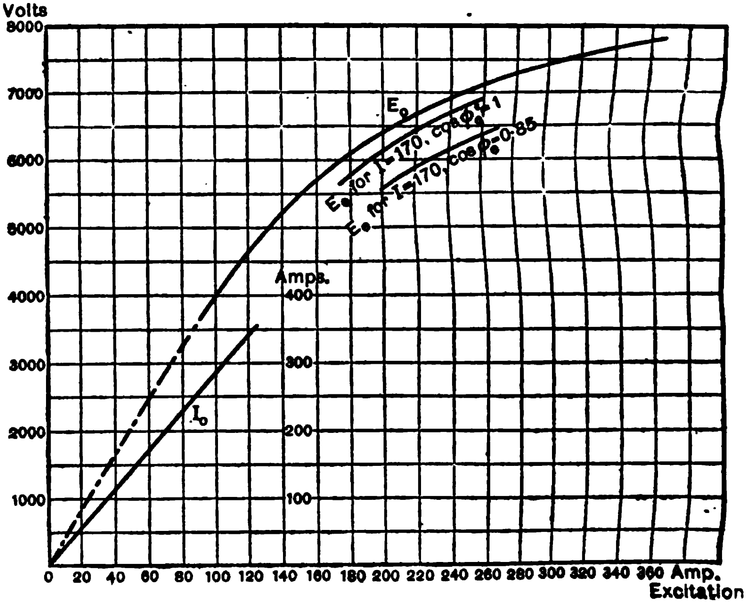


FIG. 395.—Curves of Brown 1500 K W. generator.

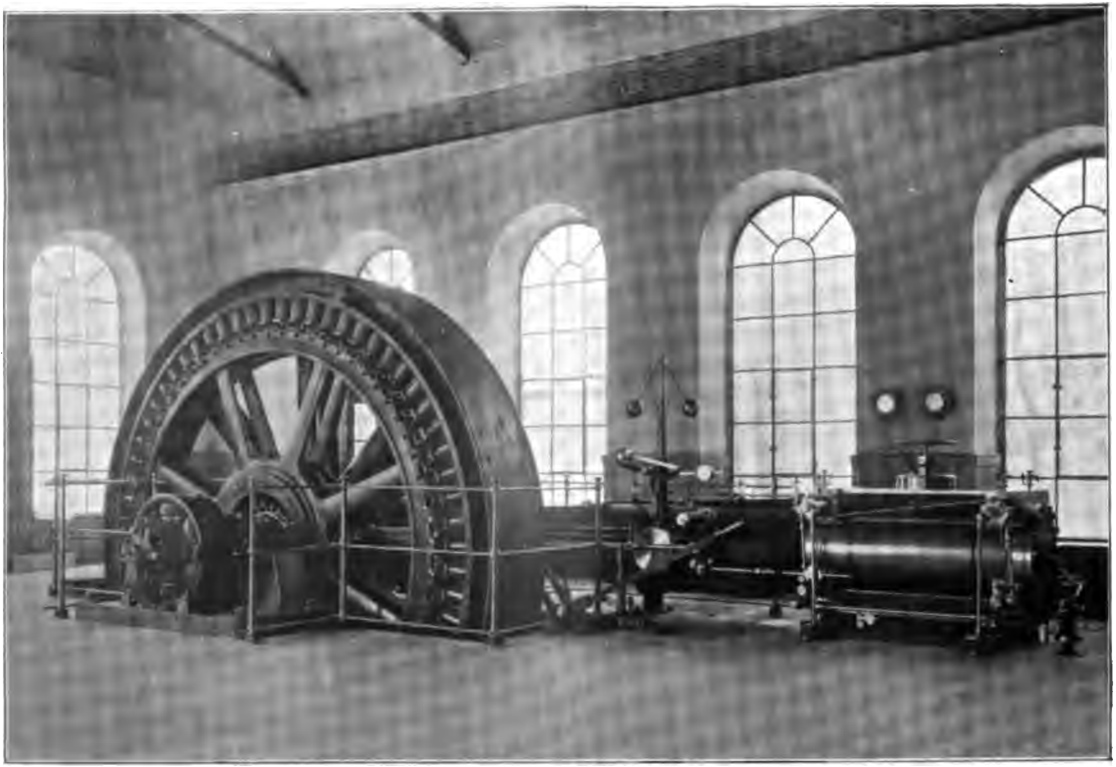


FIG. 396.—350 K W. three-phase generator by Brown, Boveri & Company.

[*To face p.* 900.

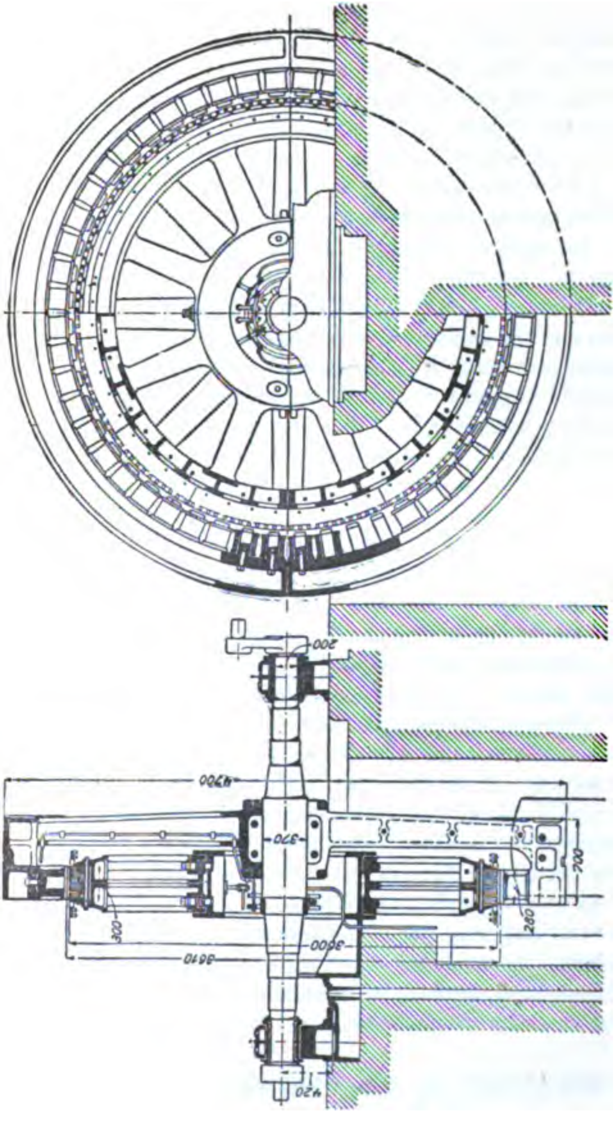


FIG. 397.—350 K W. three-phase generator.
(Brown, Boveri & Company.)

of 40. The method of fixing the poles is similar to that of the last-described machine, but the screws are lodged in a deep recess in the edge of the wheel; the flywheel is of cast iron with double arms and is fastened to the shaft by two rings shrunk on and by two keys at right angles. Its outer diameter is nearly 15 ft. 6 in., and its total width is $27\frac{1}{2}''$. The circular poles are of iron, 5·2" in diameter, and are each wound with 80 turns of rectangular cotton-covered wire, varnished on the outside with shellac; the coils are arranged in two parallels, and their joint resistance is 0·95 ohm. The pole-shoes are 11" long × 5·2" wide, or $\beta = 0{\cdot}61$ of the pitch. The bore of the pole-pieces is 142" in diameter, and the single air-gap is 6 mm. = 0·236".

The armature is 141·6" in diameter, and the core of laminated discs is 11·8" long and 6·7" deep, divided by four air-ducts. It is pierced with 156 tunnels, $1\frac{3}{16}''$ diameter, and the armature winding consists simply of 156 round copper bars, $\frac{15}{16}''$ diameter, one in each tunnel, united by flat copper end connectors to form three phases star-connected. In order to render any part of the armature winding easily accessible the wheel which carries the laminations is cast in two halves, and these are seated on two semicircular rings of cast iron through the hollow interior of which passes the flywheel shaft; the lower ring is cast in one with the outer plummer-block, and has an extended facing on it in a horizontal plane. The armature wheel can be turned round until its horizontal division coincides with the horizontal division of the two cast-iron rings; after loosening the screws which fasten together the two halves, the upper half of the armature can then be slid bodily along the planed facing on the plummer-block, until it is clear of the poles. Thus either half of the armature can be readily brought to the top, slid outwards and inspected. The magnet coils are excited at 110 volts, and the exciting current at no load is 65 ampères, and at full load with $\cos \phi_e = 0{\cdot}85$ is about 100 ampères. The exciting current required to yield the full current of the armature on short-circuit is 35 ampères. Thus the loss in excitation is about 10 kilowatts, or 2·85 per cent. of the net output.

The watts per rev. per min. being 4500, the constant of the machine is very nearly the same as in the last machine, viz.,

[illegible] The [illegible] of [illegible] fixing the poles is similar to that [illegible] the screws are lodged in [illegible] the wheel. The flywheel is of [illegible] is fastened to the shaft by two rings [illegible] at right angles. Its outer diameter is [illegible] The circular pole [illegible] diameter and are each wound with [illegible] turned on the outside [illegible] are arranged in two parallels and [illegible] The pole-pieces are [illegible] The bore of the pole-pieces is [illegible] diameter, and the [illegible] diameter, and [illegible] deep, divided by [illegible] It is [illegible] inches [illegible] diameter, and [illegible] winding [illegible] copper bars [illegible] copper end connectors [illegible] In order to render any part of the [illegible] winding easily accessible the wheel [illegible] in two halves, and these are seated [illegible] through the hollow [illegible] the lower ring is cast [illegible] and has an extended facing on [illegible] The armature wheel can be turned [illegible] coincides with the horizontal [illegible] loosening the screws which fasten [illegible] the upper half of the armature can then [illegible] along the planed facing on the plummer-block [illegible] Thus either half of the armature [illegible] outwards and inspect[illegible] 110 volts, and the exciti[illegible] at full load with cos. [illegible] The exciting current required [illegible] short-circuit is 35 [illegible] 10 kilowatts [illegible]

[illegible] per rev. per [illegible]

[illegible] is very [illegible]

y
al
ded
ns of
The
through
into four

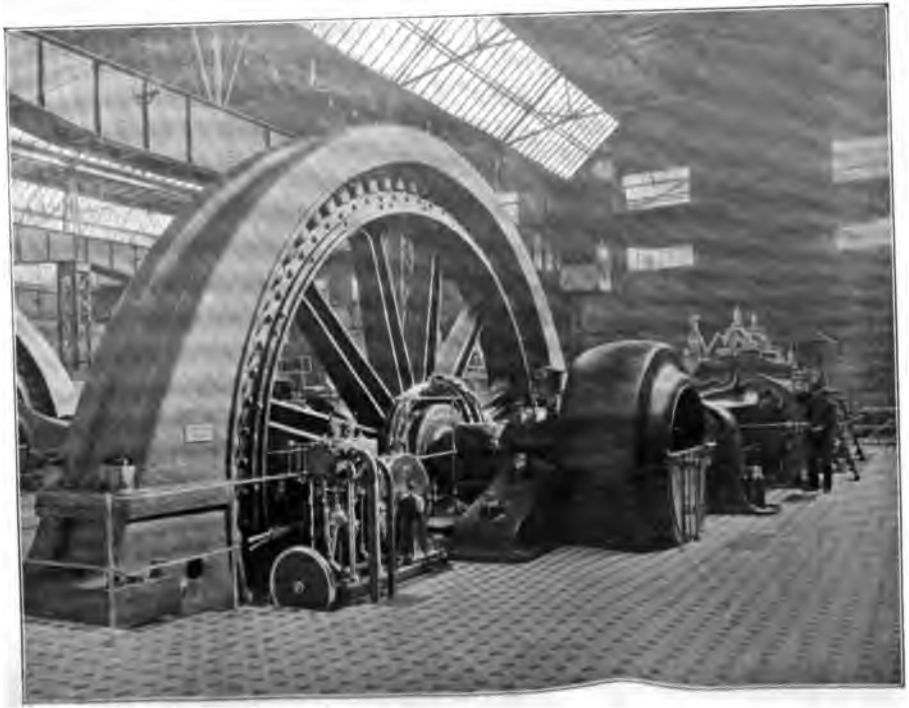


FIG. 398. 3000 K V A three-phase generator of Allgemeine Elektricitäts Gesellschaft.

[To face p. 902

$\frac{44\cdot5}{\cos\phi_e}$, a small figure due partly to the low tension of the armature.

§ 8. **3000 KVA. three-phase generator of Allgemeine Elektricitäts Gesellschaft.**—One of the largest alternators exhibited at Paris in 1900 was the 3000 kilovolt-ampère three-phase machine of the Allgemeine Elektricitäts Gesellschaft of Berlin. Eight of these alternators are now at work in the Oberspree and Moabit stations of the Berlin Electricity Works, one such being shown in fig. 398, while several more are in process of construction for the Oberspree station. The machine illustrated is directly coupled to a 4000 B.H.P. horizontal steam-engine, running at 83·3 revs. per min., and with its 72 poles gives a periodicity of 50. The winding of the stationary armature is star-connected, each phase giving 3460 volts, so that the interlinked pressure between any pair of terminals is $3460 \times \sqrt{3} = 6000$ volts, while the current per phase is 290 ampères. It is designed for a power-factor $\cos\phi_e = 0\cdot9$, so that its normal output is 2700 kilowatts, or 32,500 watts per rev. per min., while in addition the overload capacity is considerable. Although the pressure is high, yet the machine is of so large a size that massive copper bars are employed for the winding; it is further noteworthy for its large number of slots per pole, viz., 15, or 5 per pole and per phase, and also from the employment of the Hutin and Leblanc damper winding on the poles. The external diameter of the outer armature casing is 28 ft., and its over-all width nearly 4 ft. The complete lower half has already been shown in fig. 342. On the inside of the outer casting is cast a ring which forms the foundation on which the segmental laminations are built up by being threaded over a number of bolts. In the centre of the casing is cast a second ring or rib, and when half the axial width of the armature core has been built up, the laminations are tightly compressed by means of steel segments screwed to the central internal rib. When the second half of the core has been threaded over the bolts, the whole is again tightly compressed by means of segmental end-plates screwed to the inside of the casing. The central segments are grooved to allow of air being forced through behind the discs, and each half of the core is divided into four packets by metal distance-pieces placed round the circle at frequent intervals, to ensure good ventilation, while the outer surface of the casing is pierced with numerous holes to allow of the air escaping. The packets are arranged so as to break joint, and when the two halves of the armature are finally combined, cross one another and are locked together. The whole rests on three pairs of feet, one pair a little below the horizontal diameter, and the second about half-way round the lower quadrants, the two being further reinforced by a pair of smaller adjustable feet at the bottom. The gross width of the armature core is 22·8″, and the diameter of the bore is 24 ft. 4 in. nearly.

The slots in the armature, 1080 in all, are open, of rectangular shape, $1\frac{1}{2}''$ deep × $\frac{7}{16}''$ wide, and semicircular at the bottom (fig. 399.)[1] The total radial depth of the armature laminations including the slots is 10″. In each slot is a micanite tube 0·138″ thick with a rectangular copper bar threaded through it, 0·144 sq. in. in section, the whole being held in place by a wooden wedge dovetailed into grooves at the sides of the slot. The bars are united by forked or V-shaped end-connectors held well apart by the bars and nearly equal to them in sectional area. Each coil per pole consists of two lap-wound turns and one bar or half turn, by which the passage is made to the opposite side of the armature; the second coil then takes up the winding, and after describing two complete turns brings the winding back to the same side as the starting-point by means of the single bar or half-turn, so that there are ten inductors per pair of poles. Since the E.M.F. induced per bar is about ten volts, the difference between adjacent end-connectors is only some twenty volts, so that they are only lightly insulated with varnish. The resistance of each phase when hot is 0·098 ohm, whence the loss of E.M.F. is $28·5 \times 1·73 = 49·5$ volts of interlinked pressure, and the copper loss on full load is about 25 kilowatts or 0·93 per cent. of the net output. The total weight of the armature is about 90 tons.

The flywheel is cast in four quarters and united by bolts and by two rings shrunk on the nave which is keyed to the shaft by two keys at right angles (cp. fig. 400).[2] Each quadrant is carried by three

---

[1] Reproduced by permission from *Engineering*, 12th Oct. 1900.

[2] *Ibid.*

arms, and the junction of two quadrants at the rim is effected by two bolts and in addition by the introduction of a circular core held by cotters at each end. The yoke of the magnetic circuit is formed of segmental laminations, each covering the arc corresponding to three poles, traversed by bolts and compressed between a ring cast in one with the flywheel rim and segments engaging in a turned recess on the other side of the rim. The magnet cores are also laminated, and are dovetailed into the yoke-ring; each core is fastened by two wedges which are driven in from opposite ends and themselves locked by a rod passing through a half-round hole in the adjacent sides of the pair. The diameter over the pole-faces is nearly 291″ (the single air-gap being only 1 cm. = 0·394″), giving a peripheral speed of 6340 ft. per

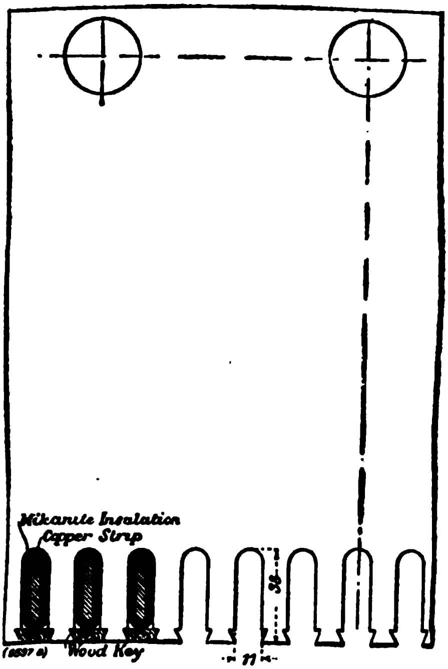


FIG. 399.—Armature lamination of 3000 K V A. generator.

min. The axial length of the pole-faces is 22″ and their width 9·45″ or nearly 75 per cent. of the pole-pitch. Each pole is wound with a rectangular cable coiled on the flat in 10 layers of 9 turns each. The excitation loss at full load amounts to about 31,000 watts or 1·15 per cent. of the net output. The damper winding at the tips of the poles is formed of six rectangular bars and two round bars at the corners of each pole, all of them riveted to two segments of copper, one on either side of the pole. The weight of the complete flywheel magnet is 70 tons, making a total weight for the machine with foundation plates of 160 tons.

The flux per pole is about 8,800,000 lines, the density averaged over the area of the air-gap opposite each pole being approximately $B_g = 6250$. The minimum area of the teeth being 610 sq. cm., the induction in them of $B_t = 14{,}500$ is low. If the leakage between the pole-tips is reckoned as 855,000 lines, and between the magnet-cores as 2,030,000, so that the total Z_m at the root of the poles $= 11{,}700{,}000 = 1{\cdot}33 Z_a$, the density in the magnet cores rises from a minimum of $B_m = 13{,}350$ to a maximum $B_m = 16{,}300$, so that the magnets are but little saturated. The constant of size is $\frac{53{\cdot}5}{\cos \phi_e}$.

§ 9. **5000 KW. Westinghouse Alternator.**—The 5000 KW. three-phase generator constructed by the Westinghouse Company for the Manhattan Railway Company is one of the largest alternators that have yet been built, and is illustrated by figs. 404–6, for which together with figs. 401–3 we are indebted to the British Westinghouse Company. The height of the great frame of the stationary armature is 42 ft., while the diameter of the rotating field is 32 ft., and its weight 166 tons out of a total for the machine of nearly 400 tons. The speed being 75 revs. per min., the peripheral speed is as much as 7500 ft. per min., and in order to reduce the length between the bearings to a minimum, the weight and design of the rotating part is such as to render any auxiliary flywheel unnecessary. The flywheel effect from the magnet system is calculated to be equal to its weight of 166 tons acting at a radius of 11·7 ft., or WD^2 in tons and ft.^{2} = 91,000. Further, the driving power is supplied by two compound engines, each with a horizontal high-pressure and a vertical low-

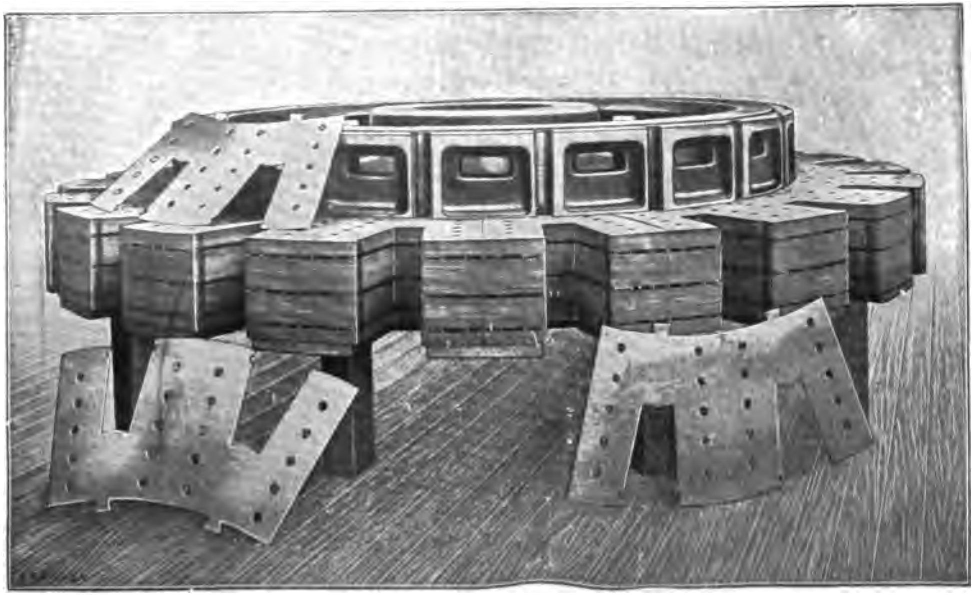


FIG. 401.--Laminated pole-pieces of Westinghouse alternator.

[*To face p.* 907.

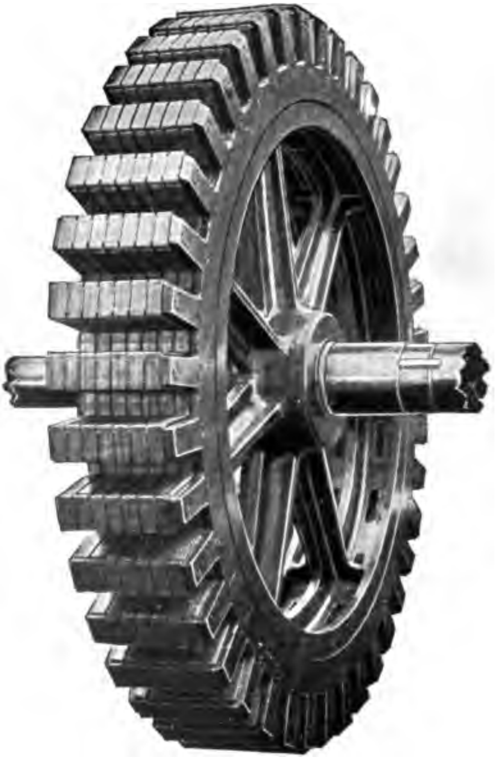


FIG. 402.—Rotating field of Westinghouse alternator, ready to receive the coils.

[*To face p.* 907.

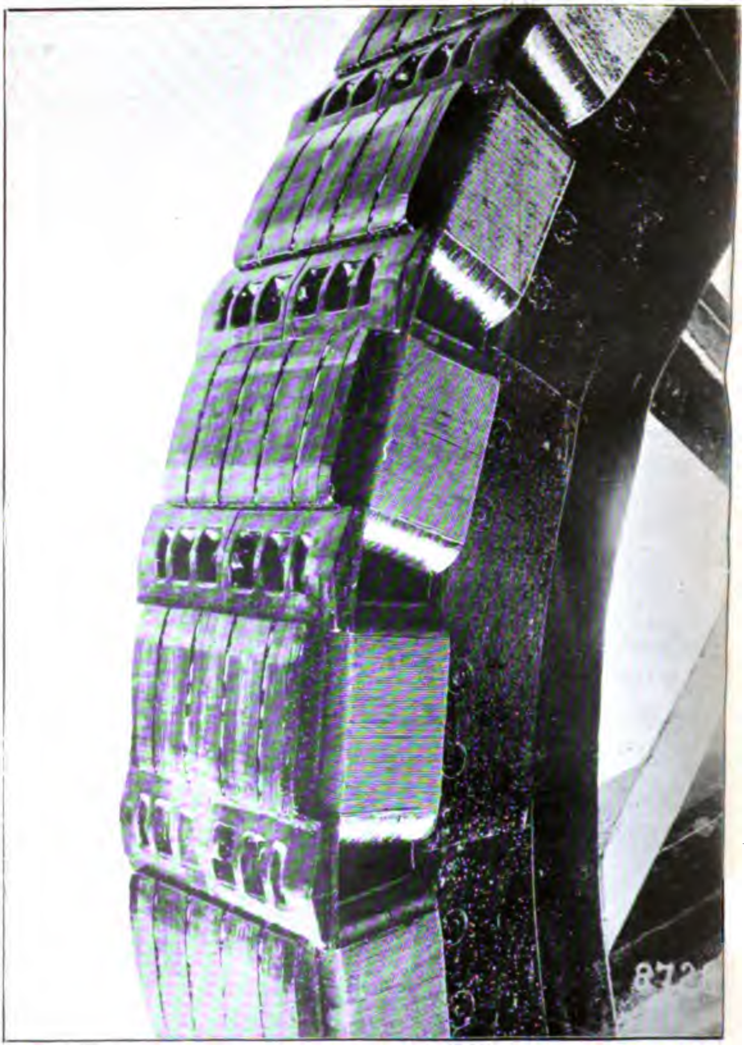


FIG. 403.—Portion of rotating field of Westinghouse alternator showing coils in place.

[*To face p.* 907.

pressure cylinder working on to a single overhung crankpin. The two cranks, one on each side of the alternator, are set at an angle of 135° to each other. Thus the four cylinders give eight impulses to the shaft at equal intervals of time in each revolution, and from the resulting uniformity of the crank effort combined with the flywheel effect from the rotor, no difficulty is anticipated in the driving even of rotary converters of 1500 kilowatts output.[1] The number of poles being 40, the periodicity is 25.

The shaft is a hollow forging of fluid-compressed steel, the external diameter being 37″, and the internal 16″. The cast steel hub of the revolving field is keyed to this shaft, and in place of arms two webs of rolled steel plates are bolted to its central flange, and at their outer edge carry a cast-iron driving rim. The poles and yoke are composed of thin sheet-steel laminations dovetailed into the cast-iron rim and further traversed by a number of bolts compressing them between two steel end-plates. The laminations overlap in neighbouring layers, and the butt joint of two adjacent laminations in the same layer is arranged to occur in the middle of a pole, the length of each lamination being equal to the width of two poles (cp. figs. 401 and 402 from photographs of a smaller Westinghouse alternator). In the laminated rim are six ventilating ducts passing from the periphery to large openings in the cast-iron rim.

The field coils are of copper strip, wound on edge, with insulating material cemented in place between the turns, the edges of the strip being exposed. No supporting case is used for the coil, but the thoroughness of the insulation provided by the internal covering is ensured by testing each coil with 2500 volts alternating potential for one minute. The coils are held in place by cast copper bridges which are slid under the bevelled edges of the poles, as shown in fig. 403; these bridges also serve the purpose to a certain extent of a damper winding, their overhanging lips retarding in virtue of induced eddy-currents any tendency of the field flux to shift across the pole-edge (cp. Chap. XXIV § 10).

---

[1] L. B. Stillwell, 'The Electric Power Plant of the Manhattan Railway Co.,' *Street Railway Journal*, Jan. 1901, from which many of the following particulars are derived.

The stationary armature frame, weighing in all some 234 tons, is supported by an exceptionally stiff cast-iron casing in six segments (figs. 404–5); the section of the casing and armature core with its six ventilating ducts opposite to those in the magnet

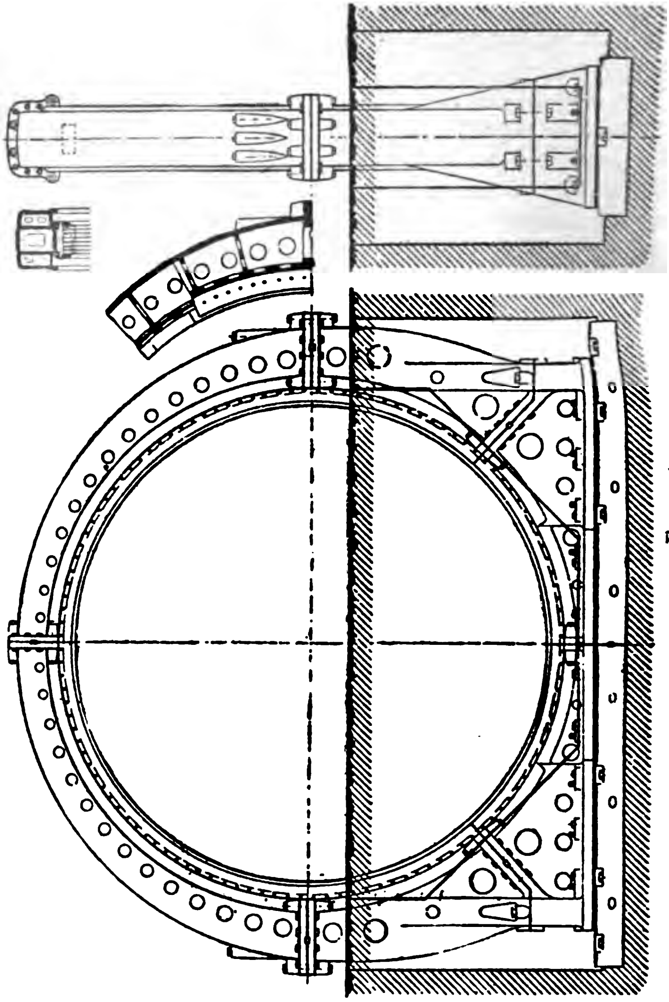


FIG. 405.

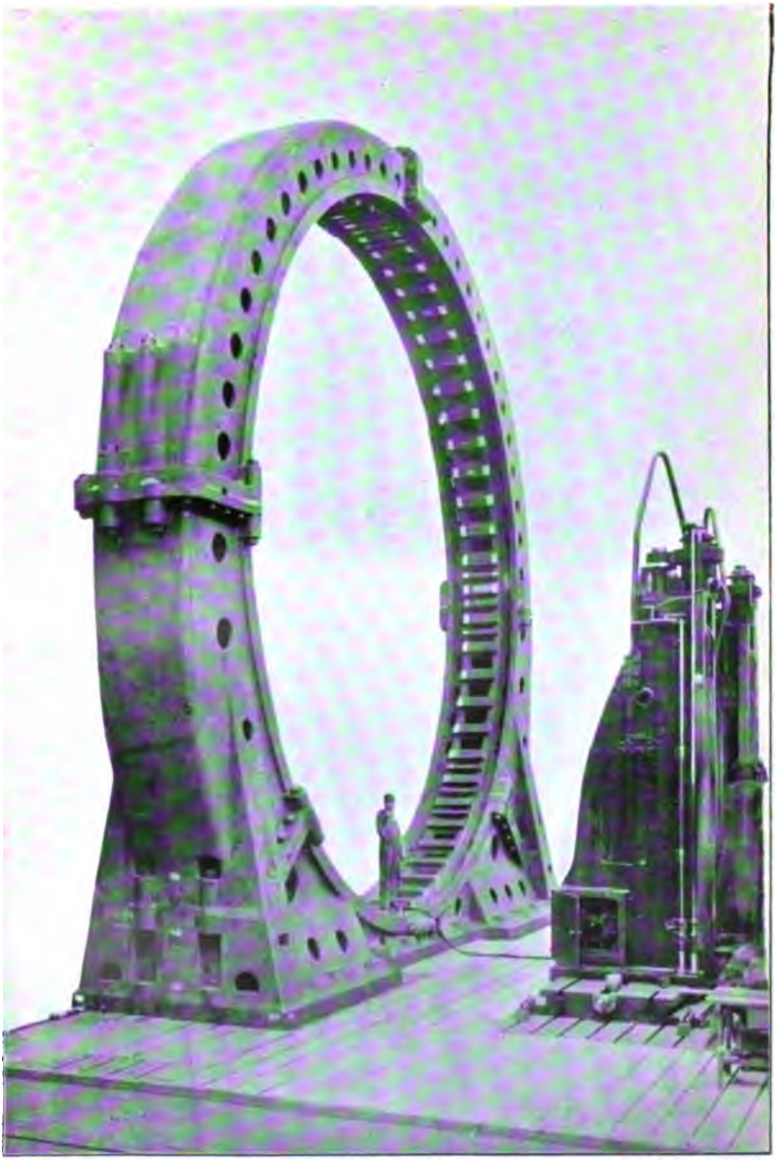


FIG. 404.—Armature frame of Westinghouse 5000 K W. alternator.

[*To face p.* 908.

el is illustrated in fig. 406. The winding is composed of s, three per slot, with separate end-connectors. As the ssure of 11,000 volts is directly generated, the insulation of armature bars is very high, and is guaranteed to stand a ncture test of 25,000 volts alternating potential for 30 minutes, 30,000 volts for one minute. The pole-tips are bevelled at edges to procure such a distribution of the field as will give inusoidal shape to the curve of E.M.F. at no load, and especial re has been taken to secure the same shape under full load. here are four slots per pole and per phase, so as to eliminate far as possible any har- onics of higher frequency. he armature slots are par- ally closed.

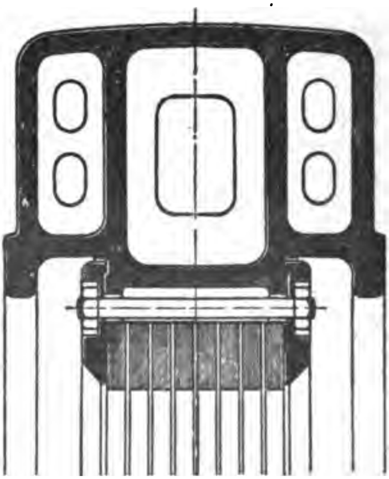


FIG. 406.

The full load output is 000 kilowatts at 75 revs. er min., or 66,600 watts er rev. per min., and is nade up of 263 ampères per phase at 11,000 volts between the terminals, the winding being star-connected, and each phase giving 6350 volts. The calculated exciting current is given as 225 ampères at 200 volts when the machine is giving its full rated current at 11,000 volts with cos $\phi = 1$ on a non-inductive load, with an increase of 15 per cent. for the full output at the normal voltage if the power-factor is 0·9. When the full load at 11,000 volts and with unity power-factor is thrown off, the potential is guaranteed not to rise more than 6 per cent. with constant speed and field excitation. The overload capacity may be taken up as high as 7500 kilowatts or 10,000 e.h.p. for a short period, and the efficiency without allowance for mechanical friction is given as 90 per cent. at one-quarter load, 94·5 at half load, 95·5 at three-quarters, 96·5 at full, and 97 per cent. at 25 per cent. overload.

On a non-inductive load the alternator is guaranteed to give its full rated load with a rise of temperature in no part of the machine exceeding 63° F. or a 25 per cent. overload with a rise not exceeding 81° F., while with a current 50 per cent. greater than the normal and with unity power-factor, the rise after two hours will not exceed 99° F., all of the temperatures being as measured by thermometer.

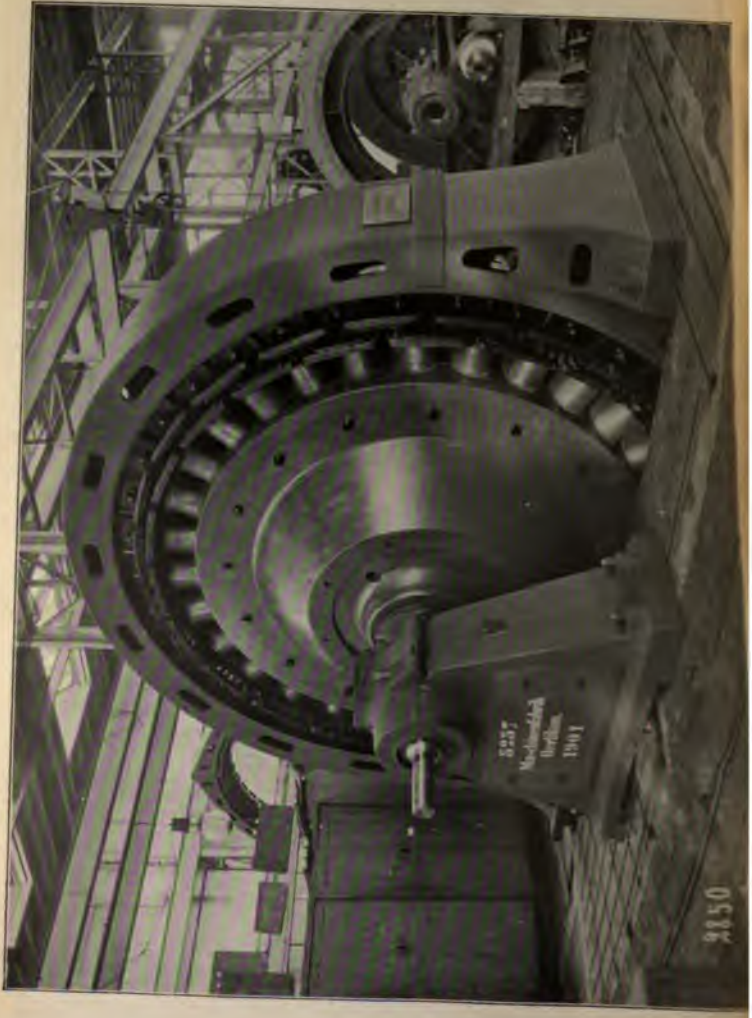
§ 10. **Oerlikon 1560 KW. three-phase generator.**—In fig. 408 is shown a three-phase revolving-field generator built by the Oerlikon Company for an output of 1560 kilowatts at 6100 volts and a periodicity of 46 when running at 184 revs. per min., the poles being 30 in number. Curves of the performance of the machine are added in fig. 409, from which it will be seen that with a voltage of 3540 per phase or an interlinked pressure of 6100 if star-connected, and a current of 175 ampères in each armature phase, its apparent power is 1840 KVA. and its real output with $\cos \phi = 0{\cdot}85$ is 1560 KW. or 8500 watts per rev. per min. The internal dimensions of the stationary armature are approximately 11′ 4″ diameter × 19¾″ width. There are 90 former-wound armature coils in 180 mica-insulated open slots, or 2 slots per pole per phase. Each coil consists of 7 turns of three wires in parallel, the diameter of the wire being 5 mm. insulated to 5·6 mm. The armature frame can be racked sideways past the magnet for cleaning or inspection of either the armature or magnet coils.

Each of the 30 field coils consists of 70½ turns of bare copper strip 1·3″ × 0·079″, with 0·5 mm. fibre insulation between the turns. The total flywheel energy is 4680 foot-tons, or 3 foot-tons per KW. of output, 2000 foot-tons being stored in the magnet wheel, and the remaining 2680 foot-tons in the separate flywheel by its side.

The directly-coupled exciter gives 200–220 volts and 140 ampères. The diameter of the armature is 21¼″, and its width $14\frac{3}{16}$″ with 115 open slots 1·53″ deep × 0·236″ wide and 2 coils per slot. Each coil consists of 2 turns of 2 wires in parallel, each wire being 3·6 mm. diam. insulated to 4 mm. There are 115 commutator sectors, insulated with 0·031″ mica. The 4 magnet coils are in series and each consists of 1450 turns of copper 2·2 mm. diameter insulated to 2·6 mm.

90

0



[To face p. 921.

Fig. 410 shows the appearance of an Oerlikon alternator of a similar rotating-field type.

§ 11. **Oerlikon 520 KW homopolar alternator.**—The class of homopolar or inductor alternators is represented by the 520 kilowatt or 700 H.P. three-phase generator exhibited by the Oerlikon Company at the Paris Exhibition. The terminal E.M.F.

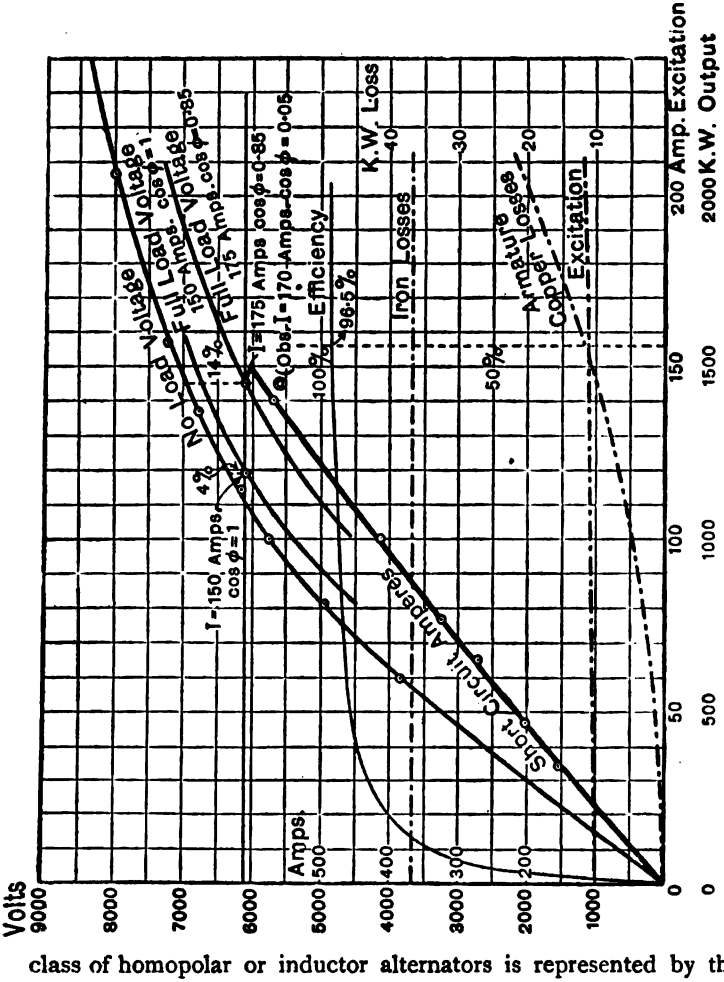


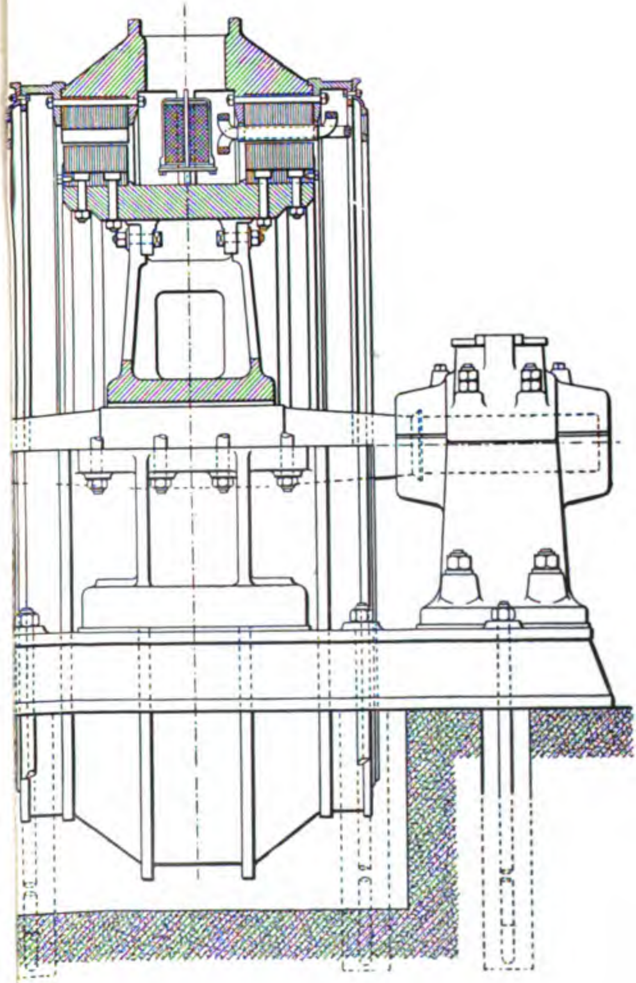
FIG. 409.—Curves of Oerlikon three-phase generator type 6170. 1560 K W. 6100 volts, 184 revs., 46 periods.

of this machine was 7500 volts, or 4330 per phase, the armature being star-connected, while the current per phase was 50 ampères, so that with an apparent power of 650 kilovolt-ampères the true output with a power-factor of 0·8 was 520 kilowatts. The speed was 250 revs. per min., and the number of polar projections in one crown 12, giving a frequency of 50. A similar machine but with 10 polar projections and arranged for 40 periods per second is shown in figs. 411 and 412, from which the ensuing description of the Paris Exhibition machine can be easily followed.

The rotating system of inductors is supported on a cast-steel yoke-ring turned on the face and bolted to a cast-iron wheel with six double arms strongly ribbed, the hub of which is keyed to the shaft. The polar projections are arranged in pairs at opposite ends of the yoke-ring, and in line with one another; they are composed of laminations with a dovetailed groove passing over a key fastened to the rim of the yoke-ring by two bolts traversing the key in a radial direction. The external diameter of the yoke-ring is 6′ 2½″, its width 3′ 0″ and its radial thickness 5″. The poles are 9½″ long axially and 9½″ wide with chamfered edges; their radial height is 5·9″, making the external diameter of the inductor wheel 7′ 2½″ nearly. The air-gap is only 4 mm. = 0·157″, the bore of the stationary armature being 7′ 2½″.

The single exciting coil is divided into two portions, held in place and concentric with the armature casing by supports of T section. Each half of the coil is composed of 83 turns of a flat copper band, 2·36″ wide × 0·079″ thick, the two halves being joined in series and giving a total resistance when warm of 0·199 ohm. The weight of the complete inductor system is 7½ tons, while that of the exciting copper is nearly 1½ tons.

The outer diameter of the cast-iron armature casing is 10′ 2″, and on its interior of 8′ 4″ diameter are bolted the armature laminations between a fixed ring on the inside of the casing and an external ring in two halves, recessed into the outer edge of the casing and carrying a protective flange pierced with numerous ventilating holes. The axial length of the laminations on each side of the centre line is slightly greater than that of the poles, and their radial depth is 6·7″. Each of the two arma-



[*To face p.* 912.

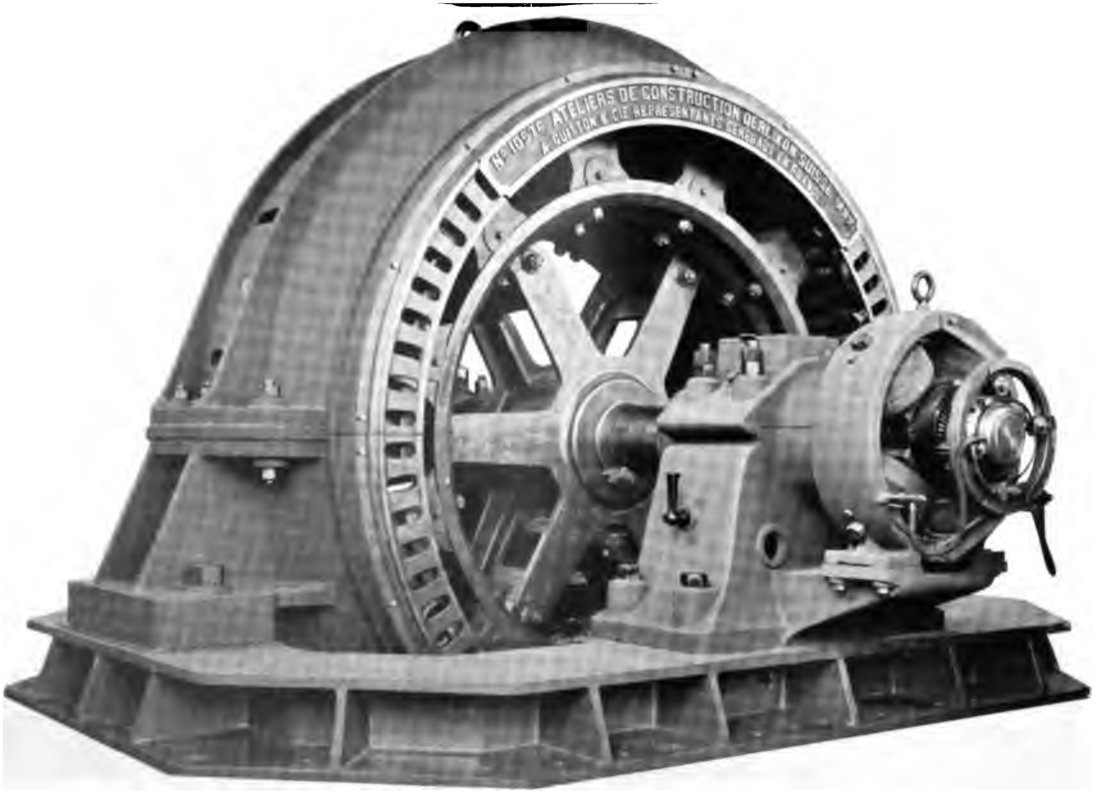

Nº 10576 ATELIERS DE CONSTRUCTION OERLIKON SUISSE
A. GUILTON & CIE REPRESENTANTS GENERAUX

FIG. 412.—Oerlikon 700 H.P. inductor alternator.

[*To face p.* 912.

tures has 36 slots, open at the top, in which are embedded three sets of 12 coils. The latter are former-wound, insulated by troughs of micanite and fastened in by wedges of fibre dovetailed into the sides of the slot. Each coil contains 33 turns of two wires, each 0·125″ diameter wound in parallel, and the corresponding phases of the two armatures are connected in series. The resistance of a complete phase is 1·37 ohms when hot, and the

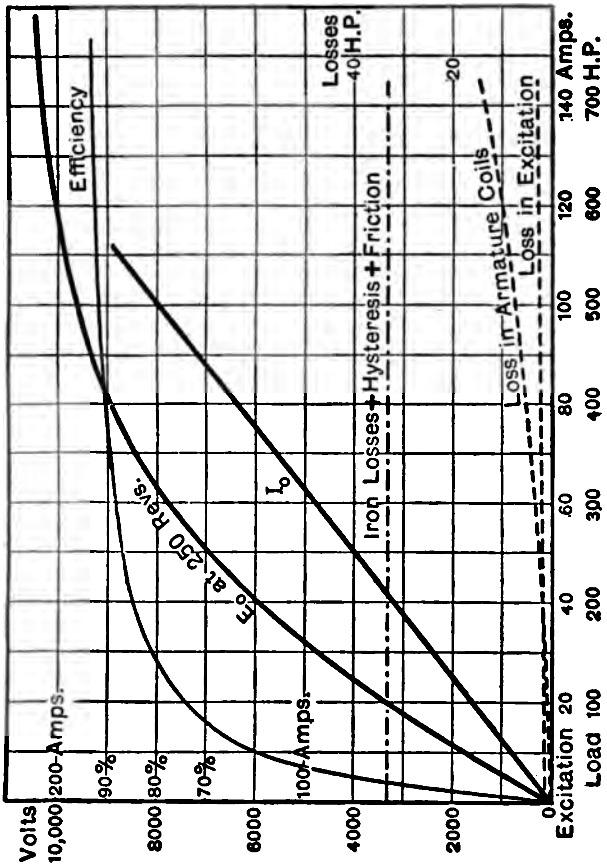


FIG. 413.—Curves of 700 H. P. Oerlikon generator 250 revs., 50 periods, 7500 volts.

total weight of copper in both armatures is nearly 12 cwt. The weight of the complete stationary armature is 15 tons.

From the curves of fig. 413 it is seen that the exciting current to give 7500 volts on open circuit is 56 ampères, and experiment showed that the current for full voltage and load with the designed power-factor of 0·8 would be about 95 ampères. The rise of volts for the same excitation on no load is then nearly 25 per cent. The exciting current for a load of 100 ampères and a power-factor of 0·7 or a real load of 910 kilowatts was 130 ampères, so that without any extravagant increase in the excitation the machine has a considerable overload capacity. The losses on full load of 700 H.P. are as follows:—

constant loss = 32·5 H.P. = 24,300 watts
copper loss in armature = 14 „ = 10,400
copper loss in exciting coil = 2·4 „ = 1,800
36,500
output = 520,000
556,500. Efficiency 93·5%.

The exciter is seen from fig. 412 to be a four-pole machine with circular magnet, and drum armature keyed on to the end of the shaft. Its capacity is 3·6 kilowatts at 30 volts.

INDEX

A

ACCUMULATORS, 500, 718.
Active E.M.F., 91, 105.
Air, permeability of, 36.
Air-ducts, 638, 811.
Air-gaps in magnetic circuit, 44, 126.
    fringe of, 45, 128, 426, 432, 748.
    induction in, 80, 815.
    distribution of, in smooth armature, 522.
    distribution of, in toothed armature, 535.
    permeance of, in smooth armatures, 425, 432, in toothed armatures, 458.
    effective area of, 435, 675.
    length of, 663.
    air-line, 458.
Air insulation in commutators, 199, 376.
Allen, H. N., on apparent inductance, 557, 586.
Allen, Son & Co's. dynamo for electroplating, 894.
Allgemeine Elektricitäts Gesellschaft's alternators, 816, 818, 824, 831, 832, 880, 903.
Alternating E.M.F. and current, measurement of, 108.
Alternators, simple forms of, 126, 133.
    types of. *Vide* Disc, Discoidal, Drum and Ring.
    constant-current, 809.
    armature reaction in, 754, 762, 778.
    differential action in, 143, 150, 155, 178, 182.
    efficiency of, 881–2, 911, 914.
    effective E.M.F. of, 173, 189, 190, 747, 758.
    characteristic curves of, 758, 760, 791, 806. *Vide* Short-circuit characteristic.
    excitation of, 835.
    regulation of, 754, 807.
    dimensions of, 847.
    induction in armature of, 811.
    ,, ,, magnets of, 829, 854.
    insulation of, 813.
    polyphase, 168, 728, 732, 735. *Vide* Quarter-phase, Three-phase and Polyphase.
    output of 116.
Alternators, uses of, 721.
    parallel working of, 860, 864, 868, 876.
    periodicity of, 160, 180, 185, 838.
    pull on inductors of, 174, 781.
    Vide Allgemeine Elektricitäts Gesellschaft; Ferranti; Brown, Boveri & Co.; Oerlikon; British Thomson Houston; Westinghouse Co.
Amortisseur, 878.
Ampères, 4; effective, 109.
Ampère-turns on electromagnet, 32.
    relation to M.M.F., 32, 424.
    relation to H., 32.
Ampère turns of armature, 519.
    back and cross in continuous-current-dynamos, 428, 523, 530.
    maximum permissible, 567.
    in alternators, 764.
Ampère-turns of field-magnet, 425, 429.
    calculation of, in bipolar magnet, 430.
    in other types, 469, 681.
    in alternators, 785, 854.
Ampère-wires on armature, 336, 568.
    maximum per pole, 568, 578.
    maximum per unit of circumference on continuous-current dynamo, 570.
    on multipolar dynamo, 572.
    in alternators, 842, 845, 846.
Analogy, hydraulic, to dynamo, 2, 4, 7, 67.
    of electro-magnetic and electro-dynamic equations, 70.
    of electro-magnet to battery, 40.
    of inertia to self-induction, 87.
    of magnetic and electric circuits, 38, 266.
Angle of lag, 114, 115, 755, 767.
    effect of, in alternator, 768.
Angle of lead of brushes, 428, 563.
    effect of, 529.
    increase of, with current, 564.
    limiting value of, 566.
    determination of, 572.
Annealing of iron, 272.
Apparent inductance of short-circuited section, 556.
Apparent power of alternator, 112, 119.
Arc lighting, 191.
    dynamos for, 500, 506.
    Brush, 199.
    Thomson-Houston, 205.
    alternators for, 809.

- Armature of electro-magnet, 40.
  - of dynamo, 8, 120.
- Armature ampère-turns. *Vide* Ampère-turns.
- Armature bars, 341, 344.
  - effect of their width, 56, 342, 628.
  - *Vide* Stranded and Twisted bars.
- Armature cores, 121.
  - function of iron in, 132.
  - non-magnetic, 139.
  - rotation of, 127.
  - construction of, from iron discs, 301, 303, 275, 811.
  - from iron tape, 309.
  - from iron wire, 308.
  - insulation of, 333.
  - lamination of, 299, 625.
  - size of, 658.
  - calculation of area of, 430.
  - *Vide* Toothed armatures.
- Armature inductors, driving of, 335.
  - pull on, 78, 336. *Vide* also Armature bars.
- Armature reaction. *Vide* Reaction.
- Armature resistance. *Vide* Resistance.
- Armature shafts, 313.
  - life of, 320.
  - diameter of, 322.
- Armatures, types of. *Vide* Closed-coil, Open-coil, Disc, Discoidal, Drum and Ring.
  - balancing of, 381.
  - speed of. *Vide* Peripheral speed.
  - proportions of, 382, 420.
  - heating of. *Vide* Heating.
- Arnold, Prof. 226, 246, 545, 560, 640, 775, 782.
- Ascending curves of induction, 267, 427.

B

- B=INDUCTION, 30.
- Back ampère-turns of armature in continuous-current dynamos, 428, 530.
  - effect on characteristic curve, 489, 499, 504, 512.
  - effect on E.M.F., 532.
  - effect on leakage, 429, 455, 458.
  - calculation of, 544.
  - experimental determination of, 545.
  - in alternators, 767, 773.
- Back E.M.F. of motor, 73, 92.
  - of self-induction, 92.
- Baily, Prof., on hysteresis, 285, 293, 294.
- Balancing of armatures, 381.
- Ballistic galvanometer, 26, 264, 453.
- Bar inductors for armatures, 341, 344.
- Bar magnet, 10, 14.
  - analogy to solenoid, 23.
  - lines of induction in, 28.
- Barlow, 176.
- Barrel-wound armatures, 353, 355, 584, 593, 674.
- Battery, voltaic, 6.
- Bearings, 324, 326, 328.
- Beattie, 288, 294.
- Behn-Eschenburg, 295.
- Behrend, B. A., 803, 806, 815, 825, 844.
- Belt, pull of, 316, 326, 718.
- Bending-moment of shafts, 314, 316, 405, 412.
  - of hub-arm, 311.
- Bending-moment, equivalent, 319.
  - at journal, 324.
- B, H curves. *Vide* Flux-density curves.
- Binding-wire of armatures, 364, 667, 680.
- Bipolar dynamos, 133, 146.
  - continuous-current, leakage in, 439, 445.
  - heating of, 618, 635.
  - maximum ampère-wires of, 570.
  - design of, 661.
- Bismuth, 34, 37.
- Blondel, A., 795.
- Bobbins of field-magnets, 477, 618.
  - *Vide* Field-magnet coils.
- Boucherot alternator, 838.
- Braiding of wires, 340.
- British Thomson Houston Co., 889.
- Brown, on heating of field coils, 611, 613.
- Brown, Boveri & Co.'s alternators, 816, 817, 823, 827, 831, 896, 899.
- Brush dynamo, 199, 205.
- Brush-holders, 604, 641.
- Brushes, 8, 126, 604.
  - position of, 195, 207, 551, 575, 607.
  - lead of, 428. *Vide* Angle of lead.
  - width of, in Thomson-Houston dynamo, 202.
  - *Vide* Carbon brushes.
  - contact-resistance of, 574, 643; effect of, 559, 595.
  - losses in, 644.
  - care of, 719.

C

- CARBON BRUSHES, 372, 605.
  - comparison with copper, 575.
  - width of, 592, 595, 605.
  - contact resistance of, 640, 575, 643.
- Carbon in iron, 270.
- Carter, F.W., 460, 648.
- Cast iron, 272.
  - for field-magnets, 385, 387.
  - induction in, 385.
- Cast-steel, 272.
  - for field-magnets, 385, 387.
- C.G.S. system, 16, 31, 49, 71, 89, 280.
- Centrifugal force, 366.
- Characteristic curves of dynamo, 486
  - shunt, 491.
  - series, 503.
  - compound-wound, 507, 512.
  - of alternators, 758, 760, 900, 911, 913.
- Charcoal iron, 275.
- Chord-winding, 256, 260, 576, 594.
- Circuit, electric, 3.
  - magnetic. *Vide* Magnetic circuit.
- Circumferential speed. *Vide* Peripheral speed.
- Classification of dynamos, 123.
- Clearance, 126, 663.
- Clinker, 288, 294.
- Clock diagram, 65, 116.
- Closed-coil armatures, 191, 206, 217.
- Closed curves of lines of induction, 21, 28, 84.
- Cobalt, 34, 37.
- Coercive intensity, 269, 272.
- Collecting rings, 133, 163.
- Commercial efficiency. *Vide* Efficiency.

Commutation, process of, 553.
  favourable conditons for, 595, 598.
Commutators, 8, 134.
  of closed-coil armatures, 211, 213, 214.
  construction of, 214, 369.
  material of sectors, 375.
  number of sectors, 214, 378, 599, 600.
  rise of potential in, 223.
  Vide Cross-connection and Sparking.
  heating of, 639, 644, 645.
  peripheral speed of, 641, 643.
  of open-coil armatures, 195.
  with air-insulation, 199, 376.
  care of, 719.
  flats on, 607.
Compass-needle, 10, 14, 19.
Compound-wound alternators, 835, 837.
Compound-wound continuous-current dynamos, 506.
  characteristic curve of, 507.
  calculation of, 509, 686.
  imperfections of, 512, 514.
  uses of, 514.
  long- and short-shunt, 508.
  parallel working of, 713, 715.
  temperature effect in, 514, 612.
  winding for, 509.
  series regulator for, 715.
Connectors. *Vide* End-connectors.
Consequent poles, 399.
Conservation of energy, 6, 73.
Constant current, for arc lighting, 506.
  regulation for, in Thomson-Houston dynamo, 205.
  alternator, 809.
Constant watt alternator, 810.
Contact resistance of brushes, 574, 640, 643.
Continuous-current dynamos, 176, 206.
  types of. *Vide* Discoidal, Disc, Drum and Ring, Closed-coil, and Multipolar.
  armature reaction in, 516.
  efficiency of, 705.
  E.M.F. of, 218.
  excitation of. *Vide* Shunt, Series, and Compound-wound.
  heating of, 617, 635.
  uses of, 206.
Conversion of energy, 7, 19, 74, 79.
Cooling surface, 81.
  of armatures, 636.
  of field-magnet coils, 391, 473, 618.
  of commutators, 645.
Copper wire, 8, 67.
  resistance of, 340, 474. *Vide* also Insulation.
Core, iron, of solenoid, 23, 32.
Cores, armature. *Vide* Armature cores.
Cores, magnet, 120, 388. *Vide* Field-magnets.
Coulomb, 4.
Creep of winding, 252.
Critical speed of shunt dynamo, 495.
Crompton twisted and stranded bars, 344, 628, 633.
Cross ampère-turns of continuous-current dynamo, 523, 530.
  reduction of flux by, in smooth armature, 528; in toothed armature, 543; maximum value of, 567.
  of alternator, 766, 776.
Cross-connection of commutator, 220, 263.
Current, flow of, 2, 4, 68.
  pull due to, 70.
  self-induction due to, 84.
  alternating, 109.
Current-density in field-magnet coils, 502, 834.
  in brushes, 574, 643.
  in alternator armatures, 814.
Curves, characteristic. *Vide* Characteristic curves.
Curves of E.M.F. of rotating inductor, 62, 128.
  of alternators, 114, 133.
  of unidirected machines, 134, 196, 197.
  of continuous-current machines, 208, 210.
Curves of induction or flux-density, 267, 273.
  use of, 427.
Curves of rate of change, 96.
Curves, sine, 63, 97, 112, 114.
Cutting, of lines, 46.
  rate of, 48.
  by a free conductor, 72.
  of conductor by its own lines, 84
Cycle, magnetic, 280.

D

DAMPER WINDING, 878.
Definition of dynamo, 1.
  electrical energy, 2.
  permeability, 36.
  induction, 30.
  pitch, 149.
  trailing and leading, 338, 554.
  hetero- and homopolar, 124.
  unit pole, 16.
Δ-connection of polyphase armatures, 733, 740.
Deptford central station, 822.
Designing of bipolar dynamo, 430.
  of multipolar dynamo, 669.
  of alternator, 849.
Dettmar, G., 458, 652.
Diameter of commutation, 516. *Vide* Sparking and Angle of lead.
Difference of potential. *Vide* Potential.
Differential action in alternators, 143, 150, 155, 178, 182.
Disc alternators, heteropolar, 138, 161, 162, 166, 168.
  ratio of coil-width to pitch, 154.
  E.M.F. equation, 173.
  hysteresis in, 292.
  Ferranti, 821, 833.
  winding for, 722, 821.
Disc alternators, homopolar, 144, 182, 184.
  ratio of coil-width to pitch, 182.
  E.M.F. equation, 189.
  hysteresis in, 292.
Disc armatures, 139. *Vide* Disc alternators, and Disc winding.
Disc Continuous-current dynamos, 139, E.M.F. equation, 218.
Disc winding, 138.
  of homopolar dynamo, 144. *Vide* Disc alternators, Disc armatures.
Discoidal-ring alternators, heteropolar, 138, 161, 162, 166.

Discoidal-ring alternators, ratio of coil-width to pitch, 150.
  E.M.F. equation, 173.
  winding of, 722, 820.
Discoidal-ring alternators, homopolar, 143, 177, 179.
  ratio of coil-width to pitch, 178.
  E.M.F. equation, 188.
Discoidal-ring armatures, 137.
  lamination of core, 300, 301.
  construction of, 309.
Discoidal-ring continuous-current dynamos, 139.
  E.M.F. equation, 218.
  magnet of, 397. *Vide* Ring continuous-current dynamos.
Discoidal-ring winding, 136.
  of homopolar dynamo, 142. *Vide* Ring winding, Discoidal-ring alternators, armatures, and continuous-current dynamos.
Discs, iron, for armatures, 276, 301.
Distortion of field by armature reaction, 517.
  in smooth armature, 525, 528.
  in toothed armature, 535.
Distribution of alternator winding, 148, 725, 749.
Drag, magnetic. *Vide* Pull.
Driving of core, 334; of inductors, 335.
Drop-forged sectors, 376.
Drum alternators, heteropolar, 133, 146, 162, 165, 166.
  ratio of coil-width to pitch, 154, 749.
  E.M.F. equation, 173, 747.
  winding of, 722, 740, 811.
  induction in armature, 811.
  field system of, 827.
Drum alternators, homopolar, 143, 182.
  ratio of coil-width to pitch, 183.
  double-armature, 184.
  E.M.F. equation, 189.
  hysteresis in, 292, 294.
  induction in armature, 811.
  field-system of, 823.
Drum armatures, 132, 214.
  lamination of core, 300.
  core-discs for, 301.
  construction of, 303, 811.
  in alternators, 816.
Drum continuous-current dynamos, 134, 214.
  E.M.F. equation, 218.
  lap-wound parallel-connected, 234, 241, 244, 254, 258, 356, 418.
  wave-wound parallel-connected, 236, 241, 244, 418.
  wave-wound series-connected, 237, 241, 244, 259, 358.
  multiplex parallel-connected, 256, 258, 601.
  multiplex series-connected, 261, 601.
  general formula, 261.
  chord-wound, 256, 260.
  rise of potential in, 222.
  equality and symmetry of parallel paths, 241, 245, 417.
  hand-wound, 346.
  split-coil armature, 348.
  former-wound, 349.
  barrel-wound, 353, 355.
  with separate end-connectors, 362.
Drum continuous-current dynamos, ratio of length to diameter of armature core, in bipolars, 382; in multipolars, 420.
  radial depth of core, 383.
  resistance of armature, 380.
  advantage of multipolars, 419.
  heating of, 635.
  maximum ampère-wires of, 567, 570, 580.
  inductance of short-circuited sections, 583, 586, 593.
  number of sectors, 599, 600.
Drum winding, 132, 214.
  hand of winding, 216, 232.
  lap-winding, 229, 230, 356, 360, 418.
  wave-winding, 234, 359, 418.
Duplex winding, 249, 601.
Dynamo, dual nature of, 1, 105.
  definition of, 1.
  action of, 7.
  efficiency of, 7.
  output of, 5.
  simple forms of, 3, 66, 123.
  hydraulic analogy to, 2, 4, 67.
  classification of, 123.
  hetero- and homo-polar, 124.
  unipolar, 124, 140.
Dynamometer, 107.
Dyne, 16, 18, 71.

E

EDDY-CURRENTS, in armature bars, 56, 341, 627.
  in armature cores, 283, 299, 306, 310, 625.
  in pole-pieces, 339, 545, 624, 646.
  loss by, in armatures, 620, 624, 631, 666, 677.
  measurement of, 624, 653, 690, 694.
  in alternators, 756, 760, 775, 884.
Edison-Hopkinson dynamo, 454.
Effective E.M.F. and current, 109, 114.
  of alternator, 114.
Efficiency of dynamos, 7, 311.
  of continuous-current dynamos, 669, 689, 705.
  of particular machines, 889, 892.
  of alternators, 881–2, 911, 913.
  measurement of, 699.
Eickemeyer, 351.
Electrical energy, 2.
  of alternator, 104.
Electro-dynamic action, 19, 68.
Electrolytic work, 500.
Electro-magnet, 23.
  analogy to voltaic battery, 39, 41.
  in dynamos, 38, 45.
Electro-magnetic induction, 50.
Electro-motive force, 2.
  how produced, 47.
  as rate of cutting, 48, 50.
  average and instantaneous, 50.
  direction of, 48, 61.
  due to rectilinear motion, 51, 55.
  of loop, 59.
  of curved inductor, 57.
  of alternator, 110, 114.
  of eddy-currents, 56.
  of self-induction. *Vide* Self-induction.
  impressed and resultant, 91.

Electro-motive force equations,
  of heteropolar alternators, 172, 747.
  of homopolar alternators, ring, 188; drum, 189, 748.
  of continuous-current dynamos, general, 218.
  of multipolar parallel-connected armature, ring, 219, 247; drum, 241, 258.
  of multipolar series-connected armature, ring, 228, 249; drum, 241, 261.
  of multiplex parallel-connected armature, ring, 247; drum, 258.
  of multiplex series-connected armature, ring, 250; drum, 261.
Electroplating, 5, 500.
  dynamo for, 804.
E.M.F. curve of alternators, 751.
End-connections of drum armatures, 156, 229, 232, 362, 584, 629, 665.
  in multipolar dynamos, 364, 674.
  in alternators, 728.
End-play of shafts, 326.
Energy, electro-magnetic, of field, 93.
Engines, steam, speed of, 655.
  want of uniformity of speed, 868, 871, 875.
English Electric Mfg. Co.'s traction generator, 885.
Equalising connections on armature, 244, 263.
  between brushes, 712.
Equivalent sine wave, 118.
Ewing, Prof., theory of magnetism, 288.
  hysteresis tester, 296.
Excitation of field-magnets, 478. *Vide* Self-excitation, Shunt, Series, Separately-excited, Compound-wound.
  of alternators, 835.
Exciting power of electro-magnet, 33.
  relation to M.M.F., 33, 425. *Vide* Ampère-turns.
Exploring coil, 25.
External characteristic. *Vide* Characteristic curves.
External circuit, 3.

F

FARADAY, 6, 25, 53, 176.
Ferranti alternator, 821, 833.
Field, magnetic, 8.
  of bar-magnet, 12, 15.
  of straight current, 19.
  of loop, 21.
  of solenoid, 22.
  of toroid, 23.
  with iron core, 23, 28.
  structureless, 17, 47.
  self-induced E.M.F., due to alteration of, 84.
  energy stored up in, 92, 483.
  of armature ampère-turns, 519.
  symmetrical distribution of, in dynamos, 411, 415.
Field-magnets of dynamos, 8, 45.
  three parts of, 120, 423.
  C-shaped single horseshoe, 122, 392.
  bipolar overtype horseshoe, 8, 392, 618.
  bipolar undertype horseshoe, 392, 430, 618.
Field-magnets, bipolar double horseshoe, 395.
  iron-clad, 398.
  Manchester, 395, 577, 618.
  of heteropolar alternators, 164, 166, 827.
  of homopolar alternators, 182, 184, 823.
  with single exciting coil, 187, 402, 827.
  material of, 273.
  comparison of forged iron and cast steel with cast iron, 385.
  composite, 388.
  with consequent poles, 395, 399.
  comparison of single and double horseshoes, 395, 400.
  induction in, 384.
  joints in, 405.
  leakage from. *Vide* Leakage.
  length of, 390.
  magnetic pull on, 403, 416.
  section of, 388.
  proportioning of areas, 407, 418.
  shaping of pole-pieces, 342, 577.
  symmetry of field, 411, 415, 577.
  throttling of lines in, 407, 577.
Field-magnet coils, 8, 120.
  of heteropolar alternators, 166, 834.
  single coil in continuous-current dynamos, 402.
  single coil in homopolar alternators, 182, 185, 187, 402, 827.
  cooling surface of, 391, 618.
  formers for, 477, 619.
  length of, 390.
  wire for, 476, 834.
  Vide Field-magnet winding, Heating.
Field-magnet winding, calculation of ampère-turns for. *Vide* Ampère-turns.
  of size of wire, 472, 501, 511, 685, 859.
  of turns, 475, 502.
  of weight of wire, 475, 484.
  Vide Self-excitation, Series, Shunt, Compound-wound, Separately-excited.
  effect of heating on, 473, 484, 611.
  loss in, 484.
  self-induction of, 483, 834.
Filings, iron, 15, 17, 19.
Fischer Hinnen, 801, 804, 878.
Flat-ring. *Vide* Discoidal ring.
Flats on commutator, 607.
Fluctuation of E.M.F. in closed-coil armatures, 209, 214.
  in open-coil armatures, 196, 202.
Flux, magnetic, 30.
  curves of, in dynamos, 456.
Flux-density, 30.
  curves of, 267, 273, 386.
  use of, 427.
Flywheel, analogy to self-induction, 88, 93.
  weight of, 316.
  effect from, required in alternators, 830, 869, 873.
Forbes's, Prof., rules for magnetic leakage, 437.
Force, lines of, 12, 14, 15, 19.
Forged ingot iron, 273, 385.
Formers, for armature coils, 348.
  for field magnet coils, 477, 619. *Vide* Field-magnet coils and Winding.
Form factor, 172.
Foucault currents. *Vide* Eddy-currents.
Foundations, 717.

Four-pole dynamo, 399.
  with two single horseshoes, 401.
  with divided circuits, 400.
  design of, 669.
Four-phase alternator, 731, 732.
Free pole, 11.
Frequency. *Vide* Periodicity.
Friction of bearings, 325, 326, 692.
  of brushes, 645, 689.
Frictional machine, 6.
Fringe of air-gap, 45, 128, 426, 432, 463.
  in alternators, 748.
Fundamental equations of dynamo, 57, 81.

G

GAS-ENGINES, 718, 873.
Gauss, 16.
General Electric Co.'s traction generator, 889.
Gramme winding, 130. *Vide* Ring winding.
Grammont alternator, 838.
Graphite brushes, 643.
Grouped distribution of alternator winding, 148.
Gun-metal, as a magnetic insulator, 37, 266, 393.
Gun-metal hub for ring armatures, 130, 306.
  for commutator sleeve, 369.

H

H, magnetising intensity, 31, 267.
Hand formulæ for direction, of lines, 21, 23.
  of magnetic pull, 69.
  of E.M.F., 51.
Hand of winding, 154, 216, 232, 368.
Hansard, A. G., 707.
Harmonic function, 64.
Hay, Dr. A., 459, 694.
Heating of dynamos at work, 67, 81, 609.
  of armatures, 610, 635.
  by copper resistance, 623.
  by eddy-currents, 299, 342. *Vide* Eddy-currents.
  by hysteresis, 283, 298, 634.
  as limiting output, 81, 571, 614.
  of field-magnet coils, 390, 473, 611.
  of pole-pieces, 339.
Hefner Alteneck, 132.
Hele-Shaw, Prof., 459.
Henry, 90.
Heteropolar dynamos, definition of, 124.
  E.M.F. of, 171.
  hysteresis in, 292.
Hird, W. B., 468.
Hobart, H. M., 585, 593, 597, 690.
Holden, 294.
Homopolar dynamo, definition of, 125.
  simple forms of, 140.
  continuous-current, 176.
Homopolar alternators, ring, 178.
  drum, 183, 823.
  disc, 184.
  E.M.F. of, 748.
  back A.T. of, 775.
  excitation of, 187.
  useless flux in, 187, 748, 776, 825.
Homopolar alternators, hysteresis in, 92, 294.
  induction in, 811.
  Oerlikon Co.'s, 911.
Hopkinson, Drs. J. and E., on magnetic circuit equation, 452.
  on leakage factors, 454.
  efficiency test, 700, 881.
Horns, on pole-piece, 415, 440.
Horsepower, 4, 6, 79.
Horseshoe magnet, 8, 392, 403.
Housman, R. H., 690.
Hub of armature, 306.
  strength of, 310.
Hysteresis, magnetic, 279.
  energy lost by, 280, 287.
  theory of, 290.
  in alternating field, 284.
  in rotating field, 293.
  tester, 296.
  in dynamos, 291, 296, 383, 634, 666, 677 691.
Hysteretic constant, 285,

I

IMPEDANCE, 118.
  synchronous, 760.
Imperfect magnetic circuit, 45.
Impressed E.M.F., 91.
  deduction of curve for, 95.
Inductance, 88.
  of loop, 89.
  of solenoid, 89.
  of short-circuited section, 556, with smooth core, 581, with toothed core, 587.
  in alternators, 796, 799.
Induction electro-magnetic, 25, 53.
Induction, magnetic, 26.
  lines of, 26, 30.
  residual, 269, 292.
  relation of hysteresis loss to, 285.
  values for, in armature cores, 383.
  apparent and real in toothed armatures, 466.
  in air-gap, 79, 80.
  distribution of, 522, 535.
  in field-magnets, 384, 829.
Inductor, 48.
  bars, 341, 344.
  effect of width, 56, 341.
Inductor alternators, 184. *Vide* Homopolar alternators.
Ingot iron, 273, 385.
Inertia, analogy to self-induction, 87.
Inherent regulation of alternators, 754, 807.
Input, 6.
Instability, magnetic, 384, 492.
Insulation, of armature cores, 333.
  between laminæ of armature cores, 302.
  of copper-wire, 340.
  of slots, 337, 673.
  loss of space by, 661.
  of commutators, 376.
  of magnet-bobbins, 477.
  resistance of armature, 378.
Insulators, magnetic, 37, 266.
Intensity, magnetising, 267.
Intensity of field, 16, 20, 31.

Interaction of two fields, 75, 83, 86.
Interferric gap. *Vide* Air-gaps.
Interlinking of phases, 728, 740.
International Electrical Engineering Co.'s dynamo, 893.
Interpolar gap, 573.
Iron, magnetic properties of, 34, 37, 265, 270.
  flux-density curves of, 267.
  wrought-iron, 270, 273.
  forged ingot-iron, 273, 385.
  cast-iron, 272, 385.
  charcoal-iron, 275.
  for armatures, 275.
  discs, 301, 304.
  tape, 301, 309.
  wire, 301, 308.
Iron-clad dynamo, 398.
Isolit, 477.

J

Johnson-Lundell dynamo, 578.
Joints, effect of, in magnetic circuits, 406.
  on armature resistance, 623.
Joule, 285.
Journals, strength of, 324.
  length of, 324, 328. *Vide* Friction, Lubrication.

K

K of alternator E.M.F. equation, 173, 747.
Kapp, G., 295, 506.
  on testing dynamos, 690, 705.
  alternator, 820.
Keeper of magnet, 121, 184.
Kiloline, 49.
Kilowatt, 5, 6.
Kirchhoff's laws, 452, 558.

L

L,=coefficient of self-induction, 89.
Lag of resultant behind impressed E.M.F., 102.
  angle of, 114, 115.
Lamination of armature cores, 300, 309.
  of armature bars, 341, 629.
  of poles, 578, 650, 828.
Lap winding, 230, 253, 418.
  in alternators, 726, 740.
Lasche, Herr O., 816, 830, 875.
Lead of brushes 428.
  effect of, 529.
  necessity for, 563.
  limiting value of, 566.
Leading and trailing, 338, 521, 554.
Leakage, magnetic, 41, 127, 409, 429.
  in heteropolar alternators, 830.
  in homopolar alternators, 825.
  in overtype single horseshoe, 409, 446–455.
  in undertype single horseshoe, 446, 454.
  in Manchester field, 395, 410, 453.
  in iron-clad dynamo, 398.
  in multipolar dynamos, 411, 447.
  relation to size of machine, 445.
  relation to exciting power, 444, 451.
Leakage, as affected by armature reaction, 546.
  Hopkinson's measurements of, 453.
  internal, in ring armatures, 310, 453.
Leakage coefficient, ν, 451.
Leakage, secondary, in alternators, 763, 796, 799, 827.
Leblanc amortisseur, 878.
Left-handed winding, 154, 214, 216, 232.
Lenz's law, 72.
Line-integral of force, 31.
Line, neutral, 195, 541, 562.
Line of maximum field, 195.
Line of symmetry, 517.
Lines of force, 12, 13, 15, 19, 21.
  unit C.G.S., 18, 49.
  as opposed to lines of induction, 27.
Lines of induction, 26.
  as closed curves, 28.
Linkages, 53, 89.
  rate of change of, 98.
Long-shunt compound-winding, 508.
Loops of armature winding, formation of, 132–145.
Losses in dynamos, 7.
  in brushes, 574, 644.
  in field-magnet coils, 484, 617.
  in armatures, 619, 635 666.
  in commutator, 639, 667.
  in pole-pieces, 648.
  measurement of, 624, 690, 694. *Vide* Hysteresis, Eddy-currents.
Lubrication of bearings, 327, 332, 719.
  of commutator, 720.

M

Magnet, bar, 10, 12.
  electro-, 23.
  permanent, 28, 38, 273.
  of dynamos. *Vide* Field-magnets.
Magnetic circuit, 30, 423.
  equation for, 34, 35, 39, 424, 452.
  in different types, 469.
  divided, 395, 401.
  joints in, 406.
Magnetic induction. *Vide* Induction.
Magnetic instability. *Vide* Instability.
Magnetic leakage. *Vide* Leakage.
Magnetic potential, 30.
Magnetic pull. *Vide* Pull, magnetic.
Magnetic saturation. *Vide* Saturation.
Magnetic screening. *Vide* Screening.
Magnetisation of iron, 264.
  curves of flux-density, 273.
  in dynamos, best degree of. *Vide* Induction.
  molecular theory of, 288.
Magnetising force, H, 267.
Magnetism, residual. *Vide* Residual magnetism.
Magneto-motive force, 30.
  of armature ampère-turns, 520.
  of field-magnet, 33, 424. *Vide* Ampère-turns.
Manchester field, 388, 395, 410, 453, 618.
Maxwell, 27.
Mechanical energy absorbed by dynamo, 6, 105, 174. *Vide* Efficiency.

Mechanical pull. *Vide* Pull, magnetic.
Megomit, 333, 337, 377.
Mesh connection of polyphase armatures, 732, 733.
Mica, 333.
  in commutators, 214, 369, 376.
Micanite, 333, 337, 369, 373, 377.
Molecular theory of magnetism, 288.
Moment of inertia, 696.
Momentum, electro-magnetic. 90.
Mordey, W. M., 881.
Motor, electric, 74, 92, 690.
μ. *Vide* Permeability.
Multiplex windings, 247, 256, 601.
Multipolar alternators, 162. *Vide* Alternators.
Multipolar continuous-current dynamos;
  with ring armature, parallel-connected, 219, 247.
  with ring armature, series-connected, 224, 249.
  with drum armature, parallel-connected, 234, 236, 254.
  with drum armature, series-connected, 237, 259.
  with drum armature, radial depth of core, 383.
  equality of parallel armature paths, 244, 263, 417.
  proportioning of magnet, 418.
  advantage of, 419, 569.
  ratio of length to diam. of armature, 420.
  leakage in, 447, 455.
  back and cross ampère-turns of, 531.
  compounding of, 514.
  ampère-wires per pole, with toothed armature, 580.
  eddy-currents in, 628, 631.
  heating of, 635.
  design of, 669.
Multipolar magnets, 161, 164, 388, 398.
  symmetry of field in, 415, 417.
  with single exciting coil, 402.
  proportioning of, 418.
  leakage in, 447.
  heating of, 619.

N

NEEDLE, magnetic, 10, 14, 19.
Negative work in alternators, 105, 175.
Neutral line, 195, 541, 562.
Nickel, 34, 37.
Nickel steel, 272.
Niethammer, Dr., 285, 295, 296, 545, 651, 787, 791, 799.
Non-overlapping winding of polyphase machines, 743.
North pole, 11, 14, 15.

O

OERLIKON alternators, 910, 911.
Oerstedt, 18.
Oil for bearings, 332.
  throwers, 330.
  on commutator, 720.
Open-coil armatures, 191.
Open-coil armatures, with two sets of coils 193.
  with four sets of coils (Brush machine), 199.
  with three coils (Thomson-Houston machine), 201.
  brushes for, 607.
Oscillatory motion, 121.
Output of dynamos, 5, 67.
  of alternators, 107.
  how limited, 80, 571, 615, 754.
  by heating, 81, 614.
  by sparking, 565, 567.
  by regulation, 754.
Over-compounding, 509, 715.
Overtype single horseshoe magnet, 8, 392.
  design of, 430, 655, 661.
  leakage in, 409, 439, 446, 455.

P

PACINOTTI, 130, 336. *Vide* Toothed armatures.
Paper in laminated cores, 302.
Parallel division of armature winding in alternators, 163, 724.
  in continuous-current dynamos, 212.
  necessity for equal E.M.F.'s and resistances, 241.
  with multipolar field, 244, 417.
Parallel working of alternators, 860, 864. 868.
  of compound-wound dynamos, 713.
  of series dynamos, 712.
  of shunt dynamos, 710.
Pedestals, 328.
Period, 63.
Periodicity of alternators, 64, 160, 838, 845.
  heteropolar, 162.
  homopolar, 180, 185.
Peripheral speed of continuous-current dynamos, 381.
  as affecting cooling surface, 622, 635.
  as affecting eddy-currents, 623.
  of alternators, 820, 822, 832.
Permanent magnets, 10, 38, 273.
Permeability, 36, 37, 265.
  of air, 36, 425.
  of iron and steel, 274, 277.
Permeance, 36.
  of air-gap in smooth armatures, 425, 432.
  in toothed armatures, 458.
  of teeth, 464.
Permeance, leakage, calculation of, 43, 429, 437.
  in bipolar dynamo, 439.
  in alternator, 856.
  empirical formulæ for, in bipolars, 445.
  in multipolars, 447.
Phase, 65.
Phase-indicator, 863.
Pitch and pitch line of alternators, 149, 178.
  of armature winding, 225, 229.
Plummer-blocks, 328.
Polar angle, 569, 573.
Polar diagrams, 110.
Pole, free magnetic, 11.
  N. and S., 15.
  unit, 16.

Pole-corners, induction at, 527, 534, 580. *Vide* Pole-pieces.
Pole-pieces, reversing, 576.
  shaping of, 342, 577.
  eddy-currents in, 339, 624, 646.
  horns on, 415, 578.
  fringe round. *Vide* Fringe.
Pole-width, ratio of, to pole-pitch in alternator, 143, 145, 148, 189.
Polyphase alternators, 168, 728.
  back ampère-turns of, 773.
Potential, difference of electric, 2.
  of magnetic, 30.
  rise of, in ring winding, 221; in commutator, 223; in alternators, 154, 723.
Potier, A., 775, 792.
Power of dynamo, 4, 5.
  of alternator, 104, 107.
  apparent, of alternator, 112.
Power-factor of alternator, 112, 115.
  of external circuit, 755.
Press-spahn, 333.
Prime movers, 6.
Pull, magnetic, on conductor, 68, 72.
  on loop in uniform field, 75.
  on armature inductors, 77, 79.
  in alternators, 174.
  on armature cores, 334, 336, 412, 414, 815.
  on field-magnets, bipolar, 403, multipolar, 416, 815.
Pulsating E.M.F. and current, 191.
Pump, analogy of, to dynamo, 7, 67,

Q

QUARTER-PHASE alternators, 168, 730.
  winding of, 735.
Quivering of field, 601.

R

RADIAL depth of armature core, 383.
Rate of change, curve of, 96.
  of self-induced lines, 98.
  of line-linkages, 53.
  of current in short-circuited section, 559.
Rate of cutting lines, 48, 54.
  average and instantaneous, 49.
Ratio of width of winding or of field to pitch in alternators, 148, 150, 155, 178, 182, 189, 749.
Reactance, 117.
  synchronous, 759.
Reactance voltage in commutator, 597.
  in alternator, 763.
Reaction of armature current on field, 86, 94, 428.
  effect on characteristic curve, 504, 512.
  as limiting output, in continuous-current dynamos, 565.
  in smooth armatures, 528.
  in toothed armatures, 534, 653.
  in open-coil armatures, 607.
  in alternators, 754.
Reciprocating motion, 121.
Rectifying commutator, 835.
Re-entrancy, 247.
Regulation of voltage, of alternators, 754, 807.
  of compound-wound dynamo, 507, 612.
  of constant-current dynamo, 205, 506.
  of shunt dynamo, 499.
Relative movement of field and conductor, 48, 51, 84.
Reluctance, 35, 38.
  of teeth, 464.
  of leakage paths, 43. *Vide* Leakage.
Reluctivity, 36.
Residual magnetism, 269, 272, 290, 481.
Resistance coils in exciting circuit, 499, 688.
Resistance, effective, 105.
Resistance of armature, loss of volts over, 67.
  loss of watts over, 623.
  calculation of, in continuous-current dynamo, 380, 665, 674.
  measurement of, 611, 623.
  increase of, by heating, 473, 610.
Retardation method of measuring losses, 694, 882.
Retentiveness of iron, 269, 272.
Reversing field, need of, 561.
  minimum value of, 567, 580, 596.
Reversing pole-pieces, 576.
Rheostat in exciting circuit, 499, 688.
Right-handed winding, 153, 216, 232.
Ring alternators, heteropolar, 133, 161, 164.
  ratio of coil-width to pitch, 150.
  E.M.F. equation, 173.
  winding of, 722.
  homopolar, 141, 177.
  ratio of coil-width to pitch, 178.
  E.M.F. equation, 188.
  hysteresis in, 294.
Ring armatures, 130, 214.
  lamination of, 300.
  core-discs for, 301
  construction of, 306, 308.
  internal leakage in, 310, 453.
Ring continuous-current dynamos, 134, 214.
  E.M.F. equation, 218.
  spirally-wound, 219, 247.
  multipolar parallel-connected, 219, 244.
  multipolar series-connected, 224, 228, 244, 249.
  multiplex parallel-connected, 247.
  multiplex wave-wound series-connected, 249.
  general formula, 252.
  rise of potential in, 221, 346.
  equality and symmetry in, 241, 246.
  armature winding, 345.
  ratio of length to diameter of armature, 382.
  radial depth of core, 383.
  maximum ampère-wires of, 568.
  inductance of short-circuited section, 583, 586, 590.
  number of sectors, 600, 601.
  sparking of, 602, 607.
  heating of, 635.
Ring winding. 129, 130, 214.
  hand of, 153, 216.
  of heteropolar dynamo, 130.
  of homopolar dynamo, 141. *Vide* Ring alternators, armatures, and continuous-current dynamos.

Rise of potential, in closed-coil armatures, 221, 223.
  in commutator, 223.
  in alternators, 154.
Rise of temperature in dynamo, 609, 616.
  of armature, 67, 81.
  of field-magnet coils, 390, 473, 611.
  effect on compounding, 514, 612.
  maximum permissible, 614.
  predetermination of, in field coils, 617.
  " " ", armature, 635.
R. M.S. values of alternating functions, 112.
Rocking-bar, 551.
Rotary converters, 722, 732, 733, 839, 870.
Rotation of inductor, 61, 121.
Rothert, A., 782, 815, 842.
Routin, 694, 882.

S

Saturation, magnetic, of iron, 269, 290.
  of pole-corners, 528, 578.
  of teeth, 534, 578.
Sayers' dynamo, 534, 576.
Schuckert dynamo, 891.
Screening, magnetic, 283, 646, 651.
Screw analogy for direction of lines, 20, 26.
  for winding, 153, 216.
Sectors of commutator. *Vide* Commutator.
Segmental discs, 302, 304.
Self-excitation, 481, 495.
Self-induction, E.M.F. of, 85.
  analogy to inertia, 87.
  of loop, 89.
  of solenoid, 89.
  as rate of change of self-induced linkages, 98.
  as limiting output, 754.
  of magnet coils, 483.
  in open-coil armatures, 197, 202.
  in short-circuited section of closed-coil armature, ring or drum. *Vide* Inductance.
Separately-excited dynamo, 478, 515.
Series-connected wave winding, ring, 225.
  drum, 237.
Series-wound dynamos, 480.
  uses of, 505.
  for high voltages, 709.
  characteristic curve of, 503.
  field-winding for, 501.
  in series, 709.
  in parallel, 712.
Shafts, armature, 313, 320, 322.
Shell, magnetic. 22, 76.
Shifting of brushes, necessity for, 563.
  effect of, 428, 529.
Short-circuit characteristic of alternator, 758, 801, 805.
Short-circuiting of section in closed-coil armature, 207, 553.
  time of, 555.
  equation of, 557.
Short-circuiting of shunt-dynamo, 491.
Short-coil winding of polyphase machines, 743, 746.
Short-shunt compound-wound dynamo, 508.
Shunt-wound dynamos, 479.
  characteristic curve of, 486, 498.
  uses of, 499, 712.
Shunt-wound dynamos, field-winding for, 483, 485.
  in parallel, 710.
  effect of short-circuiting, 491.
  instability of magnetism in, 492.
  maximum current from, 491.
  regulation of voltage, 499.
Siemens winding of drum armature, 132. *Vide* Drum armature.
Silver, 67.
Sine curve, 63.
  rate of change of, 97.
  effective value of, 114.
  assumption of, in alternators, 112, 118, 173, 752.
Single-phase alternator, 163
  windings for, 723.
  back ampère-turns of, 770, 880.
Slide rails, 717.
Smooth armature, 146.
  distortion of field in, 525.
  reduction of field by distortion, 528.
  inductance of short-circuited loops on, 581.
  maximum ampère-wires per pole, 567.
Solenoid, 22, 23.
  self-induction of, 90.
Sparking at brushes, nature of, 549.
  in closed-coil armature, 550
  in ring armature, 602.
  in open-coil armature, 199, 607.
  as limiting output, 565. *Vide* Commutator, and Commutation.
Speed of rotation, 654. *Vide* Peripheral speed.
  want of uniformity of, 871.
Spherical bearings, 330.
Spirally-wound ring, 219, 247.
Split-ring commutator, 134, 195.
Stampings, armature, 301.
Star connection of polyphase armature, 731, 733.
Steel, 276.
  castings, 272, 274, 385, 387.
  stampings, 276, 301.
Steinmetz's law, 285, 287.
Storage of energy in field, 92, 483, 523.
  in short-circuited section, 557.
Stranded armature bars, 344, 629, 633.
Subdivision of armature winding, 592, 594, 600.
Swinburne, J., on hysteresis, 293.
Symmetry in armature winding, 245.
  of field distribution, 246, 411, 415.
Synchroniser 863.
Synchronous reactance of alternator, 759, 803.

T

Tangential pull on armature, 77, 174.
Temperature, of dynamos at work, 81, 390, 609.
  maximum permissible, 613.
  measurement of, 610, 611.
  of field-coils, 473, 611.
  effect on hysteresis, 295. *Vide* Heating and Rise of temperature.
Terminal voltage, 67.
Terminals, 3.
Tesla, 122.

Testing dynamos, 616, 881.
Thermometer, 611, 617, 814.
Thomson-Houston dynamo, 205,
Three-phase alternator, 170, 732.
  windings for, 738, 746.
Throttling, magnetic, 407.
Thrust-bearings, 326.
Thury six-pole dynamo, 400.
Toothed armatures, 146, 256, 260, 302, 336, 368, 638.
  hysteresis in, 296, 635.
  force on, 338.
  solid bars on, 344.
  reluctance of teeth in, 464, 676.
  winding of, 350.
  insulation of slots, 337, 673.
  air-permeance of, 458.
  distortion of field in, 527, 534.
  back ampère-turns of, 545.
  ampère-wires per pole, 578.
  inductance of short-circuited loop on, 587, 679.
  number of sectors, 601.
  eddies in armature, 630, 633.
  eddies in pole-pieces, 624, 646.
  ratio of slot to tooth, 651.
  ratio of copper to slot area, 661.
  design of, 670.
  for alternators, 735, 738, 750, 752, 812.
Toroid, 23, 34.
Torque on armature, 78.
Traction generators, 319, 323, 339, 514, 669, 885, 889.
Trail, angle of, effect from, 533.
Trailing and leading, 338, 521, 554.
Transformation of energy, 7.
Transformer, 7, 54, 116, 721, 725, 755.
Transmission of energy, 709, 721.
Triplex winding, 249.
Tungsten steel, 273.
Tunnel armatures, 339.
Twisted armature bars, 343, 629.
Twisting moment, 310.
Two-phase alternators, 731.

U

UMBRELLA type of alternator, 823.
Undertype single horseshoe, 392, 410.
Unidirected current, 134, 191.
Uniform field, 18, 59.
Unipolar dynamo, 125.
  simple forms of, 140.
Unit pole, 16.
  field, 17.
  of flux, 27.
  of magneto-motive force, 32.
  permeability, 36.
  of inductance, 90. *Vide* also C.G.S. system.

V

VACUUM, permeability of, 36.
Varnishing of armature discs, 302.
Ventilation, internal, of armatures, 638.
Vibration, mechanical, effect on residual magnetism, 272.
  on hysteresis, 295.
  of shaft, 318.
Volt, 4, 49.
Voltage of dynamo, 5, 67. *Vide* E.M.F. equations.
Vulcasbeston, 477.

W

WATT, 4, 6.
Watts per rev. per min., 311, 315, 656.
  as determining armature size, 658.
Wattmeter, 107.
Wave winding of ring, 224, 249, 418.
  of drum, 234, 258.
  of alternator, 728, 741, 743.
Weight of armature, 315, 708.
  of machine, 707.
  of copper on field, 395, 476, 484.
Westinghouse Co.'s alternators, 818, 906.
Width coefficient, 172.
Width of brush, 604.
  of arc of contact, 595, 604.
  in Thomson-Houston dynamo, 202.
Width of coil, in alternator, 172. *Vide* Alternators and Differential action.
Width of inductor, effect of, 56, 342.
Winding of armatures, 345.
  formulæ for, 246. *Vide* Ring, Disc, Discoidal, Drum.
  of field-magnet coils, 477.
Wire, copper, resistance of, 474.
  cost of, 484.
  insulation of, 340.
  iron, for armature cores, 308.
Wöhler, 320.
Wrought iron, 270, 272, 273.

Y

Y-connection of polyphase armatures, 733, 740.
Yoke of magnet, 120.

Z

ZINC, as a magnetic insulator, 266, 393.
  for magnetic formers, 477.

PRINTED BY NEILL AND CO., LTD., EDINBURGH.